COVER SHEET FOR TECHNICAL MEMORANDUM

TITLE— Notes on the Design of Cosine Roll-Off Filters

MM 67 – 5311 – 18

DATE— August 29, 1967

CASE CHARGED— 38763

FILING CASES— 38763-17

AUTHOR— A. R. Olejniczak

FILING SUBJECTS— Filter Design
VSB Data Transmission

ABSTRACT

In data transmission systems there is a frequent need for filters with shaped amplitude response in the pass-band. In particular, cosine roll-off shaping is commonly used. A number of filters with various roll-offs have been designed in connection with the development of several options of the 203-type Data Set. This memorandum presents the various filter design and gives an outline of the design procedure. Also, the relationship between network complexity and sharpness of roll-off is demonstrated on the basis of the empirical evidence. It is shown that the complexity of a 25 percent roll-off filter is approximately 40 percent greater than that of a 50 percent roll-off filter. Similarly, a 10 percent roll-off filter is approximately 2.3 times as complex as a 50 percent roll-off filter. Almost all the additional complexity results from delay equalization.

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SUBJECT: Notes on the Design of Cosine Roll-Off Filters - Case 38763-17

DATE: August 29, 1967

FROM: A. R. Olejniczak
MM 67-5311-18

MEMORANDUM FOR FILE

I. Introduction

Raised cosine shaping of the data signal frequency spectrum is commonly used in data transmission systems because of its desirable impulse response properties. Very often it is convenient to do the shaping in the passband rather than in the baseband. A general passband spectrum is shown in Figure 1. The definitions are in terms of a VSB passband spectrum with the carrier frequency located at f_2. In the general VSB case, the high and low end of the spectrum may have different roll-off factors, since one shape provides the equivalent baseband shaping while the other gives the required odd symmetry about the carrier frequency.

The shaping function is frequently equally divided between the transmitter and receiver filters in order to maximize the signal-to-noise ratio in the presence of white noise. Hence, in terms of Figure 1, the ideal filter has the following amplitude characteristic:

$$
|H(f)| = \begin{cases} 0 & f < f_A \\ \left[\frac{1}{2} + \frac{1}{2}\sin\frac{\pi}{2} \times \frac{f-f_1}{\alpha_L f_N}\right]^{1/2} & f_A < f < f_B \\ 1 & f_B < f < f_C \\ \left[\frac{1}{2} - \frac{1}{2}\sin\frac{\pi}{2} \times \frac{f-f_2}{\alpha_H f_N}\right]^{1/2} & f_C < f < f_D \\ 0 & f_D < f \end{cases}
$$

In practice, of course, the ideal shape can only be approximated in the passband and a finite (in db) attenuation can be obtained in the stop bands.

The approximation problem is not an easy one. Although cosine roll-off filters have been designed in the past, the resulting networks sometimes required an unnecessarily large number of components. Recently, however, techniques developed in Department 4734, have been used to produce a number of economical designs. The design method is made possible by the existence of several powerful computer programs - primarily optimization programs. In essence, the method relies on manipulating component values of a bandpass filter to produce a loss characteristic approximating the ideal one. Phase equalization is done separately.

II. Description of Results

Four representative designs are depicted in Figures 2 through 17. For each of the filters, the ideal response (square root shaping), the approximating network and its amplitude and delay response are shown.

The filters, except for Filter 3, are intended for use in various options of the 203-Type Data Set. The requirements for the filters call for the amplitude response to be within +1.0 db of the ideal characteristic over the Nyquist band of frequencies. The actual computed responses, however, are usually within a few tenths of a db over the Nyquist band, with a quite good match over the remainder of the frequency range.

The filters, again with the exception of Filter 3, are delay equalized to have delay distortion within approximately ±50 microseconds over the Nyquist band. In the case of Filter 3, two bridged-tee delay equalizing sections would be necessary to bring the delay distortion within the above tolerance.

Finally, particular to all the designs is the presence of two attenuation peaks, one above and one below the data band. The peaks are located at the frequencies of two pilot tones that are transmitted to facilitate carrier recovery at the receiver. The receiver VSB filter removes these tones from the data spectrum before demodulation. The ability to control the position of the peaks demonstrates a very useful feature of the design technique.

III. The Design Procedure

As a preliminary step in the design procedure a prototype network is designed that will serve as an initial approximation in the iterative process. In general, the response of this network should be made to resemble the desired one as much as is practical since it may determine whether the iteration will be successful. Unfortunately no guidelines exist that determine what constitutes a sufficiently good approximation; hence, this step is essentially a guess. For the VSB filters, the prototype network is usually a filter with maximally flat passband, 3-db attenuation frequencies the same as those of the final network and out of band loss somewhat in excess of the required loss. The additional loss is a precaution against possible reduction of out of band loss as a result of inband shaping.

The prototype network can be designed through standard techniques. A survey of design methods and available design aids and programs is given in a recent memorandum by G. Szentirmai [1]. For the design of bandpass filters, the Bandpass Filter Design Program [2] is recommended as a very versatile and flexible tool. Theoretical background for the program is given in [3].

Shaping of the loss characteristic of the prototype network is then done by SUPRØX - A General Purpose Successive Approximation Computer Program [4]. This program matches a given function (called a "match function") to an arbitrary curve at specified values of the independent variable. A subroutine VALUE, which is needed to evaluate the match function, usually has to be supplied by the user. A number of special purpose VALUE subroutines exist, among them one that evaluates the insertion loss of a ladder network [5]. Hence, in order to run the program only data cards are needed to specify the requirement curve, frequencies, network topology, element values, and features of program to be used. Among the most useful features are the weighting of requirement points, matching the shape rather than the absolute amplitude (i.e., subtraction of a constant) and, fixing selected parameters before iteration.

When encoding the prototype network, the loss of an inductor can be entered as a series resistance which is kept fixed during iterations. If, after a successful run the inductance differs considerably from its initial value, or if great accuracy is required, the corrected resistance value can be inserted and iteration repeated.

If the prototype network has been designed through the Bandpass Filter Design Program, it usually will have the proper termination on one end only and an impedance transformation will be necessary. It may be advantageous to do this transformation before using SUPRØX, since great accuracy in computations is not required at this point. Recently a computer program [6] has been made available that does the impedance transformations.

The iteration will not always be successful during the first run. The program may encounter various difficulties or fail to come up with an acceptable solution. Remedies to some of the problems will suggest themselves once familiarity with the program is gained, others will have to be found by trial and error. The last resort is a selection of a new prototype network.

If the SUPRØX is not able to make an improvement at all, other programs might. In Department 4734 good results have been obtained with SIMPLX [7]. Although slower than SUPRØX, SIMPLX has proven to be more reliable in finding a minimum. It is being used to find crude approximation to the desired shape, with SUPRØX refining the match. Particularly useful is the version of SIMPLX written for the CDC 3100 computer, which permits manual interaction with the programs during the approximation. Although the CDC 3100 computer is in the network group area, time on it can be scheduled by outside users.

Once a sufficiently accurate amplitude shaping is obtained, a computation of network response is done to check the effect of shaping at frequencies other than the match points. Phase and delay are also computed to determine the amount of distortion. Mesh Programs [8] and [9] are well suited to do this, although other equivalent programs exist.

Delay equalization is also done on SUPRØX with the appropriate VALUE subroutine. The number of delay equalizing sections (see Reference [1], Section 3.2) has to be estimated as well as the initial values of parameters for each section. This iteration usually converges easily provided that the initial parameter values are anywhere near the correct value.

Practical delay equalizing sections produce a slight amplitude notch at the frequency of the maximum delay. The depth of the notch will depend on the Q of the inductors and the value of peak delay produced by that section. Hence, a mesh computation of the delay equalized network may be in order as a final check of the network's performance. In extreme cases it may be necessary to predistort the original amplitude requirement for the anticipated effect of delay equalizer on amplitude and repeat the iterative process until satisfactory match is obtained.

IV. Summary

The networks presented here have been designed to satisfy the same delay distortion and stop band attenuation requirements. The bandwidths of the filters are also very similar. Hence, the roll-off factor is the most significant parameter differentiating the networks and some conclusions can be drawn about the effect of roll-off on the complexity of the networks. This dependence is illustrated in the graph of Figure 18. The number of inductors in a network is used as a measure of complexity since at voice frequencies, the inductor is the largest and most expensive network element. Included in the data of Figure 18 is a filter with $\alpha_L = \alpha_H =$ 0.1, which is being designed in Department 4734. Although the final design is not available at this writing, the approximate complexity of the network is known. As might be expected, sharpening the roll-off increases the filter complexity. The amplitude shaping is accomplished quite easily; however, delay equalization becomes more difficult. Almost all of the increase in complexity is due to additional delay equalization.

