BELLCOMM, INC.
955 L'ENFANT PLAZA NORTH, S.W. WASHINGTON, D.C. 20024

B69 06044

SUBJECT: A Description of the Computer Program "STIFEIG" For Structural Dynamic Analysis - Case 320

DATE: June 12, 1969

FROM: S. N. Hou

ABSTRACT

The JPL digital computer program designated "STIFEIG" has been converted to FORTRAN V for use on the 1108 UNIVAC Computer. The program is capable of generating the stiffness matrix, and computing joint displacements and member stresses for 5 types of frame structures under specified static, dynamic, and thermal loading conditions. The types of frame structures that can be analyzed by the program are:

Three-dimensional truss, pinned joints.

Three-dimensional frame, rigid joints, axially symmetric member cross sections.

Planar frame, rigid joints, loaded in-plane.

Planar grid, rigid joints, loaded normal-to-plane.

Three-dimensional frame, rigid joints, doubly symmetric member cross sections.

The Power Method is used for eigensolution, which provides mode shapes and frequencies of a restrained system.

This memo presents a brief introduction to the program, reviews the input format, and provides example problems.

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MEMORANDUM FOR FILE

I. INTRODUCTION

The "STIFEIG" Program performs stiffness matrix structural analysis and eigenvalue solutions. Statically, it is capable of calculating joint deflections and member stresses for a frame structure under static or thermal loadings. Dynamically, it is capable of calculating natural frequencies, mode shapes, and member stresses during free vibration in normal modes.

The original program using FORTRAN II for use on an IBM 7090 was completed by Jet Propulsion Laboratory in 1962. Modifications were made by MIT under the program STEIGR in order to increase the degrees-of-freedom that could be handled. The program was further enlarged by JPL in 1965 to cover thermal analysis and incorporate Jacobi's method for eigenvalue solution of free-free systems. However, the 1965 revision used FORTRAN IV, and almost doubled the core storage requirement and was not completed its final debugging. The Bellcomm effort was to convert the program to FORTRAN V language for use on the UNIVAC 1108 computer. Due to limited storage capacity, subroutines for Jacobi's Method were not included. A comparable eigenvalue program designated "MAP" (Modal Analysis Program) is being developed independently, which will cover all the functions of the Jacobi Subroutine, but use the more efficient Householder Method allowing a larger capacity in degrees-of-freedom.

II. STRUCTURES

A. Types:

The program is capable of performing analysis of five types of structures:

1. Three-dimensional truss, pinned joints.
2. Three-dimensional frame, rigid joints, axially symmetric member cross sections.
3. Planar frame, rigid joints, loaded in-plane.

4. Planar grid, rigid joints, loaded normal-to-plane.

5. Three-dimensional frame, rigid joints, doubly symmetric member cross sections.

B. Features:

The structure should meet all the following features:

1. A restrained, stable system.

2. Uniform member cross sections.

3. Uniform elastic properties for the whole system.

4. Uniform thermal properties for each member.

5. Weightless members.

6. Lumped mass and inertia at joints for dynamic analysis.

III. LOADINGS

The program will compute joint deflections and member stresses for any of the following loadings imposed on the structure.

A. Static Loadings:

1. For all mass points, impose translational acceleration along any coordinate direction and/or rotational acceleration about any coordinate axis passing through the origin.

2. Same as above except coordinate axes pass through the centroid of the system.

3. Specific load components at specified joints.

B. Thermal Loadings:

Two input parameters, $\alpha\Delta T$, and $\frac{\alpha\delta T}{h}$ are used for describing the thermal environment of a structural member, where:

ΔT = The change in temperature through the whole member. Positive ΔT indicates increase in temperature.

δT = The change in temperature across the member cross section.

α = The coefficient of thermal expansion.

h = The height of a rectangular cross section along the direction of δT.

For each of the five types of structures, parameters are specified as follows:

1. For type 1, 2 or 5 structure, specify $\alpha\Delta T$.

2. For type 3 structure, specify $\alpha\Delta T$ and (or) $\frac{\alpha\delta T}{h}$.

3. For type 4 structure, specify $\frac{\alpha\delta T}{h}$.

C. Loadings in Free Vibration:

Specify acceleration component (g) of any mass point along one of its coordinate directions for each of the six lowest modes, the corresponding joint deflections and member stresses will be computed.

IV. SYSTEM CAPACITY

Capacity of "STIFEIG" is bounded by the size of the structural system as follows:

1. Number of degrees-of-freedom for the whole system should not exceed 130.

2. The product of the number of joint and degrees-of-freedom per joint should not exceed 180.

3. Number of joint should not exceed 60.

4. Number of structural member should not exceed 200.

5. Number of restraints should not exceed 100.

V. METHOD OF ANALYSIS

The program generates the stiffness matrix [K] for the structure of specified type based on input geometric data and elastic properties.

A. Static Cases:

$$\{F\} = [K]\{U\} \quad , \qquad (1)$$

where:

$\{F\}$ = A matrix of static loads applying at joints,

$\{U\}$ = A matrix of joint deflections.

For defined $\{F\}$ and $[K]$, these n (n = number of degrees-of-freedom) simultaneous equations can be solved for n unknown quantities of $\{U\}$ by Gaussian elimination.

$$\{U\} = [K]^{-1}\{F\} \quad . \tag{2}$$

Then, member stresses are calculated according to their relation to the displacements at the ends of a member, which are based on section properties and elastic properties of the member.

