RADIO ENGINEERING
MOBILE RADIO
RF COAXIAL TRANSMISSION LINES

CONTENTS PAGE

1. GENERAL

1.01 This section supplies information concerning the physical and electrical characteristics of coaxial transmission lines and their application in UHF and VHF radio frequency systems.

1.02 The widespread and varied use of radio equipment in the Bell System has made coaxial lines a relatively common item in telephone plant.

1.03 The primary purpose of coaxial lines is to guide electromagnetic waves from one point to another. At certain frequencies, practical use can be made of them as resonant circuits in place of coils and capacitors. Coaxial lines are often used as impedance matching devices and in many cases they are used as filters to effectively remove unwanted frequencies from a circuit.

1.04 Low transmission loss, constant impedance over a wide range of frequencies, low radiation and low susceptibility to interference from external fields, are characteristics which make coaxial lines especially useful in connection with radio systems.

2. PHYSICAL CHARACTERISTICS

2.01 Coaxial lines are constructed in a cylindrical form consisting of an inner conductor, a dielectric, and an outer conductor. Depending upon the flexibility of these two conductors and the associated dielectric, the lines are classified as flexible, semirigid, and rigid.

(A) Flexible Coaxial Lines

2.02 Flexibility in coaxial lines is obtained by using a braided outer conductor, a flexible dielectric, and an inner conductor of small gauge wire or of several small wires stranded together.

2.03 The conductor material most frequently used is copper. In some coaxial lines the copper is tinned, principally to facilitate soldering. A silver coating on the copper improves aging stability and the electrical conductivity at frequencies above about 1000 mc. To conserve copper and to provide additional tensile strength during manufacture, copper-coated steel (Copperweld) is sometimes used for the inner conductor of small size coaxial lines. Coaxial lines designed to have high attenuation may use a center conductor of Nichrome or similar high resistance material. In some coaxial lines, designed for special purposes, the inner conductor is formed by winding a small gauge wire as a helix around a cylindrical, insulating core.

2.04 The dielectric of most flexible coaxial lines consists of a solid tube of polyethylene, teflon, or rubber, enclosing the inner conductor. The dielectric gives form to the outer conductor braid and determines its diameter. Teflon is capable of withstanding relatively high temperatures in comparison with polyethylene and rubber. Rubber insulated coaxial lines have little application in Bell System radio services because of their attenuation. The dielectric used in a specific coaxial line depends upon the flexibility, temperature, voltage, and frequency requirements to be satisfied. Semisolid dielectric coaxial lines are made by wrapping a string of dielectric material around the inner conductor and then covering the helix, so formed, with a thin tube of the same dielectric material. Coaxial lines constructed in this manner retain a good degree of flexibility and provide a compromise between the electrical properties of air and solid dielectric coaxial lines. The characteristics of teflon and polyethylene dielectrics are given in Section 402-100-100.

2.05 A jacket of insulating material, such as rubber, polyethylene, or synthetic resin, is usually provided over the outer conductor of flexible coaxial lines as a protection against moisture and mechanical damage to the conductor braid. Jacket materials vary in their susceptibility to heat, cold, and sunlight. In some older types of polyethylene insulated coaxial lines with synthetic resin jackets, the vinyl plasticizer tends to migrate into the polyethylene under certain conditions, contaminating the dielectric and increasing the attenuation. Two types of noncontaminating synthetic resin jackets are now available.

2.06 One type of noncontaminating jacketed coaxial line used in early Bell System installations employed a jacket of clear or natural polyethylene. After this type jacket material had experienced aging on outside installations it was found to be susceptible to deterioration by ultraviolet rays in a relatively short time. The deterioration resulted in hair line cracks in the outer jacket. In some instances this cracking became so extensive that the copper braided outer conductor became discolored, thus resulting in an adverse effect on the transmission qualities of the cable. This experience with the above-mentioned outer jacket resulted in an improved jacket of a black polyvinylchloride material. This improved noncontaminating type jacket is expected to have better resistance to deterioration by sunlight and other conditions of weathering.

2.07 An armor of braided aluminum or galvanized steel is provided over the jackets of some flexible cables as added protection against mechanical damage.

2.08 Most flexible cables of recent manufacture are made in accordance with specifications prepared by a committee representing the manufacturers, and the armed services. Such specifications are coordinated by the Armed Services Electro Standards Agency (ASESA) and cables manufactured in accordance with those specifications are identified by an RG/U-type number which is marked along the length of the cable at suitable intervals.

2.09 The nominal characteristics of several types of coaxial lines are given in Section 402-100-100. These include dimensions, conductor materials, dielectric materials and jacket materials.

(B) Rigid and Semirigid Coaxial Lines

2.10 Lines with an outside diameter of 7/8 inch are available in both rigid and semirigid types. Larger lines are rigid and smaller lines are nearly always semirigid. Hard drawn copper is used in the manufacture of rigid lines for which the standard length is 20 feet. Semirigid lines are made of soft drawn copper and ordinarily are manufactured in lengths of 100 feet. Semirigid lines, spliced at the factory, may be shipped in continuous lengths up to 1000 or 2000 feet.

2.11 The data of different manufactures show a considerable variation in the safe bending radius for lines that are nearly the same in other respects. A safe general rule would be to avoid bends with a radius less than about 35 times the outer diameter of the lines, unless shorter bends are permitted by specific manufacturing data. Repeated flexing will result in permanent damage to all types of semirigid lines. Inner conductors, having a radius in excess of 3/8 inch are usually tubular.

2.12 The dielectric of rigid and semirigid lines is predominantly air, and separation between the conductors is maintained by insulating beads or pins of teflon or of a ceramic material such as steatite. The spacing of the beads along the conductors is one factor controlling a line's minimum bending radius. Bead spacing also may impose restrictions on the frequency band which can be transmitted satisfactorily since the beads constitute irregularities in the air dielectric. This effect is reduced in "compensated" lines by altering the diameter of one or both conductors at bead locations or by spacing the beads in sets in such a manner that the irregularities introduced by one set will be partially compensated, at specific frequencies, by the irregularities of another set.

2.13 The insulation of air dielectric lines is impaired by moisture. To prevent the entrance of moisture by breathing and condensation as a result of temperature changes, the lines must be dried and hermetically sealed after installation. As a means of maintaining moisture within air dielectric lines at an absolute minimum, it is common practice to fill the line with dry air or nitrogen. This dry air or nitrogen is maintained under approximately 15 pounds pressure per square inch.

2.14 The nominal characteristics of rigid and semirigid coaxial lines are given in Section 402-100-100.

3. ELECTRICAL CHARACTERISTICS

3.01 The transmission characteristics of uniform lines are derived from the primary constants and are given in the Transmission Engineering practices. The primary constants per unit length are the series resistance, R, of the conductors; the inductance, L; the capacitance, C, between the conductors; and the leakage conductance, G, of the dielectric. The capacitance and inductance of a given coaxial line are relatively constant with frequency; but R increases with the square root of frequency, because of the skin effect, and G increases almost directly with frequency. G is determined by losses in the dielectric and is readily obtained from the power factor of the dielectric by

$$G = 2\pi f C_1 \ . \ (\text{power factor}) \qquad (1)$$

Where C_1 is the capacitance with air as the dielectric and the power factor is the cosine of the angle between the voltage and current. With teflon and polyethylene dielectrics the power factor is very small and practically constant over the range of frequencies ordinarily used on coaxial lines.