V. Acknowledgment

Acknowledgment is given to Mr. G. Szentirmai for providing the designs of two filters and his assistance in the design of the others. The author is also indebted to Mr. G. Szentirmai for a number of valuable discussions on design techniques

W. J. Lawless for ARO

HO-5311-ARO-NG A. R. OLEJNICZAK

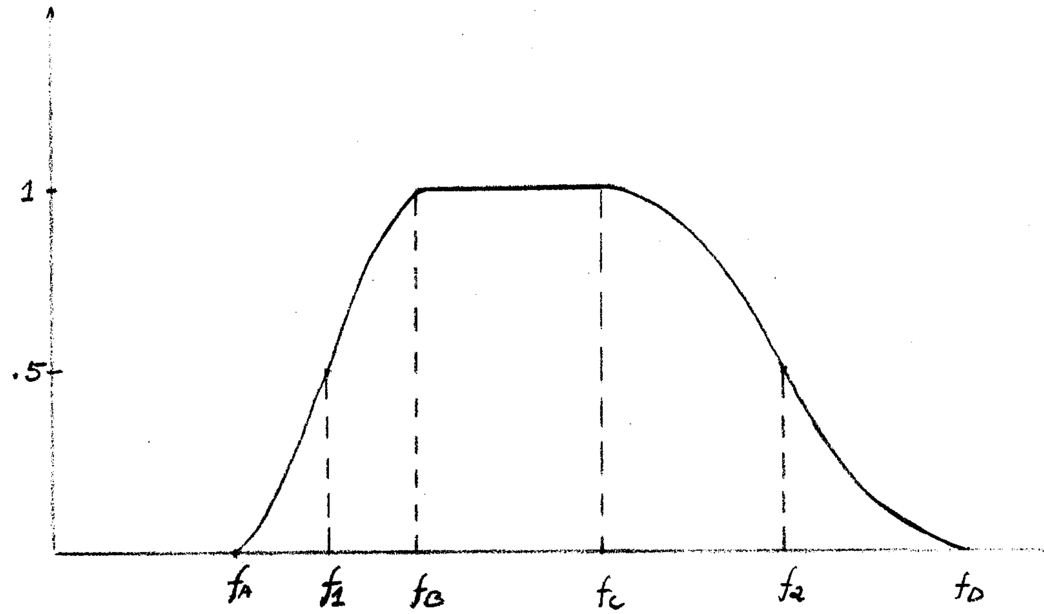
Att.
Figures 1-18

References

7 11/16

10 5/8

GENERAL COSINE SHAPED SPECTRUM



LOW END ROLL-OFF FACTOR:

$$\alpha_L = \frac{f_B - f_1}{f_2 - f_1}$$

HIGH END ROLL-OFF FACTOR:

$$\alpha_H = \frac{f_2 - f_C}{f_2 - f_1}$$

NYQUIST BANDWIDTH:

$$f_N = f_2 - f_1 = \frac{1}{T}$$ WHERE $\frac{2}{T}$ IS THE BAUD RATE

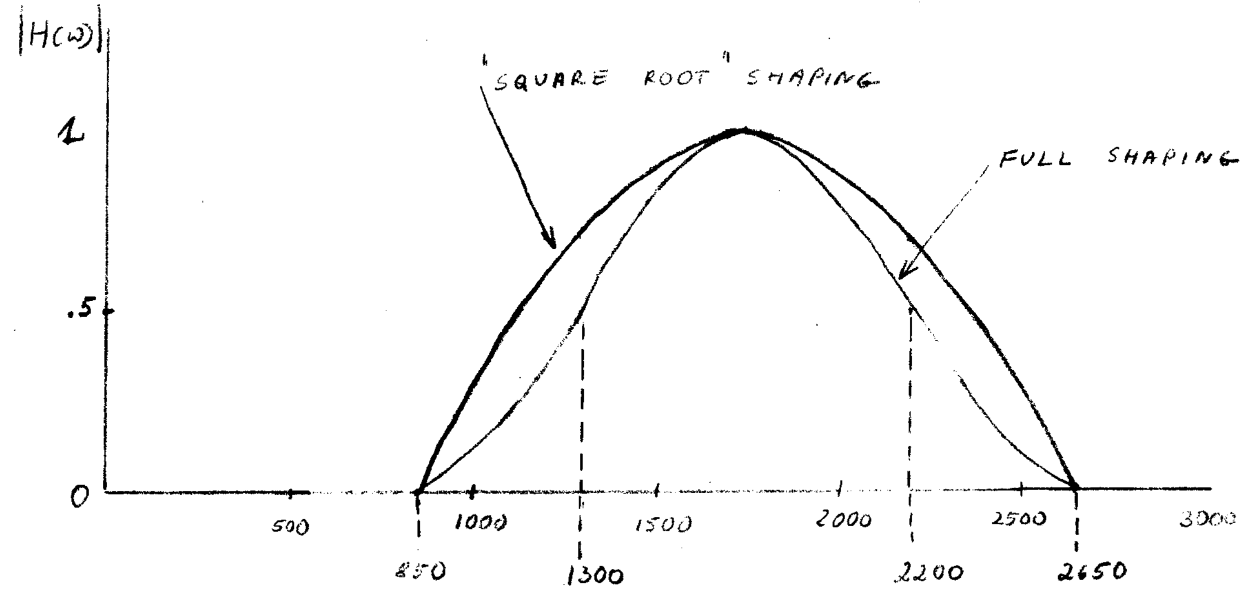
FIG. 1

ISSUE

ISSUE	ENGR ARO	TITLE	BELL TELEPHONE LABORATORIES INCORPORATED
	DRAWN		SHEET
			NO. OF SHEETS PER SET

FILTER 1

IDEAL AMPLITUDE CHARACTERISTIC

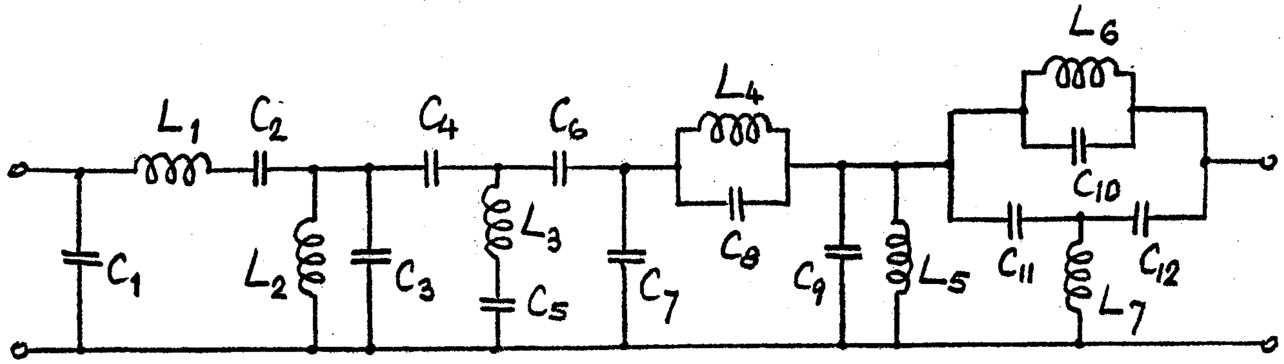


$\alpha_L = 0.50$

$\alpha_H = 0.50$

FIG. 2

ISSUE	ENGR ARO	TITLE	BELL TELEPHONE LABORATORIES INCORPORATED
	DRAWN		NO. OF SHEETS PER SET / SHEET



PASS BAND: 1300 ÷ 2200 Hz
TERMINATIONS: 2400 ohms.

ELEMENTS:

C_1 =	69.8 nF	570 GH	L_1 =	107.8 mH	
C_2 =	942. nF	535 HR	L_2 =	49.55 mH	
C_3 =	165. nF	570 DY	L_3 =	335.8 mH	(f=700Hz)
C_4 =	35.7 nF	570 HB	L_4 =	149.8 mH	(f=2700 Hz)
C_5 =	154. nF	570 FT	L_5 =	192.1 mH	
C_6 =	40.2 nF	570 EN	L_6 =	247.4 mH	(f=1993.8 Hz)
C_7 =	11.5 nF	570 HY	L_7 =	148.2 mH	(f=1993.8 Hz)
C_8 =	23.2 nF	570 PF			
C_9 =	39.2 nF	570 GU			
C_{10} =	15.0 nF	570 FP			
C_{11} =	21.5 nF	570 HK			
C_{12} =	21.5 nF	570 HK			

ALL INDUCTORS 1647 TYPE.