B. Thermal Case:

The thermal loads at the ends of a member and the member stresses are computed with all degrees-of-freedom fixed at the two ends. Then loads are applied equally, but opposite in sign, to the thermal load at the member ends, and joint deflections and member stresses are again computed. Thus, the final member stresses are obtained by superimposing the previously computed stresses.

C. Dynamic Case:

The equations of motion for a discrete system in free vibration is:

$$[M]\{\ddot{U}\} + [K]\{U\} = \{0\} \quad , \tag{3}$$

where $[M]$ is a diagonal mass matrix of the system. For any degree-of-freedom i, its component displacement in any normal mode m has the form

$$U_{im} = \phi_{im} \, \mathrm{Sin}\,\omega_m t \quad , \tag{4}$$

where:

ϕ_{im} = mode shape coordinate,

ω_m = modal frequency,

and its component acceleration is

$$\ddot{U}_{im} = - \omega_m^2 \phi_{im} \sin\omega_m t \quad . \tag{5}$$

Substituting (5) into (3), we have

$$\omega_m^2 [M]\{\phi\} = [K]\{\phi\} \tag{6}$$

$$\omega_m^2 [M]^{1/2}\{\phi\} = [M]^{-1/2}[K][M]^{-1/2}[M]^{1/2}\{\phi\} \quad . \tag{7}$$

Let

$$\{V\} = [M]^{1/2}\{\phi\} \tag{8}$$

$$[A] = [M]^{-1/2}[K][M]^{-1/2} \quad . \tag{9}$$

Equation (7) becomes

$$\omega_m^2 \{V\} = [A]\{V\} \quad . \tag{10}$$

Notice that [A] is symmetric because [K] is symmetric. The Power Method is used for solving eigenvalue and eigenvectors. Since this method always solves for the largest eigenvalue first and our interest is in the few lowest normal modes, equation (10) must be rearranged:

$$[A]^{-1}\{V\} = \frac{1}{\omega_m^2} \{V\} \quad . \tag{11}$$

Since

$$[A]^{-1} = [M]^{1/2}[K]^{-1}[M]^{1/2} \quad , \tag{12}$$

[K] should be non-singular in order to get $[K]^{-1}$. Thus the structure should be restrained. This is one of the major limitations caused by using the Power Method for a big system. After solving {V}, the mode shape {ϕ} can be computed by substituting {V} into equation (8):

$$\{\phi\} = [M]^{-1/2}\{V\} \quad . \tag{13}$$

Since the mode shape is a relative quantity, it is normalized by assigning unity to its largest coordinate.

The generalized mass and stiffness matrices are computed as:

$$[m] = [\phi]^T[M][\phi] \quad , \tag{14}$$

$$[k] = [\phi]^T[K][\phi] \quad . \tag{15}$$

Owing to the nature of orthogonality, their off-diagonal elements should vanish. Thus the order of magnitude of these elements compaired to the order of magnitude of the diagonal elements will reveal the degree of accuracy in the whole computation.

VI. INPUT FORMAT

Input to the program is provided in the same sequence as the following blocks. Each line is punched on one card. Blanks are always read as 0. Floating-point numbers must be written with a decimal point regardless of whether the decimal part is present.

A. Comment:

Alphabetic letters or numbers for comment or title information in any or all of the first 72 columns. (12A6)*

B. Identifications:

A_1	A_2

(2A6)

A_1 = Case Number (not more than 6 letters or numbers)

A_2 = Department Number (not more than 6 letters or numbers)

C. Control:

N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8	N_9	N_{10}	N_{11}	N_{12}	N_{13}

(13I6)

---

* Fortran input format.

N_1 = Structure Type (NNNN)

1 - Three-dimensional truss, pinned joints.

2 - Three-dimensional frame, rigid joints, axially symmetric member cross sections.

3 - Planar frame, rigid joints, loaded in-plane.

4 - Planar grid, rigid joints, loaded normal-to-plane.

5 - Three-dimensional frame, rigid joints, doubly symmetric member cross sections.

N_2 = Mode Shape Card Output Control (NPNCH)

0 - No output desired.

1 - Output desired. Values for the lowest six modes at the same coordinate will be punched in one card.

N_3 = Eigenvalue Control (NOPT)

1 - No output desired.

0 - Output desired.

N_4 = Quantity of Joints in Structure (NJT)

N_5 = Quantity of Members in Structure (NBAR)

N_6 = Quantity of Static Loadings (NL)

N_7 = Mass Code (NM)

0 - No mass input.

1 - Mass input included.

N_8 = Quantity of Joints Having One or More Components of Restraint (NJR)

N_9 = Degrees of Freedom Per Joint (NDFJ)

N_{10} = Normal Mode Code (NEIG)

0 - Compute no normal modes.

1 - Compute lowest six mode shapes, normalize to input accelerations, and compute dynamic member stresses.

2 - Compute lowest six mode shapes only, normalize to the largest component (U_{max} = 1.0).

N_{11} = Output Code (NOUT)

0 - No listing output of stiffness, mass, and loading matrices.