3.02 As a practical matter, the primary constants are seldom referred to in connection with coaxial lines at radio frequencies. Instead use is made of "constants" which show a more direct relationship to their effect on high-frequency signals impressed on the line. These "constants" are the characteristic impedance, Z_o, and the propagation constant α .

The propagation constant determines the attenuation and phase shift due to the line. The equations for these two constants are:

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad (2)$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \qquad (3)$$

3.03 These equations are applicable to coaxial lines transmitting electromagnetic waves in the principal mode, which is generally the case in Bell System services. At radio frequencies, where R and G are small in comparison to (ωL) and (ωC) respectively, good approximations are given with the simpler equations that follow below. However, at microwave frequencies the dielectric losses may be large and these simpler equations would not suffice.

3.04 A knowledge of the effect of terminations on low loss lines and a knowledge of limiting characteristics, such as the maximum voltage and power that can be applied to a line with safety, are of great importance.

(A) Characteristic Impedance

3.05 The characteristic impedance is a property of the line itself. It is usually defined as the impedance presented by either end of an infinitely long line or by a shorter line which is terminated by an impedance which is equal to the impedance of the infinitely long line. In the latter case, a wave arriving at the termination is completely absorbed by the termination and no terminal reflections occur. The ratio of voltage to current at any point on such a line is equal to the characteristic impedance.

3.06 When R and G of equation (2) are negligible in comparison with (ωL) and (ωC) the characteristic impedance is given by

$$Z_o = \sqrt{\frac{L}{C}} \quad \text{ohms} \qquad (4)$$

Under the same conditions of negligible R and G, the characteristic impedance of an air dielectric line is related to the diametric ratio D/d, where D = inside diameter of the outer conductor, and d = outside diameter of the inner conductor.

$$Z_o = 138 \ \text{Log}_{10} \frac{D}{d} \quad \text{ohms} \qquad (5)$$

Fig. 1 gives the approximate characteristic impedance of coaxial lines as determined by the diametric ratio of the inner and outer conductors. The characteristic impedance given in equations (4) and (5) is a pure resistance.

3.07 Dielectrics other than air affect the characteristic impedance to the extent that they contribute to the capacitance and leakage conductance of the line. Except at high microwave frequencies, the conductance of low loss dielectrics, such as teflon and polyethylene, has a negligible effect on impedance. The capacitance is directly affected, however, and in equation (4) $C = C_1 k$, where C_1 is the capacitance the line would have with an air dielectric and k is the dielectric constant (for air dielectric k = 1). A dielectric other than air, therefore, affects the impedance by a factor of $1/\sqrt{k}$, and this factor must be applied to equations (4) and (5). The characteristic impedance of coaxial lines having a dielectric other than air can be computed by multiplying the values from Fig. 1 by $1/\sqrt{k}$.

3.08 When coaxial lines have a continuous, low loss dielectric the characteristic impedance is nearly constant over the range of radio frequencies for which coaxial lines are likely to be used. In bead insulated air dielectric lines, reflections from the individual beads combine in phase at certain frequencies causing the impedance to drop as much as 10% in some cases. This effect may impose restrictions on the frequencies which can be transmitted satisfactorily and on the locations at which some bead insulated lines should be cut. In compensated lines, impedance irregularities due to beads and bead spacing are reduced.

3.09 The nominal characteristic impedance of coaxial lines most frequently used in Bell System radio services is 50 or 70 ohms, depending upon the impedance of the terminations. When the terminating impedance is the same as the characteristic impedance of the line, the impedances are said to be matched. When impedances are not matched the loss resulting from use of the line may be greater than the loss of the line itself, excessive voltages may be produced on the line, and the sending end impedance of the line may be affected to the extent that equipment is damaged or fails to operate properly. Further reference to the importance of impedance matching is contained in Part 4.

(B) Propagation Constants

3.10 For coaxial lines at radio frequencies the propagation constant given by equation (3) may be replaced by

$$\gamma = \alpha + j\beta = \left[\frac{R}{2Z_o} + \frac{GZ_o}{2}\right] + j\omega\sqrt{LC} \text{ (Approx.)} \quad (6)$$

3.11 The attenuation constant, which is the real part of the propagation constant, gives the loss in nepers per unit length. One db = 8.686 nepers. The attenuation in db per unit length is given by

$$A = 8.686\alpha = 8.686 \left[\frac{R}{2Z_o} + \frac{GZ_o}{2}\right] \text{db/unit length} \quad (7)$$

3.12 Attenuation is due principally to the conversion of electrical energy into heat in the series resistance, R. The conductance, G, is determined by the power factor of the dielectric and by the frequency. G increases almost directly with frequency; but with air, polyethylene, or teflon dielectrics the conductor losses predominate at the highest frequencies for which coaxial lines are ordinarily used. Losses in the dielectric become of relatively more importance as frequency is increased.

3.13 The attenuation of an air dielectric line, having an inner conductor of fixed size, varies with the diametric ratio D/d and is a minimum when D/d = 3.6, corresponding to a characteristic impedance of 77 ohms. A change in the diametric ratio affects the impedance more rapidly than it affects attenuation.

3.14 A loss of 3 db in a line represents a loss of about half the power delivered to the line. The use of a line with a high attenuation constant can thus waste much of the advantage gained by higher antenna towers, for example, or by increasing transmitter power.

3.15 The phase shift, velocity, and wave length of a voltage or current wave on a line are interrelated and are determined by the phase constant β, of the propagation constant in equation (6), as

$$\beta = \omega\sqrt{LC} \quad (8)$$

The phase shift, velocity, and wave length, at a given frequency, are thus determined by inductance and capacitance of the line.

3.16 The phase constant, β, is the angle in radians, by which the phase of a sinusoidal voltage or current wave is retarded in traversing unit length of line. When the wave has traveled over a length of line, l, the phase retardation will be βl and when $\beta l = 2\pi$ the phase at the distant point will be one cycle behind the phase at the near point. The distance between the two points is, therefore, one wave length and

$$\lambda\beta = 2\pi \text{ or } \lambda = \frac{2\pi}{\beta} \quad (9)$$

3.17 A phase difference of one cycle in the voltage or current wave is 2π radians, and the time required to complete one cycle is $T = \frac{1}{f}$ seconds. T is also the time in seconds required for a given point on the voltage or current wave to travel a distance, λ, on the line. The phase velocity is therefore

$$V = \frac{\lambda}{T} = \lambda f \qquad (10)$$

If λ in (10) is replaced by $\frac{2\pi}{\beta}$ from (9)

$$V = \frac{2\pi f}{\beta} = \frac{\omega}{\beta} \qquad (11)$$

If β in (11) is replaced by $\omega\sqrt{LC}$ from (8)

$$V = \frac{1}{\sqrt{LC}} \qquad (12)$$

The velocity thus will depend only on the product LC, and is independent of frequency. The wave length varies inversely with frequency and β varies directly with frequency. The above relations may be found useful in determining an unknown constant when the others are known. It should be kept in mind that the derived constant applies to the same unit length as the known constants used in the equations.

