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*John Shetz*

COVER SHEET FOR TECHNICAL MEMORANDUM

TITLE— An Introduction to the Available  
Computer Programs and Other Design  
Aids for the Design of Filters,  
Equalizers and Other Networks

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ABSTRACT

This memorandum is intended as an introduction to the design of filters, equalizers and other networks. The design of passive networks are considered only.

There is a whole range of tabulated results, computer programs, and other design aids available that enable even the occasional designer to obtain very efficient designs with a minimum of effort.

After a short introduction to the specification of the individual network types, we list and briefly describe the available tables, programs and their use.

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TABLE OF CONTENTS

The following annotated Table of Contents will indicate the scope of this memorandum.

	<u>Page</u>
1. Introduction.....	1
2. Filters.....	1
These are the most often used networks and our treatment is consequently long.	
2.1 Classification and Design Methods.....	1
Here we describe the more general filter types and the design methods presently in use.	
2.2 Specification of Filters.....	3
This section lists the electrical parameters one must specify for the designer and what practical limits one has to observe.	
2.3 Available Tables and Computer Programs.....	7
In this long section we first explain what is meant by a Butterworth, Chebyshev, Bessel, or other filter types. After that we list the available tables of element values of these and other filters. Finally we briefly describe the available computer programs one can use for the design of filters.	
2.4 Other Filter Types.....	14
In this section we mention filters that need special design techniques, among them, crystal filters, directional filters and filters with unusual specifications.	
3. Equalizers.....	16
3.1 Loss Equalizers.....	16
This section gives a very brief introduction to the design of loss equalizers, including some practical advice and listing of sources for further information.	

	<u>Page</u>
3.2 Delay Equalizers.....	20
The above treatment is repeated here for the delay equalizers.	
3.3 Time-domain (transversal) Equalizers.....	23
3.4 Iterative Approximation Programs.....	23
The programs described in this section were originally developed for loss and delay equalizer designs. They are very good at this but they are considerably more versatile as explained in the text.	
4. Other Networks.....	25
In this final chapter we briefly mention a few further types of networks one may occasionally come across like phase-splitting networks and delay lines.	
5. Conclusion.....	27
References	

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SUBJECT: An Introduction to the Available  
Computer Programs and Other Design  
Aids for the Design of Filters,  
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*Case 38565-2*

DATE: May 15, 1967

FROM: G. Szentirmai

MM-67-4734-7

MEMORANDUM FOR FILE

1. Introduction

Unlike some of the recent discoveries in the more glamorous areas of electronics, filter and network design techniques underwent a steady but quite unspectacular development and improvement in the past years. Since these networks still represent a large percentage of the expense of the physical plant, these improvements can mean quite a considerable savings, if one knows about them and uses them.

It is the object of this memorandum to acquaint the reader with the present status of our network design techniques, including available computer programs and some simple rules for deciding whether a set of requirements is realistic or not. We would like to encourage the reader to try his (or her) hand at the design of networks, use the large number of programs available and see how easy and how much fun it can be. We do not however, pretend to give all the answers, and in any doubtful case a consultation with your friendly neighborhood network designer is in order.

2. Filters

2.1 Classification and Design Methods

By far the most often used network type is the filter. Its function is to select and pass a band or bands of frequencies and suppress others.

According to the location of the band(s) of frequencies to be passed we classify filters into the following categories:

1. Low-pass filters
2. High-pass filters
3. Bandpass filters

4. Band elimination filters
5. Other (more complicated) filters

According to the types of components used, we can talk about:

1. LC filters, that use conventional inductors and capacitors only;
2. Crystal (and mechanical) filters, that also use piezoelectric or other mechanical resonant structures as components;
3. Active RC filters, that use linear amplifiers, but no inductors.

This last-mentioned type is only now reaching maturity and will not be considered here any further.

As far as structural distinctions are concerned, most filters are built in ladder form, but this is by no means the only possible configuration although it is certainly the most desirable one. Ladder networks are preferred because they are the least sensitive to component variations. Other configurations, used mainly in crystal filters, are the tandem connection of ladder and lattice sections or their equivalents.

Turning to the design methods, there are two general methods of filter design:

1. Image-parameter method
2. Insertion-loss design technique

Both methods have their advantages and disadvantages.

The image-parameter design method has been in use for about half a century, the expressions are simple and slide rule accuracy is usually adequate. Most electronics and electrical engineering handbooks contain a brief summary of this method. (See, for instance, [50]).

The disadvantage of this method is that it is approximate and does not supply the most efficient design. Recent developments in this theory permit one to come very close

to the theoretical optimum (minimum number of components), but only at the cost of greatly increased complexity in computations. Paradoxically, the older, simpler expressions have been programmed for digital computers but the more recent, very much more complicated expressions have not. Furthermore, there is no single reference that contains the up-to-date description of the technique. The main reason is that the second, exact method is slowly but surely replacing the image-parameter method in filter design.

The insertion-loss theory is an exact design method if the components to be used are assumed to be ideal (some nonideal characteristics, like homogeneous dissipation or lumped parasitic capacitances, can sometimes be taken into account). The major disadvantage of the method is that computations are quite involved and can usually be performed only on a digital computer. Tables have been prepared of some restricted classes of low-pass filters, that can also be extended to high-pass, band-elimination, and an even more restricted version of bandpass filter types. For the general cases, a library of computer programs is available. We will consider these with some of the available tables after we have described the specification of filters.

## 2.2 Specification of Filters

We will only be concerned here with the electrical specification of filters, leaving out all the mechanical and even some of the semielectrical considerations like temperature and humidity effects, shielding, etc.

A filter is generally specified by its loss- (or amplitude), phase- (or delay), and return loss characteristics in the frequency domain, and very seldom by its step- or impulse-response in the time domain.

The first thing to decide is whether a given set of requirements is theoretically compatible and practically realizable. Theoretical incompatibility can most often occur in a combination of frequency- and time-domain specification where they may be mutually exclusive. In other cases one requirement, typically the pass band return loss, may override another, in this case the pass band loss specification.

These too are related to each other and usually the required return loss implies a theoretical pass band loss ripple that is an order of magnitude smaller than the values

practically possible. For instance a modest 20 dB minimum return loss requires a theoretical pass band ripple of less than 0.043 dB that will be swamped out by dissipative and other parasitic effects.

The facts of practical life impose further limits on the realizability of a given specification. While the theoretical design can cope with any set of (compatible) requirements, the practical filter as built in the laboratory or factory, will differ from the ideal in several respects.

1. Dissipation and parasitic effects will change the performance from the ideal, although they can be partially compensated for.
2. The component values will not agree with the theoretical values because most of the time the nearest preferred value is selected and that can also vary within its tolerance range.
3. Temperature, humidity, aging effects and sometimes the signal level transmitted through the network will change the values of the components.

Going back to the actual specification, the most important, of course, is the loss versus frequency characteristic, of which a typical example is shown in Figure 1 for a low-pass filter.

