BELLCOMM, INC.
955 L'ENFANT PLAZA NORTH, S.W., WASHINGTON, D.C. 20024

COVER SHEET FOR TECHNICAL MEMORANDUM

TITLE- Statistical Analysis of Project Pyro Liquid Propellant Explosion Data

TM-69-1033-3

DATE- July 23, 1969

FILING CASE NO(S)- 320

AUTHOR(S)- P. Gunther
G. R. Andersen

FILING SUBJECT(S) (ASSIGNED BY AUTHOR(S))- Pyro
Explosions
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ABSTRACT

This study of the URS Corporation's treatment of the Project Pyro data has led to adoption of an empirical procedure for estimating explosive yield. The results may serve as a reasonable interim guide until further knowledge of self-ignition is available.

Since the Project Pyro experimental program did not explicitly investigate self-ignition characteristics of liquid propellants, URS confined its analysis merely to determining conditional predictions of yield when ignition delay time is known. Assuming further that ignition time scales geometrically leads to a URS worst case prediction of constant maximum yield independent of weight (in one case exceeding 100%).

However, real experience indicates that maximum attainable yield for large propellant weights is limited by some phenomenon causing self-ignition--and this was true also for most of Project Pyro's large scale tests. In order to obtain approximate unconditional predictions of yield, allowing for self-ignition, a statistical regression analysis was employed which combined Project Pyro's small scale, primarily controlled ignition tests, with their large scale, primarily self-ignited tests. Even though the statistical populations represented by the two groups of tests are different, the procedure is considered to be appropriate for estimating the fall-off of yield with weight, in three of the four cases where the effect of ignition delay time could meaningfully be bypassed.

Two cryogenic propellants were studied, LO_2/RP-1 and LO_2/LH_2, and two failure modes, confined by missile (CBM) and confined by ground surface (CBGS). Also analyzed were the small scale tests conducted to determine the effect of variables other than weight: tank and orifice geometry and ullage volume for CBM, and missile fall-back velocity for CBGS.

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The results of the regression analyses were mixed, and generally reflect various limitations inherent in the data, e.g., very large scatter, small number of tests relative to number of variables investigated, nonlinear behavior of the variables, and uncertainty whether optimum ignition conditions were obtained. A brief summary for each of the four propellant-failure mode combinations follows. In all cases the main conclusions apparently differ from those deduced by URS.

CBM, LO_2/RP-1. The regression on weight provides an excellent fit to the data; however, the observed Titan yield is significantly smaller than the extrapolated regression yield. Since self-ignition of the 25,000 lb. tests did not occur prematurely, there is no distinction here between conditional and unconditional prediction of yield. The effect of geometry variables turned out to be too nonlinear and the tests on ullage volume too few, to rely on the corresponding regressions for quantitative prediction.

CBM, LO_2/LH_2. The regressions are not meaningful, the data being too erratic and inconsistent. Qualitatively, the very low yield for the full scale SIV test indicates that yield decreases with weight. Whether ignition delay time for this test is atypically low, and whether a longer delay time would have increased yield, cannot be judged.

CBGS, LO_2/LH_2. The regression shows a definite fall-off of yield with weight; however, the large scatter in the data leads to large prediction limits. The regression on velocity for the 200 lb. tests is satisfactory.

CBGS, LO_2/RP-1. The regressions are not meaningful, and unfortunately so, since the observed yields were the greatest for both the small scale and large scale tests. The anomalous behavior of the data, whereby yields for 1000 lbs. substantially exceeded the yields for 200 lbs., introduces large interaction and nonlinear effects. Additional large scale testing is needed in order to estimate scaling of unconditional yield with weight.

Techniques are developed for estimating yield through a least squares fit of the peak overpressure and impulse observations to TNT calibration curves, and also for determining whether departures from such curves are significant. A detailed analysis (Appendix A) showed that over three-fourths of the tests did in fact depart significantly. The departures did not follow any recognizable pattern as a function of weight and other variables, and hence do not appear amenable to extrapolation. However, the detailed cataloging may prove useful in any future efforts to develop a blast theory for liquid propellants.

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SYMBOLS*

A - Log-log regression constant, intercept.

$A^a_{.90}$, $A^a_{.95}$ - Intercept for upper 90% (95%) prediction asymptote.

b - Slope of regression of yield on ignition time.

B - Log-log regression constant, slope.

$B_{.90}$, $B_{.95}$ - Upper 90% (95%) confidence limit on regression slope (also slope of prediction asymptote)

c - URS' proportionality constant between yield and ignition time, for CBM LO_2/RP-1.

CBGS - Confined by ground surface failure mode.

CBM - Confined by missile failure mode.

d - Distance of recording station from explosion.

d_1 - Closest distance from explosion: 23 ft. for 200 and 1000 lbs., 67 ft. for 25,000 lbs. and higher.

$d_2, \ldots, d_5$ - Successively further distances from explosion: 37, 67, 117, 200 ft. for 200 and 1000 lbs.

Δp_r - Tank differential burst pressure--burst pressure minus average initial pressure.

D_o/D_t - Orifice diameter ratio.

HVI - High velocity impact test.

k, k_P, k_I - k-factor for correcting (URS, pressure, or impulse) yield for effective ullage volume different from 10%.