3.18 The capacitance in equation (12) is $C = C_1k$.

The velocity, therefore, varies inversely with the square root of the dielectric constant, k. In air dielectric lines the wave velocity varies from about 86.6% to 99.8% of the velocity of an electromagnetic wave in free space. The higher value applies to larger lines in which the dielectric beads account for a relatively small part of the total dielectric. In solid polyethylene insulated lines, for which $k = 2.26$ and $1/\sqrt{k} = 0.665$, the wave velocity is approximately 66.5% of free space velocity.

3.19 A knowledge of wave velocity is useful in determining the length of line corresponding to some particular fraction of a wave length as in cases where a quarter wave stub is required. One wave length in free space is given by

$$\lambda = \frac{300{,}000{,}000}{f} \qquad (13)$$

The results of equation (13) are expressed in meters where the frequency is entered in cycles per second. In cutting of coaxial lines to form stubs or to construct filters, it must be remembered that velocity within the coaxial line is less than the velocity in free space. Therefore, the results of equation (13) must be multiplied by the known velocity factor of the coaxial line to obtain the wave length of a frequency within a given line.

(C) Voltage and Power Ratings

3.20 The voltage required to break down the insulation of a coaxial line depends upon the inside diameter of the outer conductor, upon the dielectric, and upon the diametric ratio D/d. A ratio D/d = 2.718, which corresponds to a characteristic impedance of 60 ohms, permits the maximum voltage between conductors. The maximum voltage that will exist on a line for a given power flow will be increased if the load impedance does not match the characteristic impedance of the line. See Part 3 (D).

3.21 The power rating of a line is that input power which may be transmitted continuously without causing injury to the cable. The power is limited by the maximum safe voltage on the line and by the permissible temperature rise. For a given line the power rating diminishes with increase in frequency, with the ambient temperature, and with the standing wave ratio. Operation of a polyethylene dielectric cable at a center conductor temperature above 175° F is likely to cause permanent damage to the cable. For polyethylene dielectric lines, particularly where the line is subject to flexing, the power rating should be large enough that the center conductor temperature will not exceed 150° F in normal operation.

3.22 Power ratings for flexible coaxial lines given in the tables attached to Section 402-100-100, are based on an ambient temperature of 104° F. For center conductor temperatures of 150° F the power ratings will be about 59% of the ratings shown in the table. For operation in an ambient temperature of 140° F and a center conductor temperature of 150° F the power ratings will be only 10% of the table values. When it is necessary for a cable to operate in an ambient temperature of 150° F it would be desirable to use teflon or air dielectric lines.

3.23 Under conditions of imperfect impedance match, a safe power rating may be determined by dividing the power rating under matched impedance conditions by the voltage standing wave ratio.

(D) Standing Waves

3.24 When a voltage source is applied to one end of a uniform line that is terminated at the far end by a matching impedance, the voltage and current will diminish continuously as the distance from the source is increased and the ratio of voltage to current taken at any point in the line will equal the characteristic impedance. If E_o is the peak value of the alternating voltage at the source, the voltage and phase, referred to E_o, at a distance l from the source will be given by

$$E_\ell = E_o\, e^{-(\alpha + j\beta)\ell} \text{ where } (\alpha + j\beta) = \gamma \qquad (14)$$

the propagation constant.

3.25 This simple condition will not prevail if the receiving end of the line is terminated by a mismatched impedance, Z_L. When the impedances are not matched the energy of the incident wave will not all be absorbed by the termination and there must be an abrupt adjustment of voltage and current that will result in a current $I_L = \frac{E_L}{Z_L}$ in the load. This adjustment is made by the reflection of part of the incident wave back toward the source. The voltage on the line at the termination will thus be $E_L = E_i + E_r$, the sum of the voltages of the incident and reflected waves. Since the two voltages result in waves traveling in opposite directions, the current in the line will be the difference between the currents represented by E_i and E_r acting on the characteristic impedance of the line. The total current at the junction of the line and termination must be the same on both sides of the junction. Therefore, $I_i - I_r = I_L$. If the currents are replaced by the equivalent voltage to impedance ratios,

$$\frac{E_i}{Z_o} - \frac{E_r}{Z_o} = \frac{E_L}{Z_L} = \frac{E_i + E_r}{Z_L} \qquad (15)$$

Equation (15) reduces to an expression for the ratio of the reflected wave voltage to the incident wave voltage. This ratio, ρ, is called the reflection coefficient and its magnitude and angle are given by

$$\frac{E_r}{E_i} = \rho = \frac{Z_L - Z_o}{Z_L + Z_o} = |\rho| \underline{/\theta} \qquad (16)$$

3.26 The total voltage at any point on a line will be the sum of the incident wave voltage and the reflected wave voltage, and the total current will be the difference between the incident current and the reflected current. At the receiving termination the total voltage will be $E_i + E_r = E_i\,(1 + |\rho| \underline{/\theta})$ and the total current will be:

$$I_i - I_r = \frac{E_i}{Z_o} - \frac{E_r}{Z_o} = \frac{E_i}{Z_o}\,(1 - |\rho| \underline{/\theta}) \qquad (17)$$

As the distance from the termination increases, the incident wave voltage increases in magnitude and advances in phase by the factor $e^{j\omega t}$ the instantaneous phase in the frequency cycle. The instantaneous voltage at a distance, ℓ, from the termination will be given by

Incident wave:

$$E_+ = E_i\, e^{j\omega t}\, e^{(\alpha + j\beta)\ell} \qquad (18)$$

Reflected wave:

$$E_- = E_i\, |\rho|\, e^{j\theta}\, e^{j\omega t}\, e^{-(\alpha + j\beta)\ell} \qquad (19)$$

Combined waves:

$$E_\ell = E_+ + E_-$$

$$= E_i\, e^{j\omega t}\, e^{(\alpha + j\beta)\ell}\left(1 + |\rho|\, e^{j\theta}\, e^{-j2(\alpha + j\beta)\ell}\right) \qquad (20)$$

The angle $e^{j\theta}$ is the angle $\underline{/\theta}$ of the reflection coefficient, since these are equivalent ways of representing an angle. Fig. 2(C) shows the instantaneous voltages on an RG59 A/U cable computed from equation (20) under the conditions that $E_i = 1$, $Z_o = 73 - j0$, $Z_L = 70 - j50.5$, and $\rho = 0.333 \underline{/74^o}$.

