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TITLE- Communications Margins for Apollo Unified S-Band Links with Phase Modulation

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ABSTRACT

A revised set of communications margin expressions is presented that is applicable to those modes of the Apollo Unified S-Band System that employ narrow-band phase modulation. This work extends that previously reported by J. D. Hill in 1965. The new model differs from that of Hill by 1) including in the circuit margin equations the effects of limiter suppression, 2) including the variation in the receiver system noise temperature as a function of received signal power, 3) expressing the turned-around signal components in the ranging channel as two truncated infinite series, 4) providing equations for the minimum allowable received signal power at both the space vehicle and the ground station that will just satisfy channel performance requirements, and 5) providing a set of expressions that describe the error in the circuit margin calculations when selected approximations are made.

A comparison is provided of circuit margins obtained using an up-dated version of Hill's model and the model presented here. The maximum difference between these two for the Apollo systems is 0.4 decibel for low signal-to-noise ratios and as large as 1.05 decibels for high signal-to-noise ratios, with the updated model giving the more accurate results.

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SUBJECT: Communications Margins for Apollo Unified S-Band Links with Phase Modulation - Case 320

DATE: December 31, 1968

FROM: N. W. Schroeder

TM: 68-2034-17

TECHNICAL MEMORANDUM

The Apollo Unified S-Band (USB) system has the capability of transmitting S-Band carriers to and from the spacecraft which are coherent and are phase modulated by voice, data and a ranging code. The capabilities of the system have been discussed by J. D. Hill[1] and W. D. Wynn[2,3] who have presented mathematical models that can be used to predict the performance of the USB communication links. The present work extends the work of Hill[1] using the system configuration presented by Wynn[2]; a new set of equations which describes the performance of the USB system is derived.

The present analysis differs from that of Hill in five respects, namely:

(1) the suppression effects of the limiters, used in both the space vehicle and MSFN ground station receivers, are included to improve the accuracy of the predicted channel performance for the "near threshold" cases.

(2) The variation of the receiver system noise temperature is included to improve the accuracy of the predicted performance for nearly all cases, but especially those whose operation is considerably above threshold. (For a discussion of this noise temperature variation see Reference 4).

(3) The turned-around signal components in the ranging channel of the down-link are expressed as two truncated infinite series. This simplifies the down-link margin expressions while improving the accuracy of the results.

(4) By equating the circuit margin to zero, the expression for the minimum allowable received signal power, "P_{MIN}" is obtained for each of the USB services; thus, a "power" margin calculation can be performed directly.

(5) Equations are provided that describe the magnitude of error of the circuit margin calculations that results when selected approximations are made. Thus, the validity of hand calculations, in which the selected approximations are often used, is easily determined.

The up and down-link circuit margin equations are derived in Appendices A and B. The power margin expressions are derived in Appendix C. The limiter equations that are used in Appendices A and B are presented in Appendix D. The equations for the error that results in the circuit margin calculations when selected approximations are used, are derived in Appendix E. A comparison of circuit margins, using Hill's original model, the up-dated model and the model presented here, is given in Tables I and II. Table I presents the results of margin calculations for the communications link between the Lunar Module (LM) and the Manned Space Flight Network (MSFN) and Table II the margins for the Command/Service Module (CSM) - to -MSFN link. The equations derived in Appendices A and B are summarized in Table III and the symbols used are presented in Table IV.

The models for the band-pass limiter and the turned-around noise that are used in the derivation of the margins equations are based on work by Drs. F. F. Carden and J. D. Hill[1]. Although these models and the approximations on which they are based are still considered to be reasonable for the Apollo cases, work is continuing to determine the differences between the results obtained from the use of these models and the more rigorous mathematical model presented by Wynn[3].

The model for the variation in the system noise temperature is presented in Reference 4. This model approximates the rise in the system noise temperature caused by an increase in the received carrier power by the following relation:

$$T_{system} = \delta + B \times (\text{Carrier Power in Watts}),$$

where δ and B are constant for a given antenna-receiver installation.

The technique of expressing the turned-around signal components as a truncated infinite series is based on unpublished work by R. L. Selden. This technique is the general analysis for which Hill's model is an approximation. In addition to simplifying the derivation of the down-link margin expressions (only one term, R_g, need be evaluated instead of the three terms, α, β, γ, in Hill's model) the use of the general analysis provides higher accuracy. This added accuracy stems from the fact that Hill's model includes, in the down-link, only the first three components of the infinite series which describe the up-link spectrum while the general analysis used in this memorandum includes the complete up-link spectrum. This permits the final infinite series to be truncated at a point that is dictated by the magnitude of the system parameters (turn-around channel gain constant and modulation indices). Hill's approximation of the turn-around channel spectrum is a reasonable one for the Apollo system. It is, however, believed that the general analysis used in this memorandum will prove useful not only in the analysis of systems where Hill's approximation is not valid, but also in the selection of modulation indices and transponder turn-around channel gain constants for communications links of future programs.

The power margin is a concept for a communications link quality factor that is presented in Reference 5. Since the system noise temperature, the limiter factor, and the interference power (code power or retransmitted noise) are functions of the received signal power, the circuit margins as defined by Hill are bounded by a well defined maximum which is determined by the parameters of a given channel. This maximum bound exists because as the received signal power increases, the interference power likewise increases. Although the circuit margins so calculated do predict accurately the performance of a given system under the selected conditions, this definition of "margin" does not directly indicate the allowable degradation in a link whose circuit margins are positive. In Appendix C, expressions are derived that describe the minimum received power, P_{MIN}, that is required under the constraint that the circuit margin equals zero. The power margin, M_p, is defined as the difference in decibels between the magnitude of the total received power, P_r, (predicted by equation C-6 of Appendix C) and the magnitude of the required received power P_{MIN} that is calculated for the selected service. The power margin therefore indicates directly the magnitude (in decibels) that the transmitter power/-antenna gain/circuit loss combination may be allowed to degrade (if $M_p > 0$) or the magnitude that this combination must be improved (if $M_p < 0$) for a communication capability to exist.

Appendix E presents several approximations that can be used in the calculation of performance margins. These approximations simplify the calculations and are attractive if margins are to be calculated without the use of a computer. This appendix also contains equations for determining the error that results in the margins when these approximations are used. In the general derivation of the equations for circuit and power margins only the approximation given in Part VI of Appendix E was used. That is, the infinite series that defines the turn-around ranging channel spectrum has been truncated after the first term. For the Apollo System this approximation can be shown to be more accurate than Hill's original model. (Truncation after one term results in very little error for the Apollo USB systems).

The comparison between the present model for the USB system that is described by the expressions in Table IV and two versions of Hill's model is presented in Tables I and II. The original Bellcomm computer program entitled, "Margin, Computation of Margins," dated December 15, 1964, which is based on the model described in Reference 1, was modified in July 1966 to include a variable system noise temperature (identical to that used in this memorandum) and a limiter factor equal to $\pi/4$ (identical to that used in this memorandum for the case of signal-to-noise ratios less than σ.035, at the limiter input). The calculations shown in Tables I and II indicate that for the links shown there is close agreement between the present model and the modified Hill model since the maximum discrepancy is 0.4 decibel. It should be noted that this "close" agreement is true only for those cases where the signal-to-noise ratio (S/N) at the limiter input is equal to or less than 0.035; for higher (S/N)'s the discrepancy could reach a maximum of 1.05 decibels with the present model giving the greater and more accurate margin prediction.

Conclusions

The revised set of margin equations presented here provide a more accurate mathematical model of the performance of the Apollo S-Band System than those models previously used.[1,4,6] For the Apollo System, however this increased accuracy results in differences in computed margins of 0.5 dB or less.

The power margin data which is now available as a result of the derivation of the P_{MIN} expressions in Appendix C will provide a much clearer description of the magnitude by which the transmitted power/antenna gains/circuit losses combination may be permitted to degrade or the magnitude by which this combination must be increased for a communications capability to exist.

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TABLE I

COMPARISON OF USB CIRCUIT MARGIN CALCULATIONS FOR PM MODES
(HILL MODEL vs PRESENT MODEL)

LUNAR RANGE = 215,000 N.M.

NOMINAL PARAMETER VALUES*
LUNAR MODULE (STEERABLE ANTENNA, HIGH POWER)
MSFN STATION (85 FT. DIAMETER ANTENNA)

SERVICE	SERVICES TRANSMITTED	PRESENT	HILL***	HILL**
UP CARRIER	FULL PM MODE	35.8 dB	35.7 dB	36.3 dB
"	VOICE AND DATA	35.6	35.6	36.1
"	VOICE OR DATA ONLY	31.5	31.5	31.7
"	RANGING ONLY	28.9	28.9	29.0
UP VOICE OR UP DATA	FULL PM MODE	8.2	8.2	8.2
"	VOICE AND DATA	23.9	23.9	24.4
"	VOICE OR DATA ONLY	28.9	28.9	29.1
DOWN CARRIER	FULL PM MODE W/51.2 KBPS TLM	37.8	37.9	38.9
"	VOICE AND 51.2 KBPS TLM	38.5	38.5	39.7
"	VOICE AND 1.6 KBPS TLM	39.1	39.1	40.4
"	1.6 KBPS TLM ONLY	39.8	39.8	41.5
"	RANGING ONLY	41.7	41.3	43.9
DOWN VOICE	FULL PM MODE W/51.2 KBPS TLM	9.6	9.7	10.7
"	VOICE AND 51.2 KBPS TLM	10.3	10.2	11.5
51.2 KBPS TELEMETRY	FULL PM MODE	8.7	8.8	9.8
"	VOICE AND 51.2 KBPS TLM	9.5	9.5	10.6
1.6 KBPS TELEMETRY	TLM ONLY	25.8	25.8	27.5
RANGING	FULL PM MODE W/51.2 KBPS TLM	19.1	18.8	19.8
"	RANGING ONLY	34.8	35.1	37.7

**THE ORIGINAL HILL MODEL - 1) A CONSTANT SYSTEM NOISE TEMPERATURE,
2) A LIMITER FACTOR k = 1

***THE MODIFIED HILL MODEL - 1) A VARIABLE SYSTEM NOISE TEMPERATURE ($\tau = \delta + B$ (CARRIER POWER))
2) A LIMITER FACTOR $k = \pi/4$

*THE SYSTEM PARAMETERS ON WHICH THE ABOVE CALCULATIONS ARE BASED ARE THOSE CONTAINED IN THE NASA-MSC/ISD MASTER PARAMETERS LIST DATED MARCH 12. 1968.

TABLE II

COMPARISON OF USB CIRCUIT MARGIN CALCULATIONS FOR PM MODES
(HILL MODEL vs PRESENT MODEL)

LUNAR RANGE = 215,000 N.M.

NOMINAL PARAMETER VALUES*
COMMAND AND SERVICE MODULE (HIGH GAIN ANTENNA-NARROW BEAM, HIGH POWER)
MSFN STATION (85 FT. DIAMETER ANTENNA)

SERVICE	SERVICES TRANSMITTED	PRESENT	HILL (MODIFIED)	HILL (ORIGINAL)
UP CARRIER	FULL PM MODE	41.3 dB	41.3 dB	41.6 dB
"	VOICE AND DATA	41.1	41.1	41.4
"	VOICE OR DATA ONLY	36.9	36.9	37.0
"	RANGING ONLY	34.2	34.2	34.3
UP VOICE OR UP DATA	FULL PM MODE	8.3	8.3	8.3
"	VOICE AND DATA	28.1	28.1	28.3
"	VOICE OR DATA ONLY	32.9	32.9	33.0
DOWN CARRIER	FULL PM MODE	41.2	41.2	43.8
"	VOICE AND 51.2 KBPS TLM	41.8	41.6	44.6
"	VOICE AND 1.6 KBPS TLM	40.9	40.9	43.2
"	1.6 KBPS TLM ONLY	39.4	39.4	40.9
"	RANGING ONLY	42.7	42.3	46.0
DOWN VOICE	FULL PM MODE W/51.2 KBPS TLM	10.4	10.5	13.0
"	VOICE AND 51.2 KBPS TLM	10.9	10.9	13.8
51.2 KBPS TELEMETRY	FULL PM MODE	8.1	8.2	10.8
"	VOICE AND 51.2 KBPS TLM	8.6	8.6	11.6
1.6 KBPS TELEMETRY	TLM ONLY	28.9	28.9	30.4
RANGING	FULL PM MODE W/51.2 KBPS TLM	22.6	22.2	24.8
"	RANGING ONLY	36.4	36.2	39.9

*THE SYSTEM PARAMETERS ON WHICH THE ABOVE CALCULATIONS ARE BASED ARE THOSE CONTAINED IN THE NASA-MSC/ISD MASTER PARAMETERS LIST DATED MARCH 12, 1968.

TABLE III*

SUMMARY OF THE COMMUNICATION MARGIN EXPRESSIONS**

FOR THE APOLLO USB PM LINKS

1. Circuit Margins

The up-carrier margin is

$$M_{uc} = 10 \log_{10}\left[\frac{k_{uc} P_{sr} J_0^2(m_1) J_0^2(m_2)\cos^2\theta}{K_o T_u B_{uc}}\right] - \left(\frac{S}{N}\right)_{ucr} \quad \text{(A-18)}$$

where:

k_{uc} = The limiter factor, defined in general by k_L a

$$k_L = \begin{cases} k_1 = \pi/4 & \text{for } \left(\frac{S}{N}\right)_L < .035 \\ k_2 = .68\left(\frac{S}{N}\right)_L + .76 & \text{for } .035 \le \left(\frac{S}{N}\right)_L \le .35 \\ k_3 = 1 & \text{for } \left(\frac{S}{N}\right)_L > .35 \end{cases} \quad \text{b}$$

For $k_L = k_{uc}$

$$\left(\frac{S}{N}\right)_L \text{ is equal to } \left(\frac{S}{N}\right)_{uc} \quad \text{c}$$

$$\left(\frac{S}{N}\right)_{uc} = \frac{P_{sr} J_0^2(m_1) J_0^2(m_2)\cos^2\theta}{K_o T_u B_{uc}} = \frac{P_{sr}\alpha_{uc}}{K_o T_u B_{uc}} \quad \text{d}$$

*The symbols used in this table are defined in Table IV.

**The numbering of the equations in this table corresponds to the numbering of the equations that appears in the derivations contained in Appendices A through E.

Table III (Contd.)

$$T_u = \delta_u + B_u P_{sr} \alpha_{uc} \qquad \text{e}$$

The up-subcarrier circuit margin is

$$M_{u\omega_i} = 10 \log_{10} \frac{k_{us} P_{sr} J_1^2(m_i) J_0^2(m_k) \cos^2\theta}{B_{\omega_i} \frac{K_o T_u}{2} + \eta_{u_i} k_{us} P_{sr}} - \left(\frac{S}{N}\right)_{u\omega_i r} \qquad \text{(A-36,37)}$$

where

$$i = 1 \text{ or } 2 \qquad \text{a}$$

$$k = 1 \text{ or } 2 \qquad \text{b}$$

$$i \neq k \qquad \text{c}$$

$$\eta ui = \frac{1}{10^6} J_0^2(m_1) J_0^2(m_2) \sin^2\theta \frac{\sin^2 K_c f_i}{(K_c f_i)^2} \qquad \text{d}$$

For $k_L = k_{us}$, $(S/N)_L$ is equal to $(S/N)_{us}$ e

$$\left(\frac{S}{N}\right)_{us} = \frac{P_{sr}}{K_o T_u B_{ts}} \qquad \text{f}$$

The down-carrier circuit margin is

$$M_{dc} = 10 \log_{10} k_{dc} \frac{P_{gr} J_0^2(R_g) J_0^2(N_u) J_0^2(m_3) J_0^2(m_4)}{K_o T_d B_{dc}} - \left(\frac{S}{N}\right)_{dcr} \qquad \text{(B-29)}$$

where

$$R_g = R_o \left[\frac{\left(\frac{S}{N}\right)_{us}}{\frac{1}{k_{us}} + \left(\frac{S}{N}\right)_{us}} \right]^{1/2} \qquad \text{a}$$

Table III(Contd.)

R_o = A constant which is defined for each spacecraft transponder b

$$N_u = R_o \left[\frac{1}{1 + k_{us}\left(\frac{S}{N}\right)_{us}}\right]^{1/2} \left[\frac{B_v}{\frac{B_{ts}}{2}}\right]^{1/2} \quad c$$

For $k_L = k_{dc}$, $(S/N)_{dc}$ d

$$\left(\frac{S}{N}\right)_{dc} = \frac{P_{gr} J_0^2(R_g) J_0^2(N_u) J_o^2(m_3) J_0^2(m_4)}{K_o T_d B_{dc}} = \frac{P_{gr} \alpha_{dc}}{K_o T_d B_{dc}} \quad e$$

The down-subcarrier circuit margin is

$$M_{d\omega j} = 10 \log_{10}\left[\frac{k_{ds} P_{gr} J_0^2(R_g) J_0^2(N_u) J_1^2(m_j) J_0^2(m_\ell)}{B_{\omega_j}\left[\frac{K_o T_d}{2} + \eta_d k_{ds} P_{gr}\right]}\right] - \left(\frac{S}{N}\right)_{d\omega jr} \quad \text{(B-48, 49)}$$

where $j = 3$ or 4 a

$\ell = 3$ or 4 b

$j \neq \ell$ c

$$\eta_d = \frac{J_0^2(R_g) J_1^2(N_u) J_0^2(m_3) J_0^2(m_4)}{B_v} \quad d$$

For $k_L = k_{ds}$, $(S/N)_L = (S/N)_{ds}$ e

$$\left(\frac{S}{N}\right)_{ds} = \frac{P_{gr}}{K_o T_d B_{gs}} \quad f$$

Table III(Contd.)