L/D - Length-to-diameter ratio of propellant tanks.

r_L, r_U - Lower (upper) 90% confidence limits on correlation between yield and ignition time.

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*Not included are specialized symbols used only in technical Appendices B and D.

σ_Y% - Scatter in yield about regression line.

S_P%, S_I% - Scatter in pressure (impulse) data about Kingery curve.

SP - Designates spurious test, i.e., abnormal test conditions.

t - Ignition delay time.

t_{max} - Maximum ignition time; for CBM LO_2/RP-1 time to reach tank burst pressure.

t* - Scaled ignition time, $t/W^{1/3}$.

t^*_{max} - Scaled maximum ignition time, $t_{max}/W^{1/3}$.

V - Impact velocity (ft/sec) for CBGS tests.

V_u - Ullage volume.

V_{u-eff} - Effective ullage volume.

W - Weight of propellants.

X - Designates external ignition source for test.

Y, Y_P, Y_I - Yield, pressure yield, impulse yield estimated from data.

$\hat{Y}$, $\hat{Y}_P$, $\hat{Y}_I$ - Expected yield from regression.

Y_{max} - Maximum observed yield.

$\hat{Y}_{max}$ - Expected regression yield corresponding to t_{max}.

$Y_{P,.95}$, $Y_{I,.95}$ - Upper 95% prediction limit for regression of pressure (impulse) yield.

Y_S, $Y_{S,P}$, $Y_{S,I}$ - Specific yield, i.e., equivalent TNT weight relative to actual propellant weight (for tanks less than full).

$Y^A_{P,.95}$, $Y^A_{I,.95}$ - Yield corresponding to 95% prediction asymptote (approximate prediction limit).

$\hat{Y}_{P,HVI}$, $\hat{Y}_{I,HVI}$ - Expected regression yield with high velocity impact (flat wall) tests included (for CBGS LO_2/LH_2).

Y_{URS} - Yield estimated by URS.

D* - Scaled pool diameter, t*V.

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FROM: P. Gunther
G. R. Andersen

TM-69-1033-3

TECHNICAL MEMORANDUM

I. INTRODUCTION

1.1 Background

Project Pyro was an experimental program designed to obtain data useful for assessing the hazards from liquid propellant rocket explosions. The program was conducted at the Air Force Rocket Propulsion Laboratory, Edwards Air Force Base. The URS Systems Corporation, Burlingame, California, established the overall design of the program, designed and constructed the test articles, and analyzed the data.

The tests were conducted primarily with small propellant weights (200 lb. and 1000 lb.) under a variety of failure modes and geometry configurations. It was intended that this data, supplemented by a limited number of 25,000 lb. tests and two full scale tests (approximately 94,000 lb.), would then provide a basis from which explosive effects for operational vehicles and weights might be predicted.

As part of a critique of URS' preliminary and final report[1], the data of Project Pyro was analyzed statistically and the results compared with those deduced by URS.

The rationale underlying the statistical approach is presented in Section II. The main results of the formal analysis are described in Section III with more complete details relegated to the appendices. Section IV provides a qualitative summary of the explosive characteristics of both the small scale and the large scale tests and notes principal areas of uncertainty. Conclusions are also presented regarding the appropriateness of the formal statistical analysis for extrapolation to large weight.

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*This study was requested by the Future Studies Office, Kennedy Space Center, NASA. However, the opinions and conclusions expressed in the report are those of the authors.

1.2 Test Setup

Two principal failure modes were investigated:

1. CBM (confined by missile). The explosion occurs while propellants are still confined within the walls of the missile. The failure corresponds to a rupture in the bulkhead between oxidizer and propellants, caused, for example, by fragmentation or overpressurization. In the actual tests a star-shaped cutter ruptures a glass diaphragm separating the two compartments. At some appropriate time a cap or squib is used to ignite the mixture. The effects of ignition time and of geometric configuration--length-to-diameter ratio of tanks, L/D, and orifice diameter ratio, D_o/D_t--were investigated for 200 lb. tests.

2. CBGS (confined by ground surface). The explosion occurs after propellants are spilled onto the ground. This type of failure might arise from tank overpressurization or fall-back of vehicle onto pad. In the actual tests the tanks are ruptured by star cutters, the propellants forming overlapping pools on the ground. Various drop heights were used, primarily at 200 lb., in order to study the effect of impact velocity. (Test data for horizontal propellant flow direction, reversed propellants, and L/D variation are not analyzed in this report, since these variables were of secondary interest.)

A few high velocity sled tests at 200 lb. were conducted to simulate missile turnaround. Subsequent impact into either a hard or soft surface was simulated by a flat wall or deep hole, respectively.

For each of the above failure modes, two cryogenic propellants, LO_2/RP-1 and LO_2/LH_2, were tested. In the former case the LO_2 tank is above the RP-1 tank. For LO_2/LH_2, the position is reversed.

For each test, pressure gauges were installed at five distances, and along three different directions. From each pressure-time trace, URS determined the peak overpressure and positive impulse (Reference 1, volume 2).

For CBM the geometry variables were constant when weight was varied, and conversely. Likewise, for CBGS, velocity and weight variations were approximately non-overlapping. As shown in Appendix D, the weight analysis and the geometry (or velocity) analysis can be made separately* and the results combined, after simple correction, into a single equation.