3.27 A physical picture of the combined voltage distributed along the line can be obtained more readily by dealing with peak values of the voltage and by assuming that the line has no attenuation. Under these conditions $t = 0$ and $\alpha = 0$ and equation (20) can be written as

$$E_\ell = E_i\, \underline{/\beta\ell}\, (1 + |\rho| \underline{/\theta - 2\beta\ell}) \qquad (21)$$

The corresponding, combined current, $I_i - I_r$, will be

$$I_\ell = \frac{E_i}{Z_o}\, \underline{/\beta\ell}\, (1 - |\rho| \underline{/\theta - 2\beta\ell}) \qquad (22)$$

The sending end impedance, at a distance, ℓ, from the termination will be

$$Z_s = \frac{E_\ell}{I_\ell} = Z_o\left[\frac{1 + |\rho| \underline{/\theta - 2\beta\ell}}{1 - |\rho| \underline{/\theta - 2\beta\ell}}\right] \qquad (23)$$

3.28 The magnitudes of voltage and current in equations (21) and (22) depend directly on the quantities in parentheses. The incident and reflected wave voltages will be in phase and add up to a maximum when the angle $\theta - 2\beta\ell$ is zero or a multiple of 2π. Then $\theta - 2\beta\ell = \pm 2\pi n$, where n is any integer. Calling the distance from the termination to the maximum voltage points ℓ_1 and substituting the value $\beta\lambda$ for 2π (see equation (9)), the above expression reduces to

$$\ell_1 = \frac{\theta}{2\beta} + n\frac{\lambda}{2} \quad \text{maximum voltage points} \qquad (24)$$

Maximum voltage points will thus recur at intervals of $\frac{\lambda}{2}$ and the first maximum will occur when n is the smallest integer that will give ℓ_1 a positive value.

3.29 The incident and reflected wave voltages will be in opposite phase and the combined voltage will therefore be a minimum when the angle $\theta - 2\beta\ell = \pm 2\pi n \pm \pi$. If ℓ_2 is the distance from the termination to the minimum voltage points, this expression reduces to

$$\ell_2 = \frac{\theta}{2\beta\ell} + \frac{\lambda}{4} + n\frac{\lambda}{2} \quad \text{minimum voltage points} \qquad (25)$$

Minimum voltage points thus recur at intervals of $\frac{\lambda}{2}$ and are displaced from maximum voltage points by a distance $\frac{\lambda}{4}$.

3.30 In a similar manner it can be shown that minimum current points coincide with maximum voltage points and maximum current points coincide with minimum voltage points. For a given frequency and reflection coefficient the maximum and minimum voltage points are stationary, and their periodic recurrence with distance along the line gives rise to the term, standing wave. The ratio of maximum to minimum voltage, occurring on a line within a distance of one-half wave length, is called the voltage standing wave ratio, VSWR.

3.31 If a line has appreciable attenuation, causing the incident wave voltage to increase and the reflected wave voltage to decrease with distance from the termination, the VSWR will vary with the position along the line at which the maximum and minimum voltages are taken. For a line with low loss, the VSWR is related to the reflection coefficient by

$$\text{VSWR} = \frac{1 + |\rho|}{1 - |\rho|} \qquad (26)$$

Only the magnitude $|\rho|$ of the reflection coefficient is involved since the + and - signs take into account the relative phases of the incident and reflected waves; the numerator representing the in-phase condition and the denominator the condition for phase opposition.

3.32 Since impedance is the ratio of voltage to current, it is evident that the sending end impedance of a line that is mismatched at the receiving end (equation (23)) may differ considerably from the characteristic impedance of the line. A general equation for the sending end impedance Z_s, of any line terminated by an impedance Z_L is

$$Z_s = Z_o \frac{Z_L \cosh\gamma\ell + Z_o \sinh\gamma\ell}{Z_o \cosh\gamma\ell + Z_L \sinh\gamma\ell} \qquad (27)$$

To avoid the use of hyperbolic functions of the complex quantity, $\gamma\ell = (\alpha + j\beta)\ell$, equation (27) can be written in the form

$$Z_s = Z_o \left[\frac{Z_L(\cosh\alpha\ell \cos\beta\ell + j\sinh\alpha\ell \sin\beta\ell) + Z_o(\sinh\alpha\ell \cos\beta\ell + j\cosh\alpha\ell \sin\beta\ell)}{Z_o(\cosh\alpha\ell \cos\beta\ell + j\sinh\alpha\ell \sin\beta\ell) + Z_L(\sinh\alpha\ell \cos\beta\ell + j\cosh\alpha\ell \sin\beta\ell)} \right] \qquad (28)$$

3.33 From the general equations (27) and (28) several cases of great practical importance involving low loss lines can be derived. When the line loss is 2 db, $\alpha\ell = \frac{2}{8.686} = 0.23$ neper. For this small quantity $\sinh\alpha\ell = \alpha\ell$, and $\cosh\alpha\ell = 1$ approximately. When lines have a loss of approximately 2 db or less, equation (28) may be replaced for practical purposes by

$$Z_s = Z_o \left[\frac{Z_L(\cos\beta\ell + j\alpha\ell \sin\beta\ell) + Z_o(\alpha\ell \cos\beta\ell + j\sin\beta\ell)}{Z_o(\cos\beta\ell + j\alpha\ell \sin\beta\ell) + Z_L(\alpha\ell \cos\beta\ell + j\sin\beta\ell)} \right] \qquad (29)$$

in which hyperbolic functions are eliminated.

3.34 When the length of a line produces a phase shift of 90° or an odd multiple of 90°, $\beta\ell = (2n+1)\frac{\pi}{2}$. Since $\beta = \frac{2\pi}{\lambda}$, $\ell = (2n+1)\frac{\lambda}{4}$; and the line is an odd multiple of a quarter wave length long. For $\beta\ell = (2n+1)\frac{\pi}{2}$, $\cos\beta\ell = 0$ and $\sin\beta\ell = \pm 1$. For a quarter wave line, therefore, equation (29) reduces to

$$Z_s = Z_o \left[\frac{Z_L\alpha\ell + Z_o}{Z_o\alpha\ell + Z_L} \right] \quad \text{quarter wave, low-loss line} \qquad (30)$$

3.35 If a quarter wave line is short-circuited, $Z_L = 0$ and equation (30) becomes

$$Z_{ss} = \frac{Z_o}{\alpha l} \qquad (31)$$

The sending end impedance of a short-circuited line that is an odd multiple of a quarter wave length long is thus very high and would be infinite if the attenuation were zero.

3.36 For an open-circuited quarter wave line $Z_L = \infty$ and equation (30) becomes

$$Z_{so} = Z_o \left[\frac{\infty\, \alpha l + Z_o}{Z_o \alpha l + \infty}\right] \text{ which reduces to:}$$

$$Z_{so} = Z_o \alpha l \qquad (32)$$

The sending end impedance of an open-circuited line which is an odd multiple of a quarter wave length long will thus be smaller than Z_o and will be zero if the line has no attenuation.

3.37 When the attenuation is so small that $Z_L \alpha l$ and $Z_o \alpha l$ can be neglected in equation (30) the sending end impedance becomes

$$Z_s = \frac{Z_o^2}{Z_L} \qquad (33)$$

Equation (33) illustrates the impedance transforming action of a quarter wave line. The impedance Z_L, seen through a quarter wave line, appears as Z_o^2/Z_L and by a proper choice of Z_o the impedance Z_s can be made to match any desired impedance. For example, to match a load Z_L to a line of characteristic impedance Z_b, the impedance Z_s of equation (33) must have the value Z_b. The lead will match the line Z_b, therefore, if it is seen through a $\lambda/4$ line having a characteristic impedance given by $Z_b = \frac{Z_o^2}{Z_L}$ from which the characteristic impedance of the quarter wave line should be $Z_o = \sqrt{Z_b Z_L}$.