1 - Listing stiffness, mass, and loading matrices.

N_{12} = Quantity of Stiffness Matrix Elements to Be Altered (NEDIT)

N_{13} = Temperature Code (NMOD)

0 - No temperature problem to be solved.

1 - Temperature problem to be solved.

D. Stiffness Matrix Punch Code (KKKK):

N_{14}

(I6)

0 - No card output of stiffness matrix.

1 - Card output of stiffness matrix desired.

E. Material Properties:

E	ν	γ

(3E12.5)

E = Elastic modulus, 10^3 lb/in^2, (E).

ν = Poisson's ratio (POIS).

γ = Specific weight, lb/in^3 (DENST).

F. Joint Coordinates:

J	X_1	X_2	X_3

(I6, 3F12.5)

J = Joint number (must be listed consecutively starting with 1)

X_1, X_2, X_3 } Joint coordinates, in.

In two dimensional problems, input X_1 and X_2 only

G. Member Properties:

Properties are entered one line (one card) per member; when temperature code N_{13} = 1, then A_5, A_6, or A_7 must be included. Values for A_i are not required unless specifically indicated.

J_A	J_B	A_1	A_2	A_3	A_4	A_5	A_6	A_7

(2I6, 7F8.0)

J_A, J_B } Member ends (enter once for each member in any order)

A_1 to A_7 are defined for each structural type as follows (all quantities to be input in inch units):

1. Structure Type 1, three-dimensional pin-jointed truss.

   A_1 = Cross section area (A).

   $A_3 \neq 0$ (let A_3 = any non-zero term).

or

   A_1 = Outside diameter of circular tube.

   A_2 = Wall thickness of circular tube.

   A_3 = 0.

   $A_5 = \alpha \Delta T$.

2. Structure type 2, three-dimensional rigid frame with axially symmetric cross sections.

   A_1 = Cross section area.

   A_2 = Section moment of inertia.

   A_3 = Section torsional stiffness.

or

   A_1 = Outside diameter of circular tube.

   A_2 = Wall thickness of circular tube.

   A_3 = 0.

   $A_5 = \alpha \Delta T$.

3. Structure type 3, two-dimensional rigid frame, loaded in-plane.

   A_1 = Section area.

   A_2 = Section moment of inertia.

$A_3 \neq 0$ (any non-zero term).

or

A_1 = Outside diameter of circular tube.

A_2 = Wall thickness of circular tube.

A_3 = 0.

A_5 = $\alpha\Delta T$.

A_6 = $\alpha\delta T/h$. δT is positive if a change in temperature will tend to rotate joint J_A of the member in positive X_3 direction. J_A is the 1st joint of a member listed in VI-7 to describe the member.

4. Structure type 4, two-dimensional grid, loaded normal-to-plane.

   A_2 = Section moment of inertia.

   A_3 = Section torsional constant.

or

A_1 = Outside diameter of circular tube.

A_2 = Wall thickness of circular tube.

A_3 = 0.

A_6 = $\alpha\delta T/h$. δT is positive if the increase in temperature across member cross-section is in positive X_3 direction.

5. Structure type 5, three-dimensional rigid frame with doubly symmetric cross section.

   A_1 = Cross section area.

   A_2 = Section torsional constant.

A_3 = Section moment of inertia about ξ_2 axis (see A_7 for definition of axes orientation).

A_4 = Section moment of inertia about ξ_3 axis.

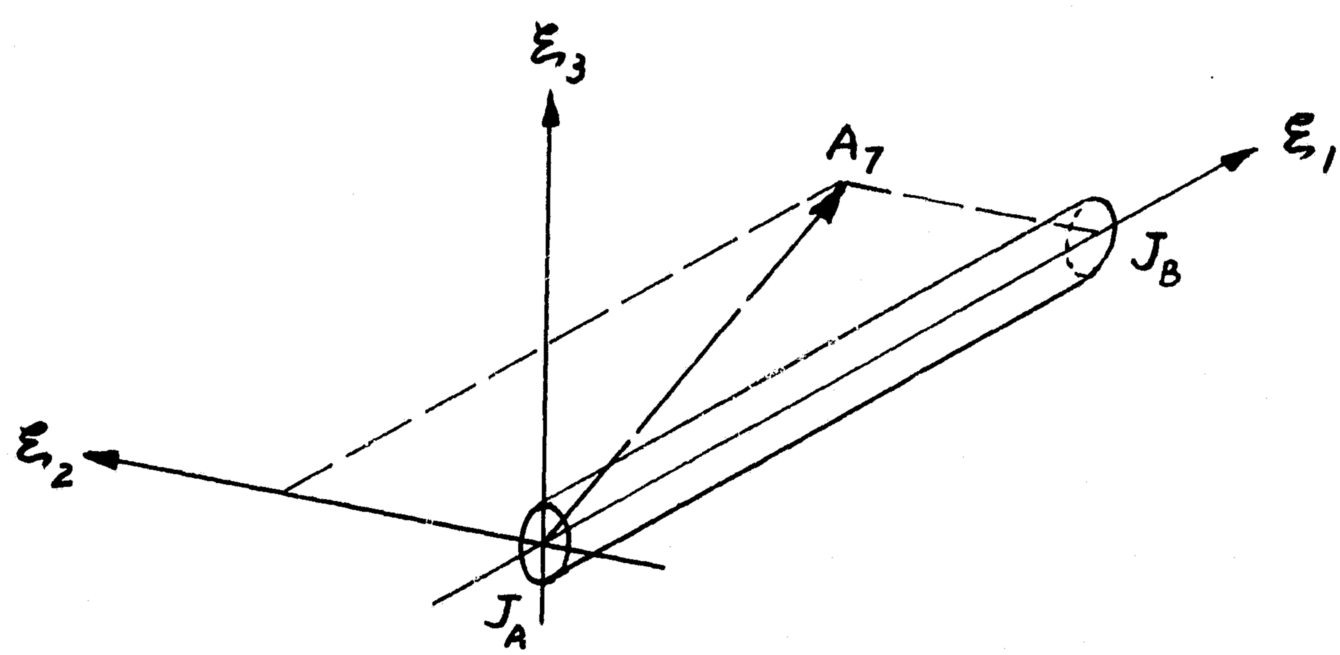
A_5 = $\alpha\Delta T$.

