LESSON 16:
SINGLE AND MULTISTAGE NETWORKS

Objective

The objective here is to learn single and multistage networks.

Introduction

Single And Multistage Networks

We have already studied about Stored program control (SPC) and various benefits offered -by it. Up to now we have discussed single stage space division networks. Here we shall turn our attention to the multistage space-division networks. In chapter 1, we have discussed about the number of cross point switches required for a large single stage network which is prohibitive, i.e. N(N -1)/2. In fact, single stage networks suffer from a number of disadvantages. These disadvantages can be overcome by adopting a multistage network configuration. A comparison between single stage and multistage Networks is given in Table 16.1.

Table 16.1 A comparison between Single Stage and Multistage Networks

S.No	Single stage	Multi stage
1.	Inlet to outlet connection is done through a single cross point.	Inlet to outlet connection is done through a multi cross point
2.	Use of single cross point per connection results in better quality link.	Use of multiple point links will degrade the quality of a connection.
3.	Each individual cross point can be used for only one inlet/outlet pair connection.	Same cross point can be used to establish connection between a number of inlet/outlet pairs.
4.	A specific cross point is needed for each specific connection	A specific connection may be established by using different sets of cross points.
5.	If a cross point fails, associated connection cannot be established. There is no redundancy.	Alternative cross points and paths are available.
6.	Cross points are inefficiently used. Only one cross point in each row or column of a square or rectangular switch matrix is ever in use, even if all the lines are active.	Cross points are used efficiently.
7.	Number of cross points is prohibitive.	Number of cross points is reduced significantly
8.	A large number of cross points in each inlet/outlet leads to capacitive loading.	There is no capacitive loading problem.
9.	The network is non-blocking in character.	The network is blocking in character.
10.	Time for establishing a call is less.	Time for establishing a call is more.

Two-Stage Networks

We state from the theorem that for any single stage network there exists an equivalent multistage network. A N x N single-stage network with a switching capacity of connections can be realised by a two-stage network of N x K and K x N stages as shown in Fig. 16.1.

A connection needs two switching elements. Any of the N inlets can be connected to any of the K outputs of the first stage. Similarly, any of the inputs of the second stage can be connected to any of the N outlets.

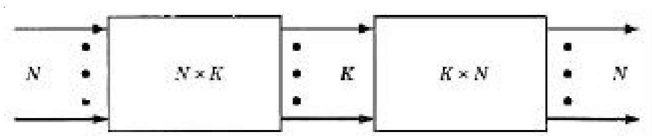

Fig. 16.1 A two-stage representation of an N x N network.

Consequently, there are K alternative paths for any inlet/outlet pair connection. This network will provide full connectivity or availability. Full availability means that any of the N inlets can be connected to any of the N outlets in the network. The term full connectivity can be distinguished from the term fully connected network. Each stage of the network has NK switching elements. We assume now that about 10% of the subscribers to be active on an average, K may be equal to N / 16. In this case, the number of switching elements, N_E in the network is $N^2 / 8$. If N = 1024

Then $$K = \frac{N}{} = \frac{1024}{} = 64$$

1.16

and $$N_E = \frac{N^2}{8} = \frac{(1024)^2}{8} = 131{,}072$$

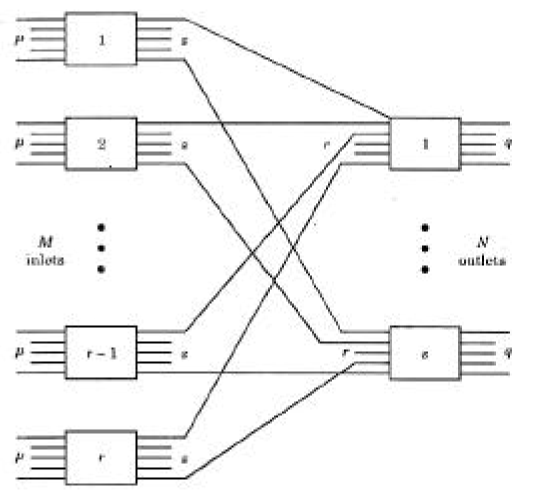

Fig. 16.2 Two stage network with multiple switching matrices in each stage.

For large N, the switching matrix N x K may still be difficult to realise practically. We always prefer architecture that use smaller sized switching matrices. Let us consider the two-stage realisation of an M x N switch using a number of smaller switching matrices. It is shown in Fig. 16.2.

Total number of M inlets is divided into r blocks and p inlets each. That is, M = p * r. Similarly, the N outlets are divided into 's' blocks of 'q' outlets each. That is N = q * s. For ensuring full availability, there must be at least one outlet from each block in the first stage terminating as inlet on every block of the second stage.