FIG. 3

ISSUE 1	6.6.67	ENGR GS. / DRAWN	TITLE VSB TRANSMITTING FILTER DDD-OPTION.	BELL TELEPHONE LABORATORIES INCORPORATED / SHEET / NO. OF SHEETS PER SET

ISSUE

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JUL15 1967CODER ARO PROB RUN OLEJNICZAK B,D-B

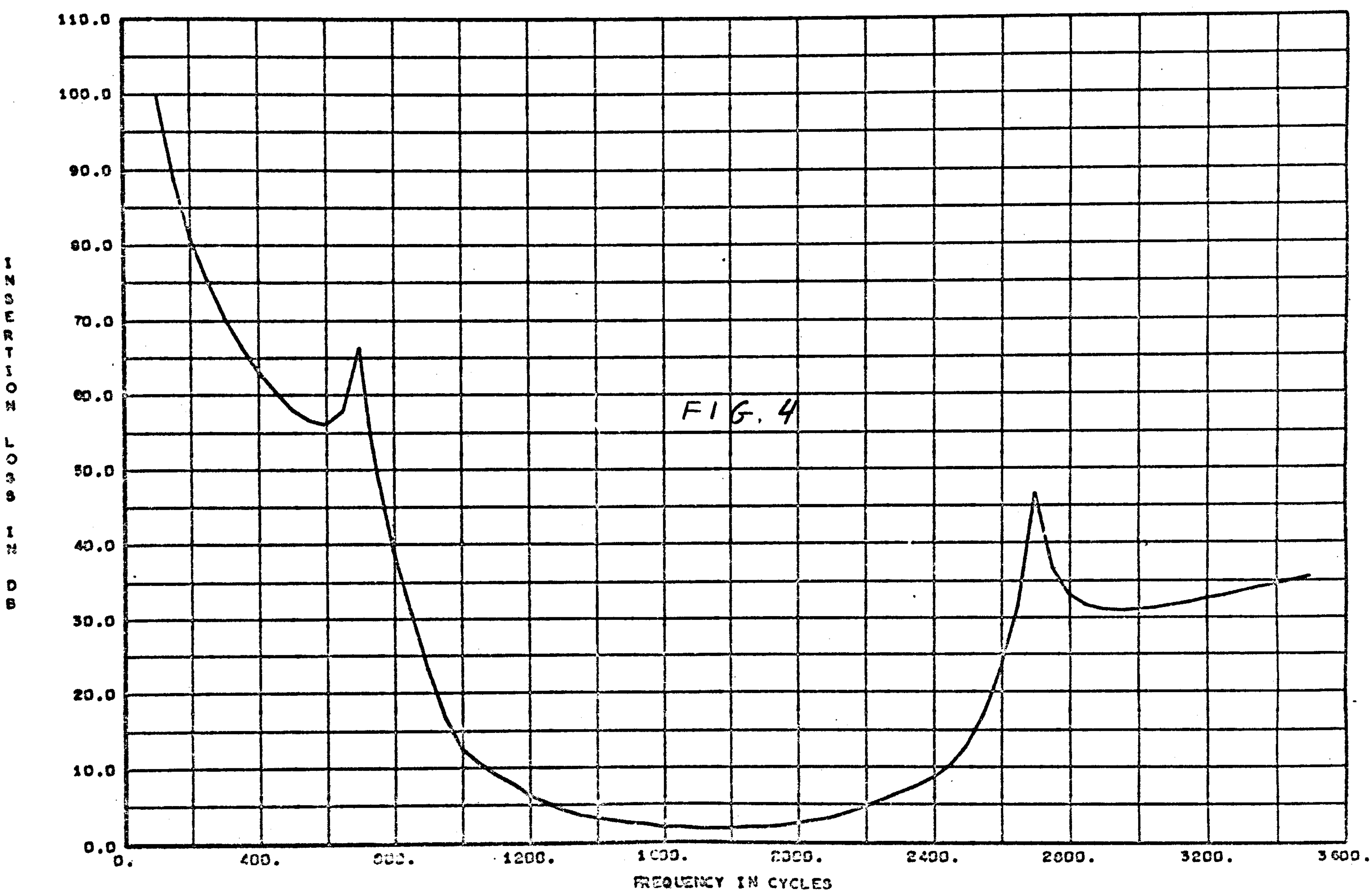


FIG. 4

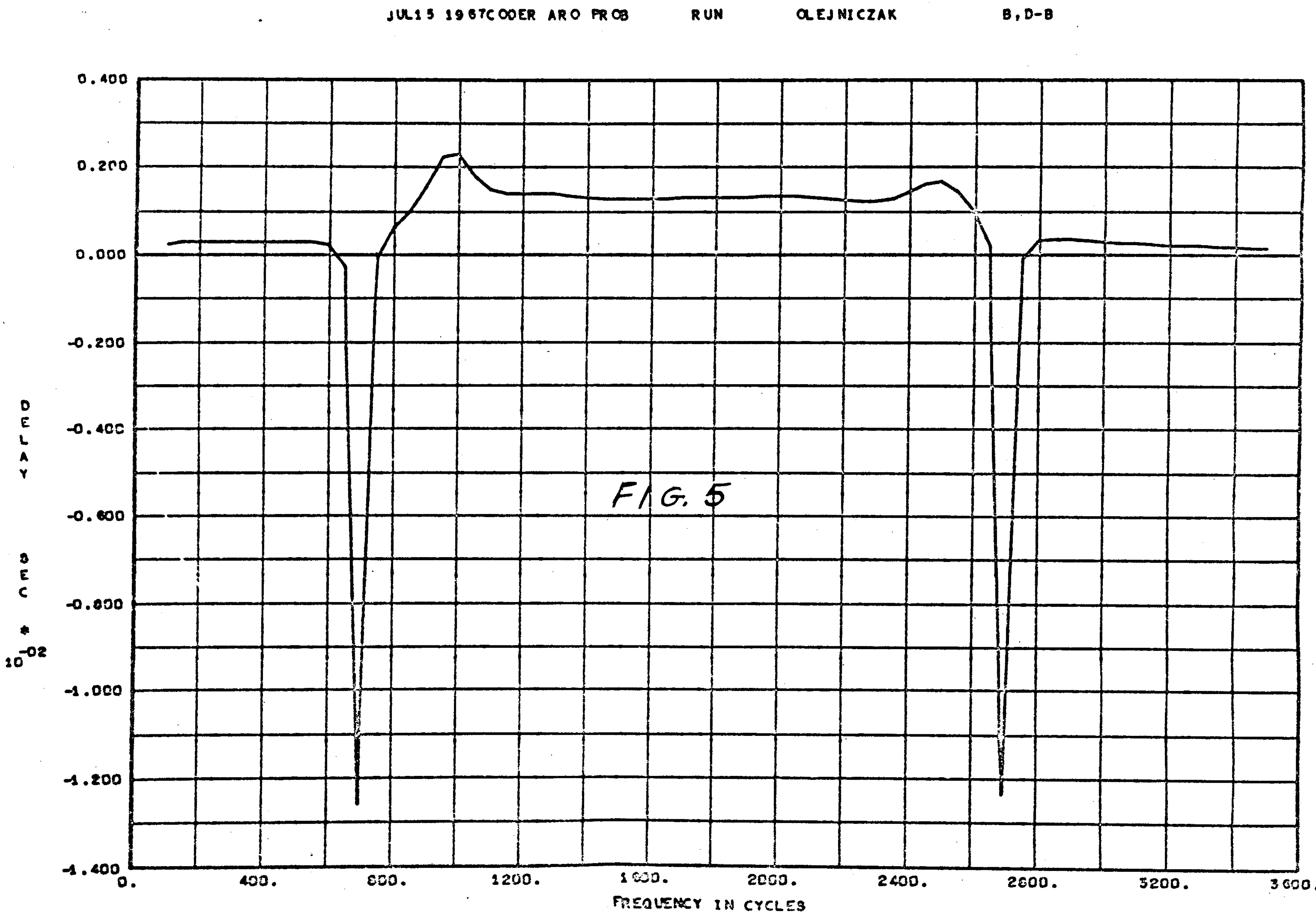
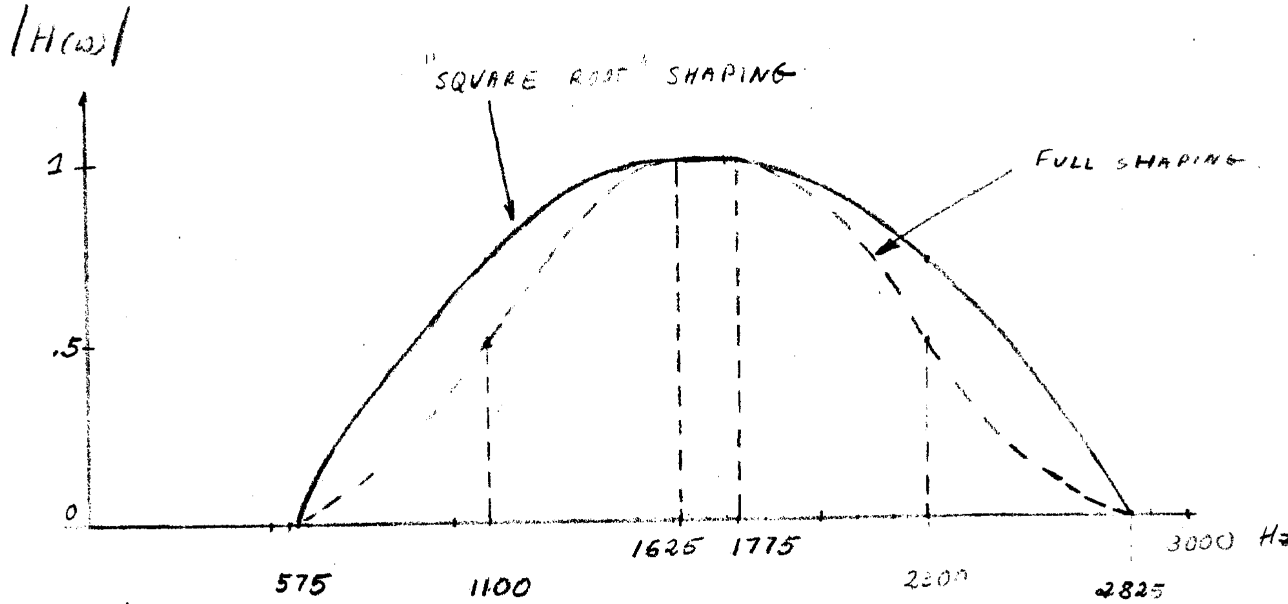