The down link range code circuit margin is

$$M_{dco} = 10 \log_{10}\left[\frac{4P_{gr}J_1^2(R_g)J_0^2(N_u)J_0^2(m_1)J_0^2(m_2)J_0^2(m_3)J_0^2(m_4)\sin^2\theta}{B_{dco}\left[K_oT_d + 2P_{gr}\eta_d\right]}\right] - \left(\frac{S}{N}\right)_{dcor}$$

(B-58)

where

$$\eta_d = \frac{1}{B_v} J_0^2(R_g)J_1^2(N_u)J_0^2(M_3)J_0^2(M_4)$$

2. Power Margins

The up-carrier power margin is

$$M_{puc} = 10 \log_{10} P_{sr} - P_{MIN_{uc}}$$

where

$$P_{MIN_{uc}} = 10 \log_{10}\left[\frac{B_{uc}\gamma_{uc}\delta_u K_o}{\alpha_{uc}\left[k_{uc} - B_u B_{uc}\gamma_{uc}K_o\right]}\right] \quad \text{(C-12)}$$

$$\gamma_{uc} = \text{Antilog}_{10}\left[\frac{1}{10}\left(\frac{S}{N}\right)_{ucr}\right]$$

Table III(Contd.)

The up-subcarrier power margin is

$$M_{pu\omega_i} = 10 \log_{10} P_{sr} - P_{MIN_{u\omega_i}}$$

where

(C-17)

$$P_{MIN_{u\omega_i}} = 10 \log_{10} \left[\frac{B_{\omega_i} \gamma_{u\omega_i} \delta_u K_o}{2k_{us} \left[\alpha_{u\omega_i} - B_{\omega_i} \gamma_{u\omega_i} \eta_{ui} \right] - \alpha_{uc} B_u B_{\omega_i} \gamma_{u\omega_i} K_o} \right]$$

$$\gamma_{u\omega_i} = \text{Antilog}_{10} \frac{1}{10} \left(\frac{S}{N} \right)_{u\omega_i}$$

$$\alpha_{u\omega_i} = J_1^2(m_i) J_0^2(m_k) \cos^2\theta$$

$i = 1$ or 2

$k = 1$ or 2

$i \neq k$

$$\eta_{ui} = \frac{1}{10^6} J_0^2(m_1) J_0^2(m_2) \sin^2\theta \sin^2 (K_c f_i)/(K_c f_i)^2$$

The down-carrier power margin is

$$M_{pdc} = 10 \log_{10} P_{gr} - P_{MIN_{dc}}$$

where

$$P_{MIN_{dc}} = 10 \log_{10} \left[\frac{B_{dc} \gamma_{dc} \delta_d K_o}{\alpha_{dc} [k_{dc} - B_d B_{dc} \gamma_{dc} K_o]} \right] \qquad \text{(C-22)}$$

Table III (Contd.)

$$\gamma_{dc} = \text{Antilog}_{10}\left[\frac{1}{10}\left(\frac{S}{N}\right)_{dcr}\right]$$

The down-subcarrier power margin is

$$M_{pd\omega_j} = 10 \log_{10} P_{gr} - P_{MIN_{d\omega_j}}$$

where

$$P_{MIN_{d\omega_j}} = 10 \log_{10}\left[\frac{B_{\omega_j}\gamma_{d\omega_j}\delta_d K_o}{2k_{ds}\left(\alpha_{d\omega_j} - B_{\omega_j}\gamma_{d\omega_j}\eta_d\right) - \alpha_{dc}B_d B_{\omega_j}\gamma_{d\omega_j}K_o}\right] \qquad \text{(C-27)}$$

$$\gamma_{d\omega_j} = \text{Antilog}_{10}\left[\frac{1}{10}\left(\frac{S}{N}\right)_{d\omega_j r}\right]$$

$$\alpha_{d\omega_j} = J_0^2(R_g)J_0^2(N_u)J_1^2(m_j)J_0^2(m_\ell)$$

$$\eta_d = \left(\frac{1}{Bv}\right)J_1^2(N_u)J_0^2(R_g)J_0^2(m_3)J_0^2(m_4)$$

The down link range code power margin is

$$M_{pdco} = 10 \log_{10} P_{gr} - P_{MIN_{dco}}$$

where

$$P_{MIN_{dco}} = 10 \log_{10}\left[\frac{B_{dco}\gamma_{dco}\delta_d K_o}{4\alpha_{dco} - B_{dco}\gamma_{dco}(\alpha_{dc}B_d K_o + 2\eta_d)}\right] \qquad \text{(C-33)}$$

Table III(Contd.)

$$\gamma_{dco} = \text{Antilog}_{10}\left[\frac{1}{10}\left(\frac{S}{N}\right)_{dcor}\right]$$

$$\alpha_{dco} = J_1^2(R_g)J_0^2(N_u)J_0^2(m_1)J_0^2(m_2)J_0^2(m_3)J_0^2(m_4)\sin^2\theta$$

3. Approximation Errors

A. A Constant System Noise Temperature ($T=\delta$)

The errors, in dB which result if the circuit margin expressions are simplified by setting the system noise temperature equal to a constant, are as follows:

1. Up-carrier

$$\eta_{Tuc} = 10 \log_{10} \left[1 + \frac{B_u}{\delta_u} P_{sr} \alpha_{uc}\right] \qquad \text{(E-2)}$$

2. Up-subcarrier

$$\eta_{Tusi} = 10 \log_{10} \left[1 + \frac{B_u K_o P_{sr} \alpha_{uc}}{\delta_u K_o + 2\eta_{ui} k_{us} P_{sr}}\right] \qquad \text{(E-4)}$$

3. Down-carrier

$$\eta_{Tdc} = 10 \log_{10} \left[1 + \frac{B_d P_{gr} \alpha_{dc}}{\delta_d}\right] \qquad \text{(E-6)}$$

4. Down-subcarrier

$$\eta_{Tds} = 10 \log_{10} \left[1 + \frac{B_d K_o P_{gr} \alpha_{dc}}{K_o \delta_d + 2k_{ds} \eta_d P_{gr}}\right] \qquad \text{(E-8)}$$

Table III(Contd.)

5. Down link range code

$$\eta_{Tco} = 10 \log_{10} \left[1 + \frac{B_d K_o P_{gr} \alpha_{dc}}{\delta_d K_o + 2P_{gr} \eta_d} \right] \quad \text{(E-10)}$$

B. A Constant Limiter Factor (k=1)

The errors, in dB which result if the circuit margin expressions are simplified by setting the limiter factor equal to unity, are as follows:

1. Up-carrier

$$\eta_{kuc} = 10 \log_{10} \frac{1}{k_{uc}} \quad \text{(E-12)}$$

2. Up-subcarrier

$$\eta_{kusi} = 10 \log_{10} \frac{1}{k_{us}} \frac{K_o T_u + 2k_{us} P_{sr} \eta_{ui}}{K_o T_u + 2P_{sr} \eta_{ui}} \quad \text{(E-14)}$$

3. Down-carrier

$$\eta_{kdc} = 10 \log_{10} \frac{1}{k_{dc}} \quad \text{(E-16)}$$

4. Down-subcarrier

$$\eta_{kds} = 10 \log_{10} \left[\left(\frac{1}{k_{ds}} \right) \left(\frac{K_o T_d + 2k_{ds} P_{gr} \eta_d}{K_o T_d + 2P_{gr} \eta_d} \right) \right] \quad \text{(E-18)}$$

Table III(Contd.)

C. Neglecting Uplink Code Interference Power ($\eta_{ui} = 0$)

The errors, in dB which result if the circuit margin expressions are simplified by setting the uplink code interference power equal to zero, are as follows:

Up-subcarrier

$$\eta_{coi} = 10 \log_{10} \left[1 + \frac{2 k_{us} P_{sr} \eta_{ui}}{K_o (\delta_u + B_u P_{sr} \alpha_{uc})} \right] \qquad \text{(E-20)}$$

D. Neglecting Modulation Loss of Turn Around Channel ($R_g = 0$)

The errors, in dB which result if the circuit margin expressions are simplified by setting the turn around channel gain for the signal equal to zero, are as follows:

1. Down-carrier

$$\eta_{Rdc} = 10 \log_{10} \frac{\delta_d + B_d P_{gr} \alpha_{dc}}{\delta_d J_0^2(R_g) + \alpha_{dc} B_d P_{gr}} \qquad \text{(E-22)}$$

2.

$$\eta_{Rds} = 10 \log_{10} \frac{K_o T_d + 2\eta_d k_{ds} P_{gr}}{K_o(\delta_d J_0^2(R_g) + \alpha_{dc} B_d P_{gr}) + 2\eta_d k_{ds} P_{gr}} \qquad \text{(E-24)}$$

Table III(Contd.)

E. Neglecting Modulation Loss of Turn Around Channel (N_u=0)

The errors, in dB which result if the circuit margin expressions are simplified by setting the turn around channel gain for the noise equal to zero are as follows:

1. Down-carrier

$$\eta_{N_u dc} = 10 \log_{10} \frac{T_d}{\delta_d J_0^2(N_u) + \alpha_{dc} B_d P_{gr}} \qquad \text{(E-26)}$$

2. Down-subcarrier

$$\eta_{N_u ds} = 10 \log_{10} \left[\frac{K_o T_d + 2\eta_d k_{ds} P_{gr}}{K_o \left(\delta_d J_0^2(N_u) + \alpha_{dc} B_d P_{gr} \right)} \right] \qquad \text{(E-28)}$$

3. Downlink Range Code

$$\eta_{N_u co} = 10 \log_{10} \frac{K_o T_d + 2\eta_d P_{gr}}{K_o \quad \delta_d J_0^2(N_u) + \alpha_{dc} B_d P_{gr}} \qquad \text{(E-30}$$

F. Approximation of Infinite Series

The expressions for the downlink circuit margins which are presented in Appendix B have been derived using only the first term of the infinite series expansions which describe the respective downlink modulation losses. The errors which result from using only the first term rather than the first two terms of the series expansions are as follows:

Table III(Contd.)

1. Downlink carrier and subcarrier

$$\eta_{c/s} = 20 \log_{10} \quad 1 + \frac{2J_2^2(R_g)J_0(2m_1)J_0(2m_2)\cos(2\ \theta)}{J_0(R_g)} \qquad \text{(E-31)}$$

2. Downlink range code

$$\eta_{cos} = 20 \log_{10} \quad 1 + \frac{J_3(R_g)J_0(3m_1)J_0(3m_2)\sin(3\ \theta)}{J_1(R_g)J_0(m_1)J_0(m_2)\ \sin\ \theta} \qquad \text{(E-32)}$$

TABLE IV

DEFINITION OF SYMBOLS USED IN COMMUNICATIONS MARGIN EXPRESSIONS

	Symbol	Definition
1.	$4\alpha_{dco} = 4J_o^2(m_1)J_o^2(m_2)J_o^2(m_3)\cdot J_o^2(m_4)J_1^2(R_g)J_o^2(N_u)\sin^2\theta$	Modulation loss of the downlink range code
2.	$\alpha_{uc} = J_o^2(m_1)J_o^2(m_2)\cos^2\theta$ $\alpha_{dc} = J_o^2(m_3)J_o^2(m_4)J_o^2(R_g)J_o^2(N_u)$	Modulation loss of the up carrier and down carrier respectively
3.	$2\alpha_{u\omega_i} = 2J_1^2(m_i)J_o^2(m_k)\cos^2\theta$ $2\alpha_{d\omega_j} = 2J_1^2(m_j)J_o^2(m_\ell)J_o^2(R_g)J_o^2(N_u)$	Modulation loss of the up and down subcarriers respectively
4.	B_{dco}	Detection bandwidth of the downlink range code in H_Z
5.	B_{gs}, B_{ts}	IF Bandwidth of the ground receiver and spacecraft receiver respectively in H_Z
6.	B_u, B_d	Multiplicative constants in the expressions for the system noise temperatures of the uplink and downlink respectively
7.	B_{uc}, B_{dc}	Loop noise bandwidths of the up and down carrier respectively in H_Z
8.	$B_{u\omega_i}, B_{d\omega_j}$	Predetection noise bandwidths of the up subcarriers and down subcarriers respectively in H_Z
9.	B_v	Video Bandwidth of spacecraft receiver

10. δ_u, δ_d — Additive constants in the expressions for the system noise temperatures (T_u, T_d) of the uplink and downlink respectively

11. $\eta_{c\&s}, \eta_{cos}$ — Approximation errors in dB resulting from the use of only the first term rather than the first two terms of the infinite series which describe the modulation loss of the down carrier and subcarrier, and the downlink range code respectively

12. η_{coi} — Approximation errors in dB resulting from the assumption that the code interference power is equal to zero when calculating circuit margins for the up voice and up data subcarriers

13. η_{kdc}, η_{kds} — Approximation errors in dB resulting from the assumption that the limiter factor is equal to unity (k=1) when calculating circuit margins for the downlink carrier and subcarriers respectively

14. η_{kuc}, $\eta_{ku\omega_i}$ — Approximation errors in dB resulting from the assumption that the limiter factor is equal to unity (k=1) when calculating circuit margins for the uplink carrier and subcarriers respectively

15. η_{Nudc}, η_{Nuds}, η_{Nuco} — Approximation errors in dB resulting from the assumption that the gain of the transponder turn around channel for noise is equal to zero when calculating circuit margins for the downlink carrier, subcarrier, and range code respectively

16. η_{Rdc}, η_{Rds} — Approximation errors in dB resulting from the assumption that the transponder turn around gain for the signal (R_g) is equal to zero when calculating circuit margins for the downlink carrier and subcarrier respectively

17.	η_{Tdc}, η_{Tds}, η_{Tco}	Approximation errors in dB resulting from the assumption that the system noise temperature is equal to a constant (δ_d) when calculating circuit margins for the downlink carrier, subcarriers and range code respectively
18.	η_{Tuc}, η_{Tus_i}	Approximation errors in dB resulting from the assumption that the system noise temperature is equal to a constant (δ_u) when calculating circuit margins for the uplink carrier and subcarrier respectively
19.	G_t, G_r	Transmit gain and receive gain of the antennas respectively in dB
20.	K_o	Boltzman's constant = 1.38×10^{-23} watts/°KH_Z
21.	K_c	Frequency normalization factor ($K_c = \pi/10^6$)
22.	k_{uc}, k_{dc}	Limiter factors of the up and down carriers respectively
23.	k_{us}, k_{ds}	Limiter factors of the up and down subcarriers respectively
24.	L_c	Circuit losses in dB
25.	L_{pol}	Polarization losses in dB
26.	L_p	Pointing losses in dB
27.	L_s	Space loss in dB
28.	m_1, m_2	Up-subcarrier peak modulation indices in radians
29.	m_3, m_4	Down subcarrier peak modulation indices in radians
30.	M_{dco}	Circuit margin for the downlink range code in dB
31.	M_{pdco}	Power margin of the downlink range code in dB

32. M_{puc}, M_{pdc} — Power margins of the up and down carrier respectively in dB

33. $M_{pu\omega_i}$, $M_{pd\omega_j}$ — Power margins of the up and down subcarriers respectively in dB

34. M_{uc}, M_{dc} — Circuit margins of the up and down carrier respectively in dB

35. $M_{u\omega_i}$, $M_{d\omega_j}$ — Circuit margins of the up and down subcarriers respectively in dB

36. $$N_u = \frac{\sqrt{2}}{2} L_{us} B_u G \left[\frac{1}{1+k_{us}\left(\frac{S}{N}\right)_{us}} \cdot \frac{B_v}{\frac{B_{ts}}{2}} \right]^{1/2}$$

Gain of transponder turn around channel for noise

37. ω_1, ω_2, (f_1, f_2) — Up-subcarrier frequencies (ω_1 & ω_2) in radians/sec, (f_1 & f_2) in H_Z

38. ω_3, ω_4 — Down subcarrier frequencies in radians/second

39. ω_{uc}, ω_{dc} — Carrier frequencies of the up and down-links respectively in radians/second

40. $P_{MIN_{dco}}$ — Magnitude of the total received power in dB for which the circuit margin of the downlink range code is zero

41. $P_{MIN_{x\omega}}$ — Magnitude of the total received power for which the circuit margins of the the subcarriers respectively are zero

42. $P_{MIN_{xc}}$ — Magnitude of the total received power in dB for which the circuit margins of the carriers respectively are zero.