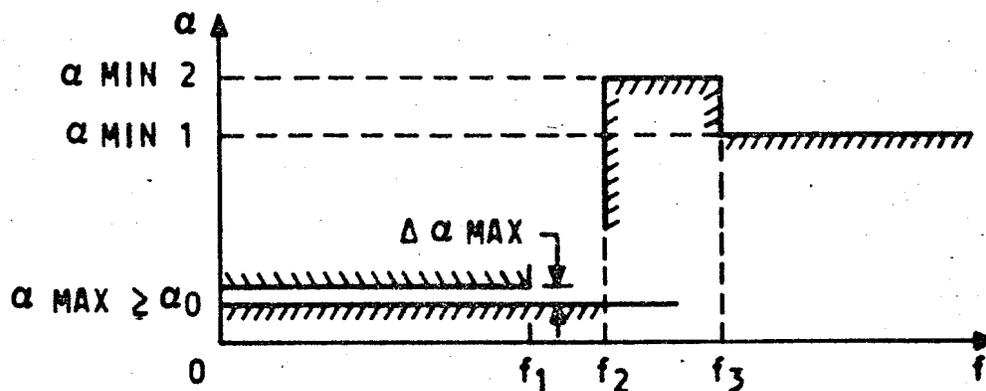


Figure 1

The pass band extends from zero to  $f_1$  where the loss should be within  $a_0$  and  $a_0 + \Delta a_{\max}$  with the further restriction  $a_0 \leq a_{\max}$  that limits the flat loss to  $a_{\max}$  in the pass band. The stop band requirements are self-explanatory.

If the phase characteristic is specified, it is usually required to be linear to within  $\Delta\phi_{\max}$  or the delay is to be constant to within a certain  $\Delta\tau_{\max}$ .

The return loss can be specified as a minimum required value in the pass band at either or both ends of the network, against the constant resistive terminations  $R_1$  and  $R_2$ .

In case of a time domain specification, the most common requirement is for the step- or impulse-response to have a ripple of limited magnitude, excluding of course, the main pulse or step.

As a rule of thumb, the practical lower limit of the pass band ripple  $\Delta a_{\max}$  is of the order of 0.1 dB and this will require additional equalization to achieve in most cases. The theoretical design value is usually selected to be lower by a factor of 2 to 3 even if the return loss requirement does not necessitate it, to leave some of it for the dissipative effects. Return loss values of up to about 40 dB are reasonable.

The other critical parameter is the steepness of the cutoff:

$$\delta_T = \frac{f_2 - f_1}{f_1}$$

The narrowest achievable transition width depends on the components to be used, particularly on the inductor  $Q_L$ .

One must have

$$Q_L \delta_T > 1$$

and preferably greater than 10.

For high-pass filters the specification and rules are similar. For bandpass filters, there is one more parameter. A typical bandpass loss requirement is shown in Figure 2.

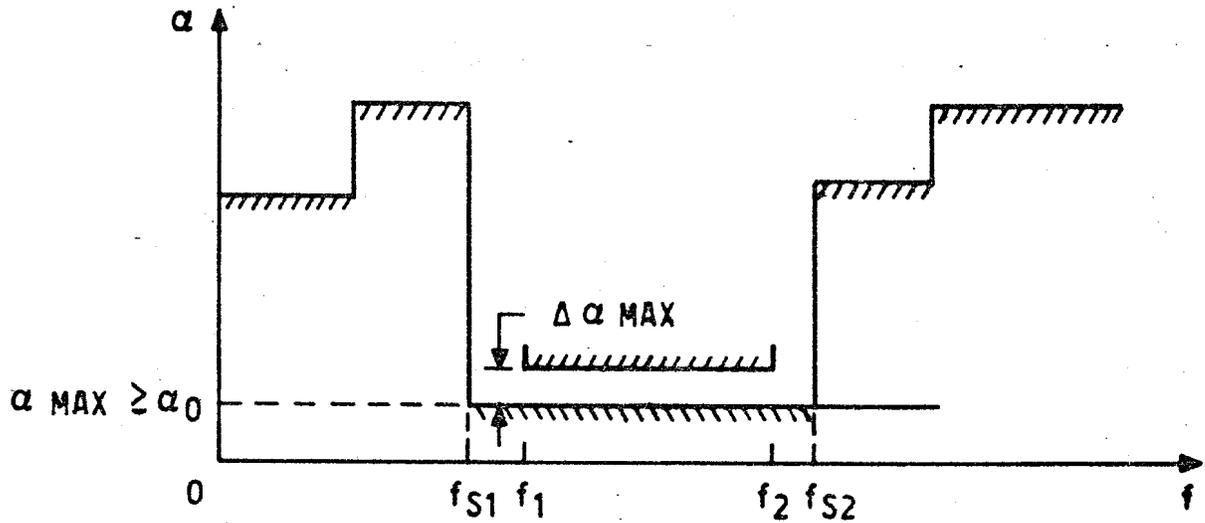


Figure 2

In this filter type, one critical parameter is the fractional bandwidth:

$$\delta = \sqrt{\frac{f_2 - f_1}{f_2 f_1}}$$

The other parameter is related to the steepness of the cutoff and for medium and narrow bandwidths can be expressed simply as:

$$\delta_T = \frac{f_{s2} - f_{s1}}{f_2 - f_1} - 1$$

Again our rule of thumb requires both

$$\delta Q_L > 1$$

$$\delta_T Q_L > 1$$

and preferably both greater than 10. In any case, the fractional bandwidth should be greater than about 0.05 if conventional components are to be used. Crystal filters can realize bandwidths up to about  $\delta = 0.05$  and there is very little overlap between the two types.

For very wide bands the two cutoff regions can be considered separately as if the filter would be a combination of a low-pass and a high-pass filter as indeed it may be.

Stop band losses range anywhere from 10 dB to 100 dB, above which it becomes very difficult to measure losses with any reliability.

In most cases, one cannot obtain the required passband flatness with available inductor Q values. One solution is to use predistortion, but this will make the filter more sensitive to element value variation and completely upsets the return loss properties of the filter.\* Furthermore it can only compensate for a certain amount of dissipation that depends on the complexity of the filter.

The other solution is a separate attenuation equalizer, that will be covered in the next chapter.

If phase or delay requirements are also specified, the only solution presently available is to design a network disregarding the phase completely, calculate its phase (or delay) and equalize it by the use of special delay equalizer sections. The design of such delay equalizers will also be covered in the next chapter.

Very little is available on meeting mixed specification in the frequency- and time-domains. A few tabulated results are at hand for special cases, to be mentioned later.

### 2.3 Available Tables and Computer Programs

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\* Predistortion works by introducing loss in areas where the dissipation had little effect, to flatten out the characteristic. This loss is produced by reflection as the only mechanism available and therefore results in large mismatch and consequently low return loss values.

### 2.31 Nomenclature

In order to enable the reader to use the available filter tables and programs we have to explain a few notations and names.