JUL15 1967CODER ARO PROB RUN OLEJNICZAK B,D-B
DELAY SEC * 10-02
0.400
0.200
0.000
-0.200
-0.400
-0.600
-0.800
-1.000
-1.200
-1.400
0.
400.
800.
1200.
1600.
2000.
2400.
2800.
3200.
3600.
FREQUENCY IN CYCLES

FIG. 5

FILTER 2

IDEAL AMPLITUDE CHARACTERISTIC

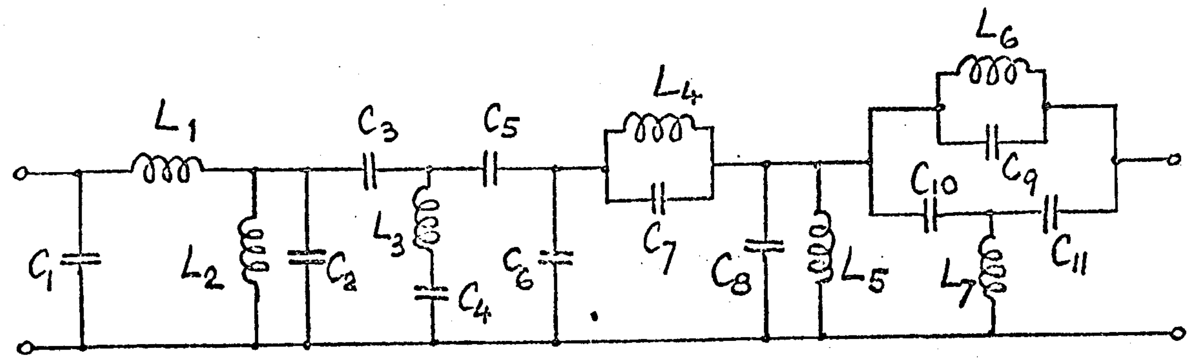


$\alpha_L = 0.44$

$\alpha_H = 0.44$

FIG. 6

ISSUE	ENGR ARO DRAWN 4/5/67	TITLE	BELL TELEPHONE LABORATORIES INCORPORATED NO. OF SHEETS PER SET	SHEET 6



PASS BAND: 1100 ÷ 2300 Hz

TERMINATIONS: 2400 ohms

ELEMENTS:

C_1 = 60.4 nF	570 PL	L_1 = 99.25 mH	
C_2 = 162. nF	570 FS	L_2 = 45.21 mH	
C_3 = 84.5 nF	570 BR	L_3 = 482.5 mH	(f = 500 Hz)
C_4 = 210. nF	570 AJ	L_4 = 136.3 mH	(f = 2900 Hz)
C_5 = 73.2 nF	570 FC	L_5 = 413.5 mH	
C_6 = 8.66 nF	570 JF	L_6 = 295.4 mH	(f = 1981.5 Hz)
C_7 = 22.1 nF	570 LY	L_7 = 126.5 mH	(f = 1981.5 Hz)
C_8 = 33.2 nF	570 HD		
C_9 = 9.09 nF	570 PD		
C_{10} = 25.5 nF	570 CH		
C_{11} = 25.5 nF	570 CH		

ALL INDUCTORS 1647 TYPE.

FIG. 7

ISSUE 2	6.2.67	ENGR GS. / DRAWN	TITLE VSB TRANSMITTING FILTER PRIVATE-LINE OPTION.	BELL TELEPHONE LABORATORIES INCORPORATED / SHEET / NO. OF SHEETS PER SET

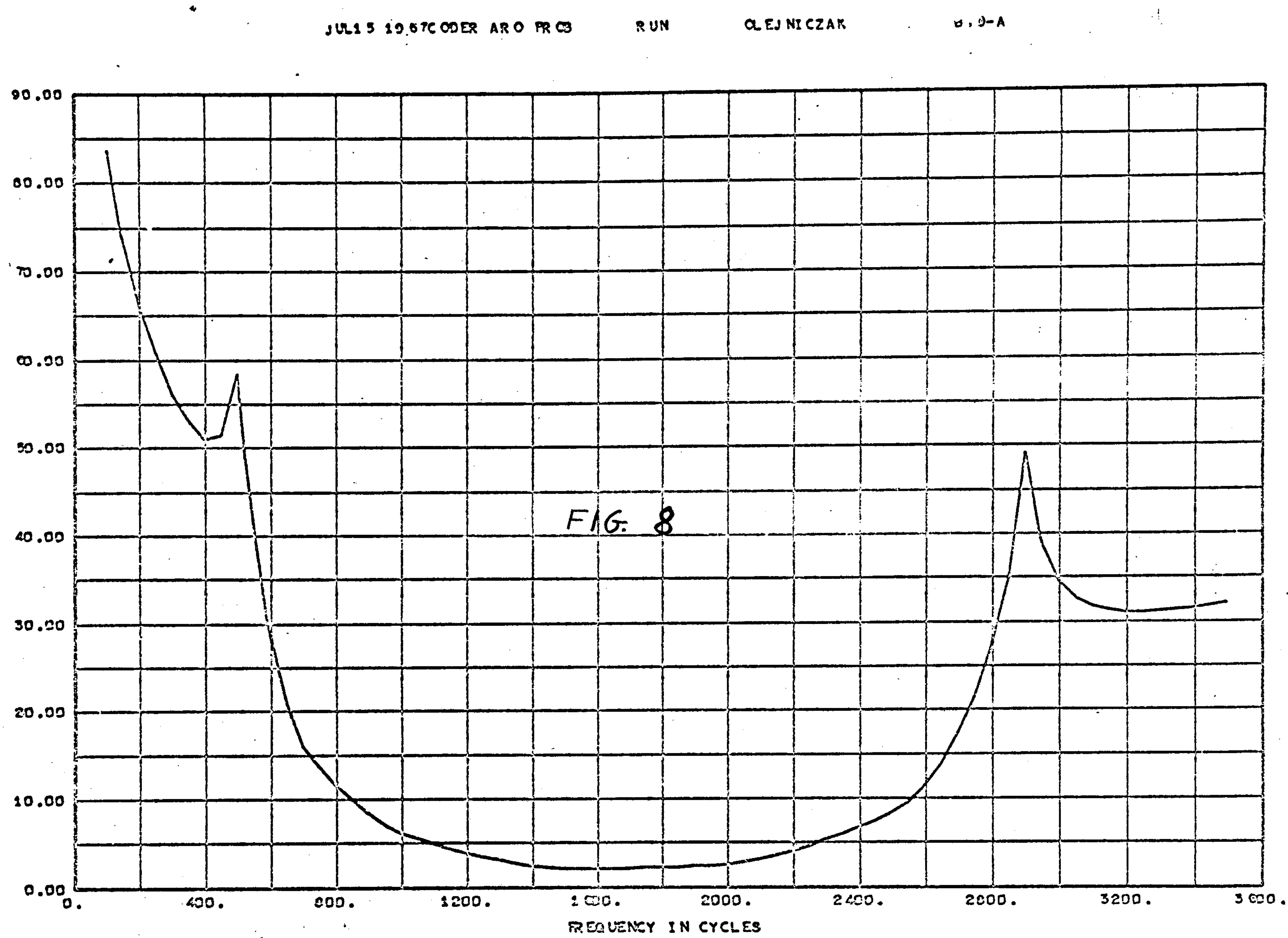

JUL15 1967CODER ARO PROB
RUN
OLEJNICZAK
90.00
80.00
70.00
60.00
50.00
40.00
30.00
20.00
10.00
0.00
0.
400.
800.
1200.
2000.
3200.
FREQUENCY IN CYCLES