43. P_{sr}, P_{gr} — Total signal power received by the spacecraft and ground receivers respectively in watts

45. R — Range in nautical miles

46. $R_g = \frac{\sqrt{2}}{2} L_{us} B_u G \cdot \left[\frac{(S/N)_{us}}{(1/k_{us}) + (S/N)_{us}} \right]^{1/2}$

Gain of transponder turn around channel for signal

47. $R_o = \sqrt{2} \; L_{us} B_u G/2$ — Transponder turn around channel constant

48. $(\frac{S}{N})_{dcor}$ — Required signal to noise ratio of the downlink range code in dB

49. $(\frac{S}{N})_{ucr}$, $(\frac{S}{N})_{dcr}$ — Required signal to noise ratios of the up and down carriers respectively in dB

50. $(\frac{S}{N})_{u\omega_i r}$, $(\frac{S}{N})_{d\omega_j r}$ — Required signal to noise ratios of the up and down subcarriers respectively in dB

51. θ — Range code modulation index in radians

52. T_u, T_d — System noise temperatures in degrees Kelvin referred to the spacecraft and ground receivers respectively [T system = δ_u + B (carrier power)]

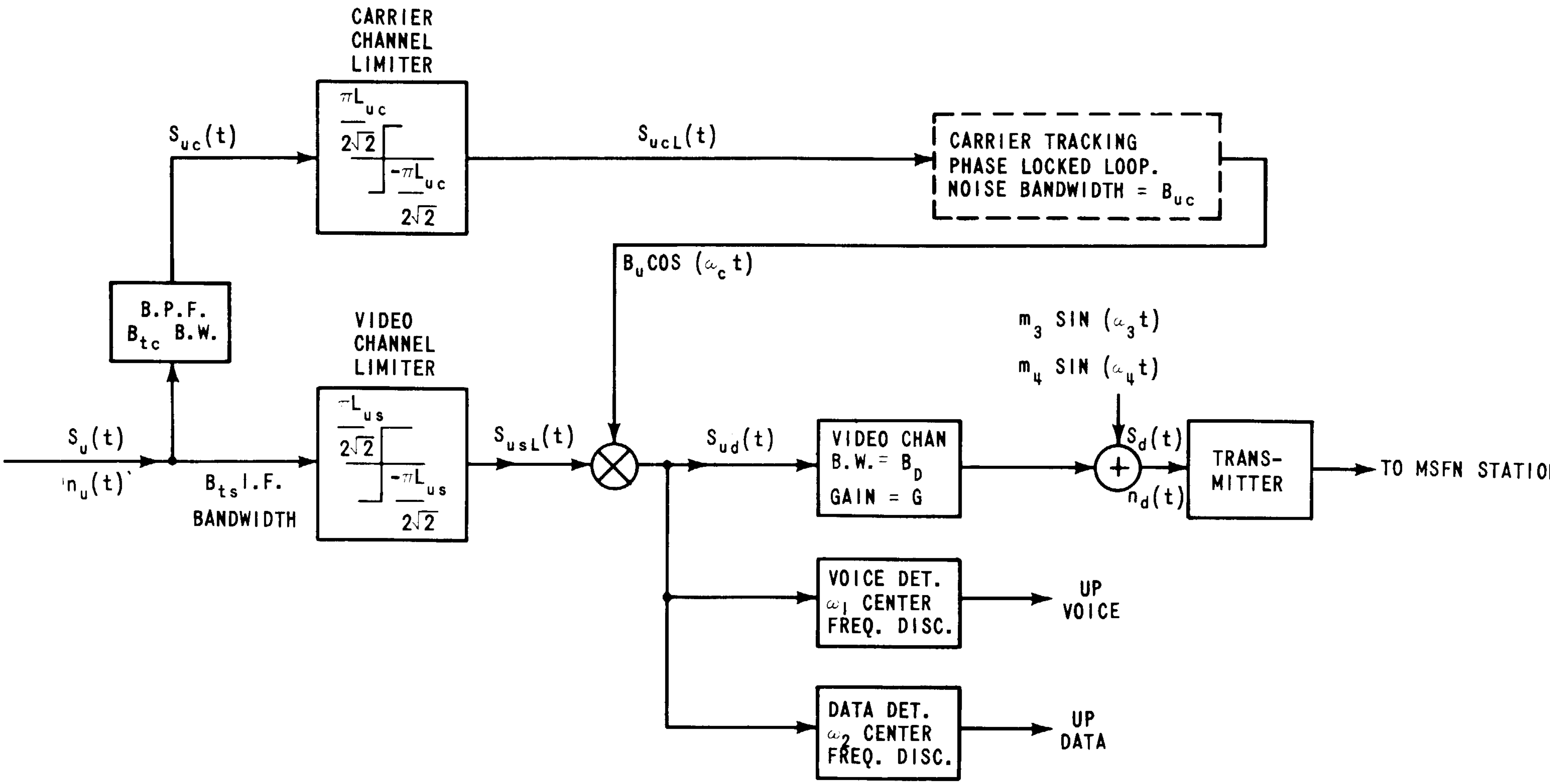

FIGURE 1 - THE TRANSPONDER STRUCTURE

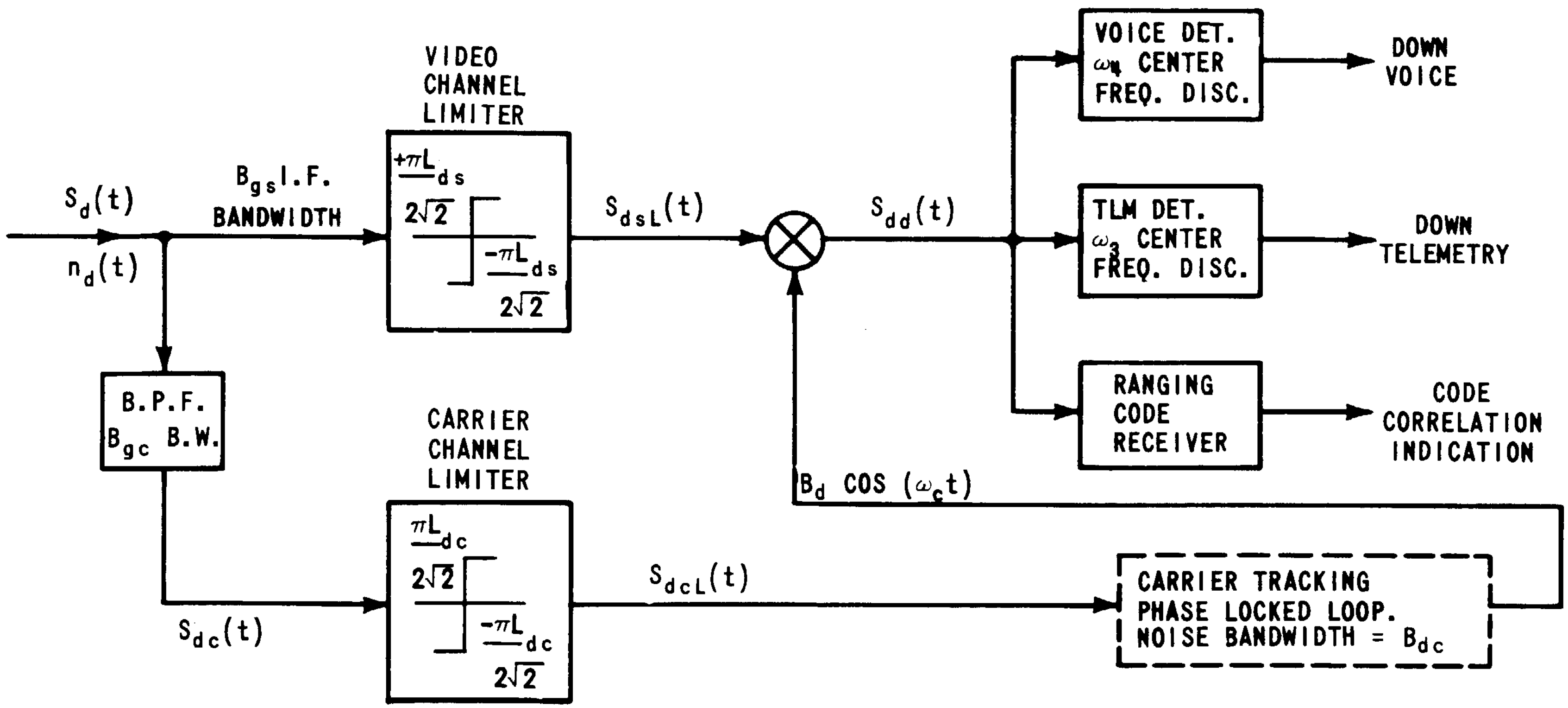

FIGURE II - MSFN STATION RECEIVER STRUCTURE

APPENDIX A

CIRCUIT MARGINS FOR USB UPLINK MODES

PURPOSE

The purpose of this appendix is to derive expressions which describe the circuit performance of the USB uplink PM modes.

The expressions for the circuit margins which are presented have been derived by direct subtraction (in dB) of the signal to noise ratio which is required "$(S/N)_r$" for a given service (Carrier Power/Noise Power in the transponder carrier detector bandwidth "B_{uc}" and Subcarrier Power/Noise Power in the transponder subcarrier detector bandwidth "$B\omega_i$") from the signal to noise ratio which is predicted "$\left(\frac{S}{N}\right)_p$" using the given circuit parameters. The margins expressions have been derived using the configuration of the Apollo uplink presented by Wynn[1] shown in Figure I; however, the expressions in effect define the communications margins for the general case of communicating via a one way link using a single carrier that is phase-modulated by two subcarriers and a pseudorandom digital range code.

The uplink signal and noise that is received by the transponder is assumed to be described by

$$S_u(t) + n_u \qquad \text{A-1}$$

$$= A_u \sin\left[\omega_{uc}t + m_1 \sin\omega_1 t + m_2 \sin\omega_2 t + \theta\right] + n_u$$

where

A-2

$\frac{A_u^2}{2}$	=	Total received signal power at the spacecraft	a
ω_{uc}	=	Up carrier frequency in radians/second	b
ω_1, ω_2	=	Up subcarrier frequencies in radians/second	c
m_1, m_2	=	Up subcarrier peak modulation indices in radians	d

θ	=	Pseudorandom range code modulation index in radians	e
$K_o T_u$	=	Power spectral density of input thermal noise n_u	f
K_o	=	1.38×10^{-23} watts/°K H_Z = Boltzman's Constant	g
T_u	=	Noise temperature referred to receiver input in °K	h

Then in general the output of a hard limiter in the transponder is defined by

$$S_{uL}(t) + n_{uL} = A_{usL} \sin\left[\omega_{uc} t + m_1 \sin\omega_1 t + m_2 \sin\omega_2 t + \theta\right] + A_{unL}\, n_u \qquad \text{A-3}$$

The coefficients A_{usL} and A_{unL} are limiter attenuation factors which are discussed in Appendix D. Using the results of Appendix D the expressions for the signal and noise components at the output of a limiter in the transponder can be written directly. The signal amplitude of the limiter output is given by

$$S_{uL}(t) = \sqrt{2}\, L_u \left[\frac{(S/N)_u}{\frac{1}{k} + \left(\frac{S}{N}\right)_u}\right]^{1/2} \sin\left[\omega_{uc} t + m_1 \sin\omega_1 t + m_2 \sin\omega_2 t + \theta\right] \qquad \text{A-4}$$

where

a) k is a function of the signal to noise power ratio $(S/N)_u$ at the input to the limiter

b) L_u^2 is the total power output of the limiter

The signal power of the limiter output is given by

$$P_s(L) = L_u^2 \left[\frac{(S/N)_u}{\frac{1}{k} + \left(\frac{S}{N}\right)_u}\right] \qquad \text{A-5}$$

The noise power spectral density of the limiter output is given by

$$p_n(L) = L_u^2 \left[\frac{1}{1 + k \left(\frac{S}{N}\right)_u} \right] \frac{1}{B} \text{ Watts/H}_Z \qquad \text{A-6}$$

The signal/noise ratio used in the above expressions is defined by

$$\left(\frac{S}{N}\right)_u = \frac{\frac{A_{uL}^2}{2}}{K_o T_u B} \qquad \text{A-7}$$

where $\frac{A_{uL}^2}{2}$ is the total signal power that is contained in the limiter input bandwidth B.

UP-CARRIER CIRCUIT MARGIN

The first step in this derivation is to describe the signal power of the up-carrier component at the input to the limiter in the transponder carrier channel. Expanding the signal term of A-1 gives the expression for the total signal received by the transponder

$$S_u(t) = A_u I_m e^{j\left[\omega_{uc} t + m_1 \sin\omega_1 t + m_2 \sin\omega_2 t + \theta\right]} \qquad \text{A-8}$$

$$= A_u I_m e^{j\left(\omega_{uc} t + \theta\right)} \sum_{N_1=-\infty}^{\infty} J_{N_1}(m_1) e^{jN_1 \omega_1 t} \sum_{N_2=-\infty}^{\infty} J_{N_2}(m_2) e^{jN_2 \omega_2 t} \qquad \text{A-9}$$

The carrier component is that portion of the signal described by A-9 which will be passed by a narrow band pass filter centered about ω_{uc}; the expression for the carrier component is obtained by letting

$$N_1 = N_2 = 0$$

Then equation A-9 can be rewritten as

$$S_{uc}(t) = A_u J_o(m_1)\, J_o(m_2) \left[\sin\omega_{uc} t \cos\theta + \cos\omega_c t \sin\theta\right] \qquad \text{A-10}$$

The first term of A-10 is the signal component of the up-carrier, and the second term is an interference signal which is caused by the code modulation "θ". The expression for the signal power at the input to the hard limiter in the transponder carrier channel then is obtained from A-10 and is

$$P_{si}(c) = \frac{A_u^2}{2} J_o^2(m_1)\, J_o^2(m_2) \cos^2\theta \qquad \text{A-11}$$

The second step is to describe the noise power added to the up-carrier by the transponder. Assuming that the noise power spectral density is a constant over the transponder carrier channel input bandwidth "B_{tc}" the noise power at the input to the limiter in the carrier channel is given by

$$K_o T_u B_{tc} \qquad \text{A-12}$$

Having derived the expressions for the signal and noise components in the carrier channel limiter input, the expressions for these components at the output of this limiter can be written directly.

a) The input signal to noise ratio is obtained from A-11 and A-12

$$\left(\frac{S}{N}\right)_{uc} = \frac{A_u^2}{2} \frac{J_o^2(m_1) J_o^2(m_2) \cos^2\theta}{K_o T_u B_{tc}} \qquad \text{A-13}$$

b) The output signal power is obtained from A-5

$$P_s(uc) = L_{uc}^2 \left[\frac{(S/N)_{uc}}{\frac{1}{K_{uc}} + \left(\frac{S}{N}\right)_{uc}}\right] \qquad \text{A-14}$$

c) The output noise power in the up carrier detector bandwidth "B_{uc}" is obtained from A-6

$$P_n(uc) = L_{uc}^2 \left[\frac{1}{1 + k_{uc}\left(\frac{S}{N}\right)_{uc}} \right] \frac{B_{uc}}{B_{tc}} \qquad \text{A-15}$$

The predicted signal to noise ratio for the up carrier is obtained from the ratio of Equations A-14 and A-15

$$\left(\frac{S}{N}\right)_{ucp} = \frac{\dfrac{L_{uc}^2 (S/N)_{uc}}{\frac{1}{kuc} + \left(\frac{S}{N}\right)_{uc}}}{L_{uc}^2 \left[\dfrac{1}{1+k_{uc}\left(\frac{S}{N}\right)_{uc}}\right] \dfrac{B_{uc}}{B_{tc}}} = k_{uc}\left(\frac{S}{N}\right)_{uc} \frac{B_{tc}}{B_{uc}} \qquad \text{A-16}$$

Substituting Equation A-13 into A-16

$$\left(\frac{S}{N}\right)_{ucp} = k_{uc} \frac{A_u^2}{2} \frac{J_o^2(m_1)J_o^2(m_2)\cos^2\theta}{K_o T_u B_{uc}} \qquad \text{A-17}$$

Assuming that the interference power in the up carrier detection bandwidth caused by the code modulation "θ" can be neglected and that the required signal to noise ratio for the up carrier in the up carrier detector bandwidth "B_{uc}" is given in decibles by $\left(\frac{S}{N}\right)_{ucr}$, then the up carrier circuit margin is described by

$$M_{uc} = 10 \log_{10} \left[k_{uc} \frac{A_u^2}{2} \frac{J_o^2(m_1)J_o^2(m_2)\cos^2\theta}{K_o T_u B_{uc}} \right] - \left(\frac{S}{N}\right)_{ucr} \qquad \text{A-18}$$

UP-SUBCARRIER CIRCUIT MARGINS

The first step in this derivation is to describe the signal powers of the up-subcarrier components at the output of the coherent carrier product detector in the transponder subcarrier channel. Expanding the signal term of A-1 gives the expression for the total signal received by the transponder

$$S_u(t) = A_u I_m e^{j\left[\omega_{uc}t+m_1\sin\omega_1 t+m_2\sin\omega_2 t+\theta\right]} \qquad \text{A-19}$$

$$= A_u I_m \left\{ e^{j\left[\omega_{uc}t+\theta\right]} \sum_{N_1=-\infty}^{\infty} J_{N_1}(m_1)e^{jN_1\omega_1 t} \sum_{N_2=-\infty}^{\infty} J_{N_2}(m_2)e^{jN_2\omega_2 t} \right. \qquad \text{A-20}$$

$$= A_u I_m \left\{ e^{j\left(\omega_{uc}t+\theta\right)} \left[J_o(m_1)+2\sum_{N_1=+1}^{\infty}\left[J_{2N_1}(m_1)\cos 2N_1\omega_1 t \right.\right.\right. \qquad \text{A-21}$$

$$\left.\left. + jJ_{2N-1}(m_1)\sin(2N-1)\omega_1 t\right]\right] \cdot$$

$$\left[J_o(m_2)+2\sum_{N_2=1}^{\infty}\left[J_{2N_2}(m_2)\cos 2N_2\omega_2 t+jJ_{2N-1}(m_2)\sin(2N-1)\omega_2 t\right]\right]$$

Neglecting the higher order and the cross frequency terms which are removed by filtering in the up-subcarrier detectors, Equation A-21 reduces to

$$S_{us}(t) = A_u \left[J_o(m_1)J_o(m_2)\sin\left(\omega_{uc}t+\theta\right) + 2J_1(m_1)J_o(m_2)\sin\omega_1 t\cos\left(\omega_{uc}t+\theta\right)\right.$$

$$\left. + 2J_o(m_1)J_1(m_2)\sin\omega_2 t\cos\left(\omega_{uc}t+\theta\right)\right] \qquad \text{A-22}$$

The second and third terms of A-22 describe the signal components of the ω_1 and ω_2 up-subcarriers respectively; the first term describes an interference signal which is caused by the code modulation "θ". The limiter in the transponder video channel causes an attenuation of the signal described by A-22. This attenuation is a function of the signal to noise "$(S/N)_{us}$" ratio at the input to the limiter, where

$$\left(\frac{S}{N}\right)_{us} = \frac{\frac{A_u^2}{2}}{K_o T_u B_{ts}} \qquad \text{A-23}$$

B_{ts} is the bandwidth of the bandpass filter preceding the limiter. Using A-4, A-22 and A-23 the signal amplitude at the output of the limiter is described by

$$S_{usL}(t) = \sqrt{2}\, L_{us} \left[\frac{(S/N)_{us}}{\frac{1}{k_{us}} + (S/N)_{us}}\right]^{1/2} \Big[J_o(m_1)J_o(m_2)\sin\left(\omega_{uc}t+\theta\right) \qquad \text{A-24}$$

$$+ 2J_1(m_1)J_o(m_2)\sin\omega_1 t\cos\left(\omega_{uc}t+\theta\right) + 2J_o(m_1)J_1(m_2)\sin\omega_2 t\cos(\omega_{uc}t+\theta)\Big]$$

The expression for the output of the coherent carrier product detector is obtained by multiplying equation A-24 by $B_u\cos\omega_c t$ as shown in Figure I. Neglecting the sum frequencies which are removed by filtering, the signal amplitude at the output of this detector is described by

$$S_{ud}(t) = \sqrt{2}\, L_{us} \frac{B_u}{2}\left[\frac{(S/N)_{us}}{\left(1/k_{us}\right) + (S/N)_{us}}\right]^{1/2} \Big[J_o(m_1)J_o(m_2)\sin\theta \qquad \text{A-25}$$

$$+ 2J_1(m_1)J_o(m_2)\cos\theta\sin\omega_1 t + 2J_o(m_1)J_1(m_2)\cos\theta\sin\omega_2 t\Big]$$

The signal powers of the up-subcarrier components at the output of the coherent carrier product detector then are described by

$$P_s(\omega_1) = L_{us}^2\, B_u^2 \left[\frac{(S/N)_{us}}{\left(1/k_{us}\right) + (S/N)_{us}}\right] J_1^2(m_1)J_o^2(m_2)\cos^2\theta \qquad \text{A-26}$$

and

$$P_s(\omega_2) = L_{us}^2\, B_u^2 \left[\frac{(S/N)_{us}}{\left(1/k_{us}\right) + (S/N)_{us}}\right] J_o^2(m_1)J_1^2(m_2)\cos^2\theta \qquad \text{A-27}$$

where $(S/N)_{us}$ is that defined by A-23.