There are two types of loss behavior in the filter passband commonly in use:

1. Maximally-flat\*
2. Equal ripple

In the stop band, there are three possible types of behavior:

1. Monotonic (no finite-frequency attenuation peaks)
2. Equal minima (all loss minima are equal)
3. General (other than 1 or 2).

Considering now low-pass filters as the only kind tabulated, the following combinations have special names:

1. Butterworth filter: maximally flat passband and monotonic stop band.
2. Chebyshev filter: equal ripple passband and monotonic stop band.
3. Inverted Chebyshev filter: maximally flat passband and equal minima stop band.
4. Elliptic (or Cauer parameter) filter: equal ripple passband and equal minima stop band.
5. General "reference" type filter: equal ripple passband, general stop band.

The last combination of maximally flat passband and general stop band has no name and is seldom used. Note also, that there is no case of general loss in the passband. This is a major drawback and will be discussed later on.

Another possible passband behavior is that of maximally flat delay. This leads us to the further types of

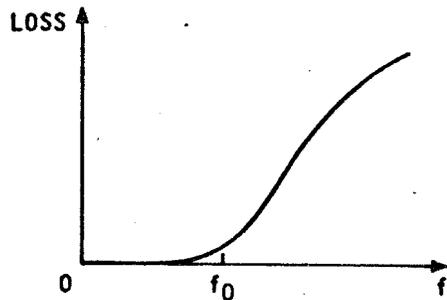
6. Bessel filter: maximally flat delay, monotonic stop band loss.

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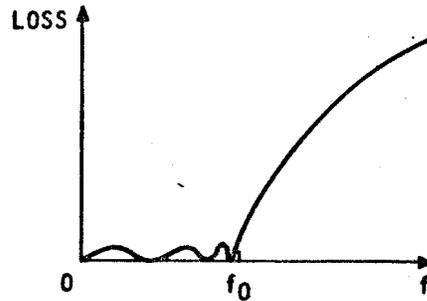
\* The first  $n-1$  derivatives of the loss as a function of  $\omega^2$  are zero for an  $n$ th degree filter at zero frequency.

7. Special Bessel filter: maximally flat delay, equal minima-type stop band loss.

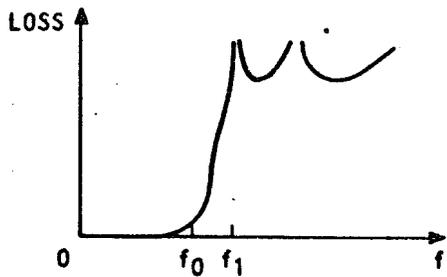
Characteristics of these filter types are illustrated in Figure 3.



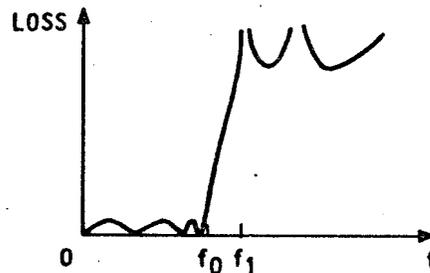
(1) BUTTERWORTH



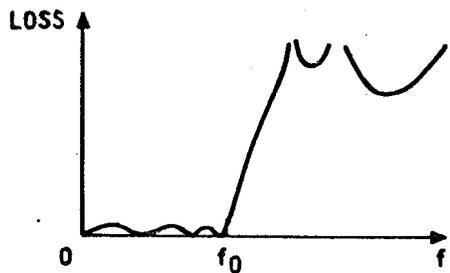
(2) CHEBYSHEV



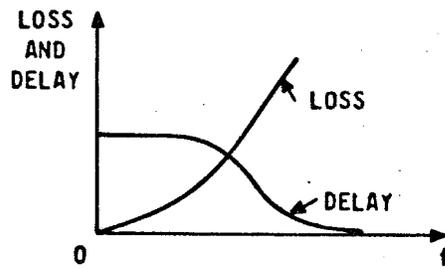
(3) INVERTED CHEBYSHEV



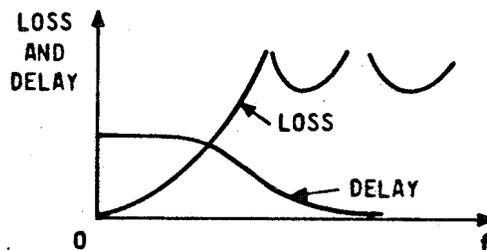
(4) ELLIPTIC (CAUER)



(5) GENERAL ("REFERENCE")



(6) BESSEL



(7) SPECIAL BESSEL

Figure 3

## 2.32 Tables

In what follows, we list tables of filter element values of some of the above types. The numbers refer to the corresponding items in the list of references. This list of available tables is fairly complete necessarily with some overlap. The only exception is Weinberg, who has published innumerable tables in equally innumerable articles, that are not included in our list, because all these tables are also repeated in [1].

Butterworth filters are tabulated in:

[1], [2], [5], [10], [12] and [16]

Chebyshev filters are available in:

[1], [2], [5], [6], [7], [10], [12] and [16]

Inverted Chebyshev filters can be found in:

[8] and [10]

Elliptic filters have extensive tabulations in:

[2], [3], [4], [9], [10], [13], [14] and [15]

General "reference" type filters cannot be tabulated because of the many variable parameters they can have.

Bessel filter tables are available in:

[1], [2], [5], and finally

Special Bessel filters are tabulated in:

[11],

Incidentally, [5] is the most complete reference available dealing with microwave filters; that is a very specialized area and is not considered further in this memorandum.

Attempting to evaluate the relative merits of the listed references, one would consider [3], [1], [4] and [6] the most useful, with [7], [8], [9], [10] and [11] supplying further tables not available in the first group. I specifically caution against items [2] and [16] on this

list. While they seem to cater for the occasional filter designers, these books contain errors and misleading statements.

High-pass, bandpass, and band-elimination filters can be obtained from the low-pass type by transformation. This is explained clearly in most of the references, and the resulting filter is as general as the original low-pass type except in the bandpass case. Naturally the Bessel and special Bessel filters will not retain their constant delay characteristics through these transformations, except approximately in the narrow bandwidth bandpass case.

As pointed out above, bandpass filters obtained from low-pass types through transformation represent a limited subclass of the most general bandpass filters and they do not represent efficient designs at that. An extremely limited attempt has been made in [2] to tabulate more general bandpass filters, but the task is a completely hopeless one, due to the large number of parameters needed to specify a filter.

It is therefore all the more imperative that computer programs be available to design bandpass, as well as other filter types.

### 2.33 Computer Programs

Most of the available filter synthesis programs are described in [21].

The theory of the low- and high-pass filter synthesis program is given in [17], the operating instructions and interpretation are described in [18].

This program can design Butterworth, Chebyshev, Inverted Chebyshev and Elliptic parameter filters directly. It can also design general "reference" type, as well as the last combination (maximally flat passband and general stop band) type filters, but for these types one has to determine from the requirements the number and location of attenuation poles needed, before the program can be used.

This preliminary step can be done either long hand, or by the use of another computer program [20] in the case of the "reference" type filter (i.e., equal ripple passband).