FIG. 8

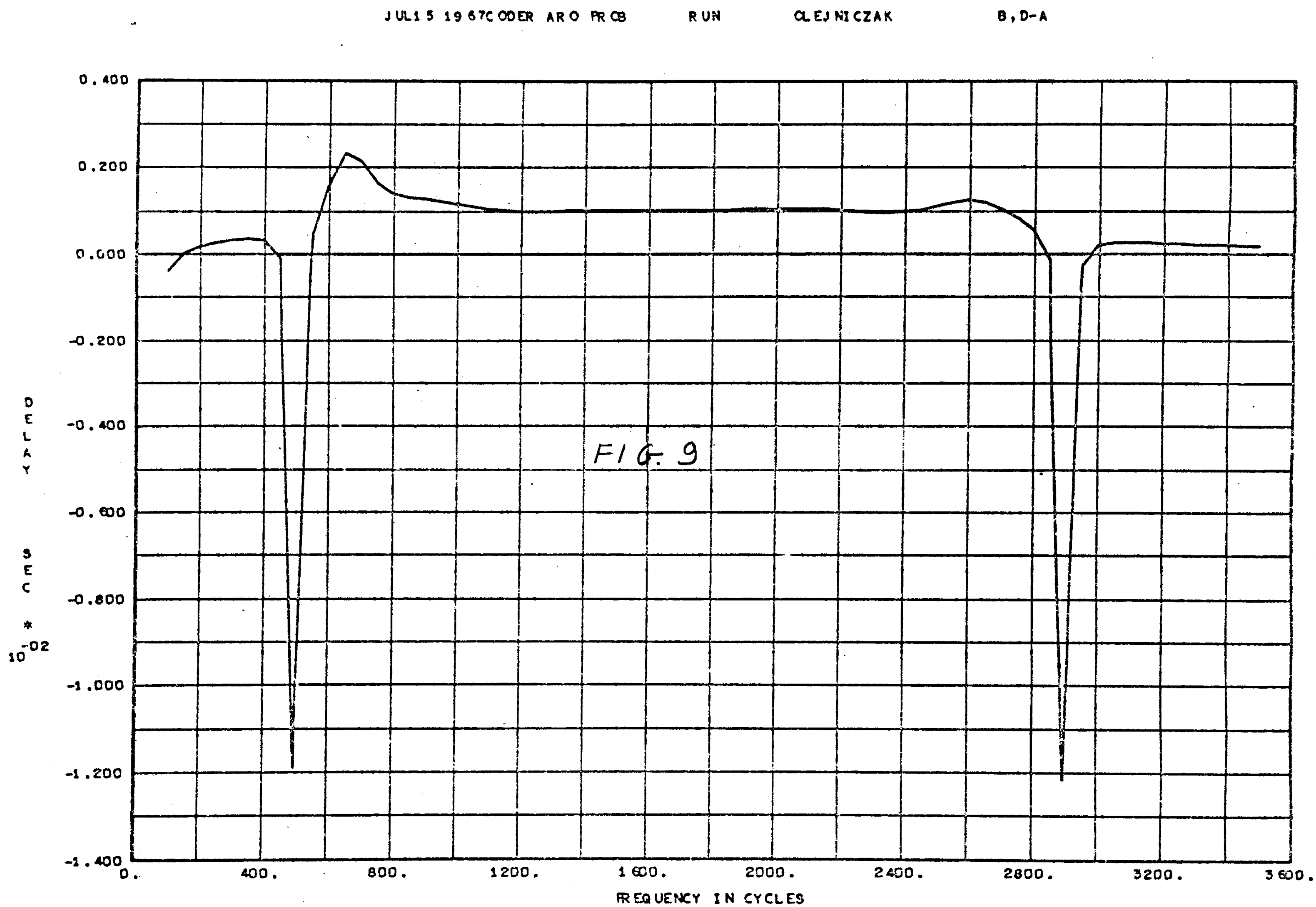
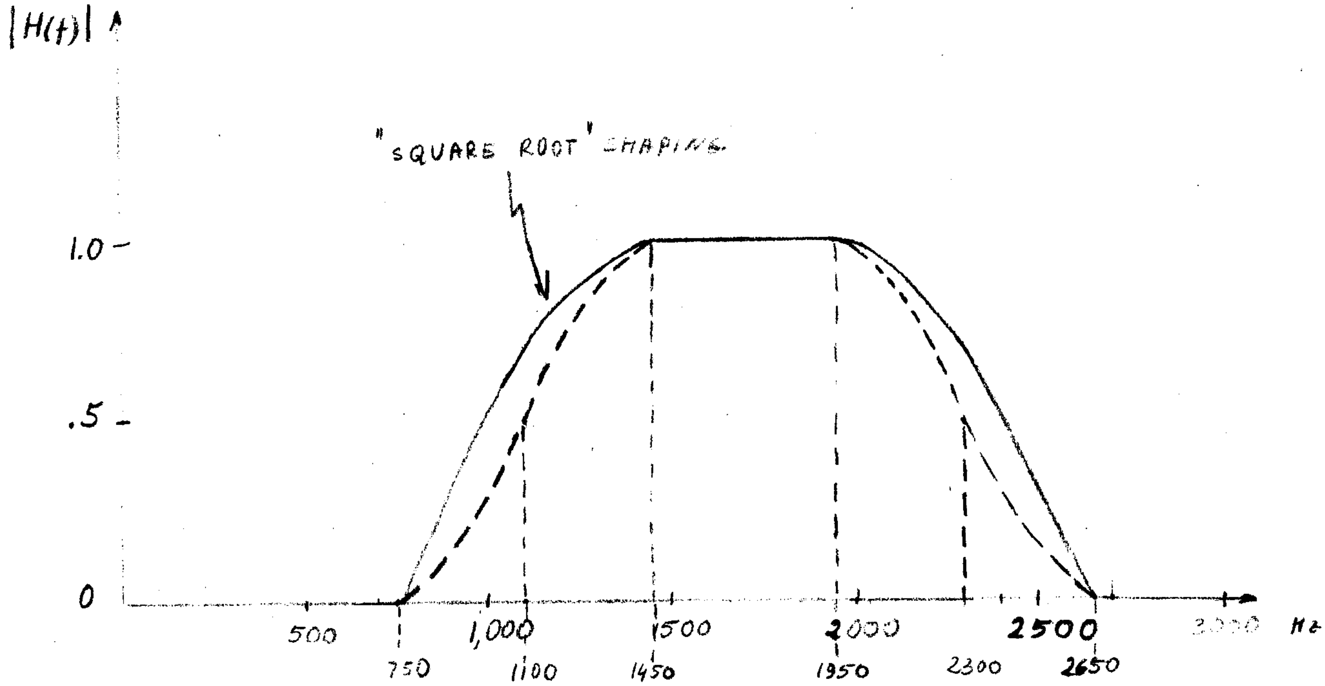

JUL15 1967CODER ARO PROB RUN OLEJNICZAK B,D-A
0.400
0.200
0.000
-0.200
-0.400
-0.600
-0.800
-1.000
-1.200
-1.400
DELAY SEC * 10-02
0.
400.
800.
1200.
1600.
2000.
2400.
2800.
3200.
3600.
FREQUENCY IN CYCLES

FIG. 9

FILTER 3

IDEAL AMPLITUDE CHARACTERISTIC



$\alpha_L = .29$

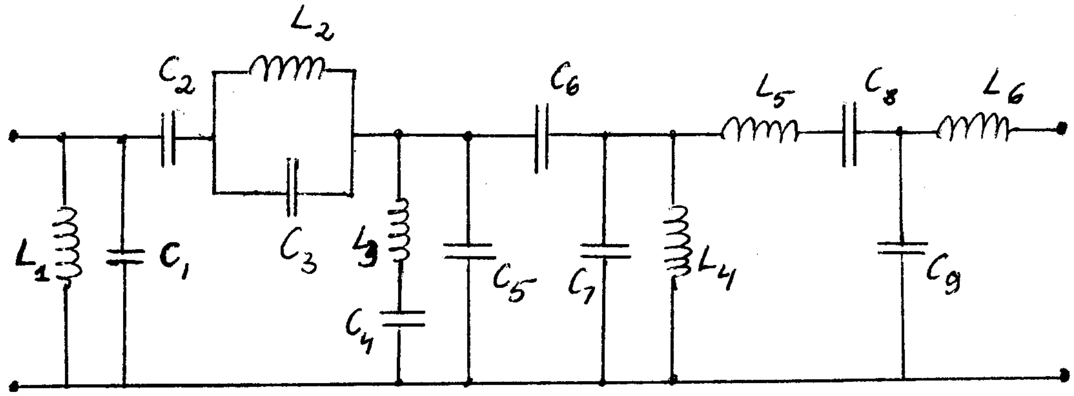
$\alpha_H = .29$

FIG. 10

ISSUE	ENGR	TITLE	BELL TELEPHONE LABORATORIES INCORPORATED
	DRAWN		SHEET
			NO. OF SHEETS PER SET

$7\frac{11}{16}$

$10\frac{5}{8}$



C_1 = 13.85 nF	L_1 = 271.83 mH
C_2 = 45.27 nF	L_2 = 35.61 mH
C_3 = 97.62 nF	L_3 = 179.6 mH
C_4 = 288.2 nF	L_4 = 385.1 mH
C_5 = 84.87 nF	L_5 = 732. mH
C_6 = 112.3 nF	L_6 = 325.2 mH
C_7 = 3.85 nF	INDUCTORS: 1647-TYPE
C_8 = 34.0 nF	CAPACITORS: 570 TYPE
C_9 = 26.7 nF	TERMINATIONS: 2400 ohms