Therefore, this determines block sizes for the first stage and second stage.

Block size of the first stage = p x s

Block size of the second stage = r x q

Total number of switching elements is given by

$$N_E = p * s * r + q * r * s$$

Since M = pr and N = qs

$$N_E = M * s + N * r \quad \text{..............Eq.1}$$

The number of simultaneous calls that can be supported by the network is equal to the number of links between the first and the second stage.

Hence,

Switching capacity

(SC) = r * s ……………………..Eq.2

For r*s connections to be simultaneously active, the active inlets and outlets must be uniformly distributed. In other words, there must be 's' active inlets in each of the 'r' blocks in the first stage and 'r' active outlets in each of the s blocks in the second stage.

The 's' active inputs in one block of the first stage must be uniformly distributed across all the 's' blocks in the second stage at the rate of one per block.

This two-stage network is blocking in nature. The blocking may occur under two conditions:

1. The calls are uniformly distributed. There are r x s calls in progress and the $(rs + 1)^{th}$ call arrives.
2. The calls are not uniformly distributed. There is a call in progress from ph block in the first stage to the J^{th} block in the second stage and another call originates in the I^{th} block destined to the J^{th} block.

Determination of the blocking probability PB for the case of calls are not uniformly distributed:

Let $ be the probability that a given inlet is active.

Then, the probability that an outlet at the I^{th} block is active is given by,

$$? = \frac{(p\phi)}{s}$$

The probability that another inlet becomes active and seeks an outlet other than the one which is already active is given by

$$\frac{(P-l)\,\phi}{(s-1)}$$

The probability that the already active outlet is sought is given by

$$P_B = \frac{PF}{S}\left[1 - \frac{(p-1)}{(S-1)}?\right]$$

Substituting P = m / r in the above equation, we get

$$PB = \frac{MF\,(S-1) - \{[M/r] - 1\}\,F}{r\,s\,(s-1)} \quad \ldots\ldots\ldots\ldots Eq.3$$

From eqn.1, we see that the number of switching elements N_E can be minimised if 'r' and 's' are as small as possible. On the other hand, if 'r' and 's' are reduced, the blocking probability P_B goes high. Therefore, we chose values of 'r' and 's' which are as small as possible but give sufficient links to provide a reasonable GOS to subscribers.

1. If N > M, the network is expanding the traffic.
2. If N < M, the network is concentrating the traffic.
3. If N = M, then it offer deserves attention.

In this case, it is reasonable to assume that a uniform matrix size is used for both the stages, i.e. r = s and p = q.

In this case., the total number of switching elements, N_E, works out to be 2Nr and switching capacity is given by

$SC = r^2$

Often square switching matrices are available as standard IC chips. It can be used as building blocks for switching networks. In such a case, $p = r = s = q = \sqrt{N}$

Thus, each first stage and second stage of the network has $\sqrt{N}$ blocks. Here each block is a square matrix of $\sqrt{N} \times \sqrt{N}$ inlets and outlets.

If N is not a perfect square, the switching matrices are chosen to have a size of $\lceil\sqrt{N}\rceil \times \lceil\sqrt{N}\rceil$. The symbol $\lceil\ \rceil$ denotes a ceiling function. It gives the smallest integer equal to or greater than N.

Substituting $r = s = \sqrt{N}$ and M= N in eqns.1 and 2,

we get

$N_E = N * \sqrt{N} + N * \sqrt{N}$

$= 2N\sqrt{N}$ ……….Eq.4

Switching capacity

SC = rs

$= \sqrt{N} * \sqrt{N} = N$ …….....Eq.5

We have already studied that N simultaneous calls can be supported on this network only if the traffic is uniformly distributed. Networks that support N simultaneous connections but under restricted traffic' distribution conditions are Called Baseline Networks.

There is only one link between a block in the first stage and a block in the second stage. As a result, a link failure would cutoff connection between p inlets and q outlets. This one-link structure gives rise to a severe blocking in the network.

The blocking in the network can be avoided by increasing the number of links between the blocks of the stages.

Then the design parameters for M = N are:

$$P = q = \sqrt{N}$$

$$s = r = \frac{K}{\sqrt{N}} \quad \text{and}$$

$N_E = 2NK\sqrt{N}$ and Switching capacity

SC=N

For making the network non blocking,

We must have

$K = \sqrt{N}$, we then get

$NE = 2N^2$ ……………… Eq.6

Switching capacity = SC = N ……….. Eq.7

Thus, a two-stage non-blocking network requires twice the number of the switching elements as the single-stage non-blocking network.