For the maximally flat passband case one can consult [24] or use guess work or trial-and-error methods in conjunction with the synthesis program.

Incidentally, since the program is so easy to use and since the computing is so fast, trial-and-error methods can be used generally for nearly all filter kinds except the most critical.

Briefly the available options of the program are as follows:

Filter type: low pass or high pass or both

Degree: up to 40

Passband: equal ripple or maximally flat

Stop Band: monotonic, equal minima or arbitrary  
(with specified peak positions)

Terminations: arbitrary at both ends

Predistortion for homogeneous dissipations: available

Configuration: Ladder, specified by computer or designer

The input data needed to specify the filter vary somewhat depending on the requirements.

The passband is specified by its end and the passband ripple amplitude (or the loss at the end of the passband if maximally flat response is required). The stop band is specified by the attenuation peaks, or the beginning of the stop band and the minimum required loss if the requirement is of the equal minima type. This is followed by the terminations, and finally the element  $Q$  if predistortion is required.

The computer output will contain, in addition to the configuration and element values, the results of a check computation that will list the loss, phase, input impedance, input admittance, return loss and (optionally) delay versus frequency, any of which can also be plotted on microfilm.

The bandpass synthesis program is quite similar with nearly identical options:

Degree: up to about 40

Passband: equal ripple or maximally flat

Stop band: monotonic or arbitrary (with specified peak positions)

Terminations: arbitrary at both ends

Predistortion for homogeneous dissipation: available

Configuration: Ladder, specified by computer or designer

The computer input is also similar to the input of the low-pass program. One first specifies the passband edge frequencies and the passband loss ripple, or the loss at the edges if the loss is of the maximally flat type. The stop band is specified by the attenuation peak locations. This is followed by the terminations, and the Q if one would like to have predistortion. Finally, we can specify our configuration in a simple code or let the computer pick one for us. The configuration supplied by the computer will have the minimum number of inductors possible.

Again the output will contain the results of a mesh computation, and any of the listed characteristics can also be plotted.

Concerning the theory of the bandpass filter synthesis program, it is available in [22], but [15] also contains a good introduction (see also [21]). Unfortunately there has been no detailed user's manual written yet, only some informal notes.

In the case of general stop band and equal ripple passband performance one has to determine the number and location of attenuation peaks needed. This can be done long hand (see Appendix I of [22]), and a computer program for this step is also under development and expected to be available shortly.

For maximally flat passband and arbitrary stop band, one may consult [24] again or use trial-and-error methods.

Bandpass filters have the unique property of being able to provide impedance transformation without transformers. This can be used not only to provide us a wide range of values for the terminations but also to change internal elements to more convenient values. A program is now available [49] to perform this type of transformation on the computer.

Finally still another computer program is under development, this one for the design of Bessel, special Bessel and other related low-pass filter types (for instance, equal ripple-type

delay behavior will also be available). For the theory, see [19], while a user's manual will be issued shortly after the program becomes operational.

## 2.4 Other Filter Types

### 2.41 Crystal Filters

The design of crystal filters represents a rather specialized field within network theory mainly because of the constraints the equivalent circuit of a crystal puts on the filter element values and thereby on the characteristics. Most crystal filters are still being designed on an image-parameter basis, principally because this constraint appears explicitly in this method, although the insertion-loss method seems to have gained considerably in recent years.

The best reference available within the Bell Laboratories is [25], and all the expressions in this book have been programmed [27]. Item [26] on our list is a fairly up-to-date review article on the subject and contains a large number of references for further study.

### 2.42 Filters with More than One Passband

We have covered the design of low-pass, high-pass and bandpass filters, all of which have only one passband. Band-elimination filters and other multiple bandpass filters still represent considerable challenge to the designers.

One can, of course, derive band-elimination filters from low-pass filters by transformation and this is explained in nearly all references. In fact, this method is recommended for general use even if it does not produce the most general and economical solution.

For a taste of the complexities involved in multiple bandpass filter design and some solutions, see [28].

### 2.43 Directional Filters

In a number of cases an incoming band of frequencies must be separated into two or more outputs (Figure 4), where the individual outputs cover separate (nonoverlapping) frequency bands.

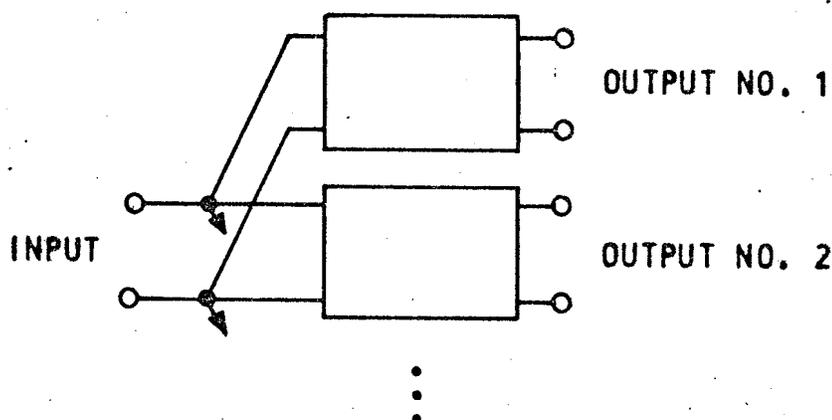


Figure 4

One can use the computer-generated filters described above, for this purpose and the details are described in [23].

#### 2.44 Filters with Shaped Passbands

With the exception of the Bessel and special Bessel filters, all other filters we have treated, are designed to have a flat (constant loss) passband.

In a number of applications one wants a filter with a passband of prescribed (nonconstant) shape. There are, in general, two approaches to this problem.

The first, conceptually simpler, approach is to design a filter with a proper stop band behavior and flat passband, and add to this equalizers (see next chapter) to provide the proper shaping. While this approach will always work, it does not usually lead to the most economical design.

The second approach is to use a more sophisticated iterative scheme. One subclass of this approach is to approximate the required characteristic by a transfer (rational) function and then synthesize the function (realize it by a network). The other subclass is to obtain a network with element values and let the iterative program work on these element values until the performance approaches the requirement closely enough.

Both of these solutions have advantages and disadvantages, the main disadvantage is that there is no guarantee of convergence in either case. Convergence depends very much on the initial starting point, and the selection of the proper starting point is a matter of art (or luck), rather than science.

We have now several computer programs to perform such iterative approximations and they will be described in Chapter 3 in more detail.

#### 2.45 Filters with Mixed Specifications

Some limited attempts have been made to design filters for simultaneous frequency- and time-domain specifications in [29] through [33]. All of these consider low-pass filters with uniform stop band requirement and uniform or uniformly tapered limit on ripples of the response of the network to impulse-, step- or square-pulse excitation. Items [29], [30], [32] and [33] contain limited tables of element values for selected parameter values. A few additional papers exist that tabulate natural modes but do not go as far as the network realization.