The second step is to describe the noise power which is added to the subcarriers by the transponder. Assuming that the noise power spectral density is a constant over the transponder video channel input band width "B_{ts}", then the noise power in the subcarrier detector bandwidths "$B(\omega_1)$", "$B(\omega_2)$" respectively (following the product detector) is obtained from Equation A-6

$$P_n(\omega_1) = \left[\frac{B_u}{2} L_{us}\left(\frac{1}{1+k_{us}\left(\frac{S}{N}\right)_{us}}\right)^{1/2}\right]^2 \frac{B(\omega_1)}{\frac{B_{ts}}{2}} \qquad \text{A-28}$$

$$P_n(\omega_2) = \left[\frac{B_u}{2} L_{us}\left(\frac{1}{1+k_{us}\left(\frac{S}{N}\right)_{us}}\right)^{1/2}\right]^2 \frac{B(\omega_2)}{\frac{B_{ts}}{2}} \qquad \text{A-29}$$

in the low pass one-sided bandwidth of $\frac{B_{ts}}{2}$.

The third step is to describe the interference to the subcarrier signal components which is due to the code modulation "θ". The expression for the total interference signal is an infinite series which includes all of the higher order and cross frequency terms of equation A-21) that are multiplied by $\sin(w_c t + \theta)$. It can however be shown that for the Apollo S-band modes only the first term of this series, the term contained in equation A-22, is significant in magnitude to consider in this analysis.

Then, using equations 22 and 25, the total first order code power which exists at the output of the coherent carrier product detector is given by

$$I_{uco}(T) = \left[\frac{B_u}{2}\sqrt{2}\, L_{us}\left(\frac{(S/N)_{us}}{\frac{1}{k_{us}} + (S/N)_{us}}\right)^{1/2}\right]^2 J_o^2(m_1) J_o^2(m_2)\sin^2\theta \qquad \text{A-30}$$

where $(S/N)_{us}$ is that defined by A-23.

It has been shown by Hill[2] that by approximating the bound of the code power with a smooth $\frac{\sin^2 x}{x^2}$ curve. Equation A-30 can be expressed by

$$I_{uco}(T) = \int_o^\infty M \frac{\sin^2 x}{x^2}\, dx = M \frac{\pi}{2} \qquad \text{A-31}$$

then

$$M = I_{uco}(T)\ \frac{2}{\pi} \qquad \text{A-32}$$

where

$x \quad = \quad K_c f_i$ (a)

$dx \quad = \quad K_c df_i = K_c B(\omega_i)$ (b)

$K_c \quad = \quad \frac{\pi}{10^6}$ for a 500K H_Z code clock frequency (c)

$B(\omega_i) \quad =$ The up-subcarrier detector bandwidth (d)

The portion of the code power "$I_{uco}(\omega_i)$" which lies in an up-subcarrier detector bandwidth "$B(\omega_i)$" can then be approximated by using A-30, A-31, and A-32

$$I_{uco}(\omega_i) = M \frac{\sin^2 K_c f_i}{\left(K_c f_i\right)^2} dx$$

$$= \left(\frac{2}{\pi}\right)\left(\frac{B_u^2 L_{us}^2}{2}\right)\left(\frac{(S/N)_{us}}{\frac{1}{k_{us}} + (S/N)_{us}}\right) J_o^2(m_1) J_o^2(m_2) \sin^2\theta\ \frac{\sin^2 K_c f_i}{\left(K_c f_i\right)^2} K_c B(\omega_i) \qquad \text{A-33}$$

The predicted signal to noise ratio for the "ω_1" up subcarrier is obtained using A-26, A-28 and A-33

$$(S/N)_{u\omega_1 p} = \frac{L_{us}^2 B_u^2 \left[\frac{(S/N)_{us}}{1/K_{us} + (S/N)_{us}}\right] J_1^2(m_1) J_o^2(m_2) \cos^2\theta}{\left\{\frac{L_{us}^2 B_u^2}{2}\left[\frac{1}{1 + k_{us}\frac{S}{N}_{us}}\right]\frac{B(\omega_1)}{B_{ts}} + \frac{B_u^2 L_{us}^2}{10^6}\left[\frac{(S/N)_{us}}{\frac{1}{k_{us}} + (S/N)_{us}}\right] J_o^2(m_1) J_o^2(m_2) \sin^2\theta \cdot \frac{\sin^2 K_c f_1}{\left(K_c f_1\right)^2} B(\omega_1)\right\}} \qquad \text{A-34}$$

where

$$(S/N)_{us} = \frac{A_u^2}{2} \Big/ K_o B_{ts} T_u$$

$$(S/N)_{u\omega_1 p} = \frac{k_{us}\frac{A_u^2}{2} J_1^2(m_1)J_o^2(m_2)\cos^2\theta}{B(\omega_1)\left[\frac{K_oT_u}{2} + \frac{k_{us}}{10^6}\ \frac{A_u^2}{2} J_o^2(m_1)J_o^2(m_2)\sin^2\theta\ \frac{\sin^2 K_c f_1}{\left|K_c f_1\right|^2}\right]} \qquad \text{A-35}$$

If the required signal to noise ratio in the "ω_i" up-subcarrier predetection bandwidth "$B(\omega_i)$" is given in dB by $\left(\frac{S}{N}\right)_{u\omega_i r}$, then the expressions for the circuit margins of the up-subcarriers are as follows:

for ω_1 A-36

$$M_{u\omega_1} = 10\ \log_{10}\left[\frac{k_{us}\frac{A_u^2}{2} J_1^2(m_1)J_o^2(m_2)\cos^2\theta}{B(\omega_1)\left[\frac{K_oT_u}{2} + \frac{k_{us}}{10^6}\ \frac{A_u^2}{2} J_o^2(m_1)J_o^2(m_2)\sin^2\theta\ \frac{\sin^2 K_c f_1}{\left|K_c f_1\right|^2}\right]}\right] - \left(\frac{S}{N}\right)_{u\omega_1 r}$$

and for ω_2 A-37

$$M_{u\omega_2} = 10\ \log_{10}\left[\frac{k_{us}\frac{A_u^2}{2} J_o^2(m_1)J_1^2(m_2)\cos^2\theta}{B(\omega_2)\left[\frac{K_oT_u}{2} + \frac{k_{us}}{10^6}\ \frac{A_u^2}{2} J_o^2(m_1)J_o^2(m_2)\sin^2\theta\ \frac{\sin^2 K_c f_2}{\left|K_c f_2\right|^2}\right]}\right] - \left(\frac{S}{N}\right)_{u\omega_2 r}$$

References for Appendix A

1. W. D. Wynn, "Spectral Analysis of a Communications Relay System Using an Apollo Unified S-Band Transponder," TM 67-2034-4, June 30, 1967.

2. J. D. Hill, "Design Philosophy of Modulation Indices for Apollo Unified S-Band Modes with Ranging," Bellcomm TM-65-2021-3 March 11, 1965.

APPENDIX B

CIRCUIT MARGINS FOR USB DOWNLINK MODES

PURPOSE

The purpose of this appendix is to derive expressions which describe the circuit margins of the USB downlink PM modes.

The expressions for the circuit margins which are presented have been derived by direct subtraction (in dB) of the signal to noise ratio which is required "$(S/N)_r$" for a given service (Carrier, Range Code, and Down Subcarriers) from the signal to noise ratio which is predicted "$(S/N)_p$" using the given circuit parameters. The configuration of the USB downlink system used for this derivation is that used by Wynn[1] and is shown in Figure II.

The downlink signal and noise that is received by the ground receiver is assumed to be described by

$$S_d(t) + n_d = A_d \sin\left[\omega_{dc}t + R_g \sin\left(m_1 \sin\omega_1 t + m_2 \sin\omega_2 t + \theta\right) + m_3 \sin\omega_3 t + m_4 \sin\omega_4 t + n_r\right] + n_d \quad \text{B-1}$$

where

B-2

$\frac{A_d^2}{2}$ = Total received signal power at the ground receiver

ω_{dc} = Down carrier frequency in radians/second

ω_1, ω_2 = The frequencies in radians/second of the up subcarriers which are retransmitted on the downlink carrier

ω_3, ω_4 = Down subcarrier frequencies in radians/second

m_1, m_2 = Up subcarrier peak modulation indexes in radians

m_3, m_4 = Down subcarrier peak modulation indexes in radians

θ = Pseudorandom range code modulation index in radians

n_r = The thermal noise component that is received by the transponder on the uplink and retransmitted on the downlink

$K_o T_d$ = Power spectral density of the input thermal noise n_d

K_o = Boltzman's Constant = 1.38×10^{-23} watts/°K H_Z

T_d = Noise temperature referred to the input of the ground receiver in °Kelvin

R_g = $\frac{\sqrt{2}}{2} L_{us} B_u G \left[\frac{(S/N)_{us}}{\frac{1}{k_{us}} + \left(\frac{S}{N}\right)_{us}}\right]^{1/2}$ = Gain of the turn around channel in the transponder (See appendix A, equation A-25)

L_{us}^2 = Power output of the limiter in the transponder video channel

$\frac{B_u}{2}$ = Gain of the transponder coherent carrier product detector

G = Combined gain of the transponder I F and modulator

$\sqrt{2}\, L_{us} \left[\frac{(S/N)_{us}}{\frac{1}{k_{us}} + \left(\frac{S}{N}\right)_{us}}\right]^{1/2}$ = Signal amplitude attenuation factor for the limiter in the transponder video channel

$(S/N)_{us} = \frac{A_u^2}{2} \Big/ K_o T_u B_{ts}$ = Signal to noise ratio at the input to the limiter in the transponder video channel

B_{ts} = Input bandwidth of the transponder video channel

T_u = Noise temperature referred to transponder input in degrees Kelvin

Then in general the signal output of a hard limiter in the ground receiver is defined by

$$S_{dL}(t) + n_{dL} = A_{dsL}\sin\left[\omega_{dc}t + R_g\sin\left(m_1\sin\omega_1 t + m_2\sin\omega_2 t + \theta\right) + m_3\sin\omega_3 t + m_4\sin\omega_4 t + n_r\right] + A_{dnL}\, n_d \qquad \text{B-3}$$

The coefficients A_{dsL} and A_{dnL} are limiter attenuation factors that are discussed in Appendix D. Using the results of Appendix D the expressions for the signal and noise components at the output of a limiter in the ground receiver can be written directly. The signal amplitude of the limiter output is given by

$$S_{dL}(t) = \sqrt{2}\, L_d \left[\frac{(S/N)_d}{\frac{1}{k} + (S/N)_d}\right]^{1/2} \sin\left[\omega_{dc}t + R_g\sin\left(m_1\sin\omega_1 t + m_2\sin\omega_2 t + \theta\right) + m_3\sin\omega_3 t + m_4\sin\omega_4 t + n_r\right] \qquad \text{B-4}$$

where

a) k is a function of the signal to noise power ratio at the input to the limiter $(S/N)_d$

b) L_d^2 is the total power output of the limiter

The signal power of the limiter output is given by

$$P_s(L) = L_d^2 \left[\frac{(S/N)_d}{\frac{1}{k} + (S/N)_d}\right] \qquad \text{B-5}$$

The noise power spectral density at the limiter output is given by

$$p_n(L) = L_d^2 \left[\frac{1}{1 + k(S/N)_d}\right] \frac{1}{B_g} \text{ watts/H}_Z \qquad \text{B-6}$$

The signal to noise ratio used in the above expressions is defined by

$$\left(\frac{S}{N}\right)_d = \frac{\frac{A_{dL}^2}{2}}{K_o T_d B_g} \qquad \text{B-7}$$

where $\frac{A_{dL}^2}{2}$ is the total signal power that is contained in the limiter input bandwidth B_g.

DOWN-CARRIER CIRCUIT MARGIN

The first step in this derivation is to describe the signal power of the down-carrier component at the input to the limiter in the ground receiver carrier channel. Expanding the signal term of B-1 gives the expression for the total signal received by the ground receiver as

$$S_d(t) = A_d I_m \left[e^{j\left[\omega_{dc} t + R_g \sin\omega_x t + m_3 \sin\omega_3 t + m_4 \sin\omega_4 t + n_r\right]} \right] \qquad \text{B-8}$$

where

$$\omega_x t = m_1 \sin\omega_1 t + m_2 \sin\omega_2 t + \theta \qquad \text{B-9}$$

$$S_d(t) = A_d I_m \left[e^{j\left(\omega_{dc} t + n_r\right)} \sum_{N=-\infty}^{\infty} J_N(R_g) e^{jN\omega_x t} \sum_{N_3=-\infty}^{\infty} J_{N_3}(m_3) e^{jN_3\omega_3 t} \cdot \right. \qquad \text{B-10}$$

$$\left. \sum_{N_4=-\infty}^{\infty} J_{N_4}(m_4) e^{jN_4\omega_4 t} \right]$$

substituting B-9 into B-10

$$S_d(t) = A_d I_m \left\{ e^{j\left(\omega_{dc} t + n_r\right)} \sum_{N=-\infty}^{\infty} \left[J_N(R_g) e^{jN\theta} \sum_{N_1=-\infty}^{\infty} J_{N_1}(Nm_1) e^{jN_1\omega_1 t} \right. \right. \cdot \qquad \text{B-11}$$

$$\left. \left. \sum_{N_2=-\infty}^{\infty} J_{N_2}(Nm_2) e^{jN_2\omega_2 t} \right] \sum_{N_3=-\infty}^{\infty} J_{N_3}(m_3) e^{jN_3\omega_3 t} \sum_{N_4=-\infty}^{\infty} J_{N_4}(m_4) e^{jN_4\omega_4 t} \right\}$$

The carrier component is that portion of the signal described by B-11 which will be passed by a narrow band pass filter centered about ω_{dc}; the expression for this component is obtained by letting

$$N_1 = N_2 = N_3 = N_4 = 0$$

Equation B-11 then reduces to

$$S_{dc}(t) = A_d \sum_{N=-\infty}^{\infty} \Big\{ J_N(R_g)\; J_o(Nm_1) J_o(Nm_2) J_o\;(m_3) J_o\;(m_4) \cdot \Big[\sin\big(\omega_{dc}t + n_r\big)\cos N\theta + \cos\big(\omega_{dc}t + n_r\big)\sin N\theta \Big] \Big\} \qquad \text{B-12}$$

The first term of Equation B-12 describes the down carrier signal component and the second term describes an interference term which is caused by the code modulation "θ"; the first term is expanded as follows:

$$S_{dcs}(t) = A_d \sum_{N=-\infty}^{\infty} \Big\{ J_N(R_g) J_o(Nm_1) J_o(Nm_2) J_o(m_3) J_o(m_4) \cos N\theta \cdot \Big[\sin(\omega_{dc}t)\cos(n_r) + \cos(\omega_{dc}t)\sin(n_r) \Big] \Big\} \qquad \text{B-13}$$

It is seen that Equation B-13 is of the same form as B-12. The first term of B-13 describes the down carrier signal component and the second term describes an interference signal which is caused by the retransmitted noise "n_r"; the signal power at the input to the limiter in the ground receiver carrier channel is obtained from B-13 and is

$$P_{si}(dc) = \frac{A_d^2}{2} \left[\sum_{N=-\infty}^{\infty} \Big\{ J_N(R_g) J_o(Nm_1) J_o(Nm_2) J_o(m_3) J_o(m_4) \cos N\theta \Big\} \right]^2 \overline{\cos^2 n_r} \qquad \text{B-14}$$

using the following identities

$$J_{-x}(m) = (-1)^x J_x(m)$$
$$J_x(-m) = (-1)^x J_x(m)$$
$$\cos(\pm x) = \cos(x)$$
$$J_o(o) = 1 \qquad \text{B-15}$$