In fact, for a non-blocking configuration, a two-stage network offers no distinct advantage over a single-stage network. It provides N alternative paths for establishing a connection. However, a standard way of designing blocking networks with full availability is to use two or more stages. Blocking networks require both concentrating and expanding network structures. These networks are easily implemented as two separate parts. The real advantages of multistage networks become if evident we consider networks of three or more stages.

Three-stage Networks

By adopting a three stage network structure in place of two-stage network structure, we can reduce significantly the blocking probability PB and number of switching elements N_E. Fig. 16.3

shows network structure of an N x N three-stage blocking the N inlets and N outlets are divided into 'r' blocks of 'p' inlets and 'p' outlets each respectively. This network is realised by matrices of size p x s in stage 1, r x r in stage 2, s x p in three-stage networks, there is a special thing than network that any arbitrary inlet in the first stage has s paths to reach any arbitrary outlet in the third stage.

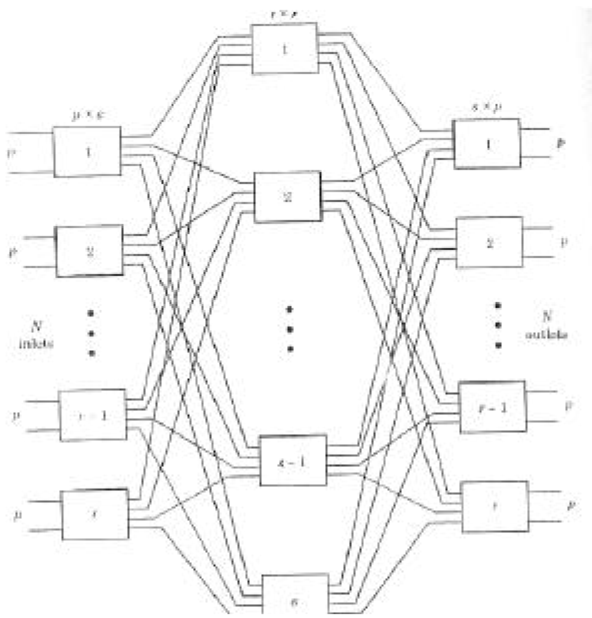

Fig.16.3 N ´ N three-stage switching network

The total number of switching elements required in three-stage network is given by

$$N_E = rps + sr^2 + spr = 2Ns + sr^2$$
$$= s\,[2N + r^2] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{Eq.8}$$

There are a variety of techniques that can be used to find the blocking probabilities of multistage switching networks. The important techniques among them widely used are:

1. One technique given by C.Y. Lee
2. Second technique given by C. Jacobaeus.

Both the techniques are approximate techniques. These techniques provide reasonable accurate results. The model proposed by Jacobaeus is somewhat more accurate than the one proposed by C.Y. Lee. But Lee's approach has an ease of modelling. There is a fact in Lee's approach that the model and the associated formulae are directly related to the underlying network structures. Here, we, use Lee's probability graphs to determine the blocking probability of multistage networks.

A Lee's probability graph of three-stage network is shown in Fig. 16.4. In the graph, circles represent the switching stages and the lines represent the inter stage links. The network graph illustrates all possible paths between a given inlet and an outlet. This graph indicates the fact that there are s alternative paths for

If we use square matrices in the first and the third stages, then we have

$$p = s = \left(\frac{N}{r}\right).$$

Substituting above values in eqn. (3.19), we get

$$N_E = \frac{2N^2}{r} + Nr \qquad \cdots\cdots \text{Eq.9}$$

We can conclude from eqn. 9 that there is an optimum value for r that would minimise the value of N_E.

For determining the value r, we differentiate eqn. 9 , set it equal to zero.

$$\frac{dN_E}{dr} = \frac{d}{dr}\left[\frac{2N^2}{r} + Nr\right]$$
$$= -\frac{-2N^2}{r^2} + N = 0$$

or $$\frac{2N^2}{r^2} = N$$

or $$r = \sqrt{2N} \qquad \cdots\cdots \text{Eq.10}$$

Taking the second derivative of eqn. 9

$$\frac{d^2N_E}{dr^2} = \frac{d}{dr}\left[\frac{dN_E}{dr}\right]$$
$$= \frac{d}{dr}\left[\frac{-2N^2}{r^2} + N\right]$$
$$= +\frac{4N^2}{r^3} + 0$$
$$= +\frac{4N^2}{r^3} \qquad \cdots\cdots \text{Eq.10}$$

$$\frac{d^2N_E}{dr^2} \text{ at } r = \sqrt{2N}$$
$$= \frac{4N^2}{\left(\sqrt{2N}\right)^3} = \frac{4N^2}{2\sqrt{2N}\,N} \qquad \cdots\cdots \text{Eq.11}$$