Basically the iterative approximation programs mentioned in the previous section, are capable of simultaneous approximation in the frequency- and time-domains, but this feature has not been tried out extensively.

### 3. Equalizers

In general, there are two types of equalizers, loss and phase (delay) equalizers. If both loss and delay equalizations are required, the loss equalization must be performed first, followed by the delay equalization. This is because loss equalizers have phase distortion of their own, while theoretically, delay equalizers have no loss. In practice, dissipation will cause delay equalizers to have some frequency dependent attenuation and this may necessitate an iterative approach to the overall equalization.

#### 3.1 Loss Equalizers

##### 3.11 Fixed Loss Equalizers

The overwhelming majority of loss equalizers consists of sections of the type shown in Figure 5.

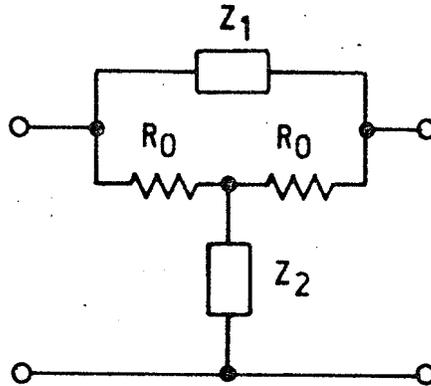


Figure 5

If  $Z_1$  and  $Z_2$  are the inverses of each other, that is if

$$Z_1 Z_2 = R_0^2$$

then this is what is called a "constant resistance" network. This means that if it is terminated by  $R_0$  at one terminal pair, the input impedance at the other is identically equal to  $R_0$  too.

The obvious advantage of this property is that any number of sections of this type (as well as one other, nonconstant impedance unit) can be connected in tandem without interaction. The overall loss will simply be the sum of the losses of the individual sections.

The loss of a single section can be easily obtained as:

$$a \text{ [dB]} = 10 \log_{10} \left| 1 + \frac{Z_1}{R_0} \right|^2$$

Evaluation of this expression for an existing equalizer is therefore fairly simple, but the design is usually of the trial-and-error type based on previous experience.

A few simple impedance forms and the associated loss behavior are shown in Table 1. Naturally, more complicated shapes can be obtained by more complicated impedances, or connecting several sections in tandem or both.

As an example, consider the equalization of a low-pass filter. Figure 6 shows the ideal characteristics (curve 1) and the dissipative computed loss (curve 2). The needed equalizer is clearly the type shown in the third row of Table 1, that can, for instance, have the loss shape indicated by the curve 3 in Figure 6. Curve 4 shows the overall loss (sum of curves 2 and 3) that may be quite satisfactory.

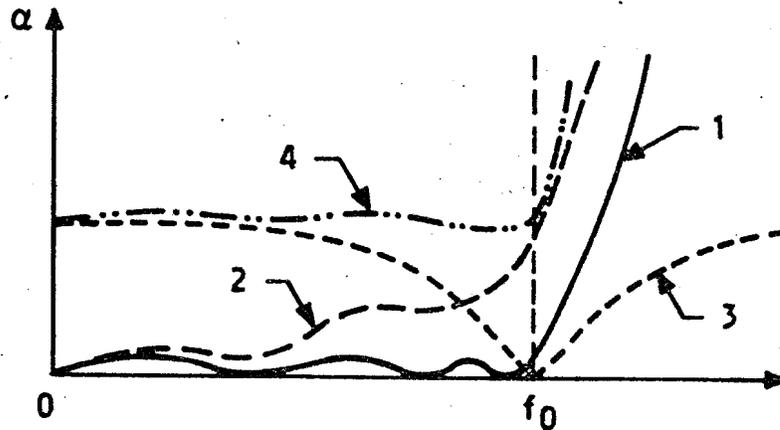


Figure 6

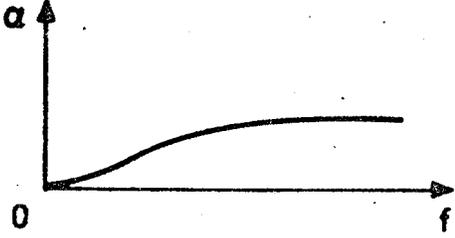
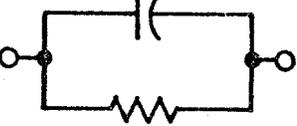
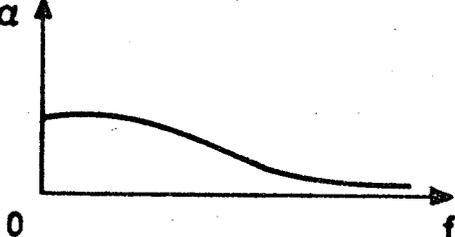
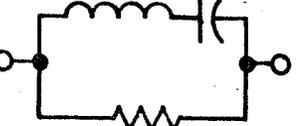
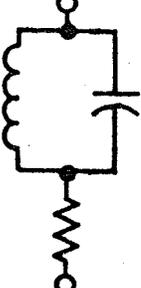
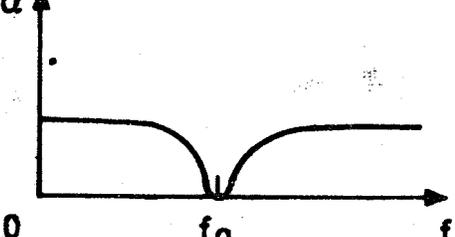
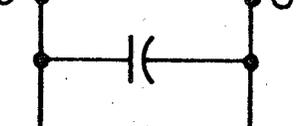
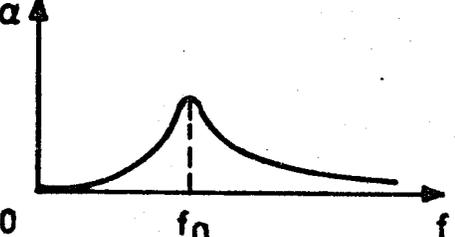
$Z_1$	$Z_2$	LOSS SHAPE
		
		
		
		

Table 1

Properties of Simple Loss Equalizers

More accurate equalization, or the equalization of more complicated shapes may not be so simple, however, and a large amount of design aids are available in the form of graphs and tables. Item [34] on our list of references contains a choice selection of such design aids with further details on how to calculate the element values once a proper shape is selected. It contains, furthermore, a large bibliography for those who want to learn more about this subject.

Iterative computer programs can be used to help design equalizers (see section 3.4), but this is only a partial help. The designer still has to select his configuration (and initial parameter values), and the computer will only optimize the parameter values but leave the configuration unchanged. We therefore, must make certain that the selected configuration is capable of producing the right loss shape and this step will still need a certain amount of experience.

Considering practical realizations, we have the following rules of thumb again.

Equalization to an accuracy of  $\pm 0.1$  dB is usually feasible, with an accuracy of about  $\pm 0.02$  dB being the ultimate limit. Beyond that one cannot guarantee reproducibility of the shapes accurately enough.

At the other end of the scale shapes of 60 dB range or more have been equalized, but naturally with considerably less accuracy.