A_7 = A chosen joint number in input list (VI-6) which is not along member axis. Thus the orientation of member coordinates axes ξ_1, ξ_2 and ξ_3 are defined as:

ξ_1 along the direction of member $\overline{J_A J_B}$.

ξ_3 along the direction of $\overline{J_A J_B} \times \overline{J_A A_7}$.

ξ_2 along the direction of $\overline{\xi_3} \times \overline{\xi_1}$.



H. Restraints:

j	r_1	r_2	r_3	r_4	r_5	r_6

(7I6)

j = Joint number (may be listed in any order).

r_i = Restraint code (integer).

0 - No restraint.

1 - $i^{\underline{th}}$ component of deflection at joint j is zero. The order of deflection components at a joint in each structure type is as follows.

1. Structure type 1, three-dimensional pin-jointed truss.

   u_{j1} = displacement in X_1 direction

   u_{j2} = displacement in X_2 direction

   u_{j3} = displacement in X_3 direction

2. Structure type 2, three-dimensional rigid frame, axially symmetric cross section.

   u_{j1} = displacement in X_1 direction

   u_{j2} = displacement in X_2 direction

   u_{j3} = displacement in X_3 direction

   u_{j4} = rotation about X_1 axis

   u_{j5} = rotation about X_2 axis

   u_{j6} = rotation about X_3 axis

3. Structure type 3, two-dimensional rigid frame, loaded in-plane.

u_{j1} = displacement in X_1 direction

u_{j2} = displacement in X_2 direction

u_{j3} = rotation about X_3 direction

4. Structure type 4, two-dimensional grid, loaded normal-to-plane.

   u_{j1} = displacement in X_3 direction

   u_{j2} = rotation about X_1 axis

   u_{j3} = rotation about X_2 axis

5. Structure type 5, three-dimensional rigid frame, doubly symmetric cross-section.

   u_{j1} = displacement in X_1 direction

   u_{j2} = displacement in X_2 direction

   u_{j3} = displacement in X_3 direction

   u_{j4} = rotation about X_1 axis

   u_{j5} = rotation about X_2 axis

   u_{j6} = rotation about X_3 axis

I. Incremental Change to Stiffness Matrix Elements:

To account for the effect of structural elements that cannot be idealized by members of the type with which an analysis is being performed, increments to elements of the stiffness may be inputed. This block may be inputed only if the control parameter $N_{12} \neq 0$.

i	j	Δk_{ij}

(2I6, F8.0)

i,j = The element k_{ij}, which is located at the i^{th} row and j^{th} column of non-deleted stiffness matrix, is changed to $k_{ij} = k_{ij} + \Delta k_{ij}$.

J. Quantity of Mass:

N_{15}

(I6)

N_{15} = Total number of mass points.

K. Mass Components:

j	m_1	m_2	m_3	m_4	m_5	m_6

(I6, 6F8.0)

j = Joint number (may be listed in any order).

m_i = The i^{th} component of mass inertia at joint j. The translational inertia (lb) and rotary inertia (lb-in^2) components are as specified for deflections in VI-8.

If normal modes are to be computed, finite (non-zero) inertia components should be specified for all degrees-of-freedom of the structure. This block of input may be written only if the mass code $N_7 = 1$. Temperature problem should be solved separately from static and dynamic cases. Thus, mass components must not be incorporated ($N_{13} = 1$, $N_7 = 0$).

L. Static Loadings:

j	A_1	A_2	A_3	A_4	A_5	A_6

(I6, 6F8.0)

j = Static loading code.

j=0 - The components of rigid body rotational acceleration about axes passing through the origin of the coordinate axes.

j=-1 - The components of rigid body rotational acceleration about axes passing through the center of mass.

j = - Joint number, loading components at joint j only.

A_i = $i^{\underline{th}}$ translational or rotational component of acceleration as specified for deflections in VI-8. For j = 0 and j = -1, A_i will specify $i^{\underline{th}}$ loading component at every joint as:

$$(\text{loading})_i = A_i M_i \qquad i = 1, 2, 3 \quad .$$

$$(\text{loading})_4 = A_4(X_2M_3 - X_3M_2) \quad .$$

$$(\text{loading})_5 = A_5(X_3M_1 - X_1M_3) \quad .$$

$$(\text{loading})_6 = A_6(X_1M_2 - X_2M_1) \quad .$$

For $j > 0$, A_i = i^{th} component of static loading at joint j only.