FIG. 11

ISSUE

ISSUE	ENGR ARO / DRAWN	TITLE	BELL TELEPHONE LABORATORIES INCORPORATED / NO. OF SHEETS PER SET	SHEET

JUN20 1967CODER FROB RUN OLEJNICZAK VSB-C

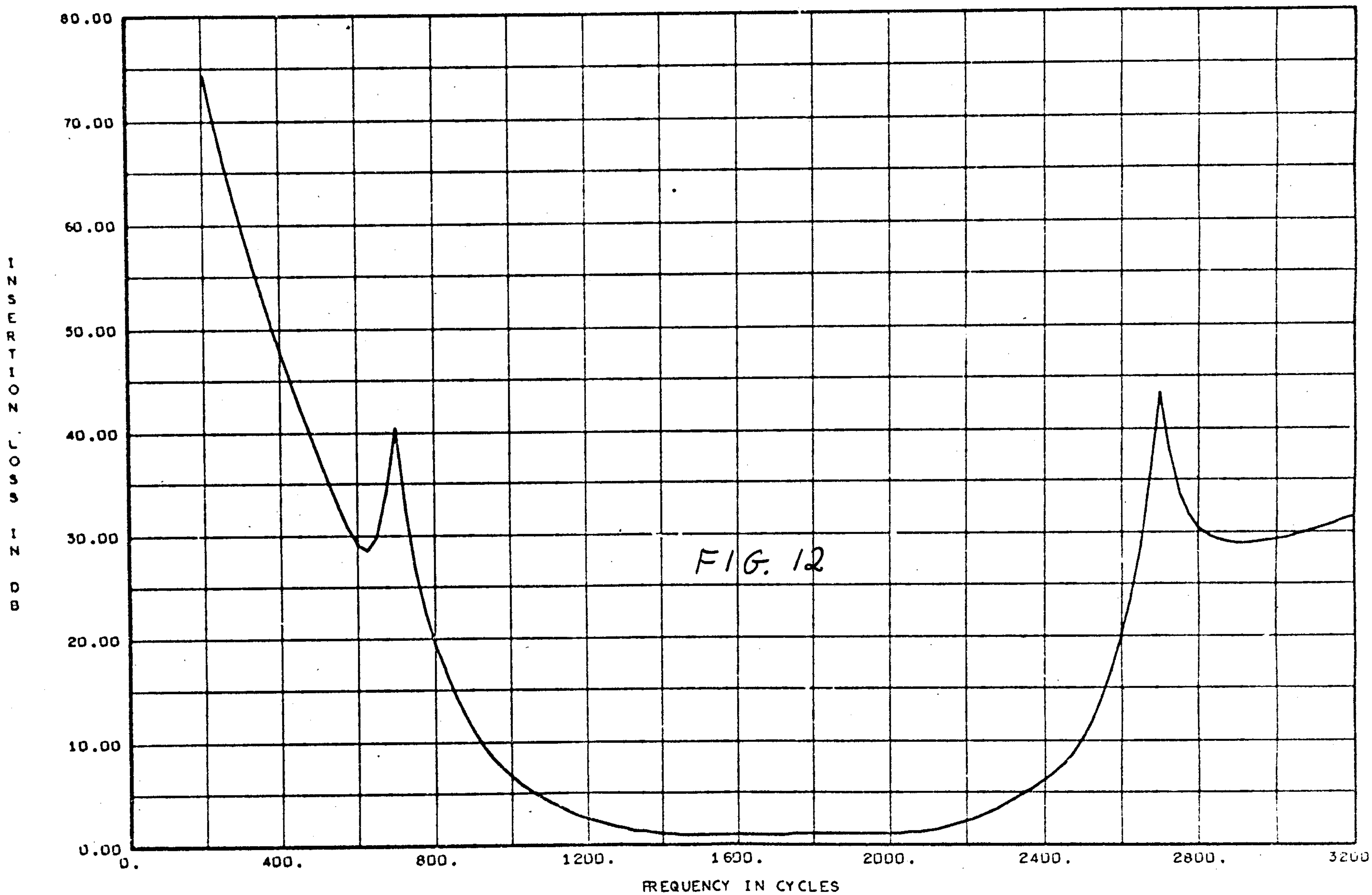


FIG. 12

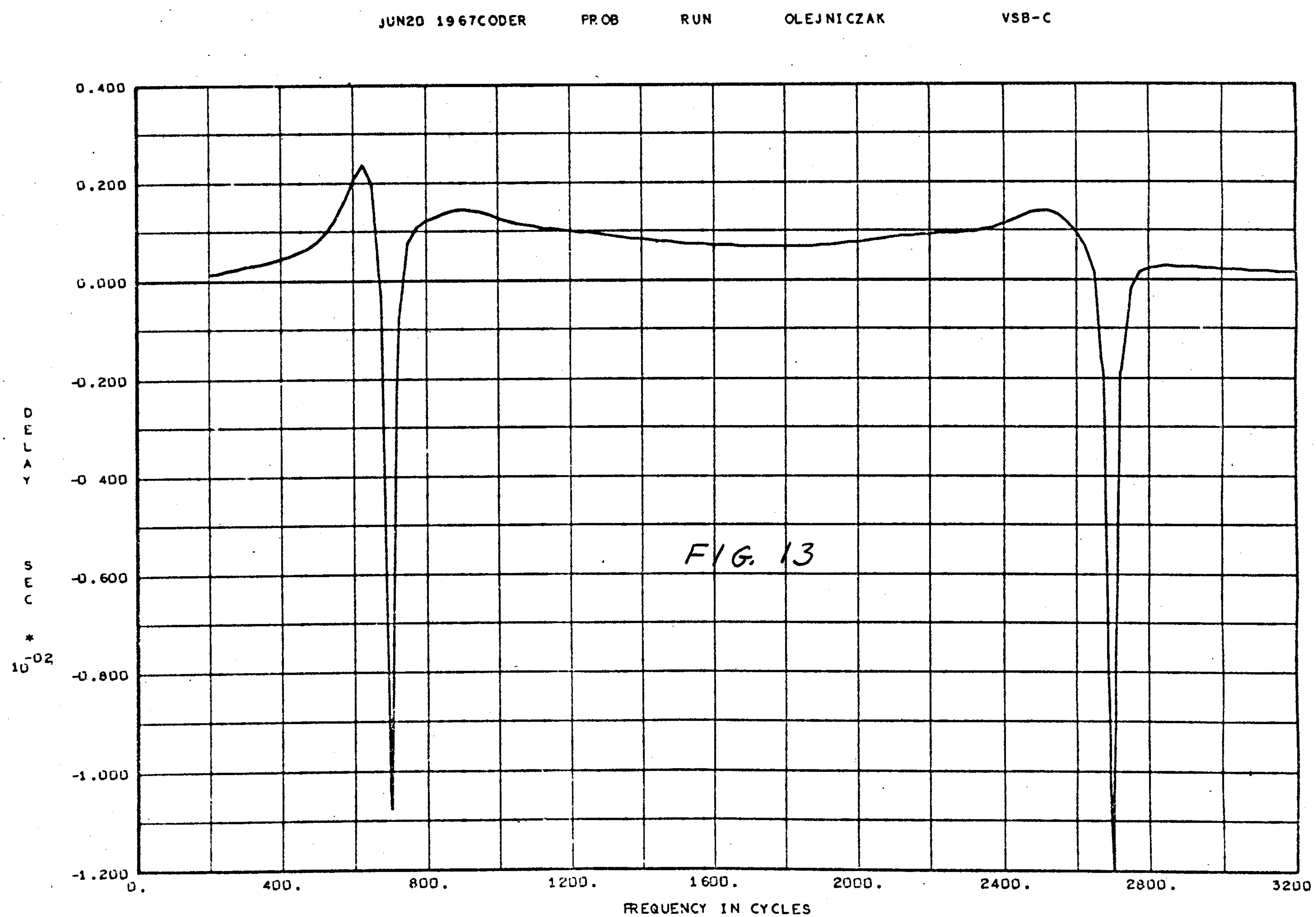
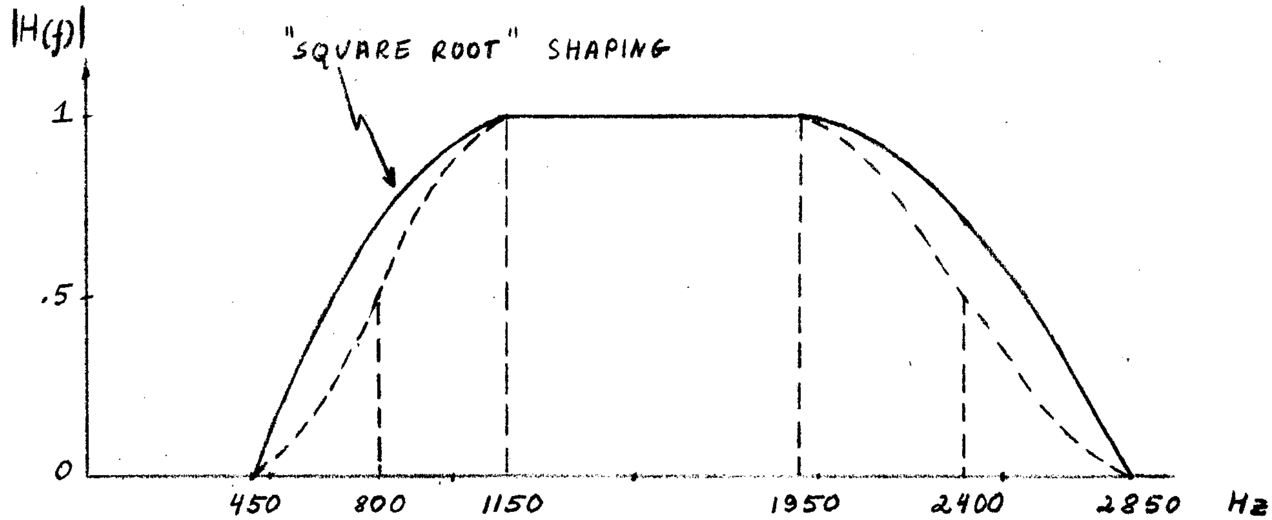