### 3.12 Adjustable (Variable) Loss Equalizers

Sometimes the loss to be equalized is not known or subject to variation, and a variable loss equalizer is required that is readjusted either manually (periodically) or automatically (through a control loop). The adjustable element should preferably be a resistor that can be varied manually or thermally (in a control loop).

The basic structure of this type is described in [44] and further references can be found in [34].

### 3.2 Delay Equalizers

Again we are looking for a constant impedance structure to enable us to connect it in tandem with other units without interaction.

The basic delay unit of this type is shown in Figure 7 in two versions.

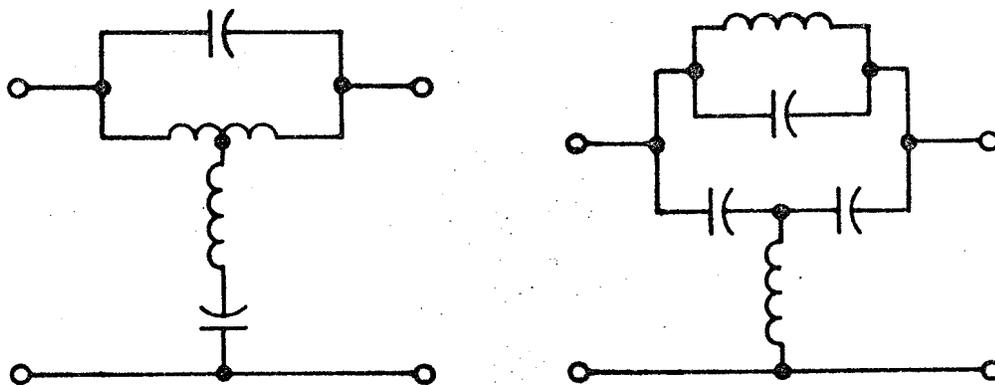


Figure 7

It can be shown, that the most general constant resistance network, that has a frequency independent loss (all-pass), can be built up of sections of the type shown, in tandem. This eliminates the problem of configuration and the only decision the designer is faced with is the number of sections needed to do a particular job.

A large amount of graphical and tabulated materials is available [35], but the problem is much simpler than the loss equalizer problem, since these sections have only two parameters and, using one for normalization, we only have to consider a single parameter set of curves. The two parameters are  $f_c$ , the frequency where the phase is 180 degrees and a steepness parameter, usually denoted as  $b$ . The general shape of this set of phase (delay) curves is shown in Figure 8 (Figure 9).

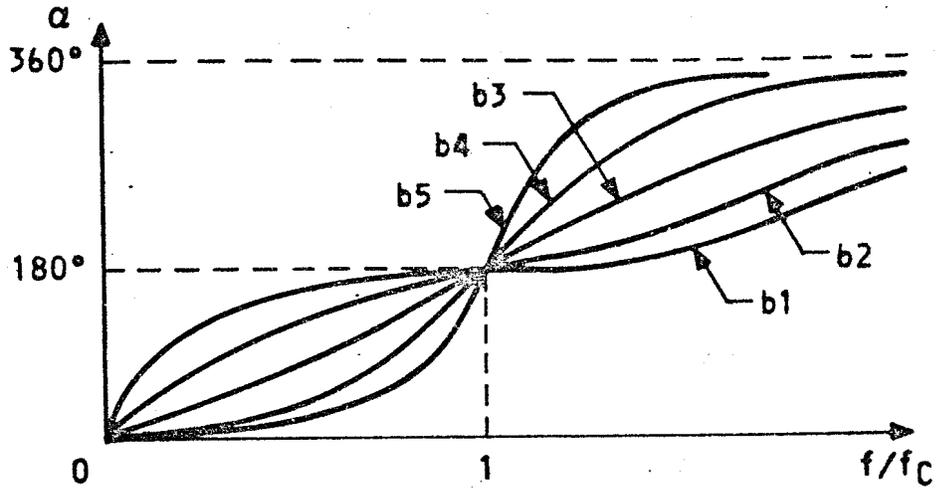


Figure 8  
Phase of Equalizer Sections

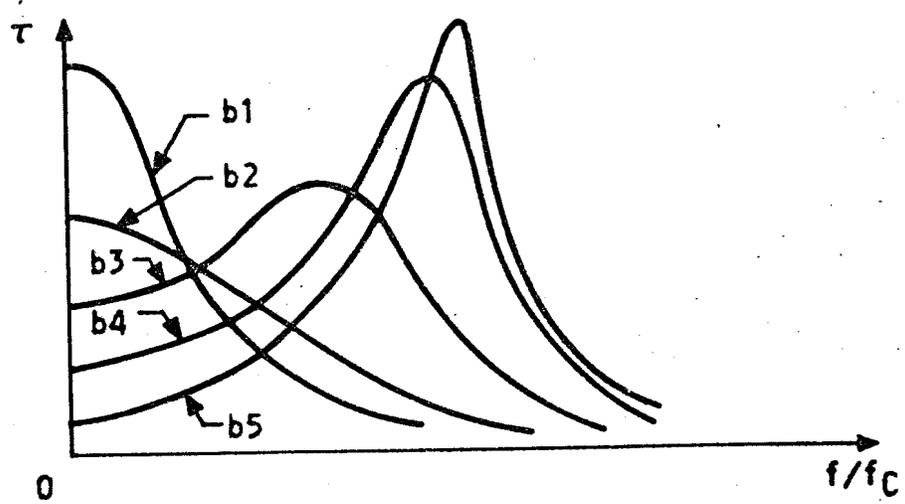


Figure 9  
Delay of Equalizer Sections

The iterative program (section 3.4) can also be used to design delay equalizers. The only difference is that while in the loss equalizer case the program works directly on the element values, in the delay equalization case it uses the  $f_c$  and  $b$  parameters. This means that after the program has come up with a solution to the equalization problem, we still have to calculate the element values of the equalizer. However, the expressions are very simple and a computer program is also available [42].

One final remark is in order. In most cases, we design for constant delay even if linear phase is requested. Naturally in the limiting case the two are identical except for the phase intercept point, which is usually immaterial. The reason for considering the delay is that it represents a more severe requirement in most cases.

The practical limits on phase or delay equalization are as follows.

The phase can usually be made linear to within  $\pm 5$  degrees with relative ease. The practical limit on phase linearity is about  $\pm 1$  degree.

Concerning delay distortion, a 5 to 1 reduction in the distortion of a filter is a feasible project, beyond that equalization can become very expensive.

Adjustable delay equalizers are not treated separately, since the only type available is where fixed delay sections are switched in or out.

### 3.3 Time-domain (Transversal) Equalizers

Direct equalization in the time domain is possible with the transversal equalizer. This consists of a delay line with taps and the signal is formed as a weighted sum of the signals on the taps. The results are very clear, simple and easily visualized for the impulse (or pulse) response and this is why the technique is used mostly in pulse transmission systems.

The idea is clearly described in [45], while [46] contains a review of the use of this technique to automatic equalization.