K. Modal Accelerations in Free Vibration:

If the normal mode code N_{10} = 0 or 2, this block must be omitted. If N_{10} = 1, modal deflections and dynamic stresses will be computed. In this case, six cards must be given in the following format (one for each lowest six modes in order):

j	i	a_m

(2I6, F8.0)

j = Joint number.

i = Translational coordinate direction number as specified for deflections (see VI-8).

a_m = Acceleration (g) of joint j in direction X_i. If j=0, a_m applies to the maximum deflection component in the mode shape. The mode shape is normalized with the factor $a_m g/\omega_m^2 u_{ji}$ before output and stress calculation. Rotary accelerations have no meaning in this equation.

VII. OUTPUT FORMAT

The following are the contents of printed output. Examples of the output format are given in sample problems, Appendix A.

1. All input data.
2. Volume (in^3) and weight (lb) of each member and sums of all members.
3. Center of mass (in).
4. Weight moment-of-inertia about center of mass ($lb\text{-}in^2$).
5. Stiffness matrix (lb/in) printed row by row.
6. Mass matrix (lb), only elements in main diagonal are printed.
7. Load matrix (lb).
8. Joint deflections due to each static or thermal loading. Each column corresponds to one loading.
9. Static or thermal member stresses.
10. Equilibrium check of static solution at all joints for each static loading. The non-zero terms represent the reactions at the restraints. The reactions are positive if they act along the positive X_i directions. The equilibrium check is not made for the thermal loads. The unrestrained joints at which elements to stiffness matrix are altered will not be zero in equilibrium check. The non-zero term $f_i = \Delta k_{ij} u_j$.

11. Convergence in each iteration cycle by Power Method for eigenvectors.

12. Six lowest normal frequencies (cps) and their associated mode shapes generated by Power Method.

13. Member stresses and joint displacements due to specified acceleration component at a joint during free vibration in a normal mode.

14. Generalized mass matrix and orthogonality check.

15. Generalized stiffness matrix and orthogonality check.

Card output of the stiffness matrix will be performed when the punch code (Section VI.D.) is specified to do so. Elements of the matrix are punched row by row in a format of 5E14.5.

VIII. ACKNOWLEDGEMENT

Sincere gratitude is extended to Mrs. A. J. Cochran, a programmer in Department 1032, who has completed this tedious conversion task with tremendous patience.

