ARITHMETIC RULES

1. ARITHMETIC

1.01 Two's complement of a number equals the number itself ***inverted*** plus one.

Example	NUMBER	BINARY	INVERT	ADD ONE = 2's complement
	8	1000	0111	1000
	7	0111	1000	1001
	A	1010	0101	0110
	3	0011	1100	1101
	C	1100	0011	0100
	5	0101	1010	1011

1.02 Signed two's complement = a two's complement number with the most significant bit representing a sign bit. It is a zero for positive and a one for negative numbers. A comma (as illustrated here) sets off the sign bit.

Example	NUMBER	POSITIVE NUMBER	SIGNED 2'S COMPLEMENT
	8	0,1000	1,1000
	7	0,0111	1,1001
	A	0,1010	1,0110
	3	0,0011	1,1101
	C	0,1010	1,0100
	5	0,0101	1,1011

- Positive numbers have a zero as a sign bit and the digits are absolute.
- Negative numbers have a one as a sign bit and the digits are in two's complement.
- Addition and subtraction are both accomplished by ***ADDING.***

Note: All of the following examples are shown using only 4 bits plus a sign bit for simplicity.

Example: Addition—Add +3 to +8

0,0011
0,1000
0,1011 = $+13_8$ = +11

Example: Subtraction—Subtract 3 from 8 (means add 2's complement of -3 to +8)

0,1000
1,1101 (2's complement of -3)
0,0101 = +5

Example: Subtraction—Subtract 8 from 3 (means add 2's complement of -8 to +3)

0,0011
1,1000
1,1011 = -5 (in 2's complement)

- ***Carry and overflow*** (or underflow)—both go beyond the most significant bit (MSB) but not sign bit of the machine (or borrow from beyond).

- Carry goes beyond due to an arithmetic operation affecting the MSB whereas overflow goes beyond because the value is larger than the capacity of the machine.

Example: Carry—Represents the propagation of a one beyond the most significant bit (MSB) of the machine as a result of an unsigned add or subtract operation.

Example: Overflow (Underflow)—Occurs when the MSB represents the sign and when the result of an add or subtract operation exceeds the capacity of the machine.

Example: Add 10 and 12—In this case, for unsigned arithmetic the carry is a 0 (ZERO) and for signed arithmetic an overflow results.

MSB
0 1010
0 1100
1 0110

Example: Add -8 and -9 (Unsigned 24 and 23)—In this case, for unsigned arithmetic the carry is a 1, and for signed arithmetic an overflow (underflow) results.

MSB
1 1000
1 0111
0 1111

2. POWERS OF TWO TABLE

N	2 TO N		1 OVER 2 TO N
0	1		1.0
1	2		0.5
2	4		0.25
3	8		0.125
4	16		0.0625
5	32		0.03125
6	64		0.015625
7	128		0.0078125
8	256		0.00390625
9	512		0.001953125
10	1024		0.0009765625
11	2048		0.00048828125
12	4096		0.000244140625
13	8192		0.0001220703125
14	16384		0.00006103515625
15	32768		0.000030517578125
16	65536	← REGISTER	0.0000152587890625
17	131072		
18	262144		
19	524288		
20	1048576	← ADDRESS	

← = Maximum value minus one

3. HEXADECIMAL ADDITION TABLE

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
1	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F	10
2	2	3	4	5	6	7	8	9	A	B	C	D	E	F	10	11
3	3	4	5	6	7	8	9	A	B	C	D	E	F	10	11	12
4	4	5	6	7	8	9	A	B	C	D	E	F	10	11	12	13
5	5	6	7	8	9	A	B	C	D	E	F	10	11	12	13	14
6	6	7	8	9	A	B	C	D	E	F	10	11	12	13	14	15
7	7	8	9	A	B	C	D	E	F	10	11	12	13	14	15	16
8	8	9	A	B	C	D	E	F	10	11	12	13	14	15	16	17
9	9	A	B	C	D	E	F	10	11	12	13	14	15	16	17	18
A	A	B	C	D	E	F	10	11	12	13	14	15	16	17	18	19
B	B	C	D	E	F	10	11	12	13	14	15	16	17	18	19	1A
C	C	D	E	F	10	11	12	13	14	15	16	17	18	19	1A	1B
D	D	E	F	10	11	12	13	14	15	16	17	18	19	1A	1B	1C
E	E	F	10	11	12	13	14	15	16	17	18	19	1A	1B	1C	1D
F	F	10	11	12	13	14	15	16	17	18	19	1A	1B	1C	1D	1E

Example:

Row		Col	
6	+	7	= D
B	+	4	= F
D	+	A	= 7 Carry 1
E	+	C	= A Carry 1