### 3.4 Iterative Approximation Programs

The iterative approximation programs were originally developed for delay equalizer design but are now used

extensively in all sorts of situations. We have substantial experience with SUPROX [36], [38] and SIMPLEX [39]; two others, developed recently, are described in [40] and [41].

Considering the first two programs, the following features and options are available.

The parameter to be approximated can be loss, phase, delay or time response or any combination of these, although we do not have experience in approximating combinations of parameters simultaneously. Other parameters can be accommodated with a minimum of programming effort.

The specification is fed in as a table of required parameter values versus frequency (or time). The required parameter can have either a single value or upper and lower limits. Weighting, which gives more importance to matching some parameters or some regions, can also be specified if desired.

The trial network is specified next by a topological description that is fixed, and element values that are individually specified as fixed or variable. Limits can also be put on the variable element values.

As mentioned above, delay equalizers are specified by two parameters per section and the number of sections, instead of configuration and element values.

The error the program attempts to minimize is the weighted sum of the squared errors at the individual frequencies. SIMPLEX can also minimize the weighted maximum absolute error (Chebyshev criterion).

Further details of the programs can be obtained from the references. Here we would like to consider our experience with these programs.

Loss or delay equalizers are designed daily by the programs and the convergence is usually fast and uneventful. Trouble arises only if the initial values are orders of magnitude wrong.

In the design of shaped filters, we have experienced considerably more difficulty. In several cases the program has not been able to converge to anything acceptable. One can play with initial parameter values, weighting and strategies to try to obtain something satisfactory, but there is no single solution to this problem.

I feel that the major difficulty is the fact that the program is not able to modify the configuration. One could, of course, overcome this difficulty by approximating through the transfer function of a network but this has the disadvantage that dissipative and parasitic effects cannot be taken into account; and the convergence properties of this approach may or may not be any better.

We have found that if SUPROX does not converge, SIMPLEX is able to, at least, start. Alternate uses of these two programs can sometimes give excellent results.

Insufficient experience prevents us from evaluating the effectiveness of these programs in time-domain approximations.

#### 4. Other Networks

There are a large number of other network types that one is occasionally required to design, and here we intend to mention a few.

##### 4.1 Phase-splitting Networks

These are networks of the all-pass type with one input and several outputs, the different outputs having a constant phase difference between each other.

The theory and design technique of such networks are well known [47], and they can be realized by LC or RC networks. The SUPROX program mentioned above, can also be used for the design of phase-splitting networks.

##### 4.2 Impedance Matching Networks

Two resistive impedances can be matched by a bandpass filter in any band of frequencies not containing zero and infinite frequencies. Matching can also be done by a network that allows direct current to pass [48].

Matching other than resistive impedances is a rather difficult job.

##### 4.3 Delay Lines

Lumped-element all-pass type delay lines are useful, among other things, for transversal equalizers. These can be designed as follows.

The computer program mentioned at the end of section 2.33 can design low-pass filters with maximally flat or equal ripple delay. The output contains, among other data, the natural modes (transfer function poles).

We put zeros in the right half of the complex frequency plane into positions that are reflections of these poles with respect to the imaginary axis (see Figure 10).

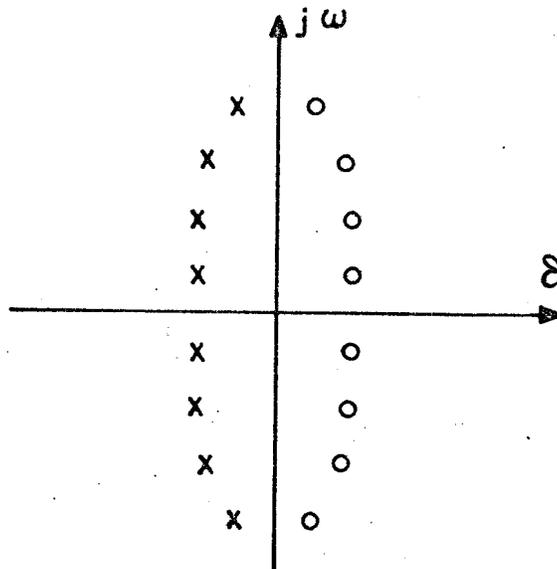


Figure 10

The resulting pole-zero configuration is that of an all-pass network that has exactly twice the delay of the original low-pass filter. Poles for maximally-flat delay (Bessel) are tabulated in [1], while the equal-ripple type solution is tabulated in [51].

A pole-zero quadruplet at  $p_i = \pm \sigma_i \pm j\omega_i$  can be converted into the corresponding  $f_c$  and  $b$  values through the following expressions:

$$\omega_{ci} = 2\pi f_{ci} = \sqrt{\sigma_i^2 + \omega_i^2}$$

$$b_i = \frac{\omega_{ci}}{\sigma_i} = \frac{\sqrt{\sigma_i^2 + \omega_i^2}}{\sigma_i}$$

The program described in [42] can again be used to obtain the element values.

## 5. Conclusion

In this memorandum we have treated the more recent developments in the design of passive networks, including available tabulated data and computer programs.

The references listed should enable the reader to make use of these programs and tabulated results and to design his own filters and other networks.

By their nature, tables and computer programs have been developed to satisfy frequent needs, hence they are adequate for routine designs, although the iterative computer programs have been developed in a form that can take unusual requests as well.

For special designs, including the active RC realization, the reader is advised to turn for help to the groups specializing in the particular design techniques.

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Att.  
References

## REFERENCES

- [1] L. Weinberg: Network Analysis and Synthesis, McGraw Hill Book Company, 1962
- [2] P. R. Geffe: Simplified Modern Filter Design, J. F. Rider, 1963
- [3] R. Saal: Der Entwurf von Filtern mit Hilfe des Kataloges normierter Tiefpässe Telefunken GMBH, 1963
- [4] J. K. Skwirzynski: Design Theory and Data for Electrical Filters, D. Van Nostrand, 1965
- [5] G. L. Matthaei, L. Young, E. M. T. Jones: Microwave Filters, Impedance-Matching Networks and Coupling Structures, McGraw Hill Book Company, 1964
- [6] H. Simon: Design Tables for Chebyshev Filters MM-61-216-5 (March 17, 1961)  
(Tables and curves are available separately as B-854584/605)
- [7] Mrs. M. D. Stoughton: Synthesis of Antimetric Tchebycheff Filters with Equal Terminations MM-62-2172-1 (January 2, 1962)  
(Tables are available separately as B-888543/6, curves as B-888765/8)
- [8] Mrs. M. D. Stoughton: Inverse and Antimetric Inverse Tchebycheff Filters, B-888604 Sheets 1-8, B-888605 Sheets 1-7
- [9] G. Fritzsche, G. Buchholz: Filterkatalog Nachrichtentechnik. Volumes 14-15, April, 1964 - February, 1965 Various Pages
- [10] G. Fritzsche, G. Buchholz: Filterkatalog 2 serie Nachrichtentechnik, Volumes 15-16 May, 1965 - August, 1966 Various Pages