---

*Assuming no interaction.

II. RATIONALE FOR THE REGRESSION ANALYSIS

Our approach to the analysis of yield as a function of weight was quite different from URS', and appears to reflect largely different conceptions of what type of result would prove to be most useful and what information can properly be extracted from the data. One might say that the URS analysis purports to provide conditional predictions of yield, given that ignition time is known. Their analysis focuses on how yield varies with ignition time (and other variables) for the small scale tests. To apply the results to large propellant weights, one would have to decide what is an appropriate ignition time to use. In contrast, our analysis purports to provide unconditional predictions of yield. The analysis focuses on how actual yield varies with weight, allowing for self-ignition.* Unfortunately, the nature of the data is such that one can obtain only rough approximations.

Project Pyro did not explicitly investigate self-ignition--almost all of their tests were small scale and employed a controlled ignition source. By postulating that ignition delay time scales with weight, URS is led to the conclusion that if explosions for large weights occurred at the most unfavorable ignition time, then the corresponding maximum yield would be a constant independent of weight. In one case this maximum exceeds 100%.

Actually the large scale Project Pyro tests resulted in yields that were clearly limited by more-or-less early self-ignition, in at least three of the four cases. Real experience suggests that this may in fact be the typical situation encountered in practice. The problem then to be faced was this: Since in general only a single representative self-ignited scale test was available, how could one use the controlled ignition small scale tests, these being the only other data available, to obtain a reasonable estimate of the dependence of (unconditional) yield upon weight.

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*Mention should be made of the work by E. Farber[2] which suggests that self-ignition would invariably occur for large weights because of electrostatic charges generated by the mixed propellants. When a critical mixed volume is reached, the difference in potential becomes sufficient to break down the gap between adjacent bubbles and the resulting discharge causes ignition. According to this theory, if all variables other than weight were held fixed, the explosive response would be a constant independent of weight, i.e., the yield would vary inversely with weight. Farber estimates the critical weight for LO_2/RP-1 at 2800 lbs. and for LO_2/LH_2 at 2300 lbs. At present, experimental confirmation of Farber's theory is lacking.

This is obviously not the type of situation one would prefer to be in, if one had any choice in the matter. There are obvious objections to combining the small and large scale tests. But the only real choice is between essentially concluding nothing (which we do in one case) and concluding something that can, at least heuristically, be considered reasonable, and may provide a useful, although rough, answer to an urgent problem.

There was another reason for adopting the approach which combines the large and small scale tests. The Project Pyro data contained severe limitations, in particular: very large scatter in the data, small number of tests in relation to the number of variables investigated, nonlinear behavior of the variables, and uncertainty whether optimum ignition conditions were obtained. Because of these limitations it was apparent that the data could not support a meaningful determination even of conditional yield. Moreover, even if such a function could be meaningfully determined, this does still not permit one to satisfactorily interpret the self-ignited large scale tests so as to obtain unconditional predictions of yield. These considerations led us to conclude that the most direct empirical statistical treatment was the best one could do with the Pyro data.

Because of the limitations noted above, URS' postulate of geometric scaling does not really help much, if indeed at all, in estimating the yield vs. ignition time relation. As it actually turns out, the data does not support geometric scaling.* For LO_2/LH_2, because of the large scatter the data contains essentially no information pertaining to geometric scaling; for CBGS LO_2/RP-1 no scaling is possible; and for CBM LO_2/RP-1, with the most consistent data, maximum yield definitely falls off with weight.

The statistical procedure adopted was simply a straightforward regression analysis. This is considered appropriate in three of the four cases where ignition time could be meaningfully bypassed. Before discussing this further, it will be useful to describe briefly, and in somewhat oversimplified fashion, the physical behavior of the propellants as it relates to ignition time t. (A more detailed discussion is found in References 1 and 2.)

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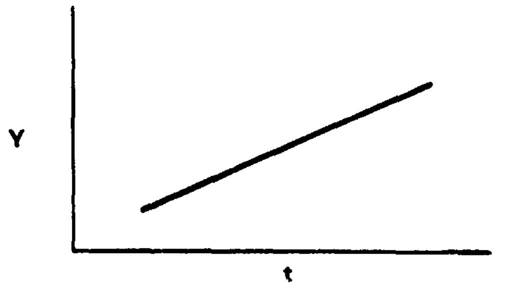
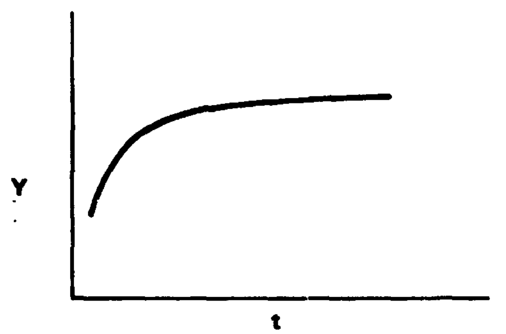
*For geometric scaling to hold, it should be possible to superimpose the 1000 lb. yields, plotted at $t/\sqrt[3]{5}$, onto the 200 lb. Y vs. t plot. Note that the conclusion that maximum yield is constant independent of weight requires merely that some scaling exists, not necessarily geometric. Superposition would then occur at kt, for some k.