Shou-nien Hou

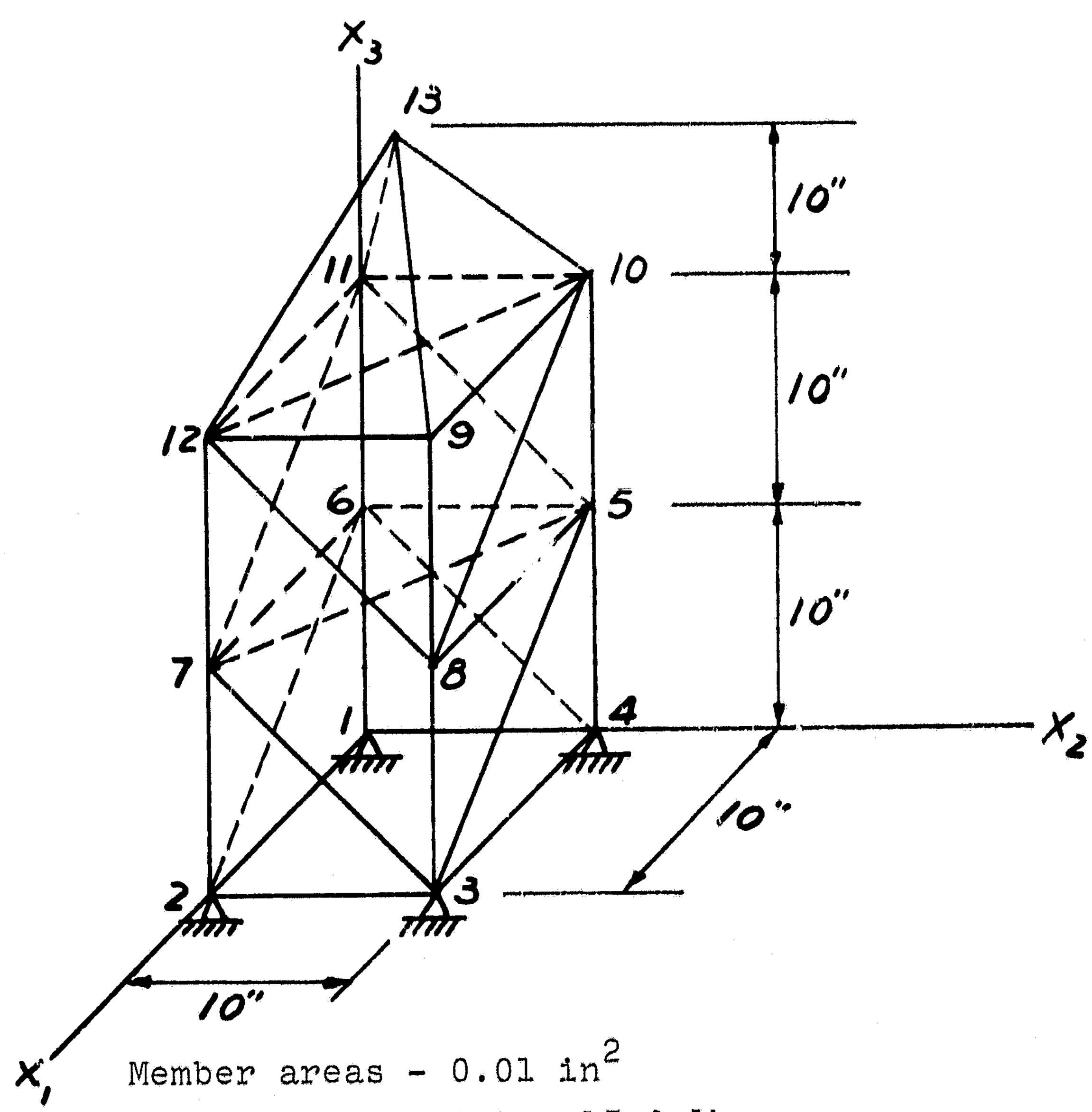
2031:SNH:ss S. N. Hou

Attachments
  Appendix
  References

APPENDIX A

Example Problem #1

Structure type 1, three-dimensional pin-jointed truss, under static loadings and dynamic modal analysis.



Member areas - 0.01 in^2

Mass at each joint - 15.0 lb

Modulus of Elasticity - 10^7 psi

Poisson's ratio - 0.3

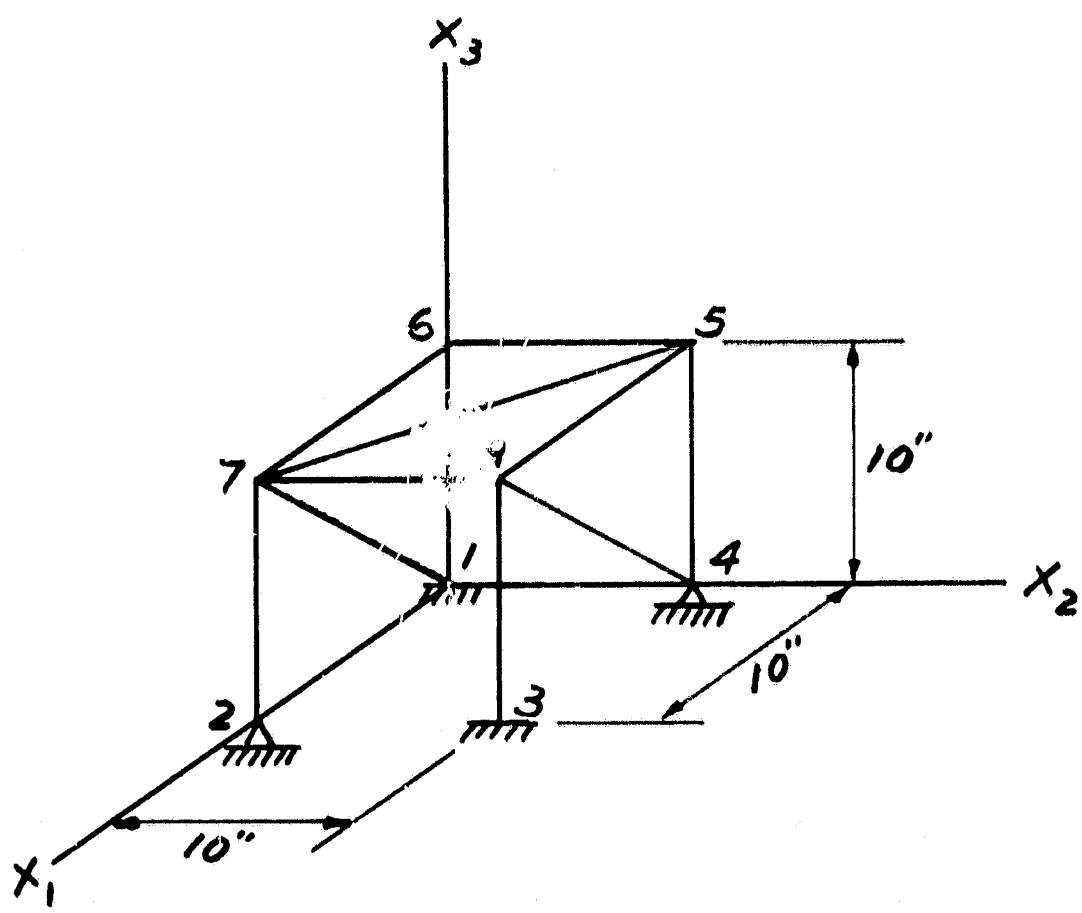
Appendix A (contd.)

Example Problem #2

Same structure as problem #1 but under thermal loading caused by elongation of member 3 (from joint 1 to joint 6).

Example Problem #3

Structure type 2, three-dimensional rigid frame, under static loading and dynamic modal analysis.