References - 2

- [11] K. H. Feistel, R. Unbehauen: Tiefpässe mit Tschebyscheff - Charakter der Betriebsdämpfung im Sperrbereich und maximal gegebener Laufzeit. Frequenz, Volume 19, August, 1965, pp. 265-282
- [12] E. Glowatzki: Katalog der Potenz und Tschebyscheff Filter Telefunken Zeitung Volume 28, March, 1955 pp. 15-22 and Volume 34, June, 1961, pp. 180-185
- [13] S. D. Bedrosian, E. L. Luke, H. N. Putsch: On the tabulation of insertion loss low-pass chain matrix coefficients and network element values Proc Nat Electronics Conf Volume 11, 1955 pp. 697-710
- [14] E. Glowatzki: Katalogisierte Filter, NTZ Volume 9, 1956, pp. 508-513
- [15] R. Saal, E. Ulbrich: On the Design of Filters by Synthesis, IRE Trans on Circuit Theory Volume CT-5 December, 1958, pp. 284-327
- [16] (No Author): A Handbook on Electrical Filters. White Electromagnetics, Incorporated, Maryland, 1965
- [17] G. Szentirmai: Theory of Low-Pass Filter Design in the Transformed Variable, MM-64-2184-1 (February 6, 1964)
- [18] J. C. Noll, Mrs. H. D. Reinecke, Miss N. K. Schellenberger, C. L. Semmelman: Low-Pass/High-Pass Filter Synthesis Program, MM-64-2112-12 (November 20, 1964)
- [19] G. Szentirmai: Synthesis of Constant Delay Low-Pass Filters, MM-65-2184-7 (February 17, 1965)
- [20] W. G. Scheerer: A Chebyshev Passband, Arbitrary Stop Band, Low-Pass Pole Placer Program with Automatic Output to the Low-Pass/High-Pass Filter Synthesis Package, Memorandum for File, Case 39881-2, April 6, 1966
- [21] G. Szentirmai: A Filter Synthesis Program, Chapter 5 in F. F. Kuo; J. F. Kaiser (Ed): System Analysis by Digital Computer J. Wiley and Sons, New York, 1966
- [22] G. Szentirmai: Theory of Bandpass Filter Design in the Transformed Variable, MM-64-2184-7 (October 6, 1964)

References - 3

- [23] G. Szentirmai: The Design of Directional Filters, MM-64-2184-5 (May 18, 1964)
- [24] G. C. Temes: Synthesis of Filters with Maximally Flat Passband and Arbitrary Stop Band Loss, Canadian Electronics Engineering, Volume 8, February 1964, pp. 29-33
- [25] E. S. Willis: Design of Quartz Crystal Filters, BTL Merrimack Valley Laboratory, 1960
- [26] A. Fettweis: Image Parameter and Effective Loss Design of Symmetrical and Antimetrical Crystal Bandpass Filters, Revue HF Volume 11, 1963, pp. 378-394
- [27] Mrs. E. R. Storrs: IBM 1620/7094 Fortran Programs for Computations of Crystal Filter Element Values, Memorandum for File, File 39881-2, October 13, 1964
- [28] G. Szentirmai: New Developments in the Theory of Filter Synthesis, Paper presented at the Conference on Electrical Networks, University of Newcastle-upon-Tyne, Newcastle-upon-Tyne, England, September, 1966
- [29] J. Jess, H. W. Schuessler: A Class of Pulse-Forming Networks, IEEE Trans on Circuit Theory, Volume CT-12 June, 1965, pp. 296-299
- [30] J. Jess, H. W. Schüssler: Über Filter mit günstigem Einschwingverhalten, AEU Volume 16, March, 1962, pp. 117-128, (English translation available from Library)
- [31] J. Jess, H. W. Schüssler: On the Design of Pulse-Forming Networks, IEEE Trans on Circuit Theory, Volume CT-12, September, 1965, pp. 393-400
- [32] M. Brockhaus, W. Schüssler: Zur Erzeugung von Impulsen für die Datenübertragung, NTZ Volume 19, September, 1966, pp. 505-508
- [33] J. Petersen: Neuere Ergebnisse beim Entwurf von Impulsformern, NTZ Volume 19, December, 1966, pp. 738-744
- [34] H. M. Thomson: Design Charts and Tables for Fixed Loss Equalizers, MM-62-2182-1 (February 28, 1962)
- [35] H. M. Thomson: Design of Delay Networks - Part I MM-60-216-16 (October 28, 1960)

References - 4

- [36] Miss J. A. Moraller (Mrs. Schilling): SUPRØX, A General Purpose Successive Approximation Computer Program, MM-64-2112-15 (March 27, 1964)
- [37] Miss P. A. Horgan: SUPLØT, A Routine for Providing Additional Tabular and Plotted Output with the SUPRØX System, MM-66-2183-2 (March 11, 1966)
- [38] Mrs. J. M. Schilling: A Subroutine for Designing Ladder Networks by Successive Approximation, Memorandum for File, File 39898-76, October 27, 1965
- [39] P. E. Fleischer, Miss D. M. Bohling: SIMPLX - A New Successive Approximation Program for Network Design, MM-67-4734-2 (January 19, 1967)
- [40] Miss B. E. Forman: Fletcher-Powell Iterative Descent Method for Minimization, Memorandum for File, File 39881-2 (January 6, 1967)
- [41] Mrs. J. F. Boone: MØPEM - A Program for Network Improvement by Element Manipulation, MM-66-6322-6 (August 17, 1966)
- [42] Mrs. M. A. Styczynski: Delay Network Element Values Using the Teletype Program "ELEMNT", Memorandum for File, File 39898-76 (July 1, 1966)
- [43] Mrs. M. A. Styczynski: An Index of Computer Programs Dealing with Transmission Networks and Systems, MM-66-4734-5 (October 17, 1966)
- [44] S. Darlington: Attenuation Equalizer, US Pat. 2,153.743 (April 11, 1939)
- [45] H. E. Kallmann: Transversal Filters, Proc IRE Volume 28, July, 1940, pp. 302-310
- [46] H. Rudin: Automatic Equalization Using Transversal Filters, IEEE Spectrum Volume 4, January, 1967, pp. 53-59
- [47] S. Darlington: Realization of a Constant Phase Difference, Bell System Tech J, January, 1950, Volume 29, pp. 94-104

References - 5

- [48] G. Szentirmai: Bandpass Matching Filter in the Form of Polynomial Low-Pass Filter, IEEE Trans on Circuit Theory, Volume CT-11, March, 1964, pp. 177-178
- [49] C. L. Semmelman: An Interaction Computer Program for Bandpass Filter Impedance Transformations, MM-67-4734-6 (April 3, 1967)
- [50] H. Pender, K. McIlwain: Electrical Engineers' Handbook, 4th Ed, 1957, J. Wiley and Sons, New York, Section 6.
- [51] E. Ulbrich, H. Piloty: Über den Entwurf von Allpässen etc. AEU Volume 14, October, 1960, pp. 451-465