Equation B-14 can be expressed by

$$P_{si}(dc) = \frac{A_d^2}{2} J_o^2(m_3)J_o^2(m_4)\overline{\cos^2 n_r}\left[J_o(R_g) + 2\sum_{N=1}^{\infty} J_{2N}(R_g)J_o(2Nm_1)J_o(2Nm_2)\cdot \cos 2N\theta\right]^2 \qquad \text{B-16}$$

For the Apollo cases the series terms can be neglected, B-16 then reduces to (See equation E-31 of appendix E)

$$P_{si}(dc) = \frac{A_d^2}{2} J_o^2(m_3)J_o^2(m_4)J_o^2(R_g)\overline{\cos^2 n_r} \qquad \text{B-17}$$

A mathematically rigorous analysis of the effect that the retransmitted noise has on the downlink signal has been presented by Wynn[1]; however, the less rigorous analysis presented by Hill[2] still is considered a reasonable approximation for the Apollo modes. It is the less rigorous treatment which will be used in this analysis although work is continuing to determine in detail the differences between the results gained from this and Wynn's work. By approximating the thermal noise power which exists in the transponder subcarrier channel input bandwidth "B_{ts}" with the power obtained from a single sinusoid whose amplitude "N_u" and frequency "ω_{nu}" are defined by (See appendix A for further definition of the terms used below):

$$N_u = \frac{\sqrt{2}}{2} L_{us}B_uG\left[\frac{1}{1+k_{us}\frac{S}{N}_{us}}\right]^{1/2}\left[\frac{B_v}{\frac{B_{ts}}{2}}\right]^{1/2} \qquad \text{B-18}$$

$$\omega_c - \frac{B_{ts}}{2} < \omega_{nu} < \omega_c + \frac{B_{ts}}{2}$$

Where B_v is the transponder video bandwidth, the retransmitted noise "n_r" can be considered as an additional subcarrier on the downlink. Using this approximation, the term $\cos(n_r)$ can be expanded as follows:

$$n_r = N_u \sin\omega_{nu} t \tag{B-19}$$

$$\cos(n_r) = \cos\left(N_u \sin\omega_{nu} t\right) \tag{B-20}$$

$$= R_e e^{jN_u \sin\omega_{nu} t}$$

$$= J_o(N_u) + 2\sum_{N_5=1}^{\infty} J_{2N_5}(N_u)\cos 2N_5\omega_{nu} t \tag{B-21}$$

For the Apollo cases the series terms can be neglected

$$\overline{\cos^2 n_r} \cong J_o^2(N_u) \tag{B-22}$$

The signal power at the input to the limiter in the ground receiver carrier channel is

$$P_{si}(dc) = \frac{A_d^2}{2} J_o^2(m_3) J_o^2(m_4) J_o^2(R_g) J_o^2(N_u) \tag{B-23}$$

The second step is to describe the noise power which is added to the down carrier signal by the ground receiver. Assuming that the noise power spectral density is a constant over the ground receiver carrier channel input bandwidth "B_{gc}" the noise power at the input to the limiter in the carrier channel is given by

$$K_o T_d B_{gc} \tag{B-24}$$

Having derived the expressions for the signal and noise components in the carrier channel limiter input, the expressions for these components in the output of this limiter can be written directly.

a) The input signal to noise ratio is obtained from B-23 and B-24

$$\left(\frac{S}{N}\right)_{dc} = \frac{A_d^2}{2} \frac{J_o^2(m_3)J_o^2(m_4)J_o^2(R_g)J_o^2(N_u)}{K_o T_d B_{gc}} \qquad \text{B-25}$$

b) The output signal power is obtained from B-5

$$P_s(dc) = L_{dc}^2 \left[\frac{(S/N)_{dc}}{\frac{1}{k_{dc}} + \left(\frac{S}{N}\right)_{dc}}\right] \qquad \text{B-26}$$

c) The output noise power in the down carrier detector bandwidth "B_{dc}" is obtained from B-6

$$P_n(dc) = L_{dc}^2 \left[\frac{1}{1+k_{dc}\left(\frac{S}{N}\right)_{dc}}\right] \frac{B_{dc}}{B_{gc}} \qquad \text{B-27}$$

If the interference to the carrier caused by the code modulation θ and the retransmitted noise can be neglected, then the predicted signal to noise ratio for the down carrier "$\left(\frac{S}{N}\right)_{dcp}$" is obtained from B-26 and B-27

$$\left(\frac{S}{N}\right)_{dcp} = \frac{L_{dc}^2\left[\frac{(S/N)_{dc}}{(1/k_{dc}) + (S/N)_{dc}}\right]}{L_{dc}^2\left[\frac{1}{1+k_{dc}\left(\frac{S}{N}\right)_{dc}}\right]\frac{B_{dc}}{B_{gc}}} = k_{dc}\left(\frac{S}{N}\right)_{dc} \frac{B_{gc}}{B_{dc}} \qquad \text{B-28}$$

where

$$\left(\frac{S}{N}\right)_{dc} = \frac{A_d^2}{2} \frac{J_o^2(m_3)J_o^2(m_4)J_o^2(R_g)J_o^2(N_u)}{K_o T_d B_{gc}}$$

$$\left(\frac{S}{N}\right)_{dcp} = k_{dc}\,\frac{A_d^2}{2}\,\frac{J_o^2(m_3)J_o^2(m_4)J_o^2(R_g)J_o^2(N_u)}{K_oT_dB_{dc}}$$

If the required signal to noise ratio for the down carrier in the down carrier detector bandwidth is given in dB by $\left(\frac{S}{N}\right)_{dcr}$, then the down carrier margin is described by

$$M_{dc} = 10\log_{10}\left[k_{dc}\,\frac{A_d^2}{2}\,\frac{J_o^2(m_3)J_o^2(m_4)J_o^2(R_g)J_o^2(N_u)}{K_oT_dB_{dc}}\right] - \left(\frac{S}{N}\right)_{dcr} \qquad \text{B-29}$$

DOWN-SUBCARRIER CIRCUIT MARGINS

The first step in this derivation is to describe the signal powers of the down subcarrier components at the output of the coherent carrier product detector in the ground receiver subcarrier channel. Equation B-11 describes the total signal that is received by the ground receiver.

$$S_d(t) = A_dI_m\left\{e^{j\left(\omega_{dc}t+n_r\right)}\sum_{N=-\infty}^{\infty}\left[J_N(R_g)e^{jN\theta}\sum_{N_1=-\infty}^{\infty}J_{N_1}(Nm_1)e^{jN_1\omega_1t}\right.\right. \cdot \qquad \text{B-11}$$

$$\left.\left.\sum_{N_2=-\infty}^{\infty}J_{N_2}(Nm_2)e^{jN_2\omega_2t}\right]\sum_{N_3=-\infty}^{\infty}J_{N_3}(m_3)e^{jN_3\omega_3t}\sum_{N_4=-\infty}^{\infty}J_{N_4}(m_4)e^{jN_4\omega_4t}\right\}$$

$$= A_dI_m\left\{\sum_{N=-\infty}^{\infty}\left[J_N(R_g)\sum_{N_1=-\infty}^{\infty}J_{N_1}(Nm_1)\sum_{N_2=-\infty}^{\infty}J_{N_2}(Nm_2)e^{j\left[\omega_{dc}t+n_r+N\theta+N_1\omega_1t+N_2\omega_2t\right]}\right]\right. \cdot \qquad \text{B-30}$$

$$\left(J_o(m_3) + 2\sum_{N_3=1}^{\infty}\left[J_{2N_3}(m_3)\cos 2N_3\omega_3t + jJ_{(2N_3-1)}(m_3)\sin(2N_3-1)\omega_3t\right]\right)\cdot$$

$$\left.\left(J_o(m_4) + 2\sum_{N_4=1}^{\infty}\left[J_{2N_4}(m_4)\cos 2N_4\omega_4t + jJ_{(2N_4-1)}(m_4)\sin(2N_4-1)\omega_4t\right]\right)\right\}$$

Neglecting the higher order, the cross frequency, and the terms resulting from the up-subcarriers "ω_1", and "ω_2" (let $N_1=N_2=0$ which are all removed by filtering in the down subcarrier detectors, Equation B-30 reduces to

$$S_{ds}(t) = A_d \sum_{N=-\infty}^{\infty}\left(J_N(R_g)J_o(Nm_1)J_o(Nm_2)\right)\Big[J_o(m_3)J_o(m_4)\sin\left(\omega_{dc}t+n_r+N\theta\right)$$

$$+ 2J_1(m_3)J_o(m_4)\cos\left(\omega_{dc}t+n_r+N\theta\right)\sin\omega_3 t \qquad \text{B-31}$$

$$+ 2J_o(m_3)J_1(m_4)\cos\left(\omega_{dc}t+n_r+N\theta\right)\sin\omega_4 t\Big]$$

The first term of Equation B-31 describes an interference signal which is caused by the code modulation "θ"; the second and third terms describe the signal components of the "ω_3" and "ω_4" subcarriers respectively. The limiter in the ground receiver subcarrier channel causes an attenuation of the signal described by B-31; this attenuation is a function of the signal to noise "$\left(\frac{S}{N}\right)_{ds}$" ratio at the input to the limiter, where

$$\left(\frac{S}{N}\right)_{ds} = \frac{\frac{A_d^2}{2}}{K_o T_d B_{gs}} \qquad \text{B-32}$$

B_{gs} is the bandwidth of the bandpass filter preceeding the limiter.

Using Equations B-4, B-31, and B-32 the signal amplitude at the output of the limiter is described by

$$S_{dsL}(t) = \sqrt{2}\, L_{ds}\left[\frac{(S/N)_{ds}}{\frac{1}{k_{ds}} + \left(\frac{S}{N}\right)_{ds}}\right]^{1/2}\left\{\left[\sum_{N=-\infty}^{\infty} J_N(R_g)J_o(Nm_1)J_o(Nm_2)\right]\cdot \right. \qquad \text{B-33}$$

$$\Big[J_o(m_3)J_o(m_4)\sin\left(\omega_{dc}t+n_r+N\theta\right) + 2J_1(m_3)J_o(m_4)\sin\omega_3 t\cos\left(\omega_{dc}t+n_r+N\theta\right)$$

$$\left. + 2J_o(m_3)J_1(m_4)\sin\omega_4 t\cos\left(\omega_{dc}t+n_r+N\theta\right)\Big]\right\}$$

The expression for the output of the coherent carrier product detector is obtained by multiplying equation B-33 by $B_d \cos \omega_{dc} t$ as shown in Figure II. Neglecting the sum frequencies which are removed by filtering, the signal amplitude at the output of this detector is described by

$$S_{dd}(t) = \sqrt{2}\, L_{ds} \frac{B_d}{2}\left[\frac{(S/N)_{ds}}{1/k_{ds}+(S/N)_{ds}}\right]^{1/2} \left[\sum_{N=-\infty}^{\infty} J_N(R_g) J_o(Nm_1) J_o(Nm_2)\right] \cdot$$

$$\Big[J_o(m_3)J_o(m_4)\sin(n_r+N\theta) + 2J_1(m_3)J_o(m_4)\sin\omega_3 t\cos(n_r+N\theta) \qquad \text{B-34}$$

$$+ 2J_o(m_3)J_1(m_4)\sin\omega_4 t\cos(n_r+N\theta)\Big]$$

Expanding the second term of Equation B-34 gives the following result

$$S\omega_3(t) = \sqrt{2}\, L_{ds} B_d\left[\frac{(S/N)_{ds}}{1/k_{ds}+(S/N)_{ds}}\right]^{1/2} \sum_{N=-\infty}^{\infty} J_N(R_g) J_o(Nm_1) J_o(Nm_2) J_1(m_3) J_o(m_4) \cdot$$

$$\sin\omega_3 t \left[\cos(n_r)\cos(N\theta) - \sin(n_r)\sin(N\theta)\right] \qquad \text{B-35}$$

The first term of B-35 describes the signal component of the "ω_3" down subcarrier and the second term is an interference term which is caused by the retransmitted noise "n_r" and the code modulation "θ". The signal power of the "ω_3" down subcarrier at the output of the product detector is given by

$$P_s(\omega_3) = L_{ds}^2 B_d^2\left[\frac{(S/N)_{ds}}{1/k_{ds}+(S/N)_{ds}}\right] J_1^2(m_3) J_o^2(m_4) \left[\sum_{N=-\infty}^{\infty} J_N(R_g)\, J_o(Nm_1) J_o(Nm_2) \cdot \cos N\theta\right]^2 \overline{\cos^2 n_r} \qquad \text{B-36}$$

Using B-15 and B-22, B-36 reduces to

$$P_s(\omega_3) = L_{ds}^2 B_d^2\left[\frac{(S/N)_{ds}}{1/k_{ds}+(S/N)_{ds}}\right] J_1^2(m_3) J_o^2(m_4) J_o^2(R_g) J_o^2(N_u) \qquad \text{B-37}$$

and similarly for the "ω_4" down subcarrier

$$P_s(\omega_4) = L_{ds}^2 B_d^2 \left[\frac{(S/N)_{ds}}{[1/k_{ds}] + (S/N)_{ds}} \right] J_o^2(m_3) J_1^2(m_4) J_o^2(R_g) J_o^2(N_u) \qquad \text{B-38}$$

where $\left(\frac{S}{N}\right)_{ds}$ is that defined by B-32

The second step is to describe the noise power which is added to the subcarriers by the ground receiver. Assuming that the noise power spectral density is a constant over the ground receiver subcarrier channel input bandwidth "B_{gs}" then the noise power in the subcarrier detector bandwidths "$B(\omega_3)$", "$B(\omega_4)$" respectively (following the product detector) is obtained from Equation B-6

$$P_n(\omega_3) = \left[\frac{B_d}{2} L_{ds} \left[\frac{1}{1+k_{ds}(S/N)_{ds}} \right]^{1/2} \right]^2 \frac{B(\omega_3)}{\frac{B_{gs}}{2}} \qquad \text{B-39}$$

and

$$P_n(\omega_4) = \left[\frac{B_d}{2} L_{ds} \left[\frac{1}{1+k_{ds}(S/N)_{ds}} \right]^{1/2} \right]^2 \frac{B(\omega_4)}{\frac{B_{gs}}{2}} \qquad \text{B-40}$$

in the low pass one sided bandwidth of $\frac{B_{gs}}{2}$

Assuming that the interference from the code modulation "θ" is negligible, then the second term of Equation B-35 can be neglected and the first term of Equation B-34 reduces to describe the interference which is caused by the retransmitted noise in the subcarrier detector bandwidth "$B(\omega_i)$" B-41

$$I_{rn}(\omega_i) = \frac{L_{ds}^2 B_d^2}{2} \left[\frac{(S/N)_{ds}}{[1/k_{ds}] + (S/N)_{ds}} \right] J_o^2(m_3) J_o^2(m_4) \left[\sum_{N=-\infty}^{\infty} J_N(R_g) J_o(Nm_1) J_o(Nm_2) \cdot \cos N\theta \right]^2 \overline{\sin^2 n_r} \; \frac{B(\omega_i)}{B_v}$$

where B_v is the video bandwidth of the transponder. Using the definition of B-19 for the retransmitted noise

$$n_r = N_u \sin\omega_{nu} t \qquad \text{B-19}$$

then

$$\sin(n_r) = \sin N_u \sin\omega_{nu} t \qquad \text{B-42}$$

$$\sin(n_r) = I_m e^{jN_u \sin\omega_{nu} t}$$

$$= 2 \sum_{N_6=1}^{\infty} J_{(2N_6-1)}(N_u)\sin(2N_6-1)\omega_{nu} t \qquad \text{B-43}$$

For the Apollo cases the higher order terms can be neglected

$$\overline{\sin^2 n_r} \cong 2J_1^2(N_u) \qquad \text{B-44}$$

Now using B-15 and B-44 Equation B-41 reduces to

$$I_{rn}(\omega_i) = L_{ds}^2 B_d^2 \left[\frac{(S/N)_{ds}}{\frac{1}{k_{ds}} + \left(\frac{S}{N}\right)_{ds}}\right] J_o^2(m_3)J_o^2(m_4)J_o^2(R_g)J_1^2(N_u)\,\frac{B(\omega_i)}{B_v} \qquad \text{B-45}$$

The predicted signal to noise ratio for the "ω_3" down subcarrier is obtained from Equations B-37, B-39 and B-45 to be

$$\left(S/N\right)_{d\omega_3 P} = \frac{L_{ds}^2 B_d^2 \left[\frac{(S/N)_{ds}}{1/k_{ds} + (S/N)_{ds}}\right] J_1^2(m_3)J_o^2(m_4)J_o^2(R_g)J_o^2(N_u)}{\left\{\frac{L_{ds}^2 B_d^2}{2}\left[\frac{1}{1+k_{ds}\left(\frac{S}{N}\right)_{ds}}\right]\frac{B(\omega_3)}{B_{gs}} + L_{ds}^2 B_d^2 \left[\frac{(S/N)_{ds}}{\frac{1}{k_{ds}} + \left(\frac{S}{N}\right)_{ds}}\right] \cdot J_o^2(m_3)J_o^2(m_4)J_o^2(R_g)J_1^2(N_u)\,\frac{B(\omega_3)}{B_v}\right\}} \qquad \text{B-46}$$

where

$$\left(\frac{S}{N}\right)_{ds} = \frac{\frac{A_d^2}{2}}{K_o T_d B_{gs}}$$

$$\left(S/N\right)_{d\omega_3 P} = \frac{k_{ds}\frac{A_d^2}{2} J_1^2(m_3)J_o^2(m_4)J_o^2(R_g)J_o^2(N_u)}{B(\omega_3)\left[\frac{K_o T_d}{2} + k_{ds}\frac{A_d^2}{2} J_o^2(m_3)J_o^2(m_4)\frac{J_o^2(R_g)J_1^2(N_u)}{B_v}\right]} \quad \text{B-47}$$