3.38 For a line which is any multiple of $\lambda/2$ the phase shift will be $\beta l = n\pi$ and equation (29) reduces to

$$Z_s = Z_o \left[\frac{Z_L + \alpha l Z_o}{Z_o + \alpha l Z_L}\right] \text{ half wave line} \qquad (34)$$

When the $\frac{\lambda}{2}$ line is short-circuited, $Z_s = \alpha l Z_o$ and when it is open-circuited, $Z_s = \frac{Z_o}{\alpha l}$. When the attenuation is zero

$$Z_s = Z_L \quad \text{zero loss, half-wave line} \qquad (35)$$

A half-wave line thus tends to act as a 1:1 transformer and the load impedance, modified by the line attenuation, recurs at the sending end at intervals of $\lambda/2$.

3.39 If $Z_L = 0$ and $Z_L = \infty$ are substituted in the general equation (27) for the sending end impedance of a line of any length, general equations for the sending end impedance of short-circuited and open-circuited lines, respectively, result. They are

$$Z_{so} = Z_o \left[\frac{\cosh \gamma l}{\sinh \gamma l}\right] = \frac{Z_o}{\tanh \gamma l} \quad \text{open-} \qquad (36)$$

circuited line for which

$$Z_o = Z_{so} \tanh \gamma l \qquad (37)$$

and

$$Z_{ss} = Z_o \left[\frac{\sinh \gamma l}{\cosh \gamma l}\right] = Z_o \tanh \gamma l \qquad (38)$$

short-circuited line.

If equation (37) is divided by equation (38)

$$\frac{Z_o}{Z_{ss}} = \frac{Z_{so}}{Z_o}\,\frac{\tanh \gamma l}{\tanh \gamma l} = \frac{Z_{so}}{Z_o}$$

for which

$$Z_o = \sqrt{Z_{so} Z_{ss}} \qquad (39)$$

This shows how the characteristic impedance of a line can be determined if the short-circuited and open-circuited impedances of a length of line are known. The only conditions are that γl be the same for the open- and short-circuited impedance determinations.

3.40 Fig. 2(A) shows the standing wave voltages and currents on a short-circuited line which has no loss. It will be observed that the voltage is zero at the termination and at intervals of $\lambda/2$ along the line. At $\lambda/4$ and at odd multiples of $\lambda/4$ the voltage is twice the incident voltage E_i. Maximum current points coincide with minimum voltage points. For an open-circuited line the E_l and I_l curves would be interchanged.

3.41 The effect of attenuation on the standing wave voltage curve (A) is shown for an exaggerated case by curve (B) of Fig. 2. It will be observed that whereas the VSWR is infinite for the zero loss line and also for the lossy line adjacent to the termination, the VSWR decreases for the lossy line as the distance from the termination is increased. A situation more commonly encountered is shown by curve (C) in which lower line attenuation produces a much less pronounced effect on voltage standing wave ratio.

3.42 Fig. 3(A) shows the sending end impedance of a zero loss, short-circuited line. It will be observed that the sending end impedance is infinite at distances which are an odd quarter wave length from the termination and zero at distances which are a multiple of $\lambda/2$. The angle of the sending end impedance is zero (pure resistance) at maximum and minimum impedance points and is 90° (pure reactance) at all other points. If the origin is moved $\lambda/4$ to the left, the same curve forms will represent the impedance conditions on a zero loss, open-circuited line.

3.43 The effect of line attenuation on the sending end impedance is illustrated by Fig. 3(B). These curves show that attenuation prevents the impedance from reaching either the zero or infinite values and as the length of the line is increased the sending end impedance approaches the characteristic impedance of the line. If the line loss is as much as 10 or 15 db, the load end impedance has little effect on the sending end impedance since attenuation prevents an appreciable amount of reflected wave from reaching such distant points. In practical cases the line attenuation does not have such high values and the sending end impedance will be represented by curves lying somewhere between the two extremes illustrated.

3.44 When the terminating impedance is a pure reactance it will absorb no energy. The reflection coefficient is, therefore, unity and the maximum and minimum voltages of the standing wave will have the same magnitudes as for a short-circuited or open-circuited line. The location of the maximum and minimum voltage points will depend upon the ratio of Z_L to Z_o. For a pure inductance termination maximum voltage will occur at a distance less than $\lambda/4$ from the termination and for a pure capacitance termination the minimum voltage point will occur at less than $\lambda/4$ from the termination. With an inductance load the angle of the sending end impedance will be nearly 90° for distances up to the first voltage maximum. Beyond that distance the angle will alternate between approximately +90° and -90° at intervals of $\lambda/4$. For a capacitance load the angle will be the reverse of the inductance load situation, being -90° until the first voltage minimum is reached. The impedance curve for a pure reactance load, terminating a line without loss, will be represented by Fig. 3(A), if the point of origin is moved appropriately. For the inductance load the origin should be moved less than $\lambda/4$ to the left while for a capacitance load the origin should be moved between $\lambda/4$ and $\lambda/2$ to the left.

3.45 From the curves on Fig. 3 it is apparent that a short-circuited line will have an inductive reactance for any length of line up to a quarter wave length. Thereafter it will alternate between capacitive and inductive reactances at intervals of $\lambda/4$. An open-circuited line will have a capacitive reactance up to a distance of $\lambda/4$ and then will alternate at $\lambda/4$ intervals. When the load has the same angle as the characteristic impedance of the line the sending end impedance tends to look like that of an open-circuited line if $Z_L > Z_o$ and like that of a short-circuited line if $Z_L < Z_o$.

4. APPLICATION

(A) Transmission of Energy

4.01 Two of the most important properties of a transmission line are attenuation and characteristic impedance. The objective is to use a transmission line which is so constructed as to have low attenuation at the radio frequencies involved, and also the characteristic impedance best suited to matching the impedance of the radio equipment and the antenna. Having chosen such a line, and assuming that the transmitter output circuit (or receiver input circuit) is properly designed to work into the antenna impedance, the line will cause no reflection losses. Hence the transfer of energy to or from the antenna will be done efficiently. Also because of the absence of reflections, the exact length of transmission line used is not critical, an important practical consideration.

4.02 Coaxial transmission lines are called unbalanced lines since the two sides of the circuit have entirely different impedances to ground because of asymmetrical construction.

4.03 Coaxial transmission lines have the advantage that they produce no appreciable field external to the line when properly used. Hence these lines can be buried or installed in close proximity to other lines or metallic material, e.g., girders or steel towers, without affecting transmission. Further the shield or outer conductor can be grounded at any suitable point for protection.

4.04 Coaxial transmission lines are used mainly between antennas and radio equipments which are unbalanced to ground. However, a coaxial line may be used to connect a balanced antenna to an unbalanced radio equipment provided that a balanced to unbalanced transformer (or its equivalent) of the proper impedance ratio is used at the junction of the antenna and the coaxial line.