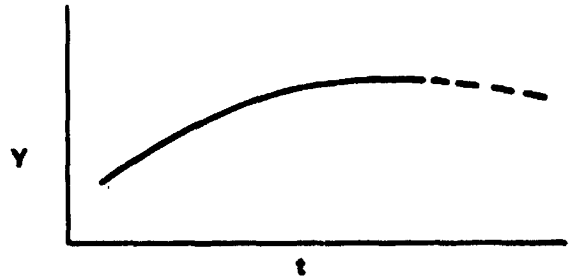
1. CBM, LO_2/RP-1: mixing increases with t while pressure builds up in tank due to evaporation of LO_2--ignition at time of tank rupture gives greatest yield Y.



2. CBM, LO_2/LH_2: mixing tends to stabilize due to freezing of LO_2, with only little increase in yield expected thereafter.*



3. CBGS, LO_2/LH_2: lower LO_2 pool spreads indefinitely so that superimposed LH_2 pool never completely overlaps the LO_2; yield tends to stabilize asymptotically (evaporation of LH_2 leads eventually to fall-off in yield).



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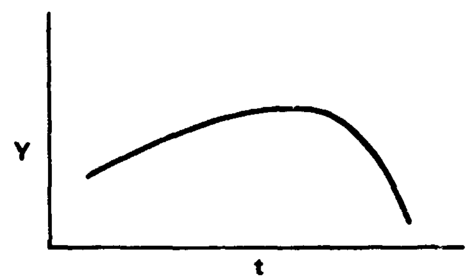
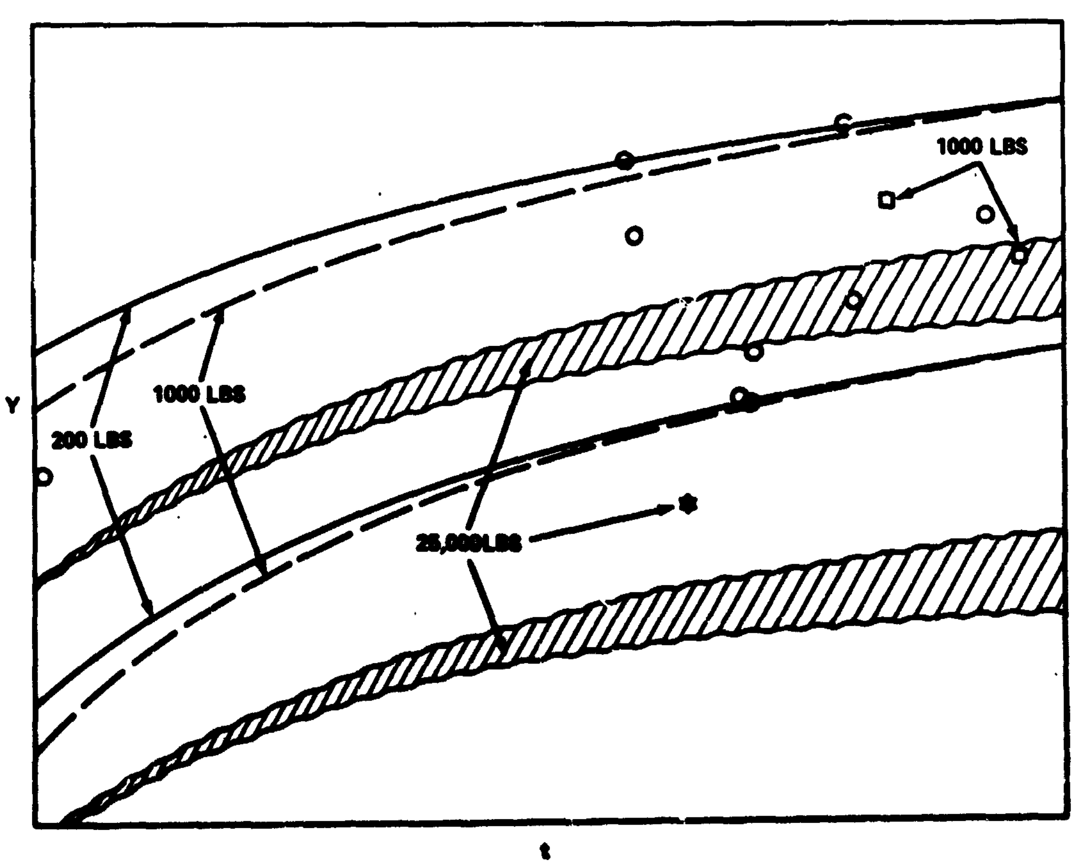
*The solidified propellants may also contribute to the blast effect.

4. CBGS, LO_2/RP-1: spreading of lower RP-1 pool is inhibited so that greatest mixing, and hence largest yield, occurs when upper LO_2 pool just overlaps the RP-1 pool.