JUN20 1967CODER PROB RUN OLEJNICZAK VSB-C
0.400
0.200
0.000
-0.200
-0.400
-0.600
-0.800
-1.000
-1.200
DELAY SEC * 10^-02
0.
400.
800.
1200.
1600.
2000.
2400.
2800.
3200.
FREQUENCY IN CYCLES

FIG. 13

FILTER 4

IDEAL AMPLITUDE CHARACTERISTIC

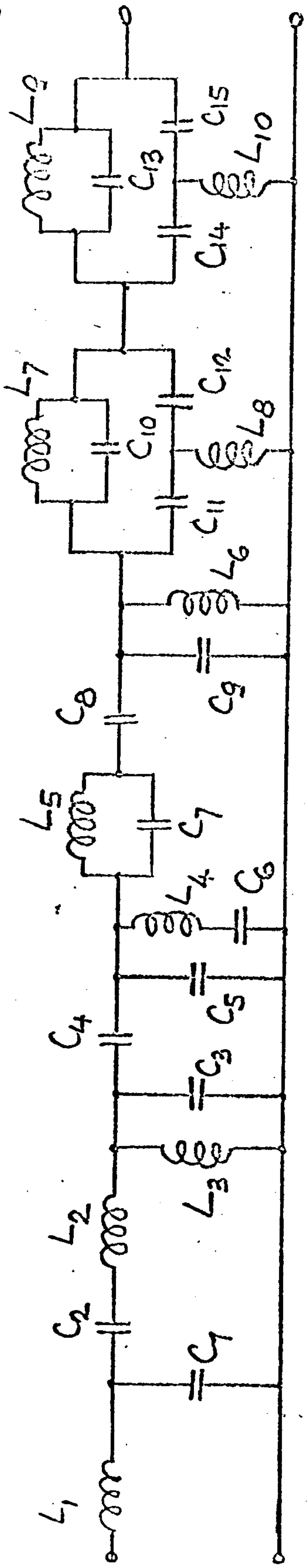


$\alpha_L = 0.22$

$\alpha_H = 0.28$

FIG. 14

ISSUE	ENGR ARO	TITLE	BELL TELEPHONE LABORATORIES INCORPORATED
	DRAWN 9/5/67		SHEET
			NO. OF SHEETS PER SET



PASS BAND: 800 HZ – 2400 HZ

TERMINATIONS: 2400 ohms

ELEMENT VALUES:

C_1 = 14.0 nF	570 EP	L_1 = 476.4 mH	
C_2 = 32.4 nF	570 HE	L_2 = 703.6 mH	
C_3 = 4.87 nF	570 NU	L_3 = 287.7 mH	
C_4 = 127. nF	570 PP	L_4 = 565.4 mH	(400 Hz)
C_5 = 59.0 nF	570 BW	L_5 = 121.0 mH	(2900 Hz)
C_6 = 280. nF	570 RB	L_6 = 334.8 mH	
C_7 = 24.9 nF	570 T	L_7 = 291.0 mH	} 1508.6 Hz
C_8 = 53.6 nF	570 BY	L_8 = 218.2 mH	
C_9 = 21.5 nF	570 HK	L_9 = 156.5 mH	} 2216.0 Hz
C_{10} = 25.5 nF	570 CH	L_{10} = 188.3 mH	
C_{11} = 25.5 nF	570 CH		
C_{12} = 25.5 nF	570 CH		
C_{13} = 26.1 nF	570 LF		
C_{14} = 13.7 nF	570 HT		
C_{15} = 13.7 nF	570 HT		

ALL INDUCTORS 1647 TYPE

WVH 21020

FIG. 15

ISSUE 1 7.17.67	ENGR ARO / DRAWN	TITLE VSB TRANSMITTING FILTER OPTION D	BELL TELEPHONE LABORATORIES INCORPORATED / SHEET / NO. OF SHEETS PER SET