If the required signal to noise ratio in the "ω_3" down subcarrier predetection bandwidth "$B(\omega_3)$" is given in dB by $\left(\frac{S}{N}\right)_{d\omega_3 r}$ then the expression for the circuit margin of the "ω_3" subcarrier is given by

$$M d\omega_3 = 10 \log_{10}\left\{\frac{k_{ds}\frac{A_d^2}{2} J_1^2(m_3)J_o^2(m_4)J_o^2(R_g)J_o^2(N_u)}{B(\omega_3)\left[\frac{K_o T_d}{2} + k_{ds}\frac{A_d^2}{2} J_o^2(m_3)J_o^2(m_4)\frac{J_o^2(R_g)J_1^2(N_u)}{B_v}\right]}\right\} - \left(\frac{S}{N}\right)_{d\omega_3 r} \quad \text{B-48}$$

and similarly for ω_4

$$M d\omega_4 = 10 \log_{10}\left\{\frac{k_{ds}\frac{A_d^2}{2} J_o^2(m_3)J_1^2(m_4)J_o^2(R_g)J_o^2(N_u)}{B(\omega_4)\left[\frac{K_o T_d}{2} + k_{ds}\frac{A_d^2}{2} J_o^2(m_3)J_o^2(m_4)\frac{J_o^2(R_g)J_1^2(N_u)}{B_v}\right]}\right\} - \left(\frac{S}{N}\right)_{d\omega_4 r} \quad \text{B-49}$$

DOWN CODE MARGIN

The first step in this derivation is to describe the signal power of the code at the input to the ranging receiver. The total code signal can be described only as an infinite series which includes all of the higher order and cross frequency terms that are generated from the expansion of equation B-11. It can however, be shown that for the Apollo system only the first term of this expansion is significant. The expression for this code signal which is considered in this first order approximation is obtained as follows:

Let $N_1 = N_2 = N_3 = N_4 = 0$

(Thus equation B-11 reduces to equation B-12). Rewriting equation 12 for convenience

$$S_{dc(t)} = A_d \sum_{N=-\infty}^{\infty} \Big\{ J_N(R_g) J_o(Nm_1) J_o(Nm_2) J_o(m_3) J_o(m_4) \cdot \Big[\sin(\omega_{dc}t + n_r)\cos N\theta + \cos(\omega_{dc}t + n_r)\sin N\theta\Big]\Big\} \qquad \text{B-12}$$

The first term of equation B-12, as was previously stated, is defined as the carrier signal component of the downlink and the second term is the first order approximation of the code signal at the input to the ranging receiver. Expanding the second term of the above expression gives

$$S_{dco(t)} = A_d\, J_o(m_3) J_o(m_4) \sum_{N=-\infty}^{\infty} \Big\{ J_N(R_g) J_o(Nm_1) J_o(Nm_2) \sin N\theta \Big\} \cdot \Big[\cos(\omega_{dc}t)\cos(n_r) - \sin(\omega_{dc}t)\sin(n_r)\Big] \qquad \text{B-50}$$

The first term of Equation B-50 describes the signal component of the code "θ" and the second term is an interference signal caused by the retransmitted noise "n_r". The code signal power can then be described by

$$P_{s(dco)} = \frac{A_d^2}{2} J_o^2(m_3) J_o^2(m_4) \left[\sum_{N=-\infty}^{\infty} J_N(R_g) J_o(Nm_1) J_o(Nm_2) \sin N\theta \right]^2 \overline{\cos^2 n_r} \qquad \text{B-51}$$

using the following identities and Equation B-22

$$J_{-x}(m) = (-1)^x J_x(m)$$

$$J_x(-m) = (-1)^x J_x(m)$$

$$\sin(\pm x) = \pm\sin(x) \qquad \text{B-52}$$

$$J_o(0) = 1$$

$$\sin(0) = 0$$

Equation B-51 becomes

$$P_{s(dco)} = \frac{A_d^2}{2} J_o^2(m_3)J_o^2(m_4)\left[2 \sum_{N=1}^{\infty}\left\{J_{2N-1}(R_g)J_o\left[(2N-1)m_1\right]J_o\left[(2N-1)m_2\right]\cdot \sin(2N-1)\theta\right\}\right]^2 J_o^2(N_u) \qquad \text{B-53}$$

For the Apollo cases the higher order terms can be neglected, the down code signal power is then described by (See equation E-32 of Appendix E)

$$P_{s(dco)} = \frac{A_d^2}{2} J_o^2(m_3)J_o^2(m_4)\left(4J_1^2(R_g)J_o^2(m_2)\sin^2\theta\right) J_o^2(N_u)J_o^2(m_1) \qquad \text{B-54}$$

The second step in this derivation is to describe the thermal noise which is added by the ground receiver. Assume a detection bandwidth of B_{dco} and a receiver noise temperature of T_d, then this noise power is given by

$$P_{n(co)} = K_o T_d B_{dco} \qquad \text{B-55}$$

The third step in this derivation is to describe the interference power in the ranging receiver which is caused by the turned around noise (n_r). It should be noted that two interference components (one in each of the two terms of the equation) are expressed by equation B-12 which we shall define as the first and second order components respectively. In effect the first order term is the interference which results from the noise (n_r) alone modulating the carrier; the second order term is the interference which results from the cross product of the noise and code signal modulating the carrier. It can be shown that for the Apollo system only the first order interference power is significant in the ranging channel. The expression for this interference power is obtained as follows. Expanding the first term of equation 12 gives, (as was shown in equation B-13)

$$S_{dcs(t)} = A_d \sum_{N=-\infty}^{\infty} \left\{ J_N(R_g) J_o(Nm_1) J_o(Nm_2) J_o(m_3) J_o(m_4) \cos N\theta \cdot \left[\sin\omega_{dc} t \cos(n_r) + \cos\omega_{dc} t \sin(n_r) \right] \right\} \qquad \text{B-13}$$

The first term of equation B-13, as was previously stated, is defined as the carrier signal component of the downlink and the second term is the first order interference component in both the carrier and the ranging channels caused by the noise n_r. Using equations B-13, B-15, and B-44 the interference power in the ranging channel can be approximated as follows:

$$I_{rn(dco)} = \frac{A_d^2}{2} \left[J_o^2(R_g) \right] J_o^2(m_3) J_o^2(m_4) \left[2J_1^2(N_u) \right] \frac{B_{dco}}{B_v} \qquad \text{B-56}$$

where B_{dco} and B_v are the range code detection bandwidth and the transponder video bandwidth respectively.

The predicted signal to noise ratio $\left(\frac{S}{N}\right)_{dcop}$ for the down code then is

$$\left(\frac{S}{N}\right)_{dcop} = \frac{A_d^2}{2} \frac{4J_1^2(R_g)J_o^2(N_u)J_o^2(m_1)J_o^2(m_2)J_o^2(m_3)J_o^2(m_4)\sin^2\theta}{K_oT_d\ B_{dco} + 2\frac{A_d^2}{2}\ J_o^2(R_g)J_1^2(N_u)J_o^2(m_3)J_o^2(m_4)\ \frac{B_{dco}}{B_v}} \qquad \text{B-57}$$

If the required signal to noise ratio in the down code detection bandwidth "B_{dco}" is given in dB by $\left(\frac{S}{N}\right)_{dcor}$ then the expression for the down code circuit margin is given by

$$M_{dco} = 10\ \log_{10}\left[\frac{4\frac{A_d^2}{2}\ J_1^2(R_g)J_o^2(N_u)J_o^2(m_1)J_o^2(m_2)J_o^2(m_3)J_o^2(m_4)\sin^2\theta}{B_{dco}\left[K_oT_d + \frac{2}{B_v}\ \frac{A_d^2}{2}\ J_o^2(R_g)J_1^2(N_u)J_o^2(m_3)J_o^2(m_4)\right]}\right] - \left(\frac{S}{N}\right)_{dcor}$$

B-58

References for Appendix B

1. W. D. Wynn, "Spectral Analysis of a Communications Relay System Using an Apollo Unified S-Band Transponder," TM67-2034-4; June 30, 1967.

2. J. D. Hill, "Design Philosophy of Modulation Indices for Apollo Unified S-Band Modes with Ranging," Bellcomm TM-65-2021-3; March 11, 1965.

APPENDIX C

POWER MARGINS FOR USB PM MODES

PURPOSE

The purpose of this appendix is to use the equations for the circuit margins which have been derived in Appendix A and B to derive expressions which permit a direct indication concerning the magnitude by which the received signal power for a given service can safely be reduced - Power Margin.

The expression for the power margins which are presented have been derived by direct subtraction (in dB) of the power which is required for the circuit margin to equal zero "P_{MIN}" from the power which is predicted "P_p" using the given circuit parameters. In deriving the expressions for the minimum required power, the variation of the system noise temperature with the magnitude of the received carrier power and the variation in the limiter suppression factor with the signal to noise ratio at the limiter inputs were considered.

VARIATION OF SYSTEM NOISE TEMPERATURE

The system noise temperature referred to a receiver input is described by[1]

$$T_s = \delta + BP_c \qquad \text{C-1}$$

where

δ, B	=	Constants which are dependent on the system (i.e., LM, CSM, SIVB or MSFN)	a
P_c	=	The narrow band carrier power which is used for the AGC function	b

Using Equations A-11 and C-1 the system temperature of the uplink is described by

$$T_u = \delta_u + B_u \left(\frac{A_u^2}{2}\right) J_o^2(m_1) J_o^2(m_2) \cos^2\theta \qquad \text{C-2}$$

Let

$$\alpha_{uc} = J_o^2(m_1)J_o^2(m_2)\cos^2\theta \qquad a$$

$$P_{sr} = A_u^2/2 \qquad b$$

Then Equation C-2 reduces to

$$T_u = \delta_u + B_u P_{sr} \alpha_{uc} \qquad C-3$$

Using Equations B-23 and C-1 the system temperature of the down-link is described by

$$T_d = \delta_d + B_d \frac{A_d^2}{2} J_o^2(m_3)J_o^2(m_4)J_o^2(R_g)J_o^2(N_u) \qquad C-4$$

Let

$$\alpha_{dc} = J_o^2(m_3)J_o^2(m_4)J_o^2(R_g)J_o^2(N_u) \qquad a$$

$$P_{gr} = \frac{A_d^2}{2} \qquad b$$

Then Equation C-4 reduces to

$$T_d = \delta_d + B_d P_{gr} \alpha_{dc} \qquad C-5$$

The terms $\frac{A_u^2}{2}$ and $\frac{A_d^2}{2}$ used in the above expressions describe the total signal power that is received by the transponder and the ground receiver respectively. In general the magnitude of the received power in a communication link can be determined as follows:

$$P_r \text{ in dB} = P_t+G_t+G_r-L_s-L_c-L_p = \text{Received Signal Power} \qquad C-6$$

where

P_t	=	Power transmitted in dB	a
G_t	=	Gain of transmitting antenna in dB	b
G_r	=	Gain of receiving antenna in dB	c
L_s	=	$20 \log_{10} R_{nm} + 20 \log_{10} f + 37.8$ in dB	d
R_{nm}	=	Range in nautical miles	e
f	=	Frequency in megacycles	f
L_c	=	Circuit losses in receiver in dB	g
L_p	=	Polarization and pointing losses between transmitting and receiving antennas in dB	h

VARIATION OF LIMITER SUPPRESSION

The relationship between the input and output signal to noise ratios of the bandpass limiter when in cascade with a phaselock loop is discussed in Appendix D. The relationship is repeated here for convenience.

$$\frac{S_o}{N_o} = k \frac{S_i}{N_i} \qquad \text{C-7}$$

where

$\frac{S_o}{N_o}$	=	Signal to noise ratio at the limiter output	a
$\frac{S_i}{N_i}$	=	Signal to noise ratio at the limiter input	b
k	=	$k_1 = \frac{\pi}{4}$ for $\frac{S_i}{N_i} < .035$	c

$$k = k_2 = .68 \left(\frac{S_i}{N_i} \right) + .76 \quad \text{for } .035 < \frac{S_i}{N_i} < 0.35 \qquad \text{d}$$

$$k = k_3 = 1 \text{ for } \frac{S_i}{N_i} > 0.35 \qquad \text{e}$$

UP-CARRIER POWER MARGIN

The circuit margin is given by Equation A-18

$$M_{uc} = 10 \log_{10} \left[k_{uc} \frac{A_u^2}{2} \frac{J_o^2(m_1) J_o^2(m_2) \cos^2\theta}{K_o T_u B_{uc}} \right] - \left(\frac{S}{N} \right)_{ucr} \qquad \text{C-8}$$

Let

$$M_{uc} = 0 \qquad \text{a}$$

$P_{sr} = \frac{A_u^2}{2}$ = Total signal power at the transponder input b

$$\alpha_{uc} = J_o^2(m_1) J_o^2(m_2) \cos^2\theta \qquad \text{c}$$

$$10 \log_{10} \left(\gamma_{uc} \right) = \left(\frac{S}{N} \right)_{ucr} \qquad \text{d}$$

$T_u = \delta_u + B_u P_{sr} \alpha_{uc}$ (from Equation C-3) e

With these definitions Equation C-8 can be rewritten as

$$\frac{k_{uc} P_{sr} \alpha_{uc}}{K_o \left(\delta_u + B_u P_{sr} \alpha_{uc} \right) B_{uc}} = \gamma_{uc} \qquad \text{C-9}$$

The magnitude of P_{sr} which satisfies C-9 is the minimum power which will satisfy the required signal to noise ratio; therefore, let $P_{sr} = P'_{M1N_{uc}}$ then C-9 can be rewritten as

$$P'_{M1N_{uc}} \cdot \left[\alpha_{uc} k_{uc} - \alpha_{uc} B_u B_{uc} \gamma_{uc} K_o\right] = B_{uc} \gamma_{uc} \delta_u K_o \qquad \text{C-10}$$

$$P'_{M1N_{uc}} = \frac{B_{uc} \gamma_{uc} \delta_u K_o}{\alpha_{uc}\left[k_{uc} - B_u B_{uc} \gamma_{uc} K_o\right]} \qquad \text{C-11}$$

and

$$P_{M1N_{uc}} = 10 \log_{10} P'_{M1N_{uc}} \qquad \text{C-12}$$

It was shown previously that the limiter factor "k_{uc}" has three possible expressions (c, d, and e of C-7); therefore, an iterative process is required to solve for P'_{M1N} and P_{M1N}. This iterative process is described as follows:

Steps

1. Calculate P'_{M1N} by setting k_{uc} equal to unity and using C-11

2. Use this magnitude of P'_{M1N} for P'_{iM1N} and calculate T_i from

$$T_i = T_{iu} = \delta_u + B_u \, P'_{i\,M1N} \, \alpha_{uc}$$

3. Use these magnitudes of T_{iu} and P_{iM1N} to calculate $\left(\frac{S}{N}\right)_{iL}$ from $\left(\frac{S}{N}\right)_{iL} = \frac{P'_{iM1N} \, \alpha_{uc}}{K_o \, B_{uc} \, T_{iu}}$

4. If the magnitude of $\left(\frac{S}{N}\right)_{iL} > .35$ the solution is complete, but if $\left(\frac{S}{N}\right)_{iL} < .35$ calculate k_{iuc} from

$$k_{iuc} = .68 \left(\frac{S}{N}\right)_{iL} + .76;$$

5. Use k_{iuc} and C-11 to calculate the new P'_{M1N}
6. Repeat steps 2 and 3 above
7. If the new $\left(\frac{S}{N}\right)_{iL}$ satisfies the relation

$$.035 \leq \left(\frac{S}{N}\right)_{iL} \leq .35$$

the solution is complete, but more accuracy can be obtained by repeating steps 4, 5, and 6:

8. If in step 4 the

$$\left(\frac{S}{N}\right)_{iL} < .035$$

use $k_{uc} = \frac{\pi}{4}$ and C-11 to calculate a new P'_{M1N} and the solution is complete if the resulting calculations satisfy

$$\left(\frac{S}{N}\right)_{iL} < .035$$

The expression for the up carrier power margin then is

$$M_{puc} = 10 \log_{10} P_{sr} - P_{M1N_{uc}} \qquad \text{C-13}$$

UP-SUBCARRIER POWER MARGINS

The circuit margin for the "ω_1" up-subcarrier is given by A-36

$$M_{u\omega_1} = 10 \log_{10}\left[\frac{k_{us} \frac{A_u^2}{2} J_1^2(m_1)J_o^2(m_2)\cos^2\theta}{B\omega_1\left[\frac{K_oT_u}{2} + \frac{k_{us}}{10^6}\frac{A_u^2}{2} J_o^2(m_1)J_o^2(m_2)\sin^2\theta\sin^2\frac{K_cf_1}{\left(K_cf_1\right)^2}\right]}\right]$$

$$-\left(\frac{S}{N}\right)_{u\omega_1 r} \qquad \text{C-14}$$

Let

$$M_{u\omega_1} = 0$$

$$P_{sr} = \frac{A_u^2}{2} \qquad \text{a}$$

$$\alpha_{u\omega_1} = J_1^2(m_1)J_o^2(m_2)\cos^2\theta \qquad \text{b}$$

$$\eta_u = \frac{J_o^2(m_1)}{10^6} J_o^2(m_2)\sin^2(\theta)\sin^2\frac{K_cf_1}{\left(K_cf_1\right)^2} \qquad \text{c}$$

$$10 \log_{10}(\gamma)_{u\omega_1} = \left(\frac{S}{N}\right)_{u\omega_1 r} \qquad \text{d}$$

$$T_u = \delta_u + B_u P_{sr} \alpha_{uc} \qquad \text{e}$$

With these definitions Equation C-14 can be rewritten as

$$\frac{k_{us}P_{sr}\alpha_{u\omega_1}}{B\omega_1\left[\frac{K_o}{2}\left(\delta_u + B_uP_{sr}\alpha_{uc}\right) + k_{us}P_{sr}\eta_u\right]} = \gamma_{uw_i} \qquad \text{C-15}$$

The magnitude of P_{sr} which satisfies C-15 is the minimum power which will satisfy the required signal to noise ratio; therefore, let $P_{sr} = P'_{M1N\omega_1}$ and C-15 can be rewritten as

$$P'_{M1N\omega_1}\left[2\alpha_{u\omega_1}k_{us} - 2B\omega_1\gamma_{u\omega_1}\eta_u k_{us} - \alpha_{uc}B_u B\omega_1\gamma_{u\omega_1}K_o\right] = B\omega_1\delta_u\gamma_{u\omega_1}K_o \qquad \text{C-16}$$

$$P'_{M1N\omega_1} = \frac{B\omega_1\ \delta_u\ \gamma_{u\omega_1}\ K_o}{2k_{us}\left[\alpha_{u\omega_1} - B\omega_1\gamma_{u\omega_1}\eta_u\right] - \alpha_{uc}B_u B\omega_1\gamma_{u\omega_1}K_o}$$

and $P_{M1N\omega_1} = 10\log_{10} P'_{M1N\omega_1}$ C-17

The iterative process which is required for the solution of equation C-16 is identical to that described for the solution of equation C-11 except that the following relation must be used:

$$\left(\frac{S}{N}\right)_{iL} = \frac{P'_{M1N_{\omega 1}}}{K_o\ T_{iu}\ B_{ts}}$$

The expression for an up subcarrier power margin then is

$$M_{puwi} = 10\log_{10}P_{sr} - P_{M1N\omega_i} \qquad \text{C-18}$$

where i = 1 or 2 for the upvoice and updata subcarriers respectively.