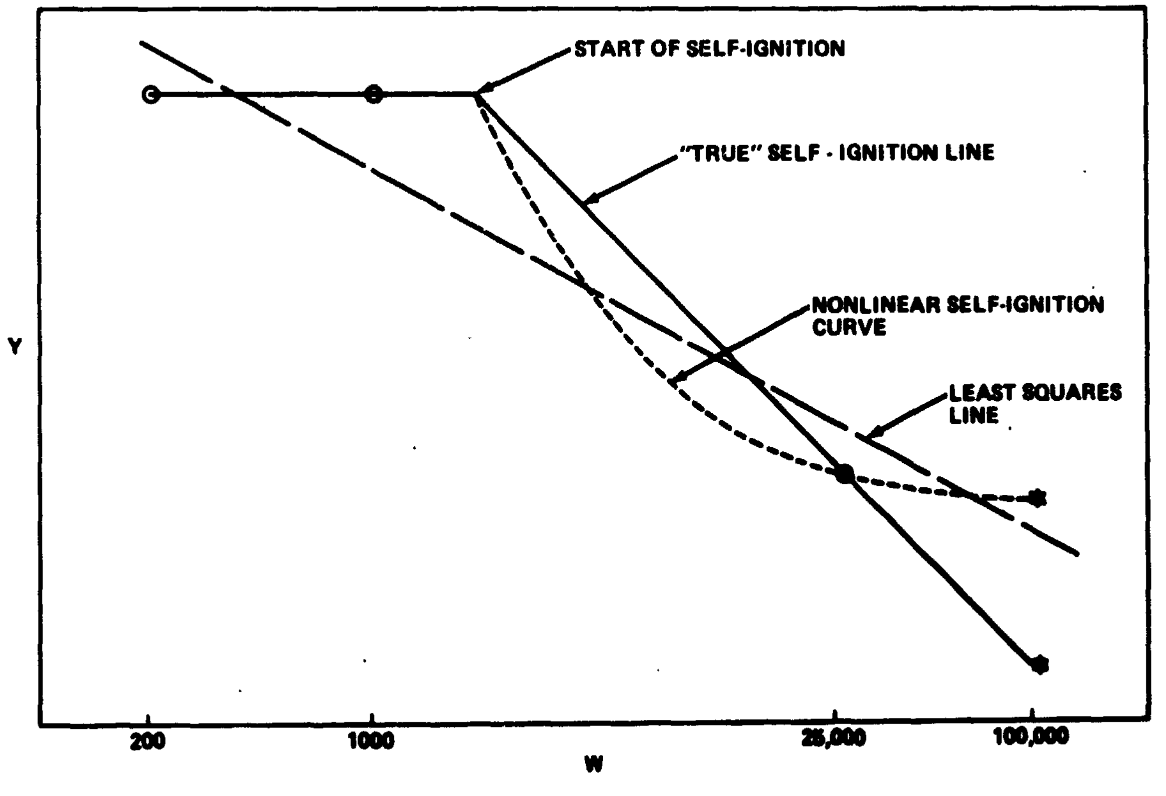
A heuristic justification of the regression analysis can be given to show that it provides meaningful, though rough, unconditional estimates of yield, which moreover are likely to be conservative. The discussion can best be carried out in the specific context of CBGS LO_2/LH_2. We are given in this case one "representative" large scale 25,000 lb. self-ignition test. The question is whether the small scale tests can be used to provide a rough estimate of a second "point" so as to provide some estimate of fall-off of yield.

The diagram below illustrates conceptually a plausible situation that one would infer from the actual data and from the



a priori physical considerations previously noted. Y increases with t; however, the band, representing the large scatter in the 200 lb. data, is so wide as to largely overshadow this effect. Assuming similar behavior at 1000 lbs., the two 1000 lb. tests would then be included within a similar band. One can envision also a third such band corresponding to the single 25,000 lb. test but with the added uncertainty relating to the shape of the band. The diagram shows quite clearly that it is hardly more meaningful to draw any inference from the data concerning how ignition time scales and where on the Y-t curve the 25,000 lb. test actually lies, than it is to treat the small scale tests as representative while at the same time ignoring ignition time for the 25,000 lb. test.

Consider now the question of bias in the regression. The diagram below plots the 200 and 1000 lb. observed yields (taking the average for simplicity) and also the single 25,000 lb. test. The solid line represents a hypothetical, but plausible, "true" relation between "representative" yield and



weight. If, after self-ignition "takes over", the behavior of self-ignited tests were approximately linear with weight (on log paper, say) then a hypothetical "representative" yield for 100,000 lb. would lie on this line. It is evident that then the least squares (dashed) line through the three observed "representative" points lies above the "true" line for large weights. In order for the 100,000 lb. yield to be such that the least squares line were below the self-ignition line, the self-ignition relation would have to be exceedingly nonlinear and convex, as shown by the dotted curve. This argument is admittedly heuristic, and clearly depends upon how one chooses to view the heuristics of self-ignition.

For CBM LO_2/LH_2, the situation is essentially the same, except that the small scale tests are so erratic that the resulting "representative" values for 200 and 1000 lbs. are very dubious.

For CBCS LO_2/RP-1 the scatter is much less, and the effect of ignition time in determining a maximum yield is more evident. Because of this, together with the fact that the 1000 lb. maximum yield greatly exceeds that for 200 lb., a representative value for the small scale tests cannot be selected. The regression is not considered meaningful.

For CBM LO_2/RP-1, no problem arises since the large scale tests all achieved tank rupture, and one need only include in the analysis the most closely related small scale tests.