MARS 1967CODER ARO PROB 5123 RUN 1 OLEJNICZAK39027 D,D-D

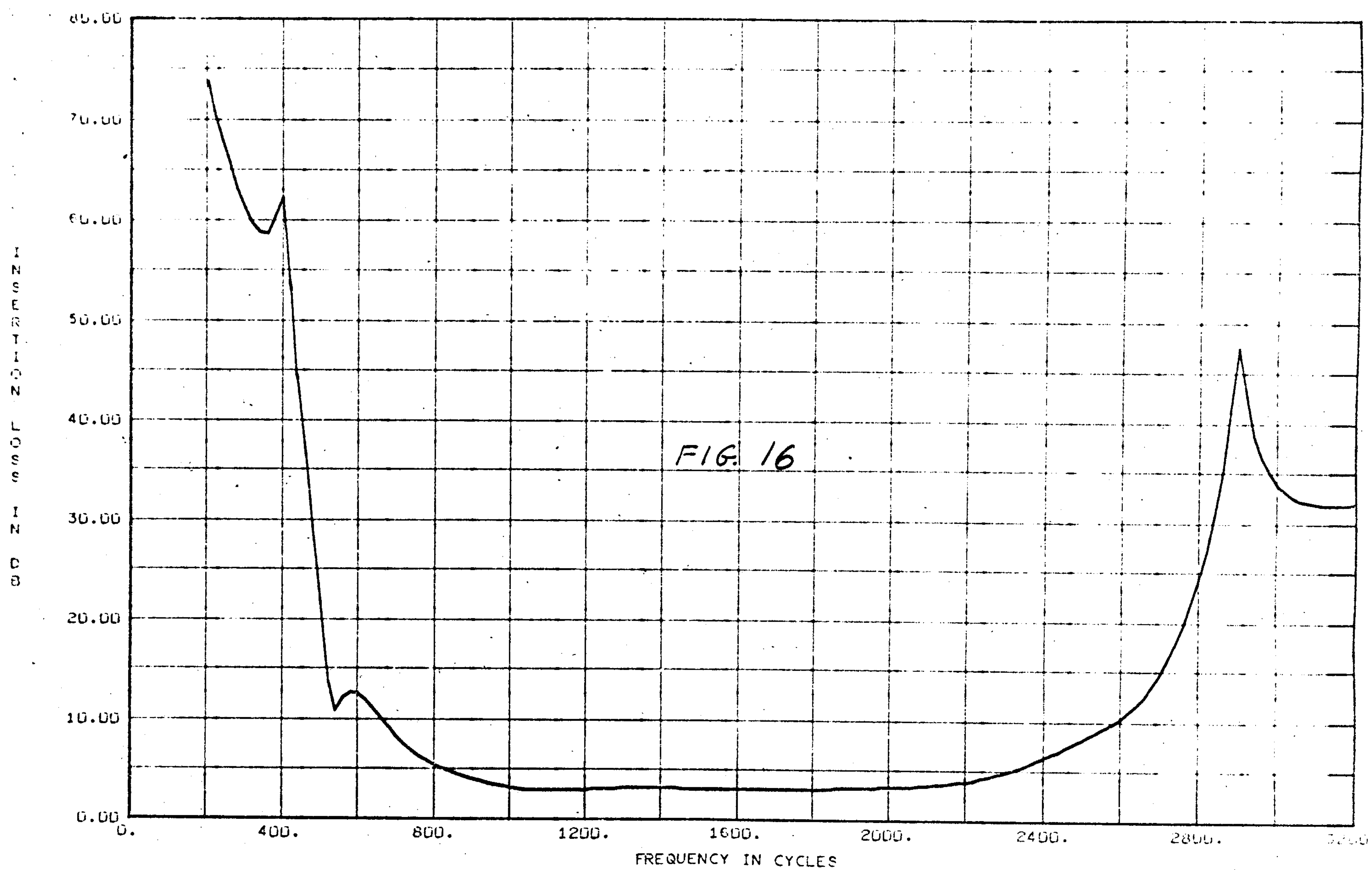


FIG. 16

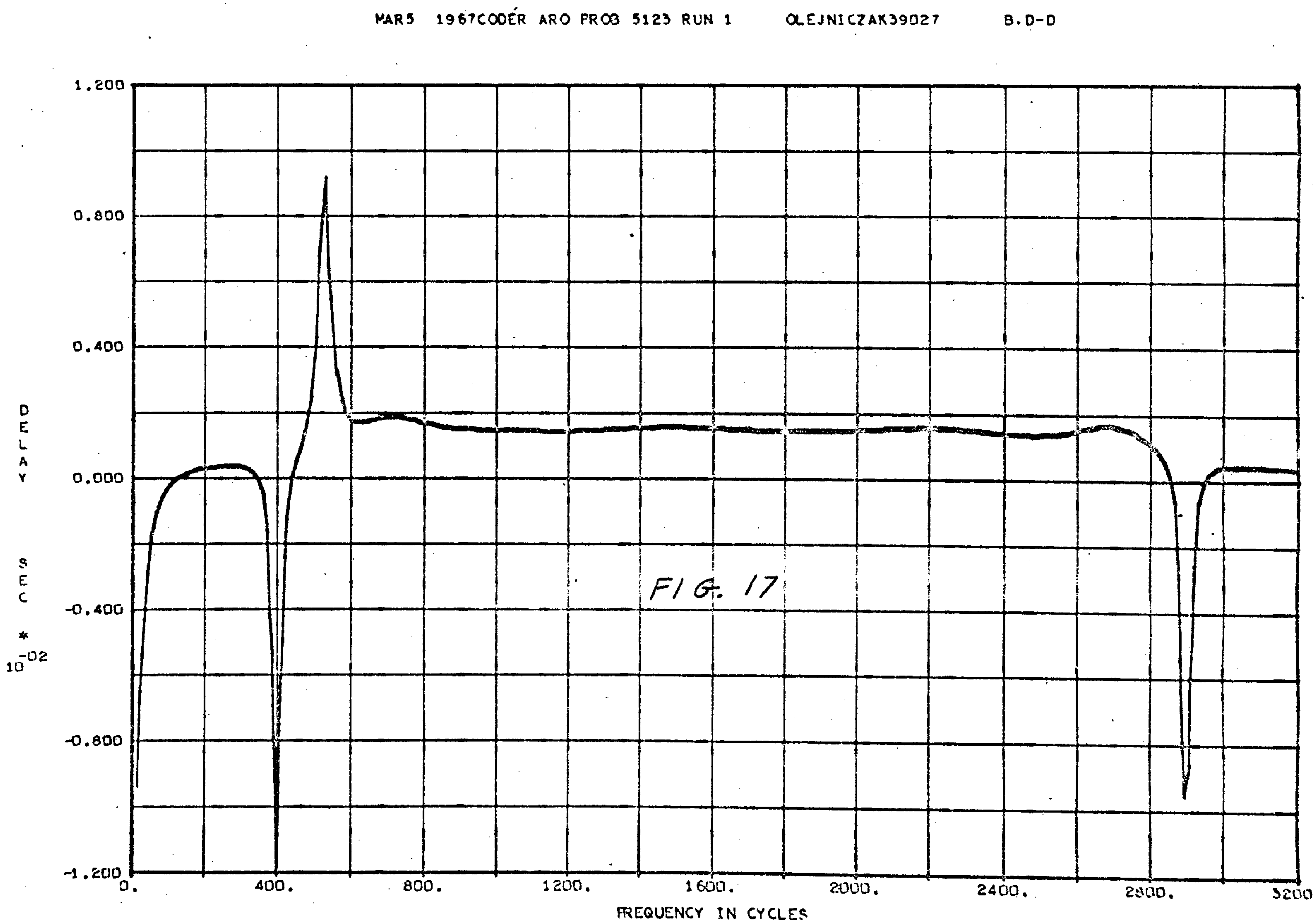

MARS 1967CODER ARO PROB 5123 RUN 1 OLEJNICZAK39027 B.D-D
1.200
0.800
0.400
0.000
-0.400
-0.800
-1.200
DELAY SEC * 10^-02
0.
400.
800.
1200.
1600.
2000.
2400.
2800.
3200.
FREQUENCY IN CYCLES

FIG. 17

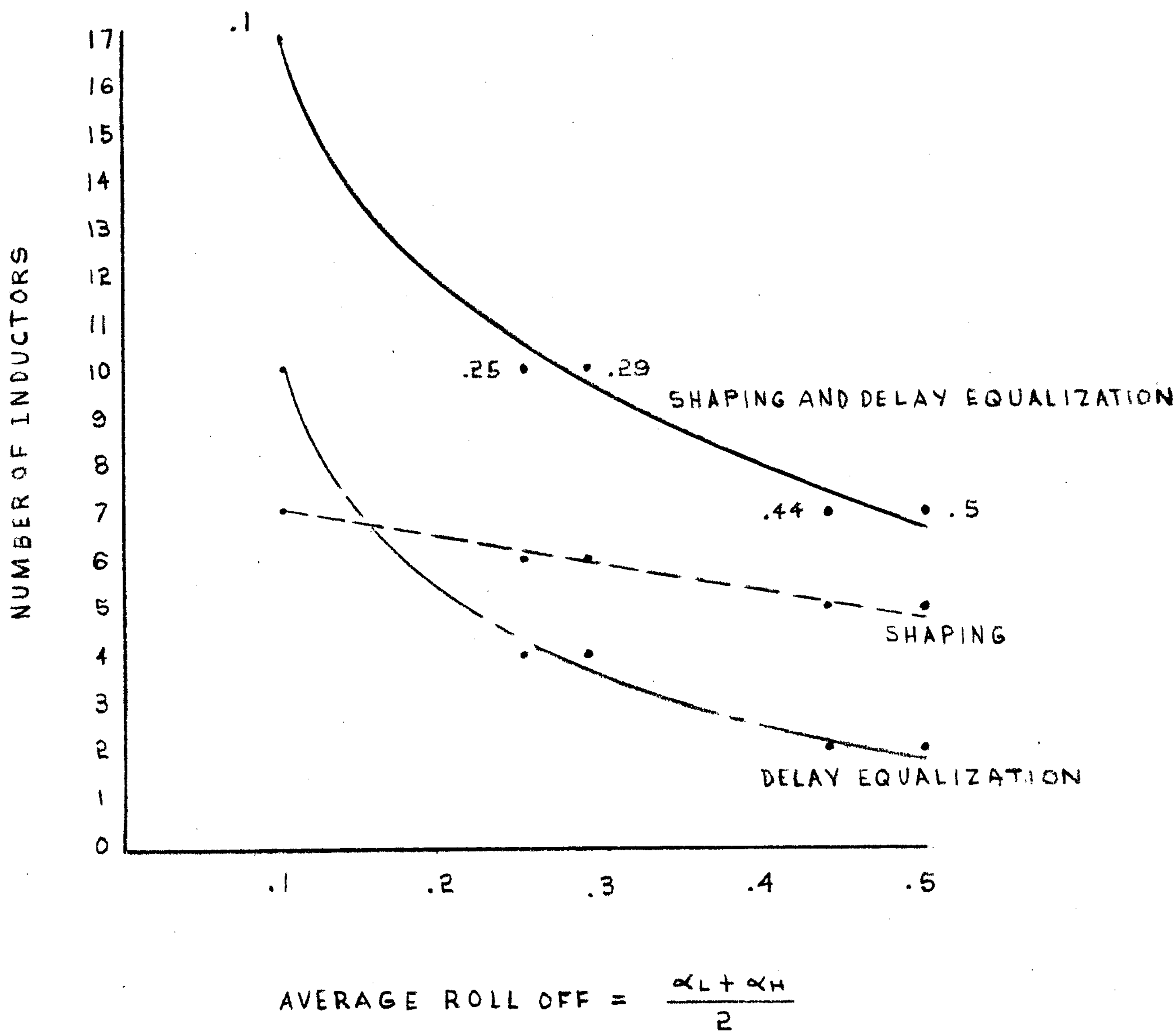


FIGURE 18

ISSUE	ENGR	TITLE	BELL TELEPHONE LABORATORIES INCORPORATED	
9/12/67	ARO	FILTER COMPLEXITY VS. ROLL OFF	1	SHEET 1
	DRAWN MJF		NO. OF SHEETS PER SET	

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