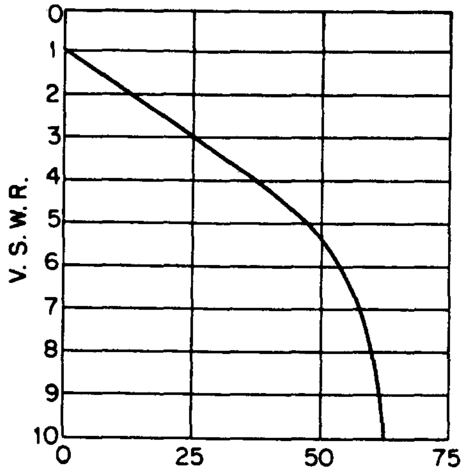
(B) Impedance Matching

4.05 When a transmission line is not matched by the load, the impedance looking into the line toward the load varies with the distance from the load. If the input impedance is considered equal to a resistance in parallel with the reactance, the input resistance will be equal to the resistive part of Z_o at some distance along the line. If at this point a reactance equal in magnitude and opposite in sign to the reactive part of the input impedance is connected across the line, the reactances will cancel and leave only the resistive component. From this point back toward the generator, the line will be matched.

4.06 The reactances used for matching in this way are usually linear reactance sections of transmission line called stubs. Stubs may be open or closed, depending on whether the free end is left open or short-circuited according to the type of reactance required in a particular case.

4.07 The length of the stub, as well as the point at which it should be attached to the line, can be found without any knowledge of the antenna input impedance, provided that the VSWR on the line can be measured before the stub is attached, and provided the position of the maximum voltage points can be determined under the same condition.

4.08 The amplitude of the voltage standing wave pattern, assuming the angle of the load impedance is the same as that of the characteristic impedance, is proportional to the ratio of the terminating resistance to the characteristic impedance of the line, or vice versa, whichever is greater than unity. Thus, a 50-ohm line terminated by either a 10-ohm or a 250-ohm resistive load will show a voltage standing wave ratio (VSWR) of 5:1. The standing wave ratio thus serves as a convenient measure of the degree of match existing between the line and its load. The percentage of power lost due to reflections from a mismatched load, plotted versus VSWR is shown below.



Percent of Power Reflected vs VSWR

4.09 When the VSWR and the position of the maximum voltage points are known, Fig. 14 gives the information necessary for placing a short-circuited stub in the transmission line for the purpose of matching it to the load. The data in Fig. 4 is based on the assumption that both the line and the stub have the same Z_o.

4.10 Curve A of Fig. 4 gives the location of matching stubs in wave lengths from the maximum voltage point. If an inductive stub is used, the location of the stub should be measured from a maximum voltage point toward the generator. Curve B gives the length of short-circuited line for the inductive stub. If a capacitive stub is used, the location of the stub should be measured from a maximum voltage point toward the load. Curve C gives the length of short-circuited line for the capacitive stub.

(C) Resonant Line Transformers

4.11 A radio frequency transmission line with standing waves is commonly called a resonant line, although the term "resonant" is often used more strictly in referring to a line that is resonant at a specific line length.

4.12 One use of resonant line lengths is to produce a step-up or step-down transformer effect. For example, suppose it is desired to transfer energy from a source whose impedance is 350 ohms, to a load which has an impedance of 70 ohms. To do this efficiently requires that the impedances should be matched by the use of a 350:70 ratio transformer. As shown by equation (33), if a quarter wave length (or odd multiples of a quarter wave length) of transmission line which has a characteristic impedance of $Z_o = \sqrt{70 \times 350} = 157$ ohms is used between the load and the source, the impedance as seen from the source is 350 ohms, as expressed by the relation $Z_s = \frac{Z_o^2}{Z_L} = \frac{157^2}{70} = 350$. In other words the quarter-wave line section transforms the load impedance to match the source impedance. As in the example above, this relationship holds only for the stated critical line lengths, thus limiting its practicability in many situations.

4.13 Under certain conditions the use of transmission lines which do not match the load impedance will not impair the efficient transfer of energy. For example, if a low loss line of any characteristic impedance is adjusted to exactly half-wave length at the frequency in use, the impedance of a load as seen through this line will be the same as if measured at the load itself. Refer to equation (35). If the length is altered from a half wave length (or from multiples of a half wave length) the above relationship will not hold. The 1:1 transformer arrangement, therefore, is not as flexible as nonresonant line operation, where the line length is not critical.

(D) Lightning Protection

4.14 Coaxial transmission line stubs are also used to provide additional lightning protection for the connected radio equipment. Shorted stubs of a length equal to a quarter wave length at the operating frequency are connected at either end of the transmission line. Shorted stubs when used as a coaxial line filter also serve this purpose. At the operating frequency this quarter wave length stub looks like an open circuit and therefore does not add attenuation to the transmission line. To lightning charges picked up by the antenna, however, the stub appears as a short circuit with its shield conductor grounded.

(E) Coaxial Line Filters

4.15 Resonant coaxial line stubs may be connected to the transmission line to form coaxial line filters. The filter will consist of two parts, a wave trap stub and a compensating stub. The wave trap stub introduces a low impedance (high loss) across the transmission line at the unwanted frequency. The compensating stub introduces a reactance to resonate the wave trap stub at the operating frequency and reduce the loss at that frequency. A description of coaxial line filters and their application will be found in Section 402-307-100.