III. SUMMARY OF ANALYSIS

1. CBM, LO_2/RP-1

1.1 Weight

The tests performed under nominal geometry (L/D=1.8, D_o/D_t=.45) consisted of seven 200 pound tests, four 1,000 pound tests, and three 25,000 pound tests. Ignition of the two non-spurious* 25,000 lb. tests was the result of tank overpressurization and failure. A 94,000 pound Titan with L/D=4 and D_o/D_t=.1375 was also tested, but the ignition source was unknown.**

The quality of the data is comparatively good. This can be seen from Figures 1P and 1I (upper portion for nominal geometry tests) which plot yield (Y_P or Y_I) versus ignition time t. For 200 lbs., although yield definitely increases with t, substantial scatter occurs for t $\gtrsim$ 125 msec. The effect of this is to provide a natural grouping of the data into high yield and low yield tests. The three high yield tests also correspond to tank rupture and/or maximum ignition time. For 1000 lbs., the scatter is considerable and the dependence upon t uncertain. For 25,000 lbs. the two high yield tests were almost identical, showing excellent reproducibility. Overall, the high yield tests, including the maximum yield 1000 lb. test, appear to constitute a statistically meaningful grouping.

For 200 lbs. the average impulse yield is about .9 of the pressure yield; for larger weights the two are approximately equal.

Yield versus propellant weight W is plotted in Figures 2P and 2I. The high yield tests are again seen to group naturally. The least squares log log regression lines for these tests has been determined both for Titan excluded as well as Titan included. The Titan yield was significantly low (3.3%). Although the geometry was different from the other tests, the 200 lb. tests conducted at non-nominal geometry and ullage did not establish valid

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*Spurious tests are those which encountered abnormal test conditions, usually resulting in premature self-ignition, e.g., diaphragm rupture, fire, possibly small orifice, etc.

**R. L. Thomas, AFRPL, Project Manager for Project Pyro, believes that ignition occurred after tank rupture.

correction factors for the Titan conditions (see 1.2 below). The regressions are given by:

	Titan Excluded	Titan Included
$\hat{Y}_P =$	$93.9W^{-.213}$	$150W^{-.290}$
$\hat{Y}_I =$	$66.6W^{-.173}$	$110W^{-.256}$

When the Titan is excluded, the prediction limits for the regression equations are small because of the comparatively good reproducibility of the high yield tests. When the Titan is included, the expected yields are of course decreased. However, the sharp falloff in yield between 25,000 and 94,000 lbs. leads to a poor linear fit. This increases the scatter about the line and hence also the prediction limits. For 95% probability, these limits can be written approximately (for large W) as*

	Titan Excluded	Titan Included
$Y^A_{P,.95} \sim$	$72.6W^{-.177}$	$71.0W^{-.194}$
$Y^A_{I,.95} \sim$	$45.9W^{-.121}$	$48.3W^{-.150}$

A comparison of observed and calculated yields for Titan and SIC, for 90% and 95% probabilities, is shown in the following table.

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*See Appendix D for the exact expression and for improved approximations. The exact prediction limits are asymptotic to the equations shown.

		Actual	Expected Titan Out	Expected Titan In	90% Prediction Limit Titan Out	90% Prediction Limit Titan In	95% Prediction Limit Titan Out	95% Prediction Limit Titan In
Titan	P	3.27	8.2	5.4	9.9	9.5	10.6	11.7
(94,000 lb.)	I	3.42	9.2	5.9	12.0	10.9	13.3	13.7
SIC	P	--	3.6	1.8	4.7	3.7	5.2	4.8
(4.6×10^6 lb.	I	--	4.7	2.2	6.8	4.9	7.8	6.5

The URS prediction equation for maximum ignition time is:

$$Y_{URS} = 12.5\ (1+217/W)$$

For W greater than 10,000 pounds, the yield is constant at 12.5%, with upper 90% prediction limit of 16%.