The second derivative; $\frac{d^2N_E}{dr^2}$ is being positive at $r = \sqrt{2N}$. Therefore it indicates that the value of N_E is minimum.

$$(N_E)_{min} = 2N\sqrt{2N} \qquad \cdots\cdots \text{Eq.12}$$

and $$p = \frac{N}{r} = \sqrt{\frac{N}{r}} \qquad \cdots\cdots \text{Eq.13}$$

The optimum ratio of the number of blocks to the number of inputs per block is given by

$$\frac{r}{p} = \frac{\sqrt{2N}}{\sqrt{(N/2)}} = 2 \qquad \ldots\ldots\ldots\ldots\text{Eq.14}$$

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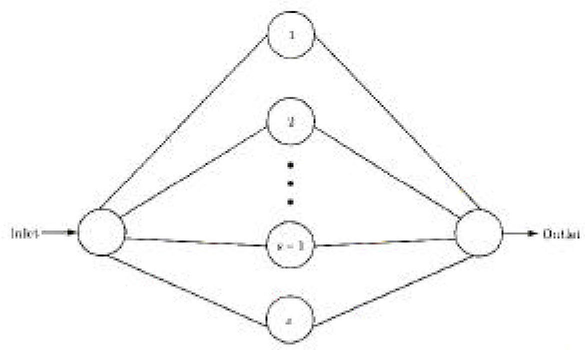

A Lee's probability graph of three-stage network is shown in Fig. 16.4. In the graph, circles represent the switching stages and the lines represent the inter stage links. The network graph illustrates all possible paths between a given inlet and an outlet. This graph indicates the fact that there are s alternative paths for any particular connection, one through each block in the second stage.

The factor K represents either space expansion or concentration.

If $s > p$, the first stage provides expansion

If $s < p$, the first stage provides concentration.

From eqn.17 we see that in order to have a low value for the blocking probability the factor must be small.

1. If α is large, K must be large. In other words, if the inlets are well loaded, we need an expanding first stage. This is usually the case with the transit exchanges. In transit exchanges, the incoming trunks are heavily loaded and we need an expansion to provide adequately low blocking probabilities.
2. In other case, if $\alpha$is small, K may be small. It means that if the inputs are lightly loaded, the first stage may be concentrating one. Such type of situation occurs with the end offices or PBX switches.

Fig. 16.4 Illustration of Lee's probability graph for a three-stage network.

Blocking probabilities in the network may be estimated by decomposing a graph into serial and parallel paths.

Let

β = Probability that a link is busy

β' = Probability that a link is free i.e. not busy

Then

$$\beta = 1 - \beta'$$

If there are s parallel links, then the blocking probability

= Probability that all the links are busy

$$P_B = \beta^s$$

$$Q_B = 1 - P_B = 1 - \beta^s$$

When a series of s links are needed to complete a connection, the r blocking probability is easily determined as one minus the probability t that they are available.

$$P_B = 1 - (\beta')^s$$

$$= 1 - (1 - \beta)^s, \qquad \text{since } \beta' = 1 - \beta$$

For a three-stage network, there are two links in series for every path and there are s parallel paths.

Therefore,

$$PB = [1 - (\beta')^2]^s$$

$$= [1 - (1 - \beta)^2]^s \text{ ....Eq.15}$$

If α is the probability that an inlet at the first stage is busy, then

$$\beta = \frac{p\alpha}{s} = \frac{\alpha}{K} \text{ ....Eq.16}$$

Putting the value of 13 from eqn.16 in eqn.17, we get the complete expression for the blocking probability of a three-stage switch in terms of its inlet utilisation as

$$P_B = [1 - (1 - (\alpha / K))^2]^s \text{ ....Eq.17}$$

Multistage Networks Design (Non-blocking type)

Such networks were first studied by C.Clos in 1954. He showed that it is possible to construct multistage non blocking networks which have less number of switching elements than the corresponding single stage networks. Multistage non-blocking networks with full availability are known as Clos Networks. A three-stage switching network can be made non-blocking by providing adequate number of blocks in the second stage (i.e.; by increasing the value of B). Worst case situation occurs for multistage non-blocking networks when the following conditions prevail:

1. (p -1) inlets in a block I in the first stage are busy.
2. (P -1) outlets in a block 0 in the third stage are busy.
3. The (p -1) second-stage blocks (on which the (p -1) outlets from block I are terminated are different from the (p -1) second-stage blocks from which the links are established to the block O.
4. The free inlet of block I needs to be terminated on the free outlet of block O.

Above condition are shown in Fig. 16.5. Under these circumstances we require additional block in the second stage.

Notes