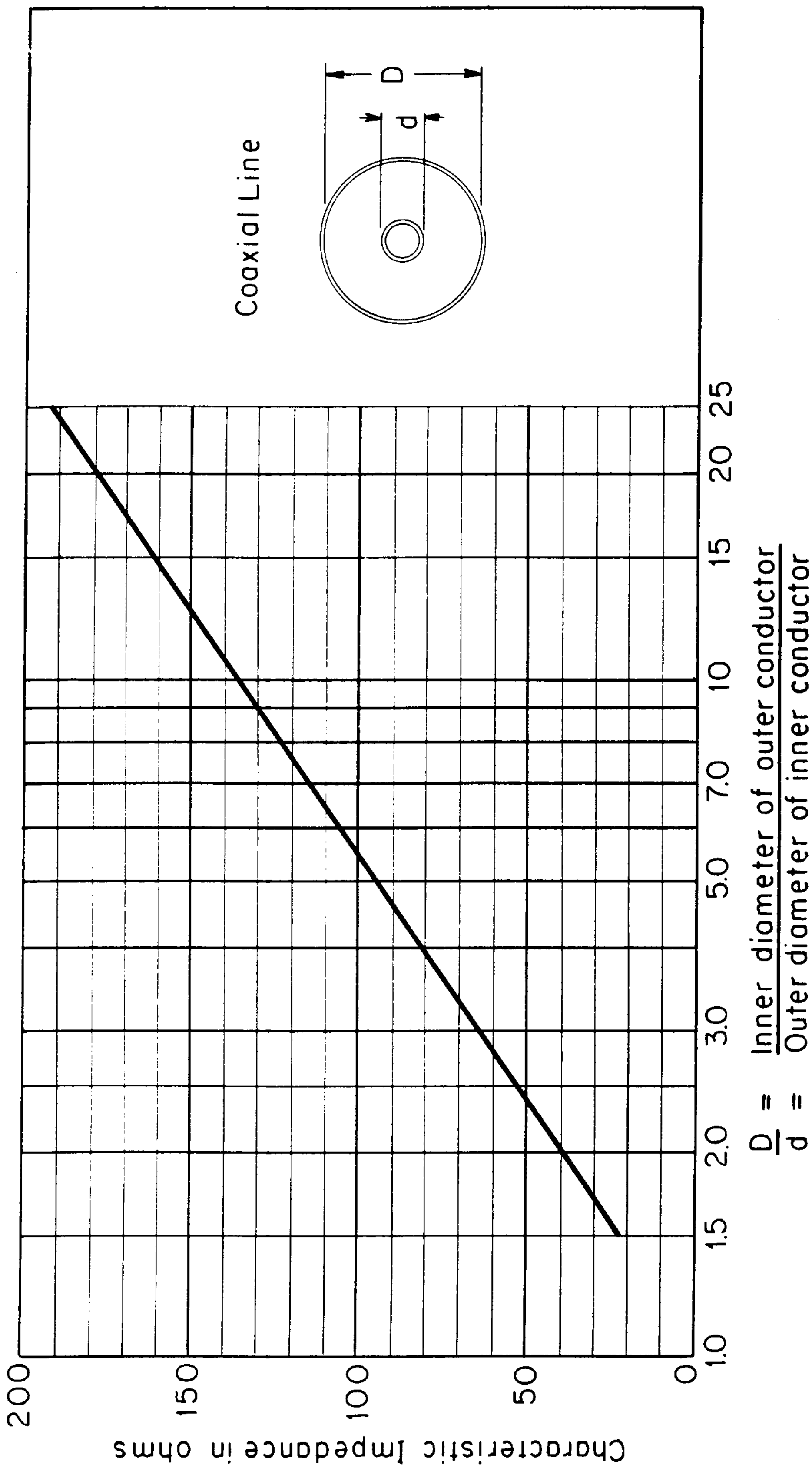


Fig. 1 – Characteristic Impedance of Air Dielectric Coaxial Lines as a Function of Diametric Ratio $\frac{D}{d}$