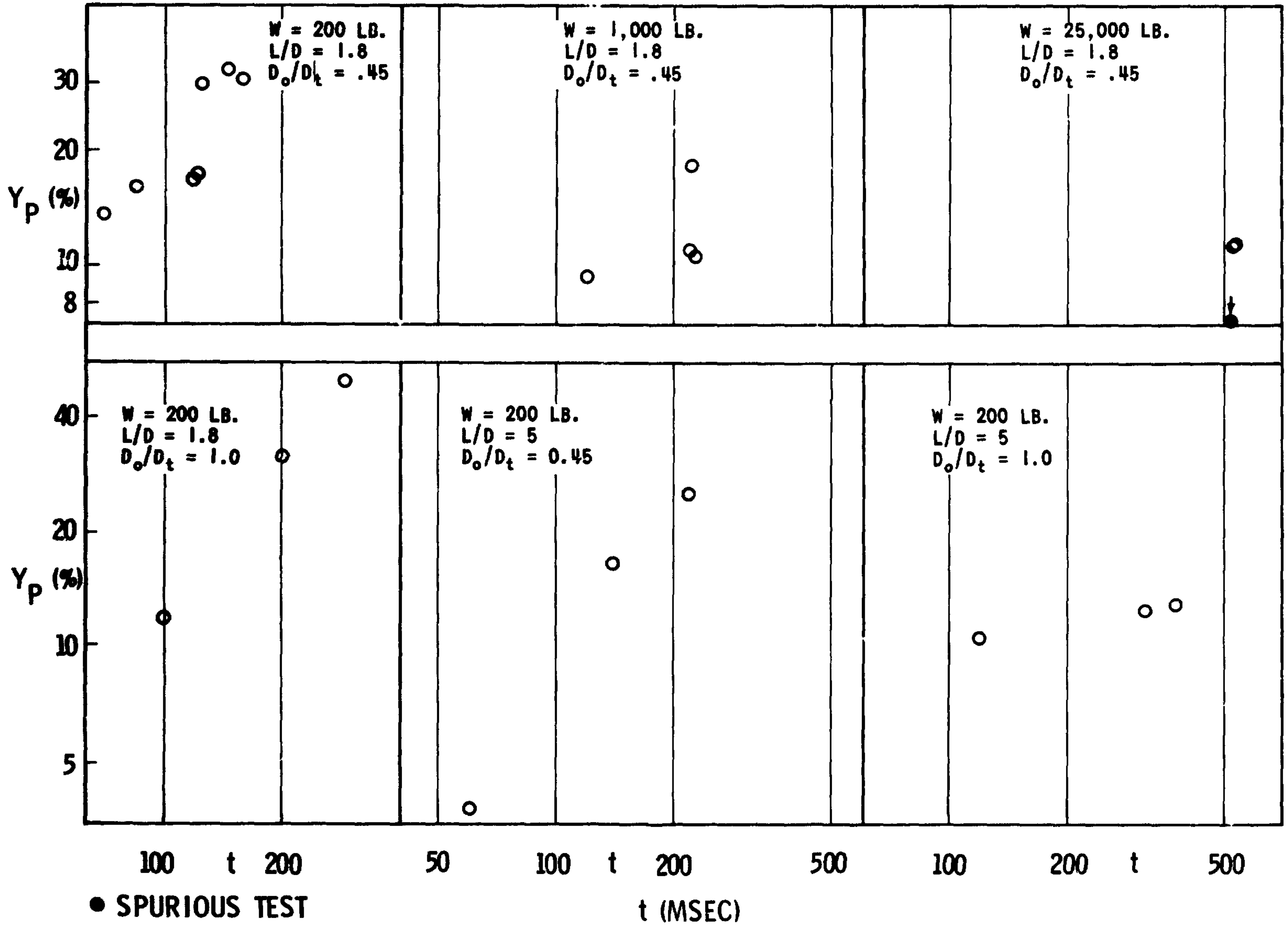

FIGURE 1P - CBM, LO_2/RP-1

OVERPRESSURE YIELD vs. IGNITION TIME t

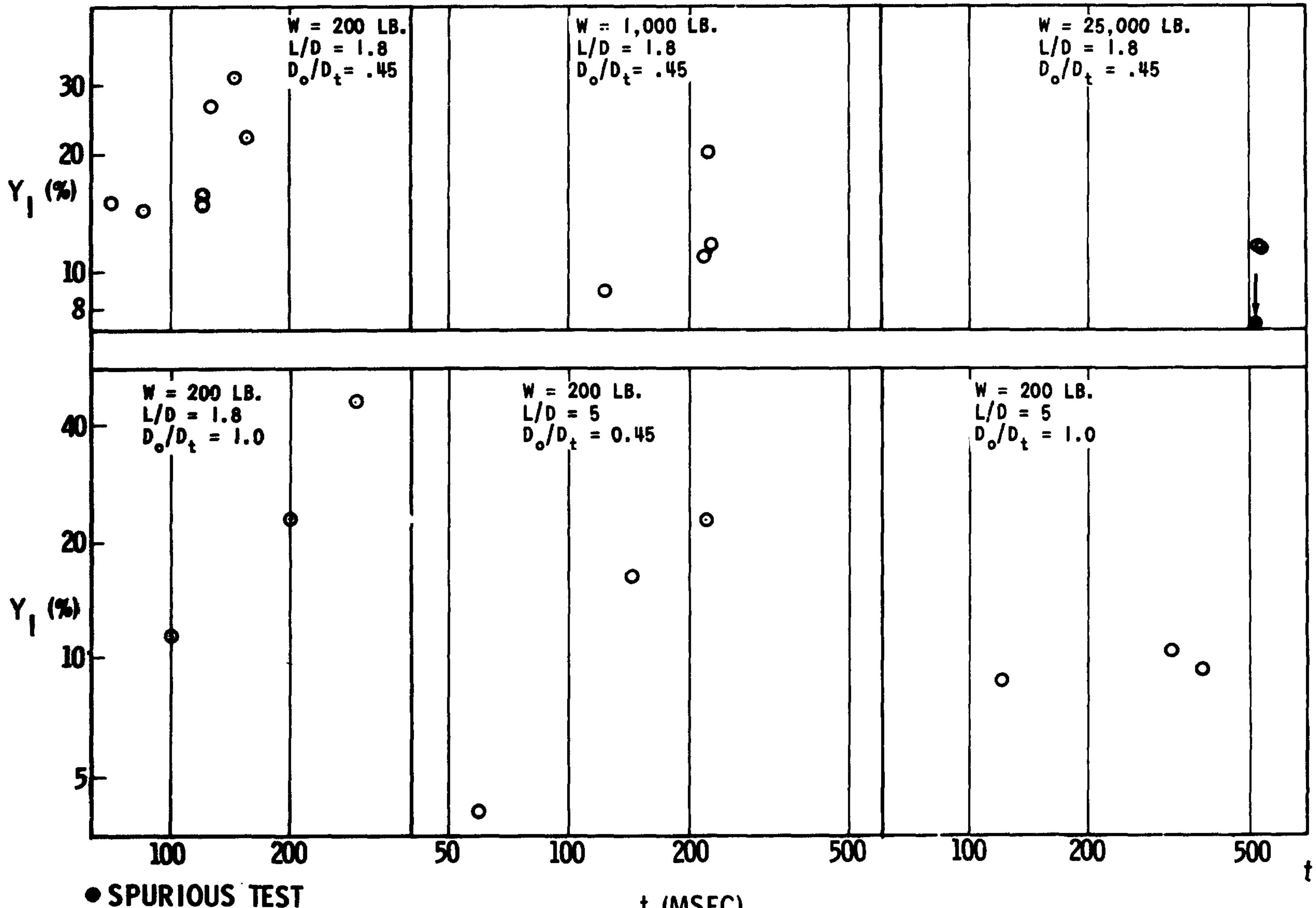

FIGURE 11 - CBM, LO_2/RP-1