Section moment of inertia = 0.1

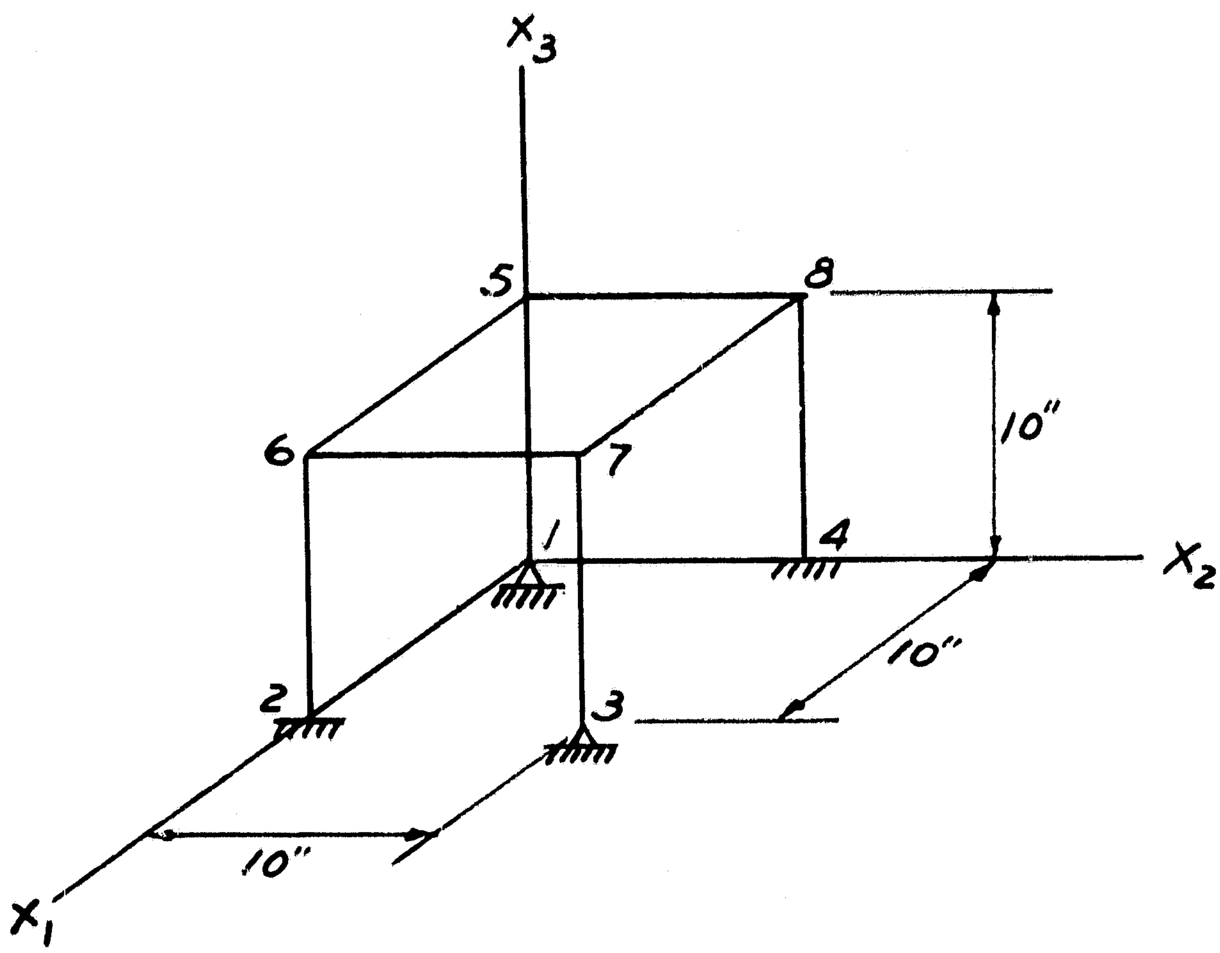
Section torsional stiffness = 0.2

Elastic modulus = 10^3

Appendix A (contd.)

Example Problem #4

Structure type 2, three-dimensional rigid frame, under thermal loading caused by elongation of member 1 (from joint 1 to joint 5) and member 3 (from joint 3 to joint 7)



Member area = 0.01

Section moment of inertia = 0.1

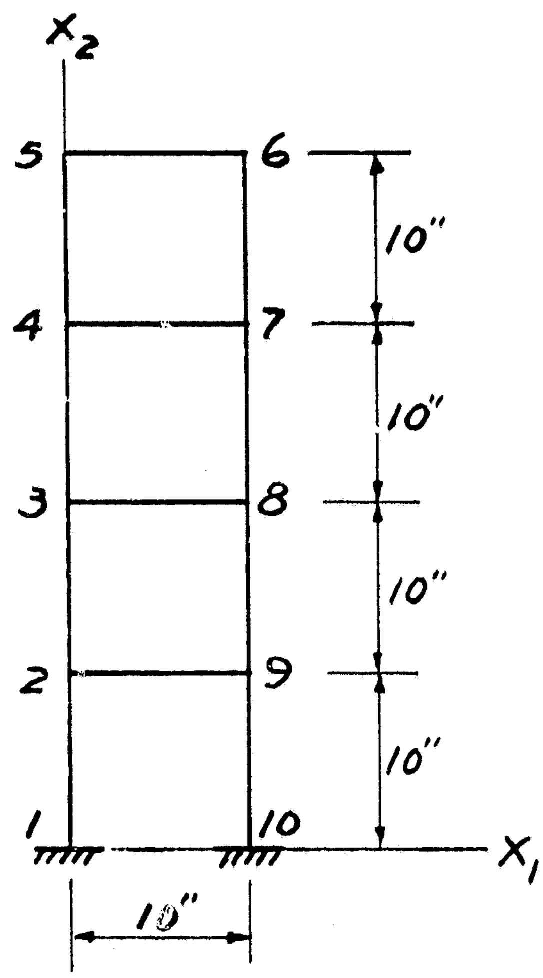
Section torsional stiffness = 0.2

$\alpha\Delta T$ = 0.02

Appendix A (contd.)