DOWN-CARRIER POWER MARGIN

The circuit margin is given by Equation B-29

$$M_{dc} = 10\log_{10}\left[k_{dc}\ \frac{A_d^2}{2}\ \frac{J_o^2(m_3)J_o^2(m_4)J_o^2(R_g)J_o^2(N_u)}{K_oT_dB_{dc}}\right] - \left(\frac{S}{N}\right)_{dcr} \qquad \text{C-19}$$

Let

$$M_{uc} = 0 \qquad \text{a}$$

$$P_{gr} = \frac{A_d^2}{2} = \text{Total signal power at the ground receiver} \qquad \text{b}$$

$$\alpha_{dc} = J_o^2(m_3)J_o^2(m_4)J_o^2(R_g)J_o^2(N_u) \qquad \text{c}$$

$$10\log_{10}(\gamma)_{dc} = \left(\frac{S}{N}\right)_{dcr} \qquad \text{d}$$

$$T_d = \delta_d + B_d P_{gr} \alpha_{dc} \qquad \text{e}$$

With these definitions Equation C-19 can be rewritten as

$$\frac{k_{dc} P_{gr} \alpha_{dc}}{K_o B_{dc} \left(\delta_d + B_d P_{gr} \alpha_{dc}\right)} = \gamma_{dc} \qquad \text{C-20}$$

Since the magnitude of P_{gr} which satisfies C-20 is the minimum power which will satisfy the required signal to noise ratio, let

$$P_{gr} = P'_{MIN_{dc}}$$

then solving for $P'_{MIN_{dc}}$ gives

$$P'_{MIN_{dc}} \left(\alpha_{dc} k_{dc} - \alpha_{dc} B_d B_{dc} \gamma_{dc} K_o\right) = B_{dc} \gamma_{dc} \delta_d K_o \qquad \text{C-21}$$

$$P'_{MIN_{dc}} = \frac{B_{dc} \gamma_{dc} \delta_d K_o}{\alpha_{dc} \left[k_{dc} - B_d B_{dc} \gamma_{dc} K_o\right]}$$

and

$$P_{MIN_{dc}} = 10 \log_{10} P'_{MIN_{dc}} \qquad \text{C-22}$$

The iterative process which is required for the solution of equation C-21 is identical to that described for the solution of equation C-11 except that the following relations must be used:

1. $T_i = T_{id} = \delta_d + B_d P'_{M1N_{dc}} \alpha_{dc}$

2. $\left(\frac{S}{N}\right)_{iL} = \frac{P'_{iM1N_{dc}} \alpha_{dc}}{K_o T_{id} B_{dc}}$

The expression for the down carrier power margin then is:

$$M_{pdc} = 10 \log_{10} P_{gr} - P_{M1N_{dc}} \qquad \text{C-23}$$

DOWN SUBCARRIER POWER MARGIN

The circuit margin for the ω_3 down subcarrier is given by Equation B-48

$$M_{d\omega_3} = 10 \log_{10} \left[\frac{k_{ds} \frac{A_d^2}{2} J_1^2(m_3) J_o^2(m_4) J_o^2(R_g) J_o^2(N_u)}{B(\omega_3) \left[\frac{K_o T_d}{2} + k_{ds} \frac{A_d^2}{2} \frac{J_o^2(m_3) J_o^2(m_4) J_o^2(R_g) J_1^2(N_u)}{B_v} \right]} \right] - \left(\frac{S}{N}\right)_{d\omega_3 r}$$

C-24

Let

$M_{d\omega_3} = 0$ a

$P_{gr} = A_d^2/2$ = Total signal power at the ground receiver b

$\alpha_{d\omega_3} = J_1^2(m_3) J_o^2(m_4) J_o^2(R_g) J_o^2(N_u)$ c

$10 \log_{10} (\gamma)_{d\omega_3} = \left(\frac{S}{N}\right)_{d\omega_3 r}$ d

$T_d = \delta_d + B_d P_{gr} \alpha_{dc}$ e

$\eta_d = J_o^2(m_3) J_o^2(m_4) J_o^2(R_g) J_1^2(N_u)/B_v$ f

With these definitions Equation C-24 can be rewritten as

$$\frac{k_{ds} P_{gr} \alpha_{d\omega_3}}{B\omega_3 \frac{K_o}{2} \left(\delta_d + B_d P_{gr} \alpha_{dc}\right) + k_{ds} B\omega_3 P_{gr} n_d} = \gamma_{d\omega_3} \qquad \text{C-25}$$

Since the magnitude of P_{gr} which satisfies Equation C-22 is the minimum power, which will satisfy the required signal to noise ratio, let $P_{gr} = P'_{M1N_{d\omega_3}}$ then solving for $P'_{M1N_{d\omega_3}}$ gives

$$P'_{M1N_{d\omega_3}} \cdot \left[2\alpha_{d\omega_3} k_{ds} - \alpha_{dc} B_d B_{\omega_3} \gamma_{d\omega_3} K_o - 2B\omega_3 \gamma_{d\omega_3} n_d k_{ds}\right] = B\omega_3 \gamma_{d\omega_3} \delta_d K_o$$

$$P'_{M1N_{d\omega_3}} = \frac{B\omega_3 \gamma_{d\omega_3} \delta_d K_o}{2k_{ds}\left[\alpha_{d\omega_3} - B\omega_3 \gamma_{d\omega_3} n_d\right] - \alpha_{dc} B_d B\omega_3 \gamma_{d\omega_3} K_o} \qquad \text{C-26}$$

and

$$P_{M1N_{d\omega_3}} = 10 \log_{10} P'_{M1N_{d\omega_3}} \qquad \text{C-27}$$

The iterative process which is required for the solution of equation C-26 is identical to that described for the solution of equation C-11 except that the following relations must be used:

1. $T_i = T_{id} = \delta_d + B_d P'_{M1N} \alpha_{dc}$

2. $\left(\frac{S}{N}\right)_{iL} = \frac{P'_{M1N_{d\omega_3}}}{K_o T_{id} B_{gs}}$

The expression for a down subcarrier power margin then is:

$$M_{pd\omega j} = 10 \log_{10} P_{gr} - P_{M1Nd\omega j} \qquad \text{C-28}$$

where j = 1 or 2 for the down voice and down telemetry subcarriers respectively.

DOWN CODE POWER MARGIN

The circuit margin is given by Equation B-58 C-29

$$M_{dc} = 10\log_{10}\left[\frac{4\frac{A_d^2}{2} J_o^2(m_1)J_o^2(m_2)J_o^2(m_3)J_o^2(m_4)J_o^2(N_u)J_1^2(R_g)\sin^2\theta}{B_{dco}\left[K_oT_d + \frac{2}{B_v}\frac{Ad^2}{2} J_o^2(m_3)J_o^2(m_4)J_o^2(R_g)J_1^2(N_u)\right]}\right] - \left(\frac{S}{N}\right)_{dcor}$$

Let

$$M_{dco} = 0 \qquad \text{a}$$

$$P_{gr} = \frac{A_d^2}{2} = \text{Total signal power at the ground receiver} \qquad \text{b}$$

$$\alpha_{dco} = J_o^2(m_1)J_0^2(m_2)J_o^2(m_3)J_o^2(m_4)J_o^2(N_u)J_1^2(R_g)\sin^2\theta \qquad \text{c}$$

$$10\log_{10}(\gamma)_{dco} = \left(\frac{S}{N}\right)_{dcor} \qquad \text{d}$$

$$T_d = \delta_d + \alpha_{dc}B_dP_{gr} \qquad \text{e}$$

$$\eta_d = J_o^2(m_3)J_o^2(m_4)J_1^2(N_u)J_o^2(R_g)/B_v \qquad \text{f}$$

With these definitions Equation C-29 can be rewritten as

$$\frac{4P_{gr}\alpha_{dco}}{B_{dco}K_o\left[\delta_d+\alpha_{dc}B_dP_{gr}\right] + 2B_{dco}P_{gr}\eta_d} = \gamma_{dco} \qquad \text{C-30}$$

Since the magnitude of P_{gr} which satisfies C-30 is the minimum power which will satisfy the required signal to noise ratio, let $P_{gr} = P'_{MINdco}$ then solving for P'_{MINdco} gives:

$$P'_{M1N_{dco}}\left(4\alpha_{dco} - \gamma_{dco}B_{dco}K_o\alpha_{dc}B_d - 2\gamma_{dco}B_{dco}n_d\right) = \gamma_{dco}B_{dco}K_o\delta_d \qquad \text{C-31}$$

$$P'_{M1N_{dco}} = \frac{B_{dco}\gamma_{dco}\delta_d K_o}{4\alpha_{dco} - B_{dco}\gamma_{dco}\left(\alpha_{dc}B_d K_o + 2n_d\right)} \qquad \text{C-32}$$

and

$$P_{M1N_{dco}} = 10\log_{10} P'_{M1N_{dco}} \qquad \text{C-33}$$

The above expression for the down code power margin is much simpler than those presented previously because the limiter effects are not considered in the down code channel.

The expression for the down code power margin then is:

$$M_{pdco} = 10\log P_{gr} - P_{M1N_{dco}} \qquad \text{C-34}$$

References for Appendix C

1. G. P. Arndt and G. A. Jaegers, "A Computer Program and Math Model for the Unified S-Band System," MSC-Information Systems Division - Systems Analysis Branch; September 19, 1966

APPENDIX D

SIGNAL TO NOISE POWER RATIOS IN BANDPASS LIMITERS

PURPOSE

The purpose of this appendix is to present expressions for the signal and noise components of the bandpass limiter output for the case where the input signal is degraded by thermal noise.

Given:

$s_i(t)$ = The signal amplitude at the input to the limiter D-1

= $A\sin\omega t$ a

$n_i(t)$ = The thermal noise amplitude at the input to the limiter

= $K_o T B_i$ b

where

K_o = 1.38×10^{-23} watts/°K H_Z = Boltzman's Constant

T = Noise temperature

B_i = Bandwidth preceeding limiter

Viterbi[1] shows that the signal and noise power ratios of the bandpass limiter can be expressed by

$$\frac{S_o}{N_o} = k\,\frac{S_i}{N_i} \qquad \text{D-2}$$

where

S_i/N_i = The input signal to noise power ratio a

S_o/N_o = The output signal to noise power ratio b

= $\frac{\pi}{4} \leq k \leq 2$ c

$\pm\pi L/2\sqrt{2}$ = Peak excursions of the limiter output d

$L^2 \quad = \quad S_o + N_o$ = Total power output of limiter e

$s_o(t) \quad = \quad \sqrt{2}\left(S_o\right)^{1/2} \sin\omega t$ = Signal amplitude at the output of the limiter f

Since the limiter power output is defined by

$$L^2 = S_o + N_o \tag{D-3}$$

The expression for the noise power output can be substituted into D-2 to give

$$\frac{S_o}{L^2 - S_o} = k\,\frac{S_i}{N_i} \tag{D-4}$$

Solving for S_o, the signal power output of a limiter is given by

$$S_o = \frac{L^2\; S_i/N_i}{\frac{1}{k} + \frac{S_i}{N_i}} \tag{D-5}$$

Using D-2 f and D-5 the signal amplitude at the output of a bandpass limiter has the following form:

$$s_o(t) = \sqrt{2}\,L\left[\frac{S_i/N_i}{(1/k) + S_i/N_i}\right]^{1/2} \sin\omega t \tag{D-6}$$

In a similar manner the noise power output of the limiter is determined from D-2 and D-3 to be

$$N_o = \frac{L^2}{k\,\frac{S_i}{N_i} + 1} \tag{D-7}$$

Arndt and Jaegers,[2] based on an analysis by Dr. F. F. Carden, indicate that for the Apollo system the relationship between the input and output signal to noise power ratios of the bandpass limiter when in cascade with a phase lock loop can be approximated by the following values of k and Equation D-2

$$\frac{S_o}{N_o} = k \frac{S_i}{N_i} \qquad \text{D-2}$$

where

$k = \pi/4$ for $0.035 > S_i/N_i$ (a)

$k = .68 \left|S_i/N_i\right| + .76$ for $.035 < S_i/N_i < .35$ (b)

$k = 1$ for $S_i/N_i > .35$ (c)

Wynn[3] has presented a mathematically more rigorous analysis for the limiter-phase detection cascade. Although the approximation by Carden is still considered to be a reasonable approximation for the Apollo cases, work is continuing to determine in detail the differences between the results gained from the approximation and from Wynn's work.

References for Appendix D

1. A. J. Viterbi, Principles of Coherent Communication, McGraw-Hill 1966, p.41.

2. G. D. Arndt and G. A. Jaegers, "A Computer Program and Math Model for the Unified S-Band System." EB-66-2009-U, MSC.

3. W. P. Wynn, "A Statistical Analysis of a Bandpass Nonlinearity - Phase Detector Cascade," TM 68-2034-14, August 19, 1968.

APPENDIX E

APPROXIMATION ERRORS

PURPOSE

The purpose of this appendix is to suggest means by which the circuit margin expressions that were derived in Appendix A and B can be approximated, to derive expressions for the resulting approximation errors, and to derive expressions for the errors resulting from truncating the infinite series of equations B-16 and B-53.

The approximations which are considered in this Appendix result in circuit margins that are of greater magnitude than those that would have been obtained from the expressions in Appendix A and B. The expressions for the approximation errors are therefore obtained by describing this erroneous increase in the magnitude of the calculated margin.