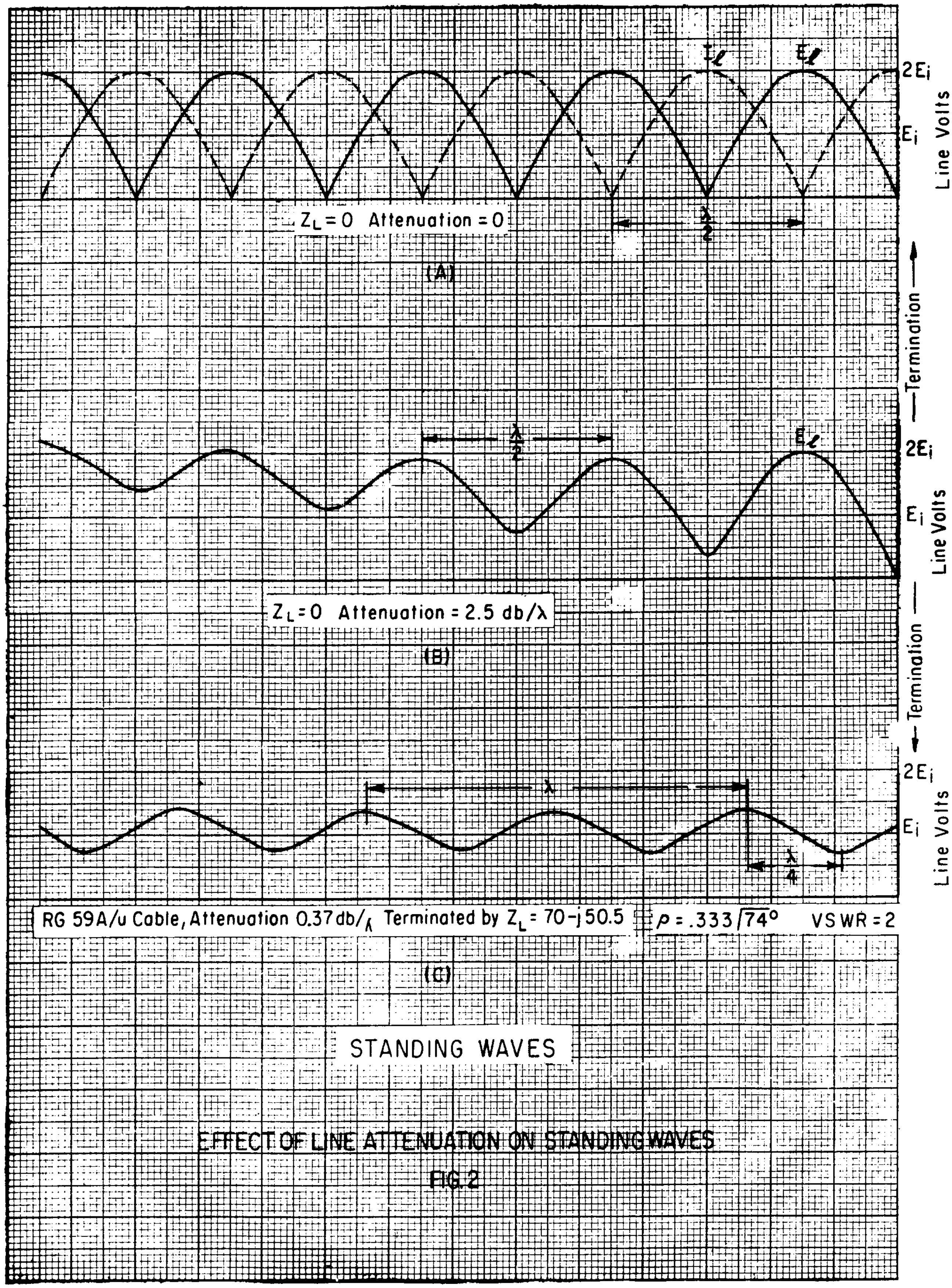


Fig. 2 – Effect of Line Attenuation on Standing Waves

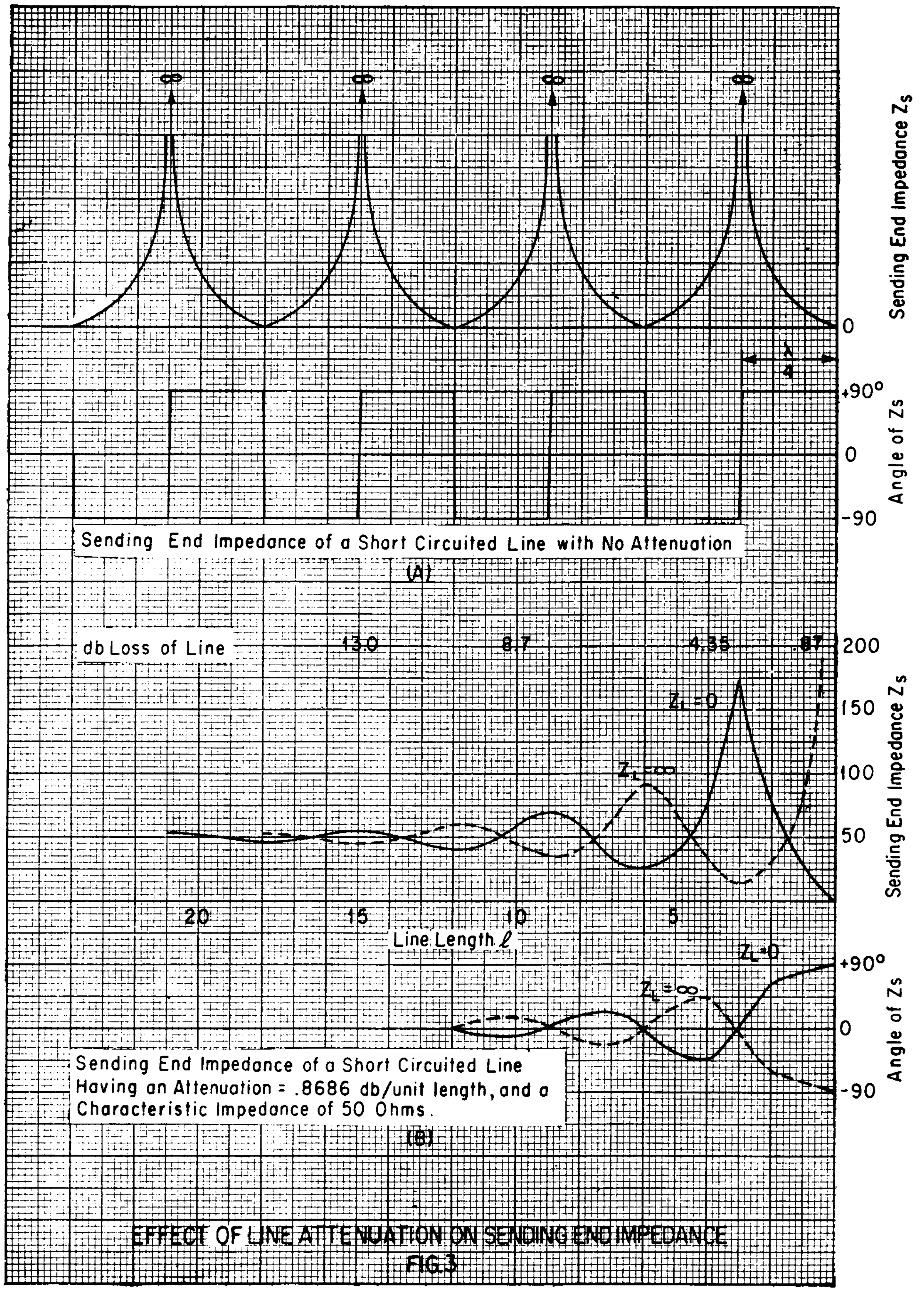


Fig. 3 – Effect of Line Attenuation on Sending End Impedance

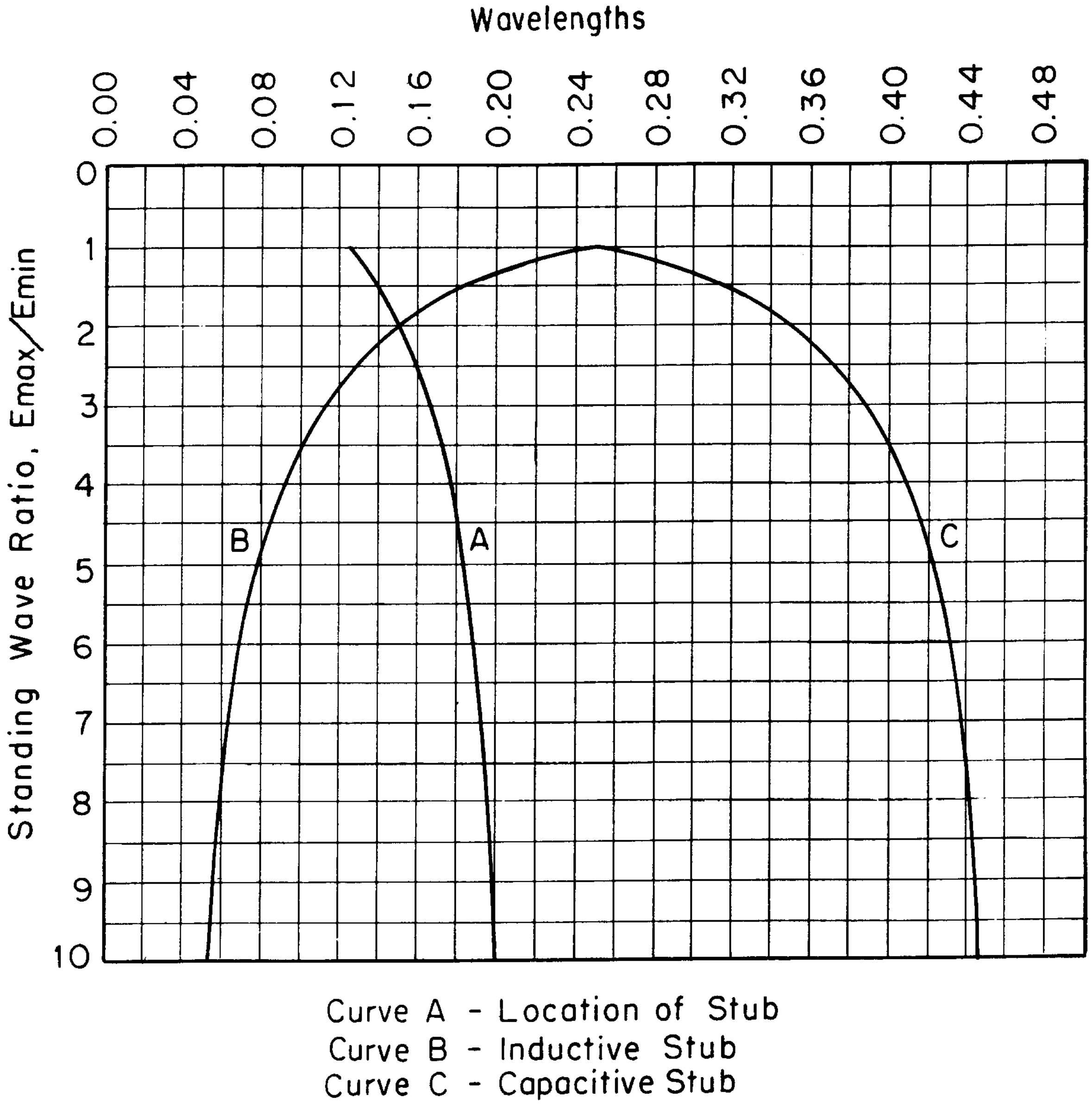


Fig. 4 — Location of Matching Stubs in Wavelengths from a Maximum Voltage Point