Example Problem #5

Structure type 3, two-dimensional rigid frame, under planar static loading and dynamic modal analysis.



Member area = 0.01

Section moment of inertia = 1.0

Elastic modulus = 10^3

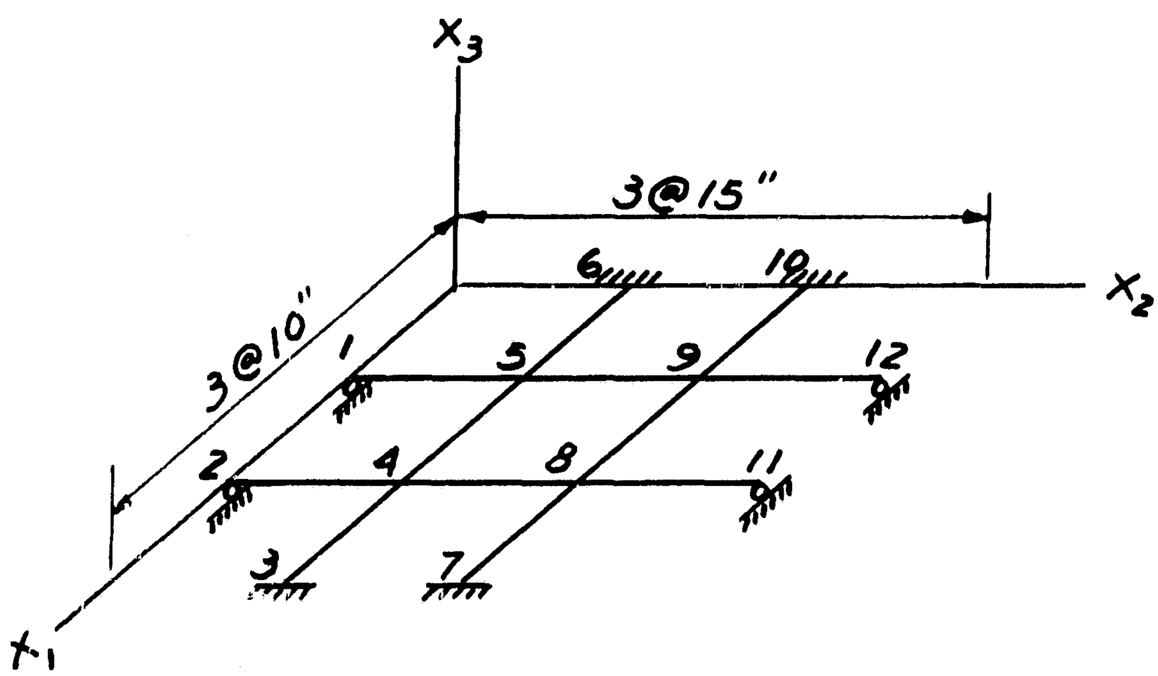
Example Problem #6

Same structure as problem #5 but under thermal loading caused by elongation of member 3 (from joint 2 to joint 9) and bending of member 5 (from joint 3 to joint 8).

Appendix A (contd.)

Example Problem #7

Structure type 4, two-dimensional rigid grid, under static loading perpendicular to the plane, and dynamic modal analysis.



Section moment of inertia = 10

Section torsional constant = 20

Elastic modulus = 10^3

Example Problem #8

Same structure as problem #7 but under thermal loading caused by bending of member 5 (from joint 4 to joint 8) and member 7 (from joint 5 to joint 9).

Appendix A (contd.)

Example Problem #9

Structure type 5, three-dimensional rigid frame with doubly symmetric cross section, under static loading and modal analysis.

Notice that this problem has the same structure and loadings as given in problem #3, but their input format are different. However, they yield same results, as expected.

Example Problem #10

Structure type 5, same structure and thermal loading as shown in problem #4 but with different input format.

Notice that the results from both problems are identical.

Example Problem #11

Same structure as given in problem #3 but under free vibration.

Notice that the mode shapes are normalized to specified acceleration at certain coordinates. Dynamic stresses in each mode are computed.

NOTE: List of program and output of numerical examples, which are bound separately, are available in Department 2031.

REFERENCES

1. JPL Technical Report No. 33-75, "Stiffness Matrix Structural Analysis", R. R. Batchelder and B. K. Wada, February 12, 1962, Pasadena, California.

2. JPL Technical Report No. 32-774, "Stiffness Matrix Structural Analysis", B. Wada, October 31, 1965, Pasadena, California.

BELLCOMM, INC.

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