I. APPROXIMATION OF SYSTEM NOISE TEMPERATURE

If the system noise temperature is assumed to be equal to δ_u and δ_d for the uplink and downlink respectively the error expressions, defined as η_{Ti}, are obtained as follows:

A. Uplink Carrier

From equation A-18 the circuit margin is expressed by:

$$M_{uc} = 10 \log_{10}\left[\frac{k_{uc}\ P_{sr}\ \alpha_{uc}}{K_o\ T_u\ B_{uc}}\right] - \frac{S}{N}\Big|_{ucr} \qquad \text{E-1}$$

Where

$$P_{sr} = \frac{A_u^2}{2} \qquad \text{a}$$

$$\alpha_{uc} = J_o^2(m_1)J_o^2(m_2)\ \cos^2\theta \qquad \text{b}$$

$$T_u = \delta_u + B_u\ P_{sr}\ \alpha_{uc} \qquad \text{c}$$

Let

$$\beta_{uc} = \frac{k_{uc}\ P_{sr}\ \alpha_{uc}}{K_o\ T_u\ B_{uc}} \qquad \text{d}$$

$$\beta_{Tuc} = \frac{k_{uc}\ P_{sr}\ \alpha_{uc}}{K_o\ \delta_u\ B_{uc}} \qquad \text{e}$$

The error in the upcarrier circuit margin (for $T_u = \delta_u$) is:

$$\eta_{Tuc} = 10 \log \beta_{Tuc} - 10 \log \beta_{uc} \qquad \text{E-2}$$

$$= 10 \log \left[1 + \frac{B_u \, P_{sr} \, \alpha_{uc}}{\delta_u} \right]$$

B. Uplink Subcarrier

From equations A-36 and A-37 the circuit margin is expressed by:

$$M_{u\omega i} = 10 \log_{10} \left[\frac{k_{us} \, P_{sr} \, \alpha_{u\omega i}}{B_{\omega_i} \left[\frac{K_o \, T_u}{2} + k_{us} \, P_{sr} \, \eta_{ui} \right]} \right] - \left(\frac{S}{N} \right)_{u\omega_i r} \qquad \text{E-3}$$

where

$i = 1$ or 2 a

$k = 1$ or 2 b

$i \neq k$ c

$$\eta_{ui} = \frac{1}{10^6} J_o^2(m_1) \, J_o^2(m_2) \, \sin^2\theta \, \frac{\sin^2 K_c \, f_i}{(K_c \, f_i)^2} \qquad \text{d}$$

$$P_{sr} = A_u^2/2 \qquad \text{e}$$

$$\alpha_{u\omega i} = J_1^2(m_i) \, J_o^2(m_k) \cos^2\theta \qquad \text{f}$$

Let

$$\beta_{u\omega i} = \frac{k_{us} \, P_{sr} \, \alpha_{u\omega i}}{B_{\omega_i} \left[\frac{K_o \, T_u}{2} + k_{us} \, P_{sr} \, \eta_{ui} \right]} \qquad \text{g}$$

$$\beta_{Tu\omega i} = \frac{k_{us} \, P_{sr} \, \alpha_{u\omega i}}{B_{\omega_i} \left[\frac{K_o \, \delta_u}{2} + k_{us} \, P_{sr} \, \eta_{ui} \right]} \qquad \text{h}$$

The error in the up subcarrier circuit margin (For $T_u = \delta_u$) is:

$$\eta_{Tusi} = 10 \log_{10} \beta_{Tu\omega i} - 10 \log_{10} \beta_{u\omega i}$$

$$= 10 \log_{10} \left| 1 + \frac{\alpha_{uc} B_u K_o P_{sr}}{\delta_u K_o + 2\eta_{ui} k_{us} P_{sr}} \right| \qquad \text{E-4}$$

C. Downlink Carrier

From equation B-29 the circuit margin is expressed by:

$$M_{dc} = 10 \log_{10} \left[\frac{k_{dc} P_{gr} \alpha_{dc}}{K_o T_d B_{dc}} \right] - \left(\frac{S}{N} \right)_{dcr} \qquad \text{E-5}$$

where

$$P_{gr} = A_d^2/2 \qquad \text{a}$$

$$\alpha_{dc} = J_o^2(R_g) J_o^2(N_u) J_o^2(m_3) J_o^2(m_4) \qquad \text{b}$$

$$T_d = \delta_d + B_d P_{gr} \alpha_{dc} \qquad \text{c}$$

Let

$$\beta_{dc} = \frac{k_{dc} P_{gr} \alpha_{dc}}{K_o T_d B_{dc}} \qquad \text{d}$$

$$\beta_{Tdc} = \frac{k_{dc} P_{gr} \alpha_{dc}}{k_o \delta_d B_{dc}} \qquad \text{e}$$

The error in the down carrier circuit margin (for $T_d = \delta_d$) is:

$$\eta_{Tdc} = 10 \log_{10} \beta_{Tdc} - 10 \log_{10} \beta_{dc}$$

$$= 10 \log_{10} \left[1 + \frac{B_d}{\delta_d} P_{gr} \alpha_{dc} \right] \qquad \text{E-6}$$

D. Downlink Subcarriers

From equations B-48 and 49 the circuit margin is expressed by:

$$M_{d\omega j} = 10 \log_{10}\left[\frac{k_{ds}\ P_{gr}\ \alpha_{d\omega j}}{B_{\omega_j}\left[\frac{K_o T_d}{2} + \eta_d k_{ds} P_{gr}\right]}\right] - \left.\frac{S}{N}\right|_{d\omega_j r} \qquad \text{E-7}$$

where

$$P_{gr} = A_d^2/2 \qquad a$$

$$j = 3 \text{ or } 4 \qquad b$$

$$\ell = 3 \text{ or } 4 \qquad c$$

$$j \neq \ell \qquad d$$

$$\alpha_{d\omega j} = J_o^2(N_u) J_o^2(R_g) J_1^2(m_j) J_o^2(m_\ell) \qquad e$$

$$\eta_d = J_1^2 \frac{(N_u)\ J_o^2\ (R_g)\ J_o^2\ (m_3)\ J_o^2\ (m_4)}{B_v}$$

Let

$$\beta_{d\omega j} = \frac{k_{ds}\ P_{gr}\ \alpha_{d\omega j}}{B_{\omega_j}\left[\frac{K_o T_d}{2} + \eta_d\ k_{ds}\ P_{gr}\right]} \qquad f$$

$$\beta_{Td\omega j} = \frac{k_{ds}\ P_{gr}\ \alpha_{d\omega j}}{B_{\omega_j}\left[\frac{K_o \delta_d}{2} + \eta_d\ k_{ds}\ P_{gr}\right]} \qquad g$$

The error in the down subcarrier circuit margin (for $T_d = \delta_d$) is:

$$\eta_{Tds} = 10 \log_{10}\beta_{Td\omega j} - 10 \log_{10}\beta_{d\omega j}$$

$$\eta_{Tds} = 10 \log_{10}\left[1 + \frac{\alpha_{dc}\ B_d\ K_o\ P_{gr}}{K_o \delta_d + 2k_{ds}\ P_{gr}\ \eta_d}\right] \qquad \text{E-8}$$

E. Downlink Range Code

From equation B-58 the circuit margin is expressed by:

$$M_{dco} = 10 \log_{10} \left[\frac{4P_{gr} \alpha_{dco}}{B_{dco}\left[K_o T_d + 2P_{gr} \eta_d\right]} \right] - \left(\frac{S}{N}\right)_{dcor} \qquad \text{E-9}$$

where

$$P_{gr} = \frac{A_d^2}{2} \qquad \text{a}$$

$$\alpha_{dco} = J_o^2(m_1)\, J_o^2(m_2)\, J_o^2(m_3)\, J_o^2(m_4)\, J_o^2(N_u)\, J_1^2(R_g)\, \sin^2\theta \qquad \text{b}$$

$$\eta_d = J_1^2(N_u) J_o^2(R_g) J_o^2(M_3) J_o^2(M_4) \left(\frac{1}{B_v}\right) \qquad \text{c}$$

Let

$$\beta_{co} = \frac{4P_{gr} \alpha_{dco}}{B_{dco}\left[K_o T_d + 2P_{gr} \eta_d\right]} \qquad \text{d}$$

$$\beta_{Tco} = \frac{4P_{gr} \alpha_{dco}}{B_{dco}\left[K_o \delta_d + 2P_{gr} \eta_d\right]} \qquad \text{e}$$

The error in the downlink range code circuit margin (for $T_d = \delta_d$) is:

$$\eta_{Tco} = 10 \log_{10} B_{Tco} - 10 \log_{10} B_{co}$$

$$= 10 \log_{10}\left[1 + \frac{\alpha_{dc}\, B_d\, K_o\, P_{gr}}{K_o \delta_d + 2P_{gr} \eta_d}\right] \qquad \text{E-10}$$

II. APPROXIMATION OF LIMITER FACTOR

If the limiter factor is assumed to be equal to unity the error expressions, defined as η_{ki}, are obtained as follows:

A. Uplink Carrier

Let

$$\beta_{kuc} = \frac{P_{sr}\ \alpha_{uc}}{K_o\ T_u\ B_{uc}} \qquad \text{E-11}$$

From equations E-1 and E-11 the error in the up carrier circuit margin (for $k_{uc} = 1$) is:

$$\eta_{kuc} = 10\ \log_{10}\beta_{kuc} - 10\ \log_{10}\beta_{uc}$$

$$= 10\ \log_{10}\left(\frac{1}{k_{uc}}\right) \qquad \text{E-12}$$

B. Uplink Subcarriers

Let

$$\beta_{ku\omega i} = \frac{P_{sr}\ \alpha_{u\omega i}}{B_{\omega_i}\left[\frac{K_o T_u}{2} + P_{sr}\ \eta_{ui}\right]} \qquad \text{E-13}$$

From equations E-3 and E-13 the error in the up subcarrier circuit margin (for $k_{us} = 1$) is:

$$\eta_{ku\omega i} = 10\ \log_{10}\beta_{ku\omega i} - 10\ \log_{10}\beta_{u\omega i}$$

$$= 10\ \log_{10}\left[\left(\frac{1}{k_{us}}\right)\left(\frac{K_o\ T_u + 2k_{us}\ P_{sr}\ \eta_{ui}}{K_o\ T_u + 2P_{sr}\ \eta_{ui}}\right)\right] \qquad \text{E-14}$$

C. Downlink Carrier

Let

$$\beta_{kdc} = \frac{P_{gr}\ \alpha_{dc}}{K_o\ T_d\ B_{dc}}$$

From equations E-5 and E-15 the error in the down carrier circuit margin (for $k_{dc} = 1$) is:

$$\eta_{kdc} = 10\ \log_{10}\beta_{kdc} - 10\ \log_{10}\beta_{dc} \qquad \text{E-16}$$

$$= 10\ \log_{10}\left[\frac{1}{k_{dc}}\right]$$

D. Downlink Subcarriers

Let

$$\beta_{kd\omega j} = \frac{P_{gr}\ \alpha_{d\omega j}}{B_{\omega_j}\left[\frac{K_o T_d}{2} + \eta_d\ P_{gr}\right]} \qquad \text{E-17}$$

From equations E-7 and E-17 the error in the down subcarrier circuit margin (for $k_{ds} = 1$) is:

$$\eta_{kds} = 10\ \log_{10}\beta_{kd\omega j} - 10\ \log_{10}\beta_{d\omega j}$$

$$= 10\ \log_{10}\left[\left(\frac{1}{k_{ds}}\right)\frac{K_o T_d + 2\eta_d k_{ds} P_{gr}}{K_o T_d + 2\eta_d P_{gr}}\right] \qquad \text{E-18}$$

III. APPROXIMATION OF THE RANGE CODE INTERFERENCE IN THE UP SUBCARRIER CHANNEL

If the range code interference power in the up subcarrier channel is assumed to be equal to zero, the error expressions defined as η_{coi} are obtained as follows:

Let

$$\beta_{cu\omega i} = \frac{k_{us}\ P_{sr}\ \alpha_{u\omega i}}{B_{\omega_i}\left[\frac{K_o T_u}{2}\right]} \qquad \text{E-19}$$

The error in the up subcarrier circuit margins (for $\eta_u = 0$) is:

$$\eta_{coi} = 10\ \log_{10}\beta_{cu\omega i} - 10\ \log_{10}\beta_{u\omega i}$$

$$= 10\ \log_{10}\left[1 + \frac{2k_{us}\ P_{sr}\ \eta_{ui}}{K_o\left[\delta_u + B_u\ P_{sr}\ \alpha_{uc}\right]}\right] \qquad \text{E-20}$$

IV. APPROXIMATION OF THE SIGNAL GAIN OF THE TRANSPONDER TURN AROUND CHANNEL

If the signal gain of the transponder turn around channel is assumed to be equal to zero the error expressions, defined as η_{Rj}, are obtained as follows:

A. Down Carrier

Let

$$\beta_{Rdc} = \frac{k_{dc}\ P_{gr}\ \alpha_{dc} \Big/ J_o^2(R_g)}{B_{dc}\ K_o\left[\delta_d + \left(B_d\ P_{gr}\ \alpha_{dc}\right) \Big/ J_o^2(R_g)\right]} \qquad \text{E-21}$$

From equations E-5 and E-21 the error in the down carrier circuit margin (for $R_g = 0$) is:

$$\eta_{Rdc} = 10\log_{10}\beta_{Rdc} - 10\log_{10}\beta_{dc}$$

$$= 10\log_{10}\left[\frac{\delta_d + B_d\ P_{gr}\ \alpha_{dc}}{\delta_d\ J_o^2(R_g) + \alpha_{dc}\ B_d\ P_{gr}}\right] \qquad \text{E-22}$$

B. Down Subcarriers

Let

$$\beta_{Rd\omega j} = \frac{k_{ds}\ P_{gr}\ \alpha_{d\omega j} \Big/ J_o^2(R_g)}{B_{\omega_j}\left[\frac{K_o}{2}\left(\delta_d + \frac{\alpha_{dc}\ B_d\ P_{gr}}{J_o^2(R_g)}\right) + \frac{\eta_d\ k_{ds}\ P_{gr}}{J_o^2(R_g)}\right]} \qquad \text{E-23}$$

From equations E-7 and E-23 the error in the down subcarrier circuit (for $R_g = 0$) is:

$$\eta_{Rds} = 10\ \log_{10}\beta_{Rd\omega j} - 10\ \log_{10}\beta_{d\omega j}$$

$$= 10\ \log_{10}\left[\frac{K_o\left(\delta_d + \alpha_{dc}\ B_d\ P_{gr}\right) + 2\eta_d\ k_{ds}\ P_{gr}}{K_o\left(\delta_d\ J_o^2(R_g) + \alpha_{dc}\ B_d\ P_{gr}\right) + 2\eta_d\ k_{ds}\ P_{gr}}\right]$$

E-24

V. APPROXIMATION OF THE NOISE GAIN OF THE TRANSPONDER TURN AROUND CHANNEL

If the noise gain of the transponder turn around channel is assumed to be equal to zero the error expressions, defined as η_{Nuj}, are obtained as follows:

A. Down Carrier

Let

$$\beta_{N_u dc} = \frac{k_{dc} \, P_{gr} \, \alpha_{dc} \Big/ J_o^2(N_u)}{K_o B_{dc} \left[\delta_d + \dfrac{\alpha_{dc} \, B_d \, P_{gr}}{J_o^2(N_u)}\right]} \qquad \text{E-25}$$

From equations E-5 and E-25 the error in the down carrier circuit margin (for $N_u = 0$) is:

$$\eta_{N_u dc} = 10 \log_{10} \beta_{N_u dc} - 10 \log_{10} \beta_{dc}$$

$$= 10 \log_{10} \left[\frac{\delta_d + \alpha_{dc} \, B_d \, P_{gr}}{\delta_d J_o^2(N_u) + \alpha_{dc} \, B_d \, P_{gr}}\right] \qquad \text{E-26}$$

B. Down Subcarrier

Let

$$\beta_{N_u d\omega j} = \frac{k_{ds} \, \alpha_{d\omega j} \, P_{gr} \Big/ J_o^2(N_u)}{\dfrac{B_{\omega j}}{2} \, K_o \left[\delta_d + \dfrac{\alpha_{dc} \, B_d \, P_{gr}}{J_o^2(N_u)}\right]} \qquad \text{E-27}$$

From equations E-7 and E-27 the error in the down subcarrier circuit margin (for $N_u = 0$) is:

$$\eta_{N_u ds} = 10 \log_{10} \beta_{N_u d\omega j} - 10 \log_{10} \beta_{d\omega j}$$

$$= 10 \log_{10} \left[\frac{K_o T_d + 2\eta_d \, k_{ds} \, P_{gr}}{K_o \left(\delta_d \, J_o^2(N_u) + \alpha_{dc} \, B_d \, P_{gr}\right)}\right] \qquad \text{E-28}$$

C. Down Link Range Code

Let

$$\beta_{Nuco} = \frac{4 P_{gr} \; \alpha_{dco} \; J_o^2(N_u)}{B_{dco} K_o \left[\delta_d + \frac{\alpha_{dc} \; B_d \; P_{gr}}{J_o^2(N_u)} \right]} \qquad \text{E-29}$$

The error in the down link range code circuit margin (for Nu = 0) is:

$$\eta_{Nuco} = 10 \log_{10} \beta_{Nuco} - 10 \log_{10} \beta_{co}$$

$$= 10 \log_{10} \left[\frac{K_o \; T_d + 2 \eta_d \; P_{gr}}{K_o \; \delta_d \; J_o^2(N_u) + \alpha_{dc} \; B_d \; P_{gr}} \right] \qquad \text{E-30}$$

VI. APPROXIMATION OF INFINITE SERIES

The expressions for the downlink circuit margins which are presented in Appendix B have been derived using only the first term of the infinite series expansions which describe the respective downlink modulation losses. The expressions for the errors which result from using only the first term rather than the first two terms of these series expansions are obtained as follows:

A. Downlink Carrier and Subcarriers

In describing the modulation losses of the down carrier and subcarriers the following series expansion is used:

$$\left[J_o(R_g) + 2 \sum_{N=1}^{\infty} J_{2N}(R_g) \; J_o(2Nm_1) \; J_o(2Nm_2) \cos 2N\theta \right]^2$$

The resulting error in the circuit margins that are calculated using the expressions that are contained in Appendix B is:

$$\eta_{c\not{c}s} = 10 \log_{10} \left[\left(\frac{J_o(R_g) + 2J_2(R_g) \; J_o(2m_1) \; J_o(2m_2) \cos 2\theta}{J_o(R_g)} \right)^2 \right]$$

$$\eta_{c/s} = 20 \log_{10}\left[1 + \frac{2J_2(R_g)\ J_o(2m_1)\ J_o(2m_2)\ \cos 2\theta}{J_o(R_g)}\right]$$

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B. Downlink Range Code

In describing the modulation losses of the downlink range code the following series expansion is used:

$$\left[\sum_{N=1}^{\infty} J_{2N-1}(R_g)\ J_o\left[(2N-1)\ m_1\right] J_o\left[(2N-1)m_2\right] \sin\left[(2n-1)\theta\right]\right]^2$$

The resulting error in the circuit margins that are calculated using the expression that is contained in Appendix B is:

$$\eta_{cos} = 20 \log_{10}\left[\frac{J_1(R_g)J_o(m_1)J_o(m_2)\sin\theta + J_3(R_g)\ J_o(3m_1)J_o(3m_2)\sin 3\theta}{J_1(R_g)J_o(m_1)J_o(m_2)\sin\theta}\right]$$

$$= 20 \log_{10}\left[1 + \frac{J_3(R_g)J_o(3m_1)J_o(3m_2)\sin 3\theta}{J_1(R_g)J_o(m_1)J_o(m_2)\sin\theta}\right]$$

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