IMPULSE YIELD vs. IGNITION TIME t

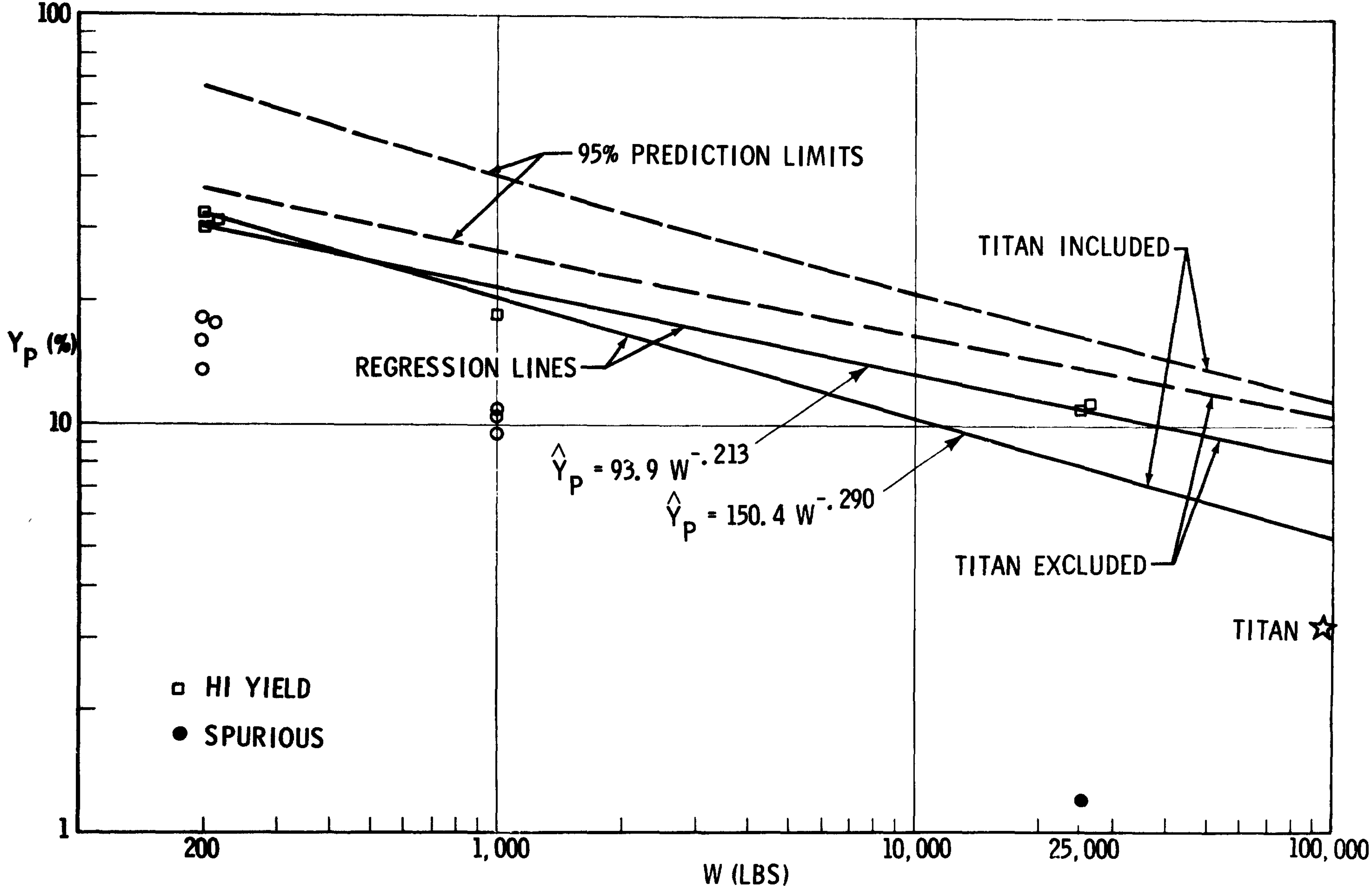

FIGURE 2P - CBM, LO_2/RP-1. REGRESSION OF PRESSURE YIELD ON WEIGHT--HIGH YIELD TESTS ONLY

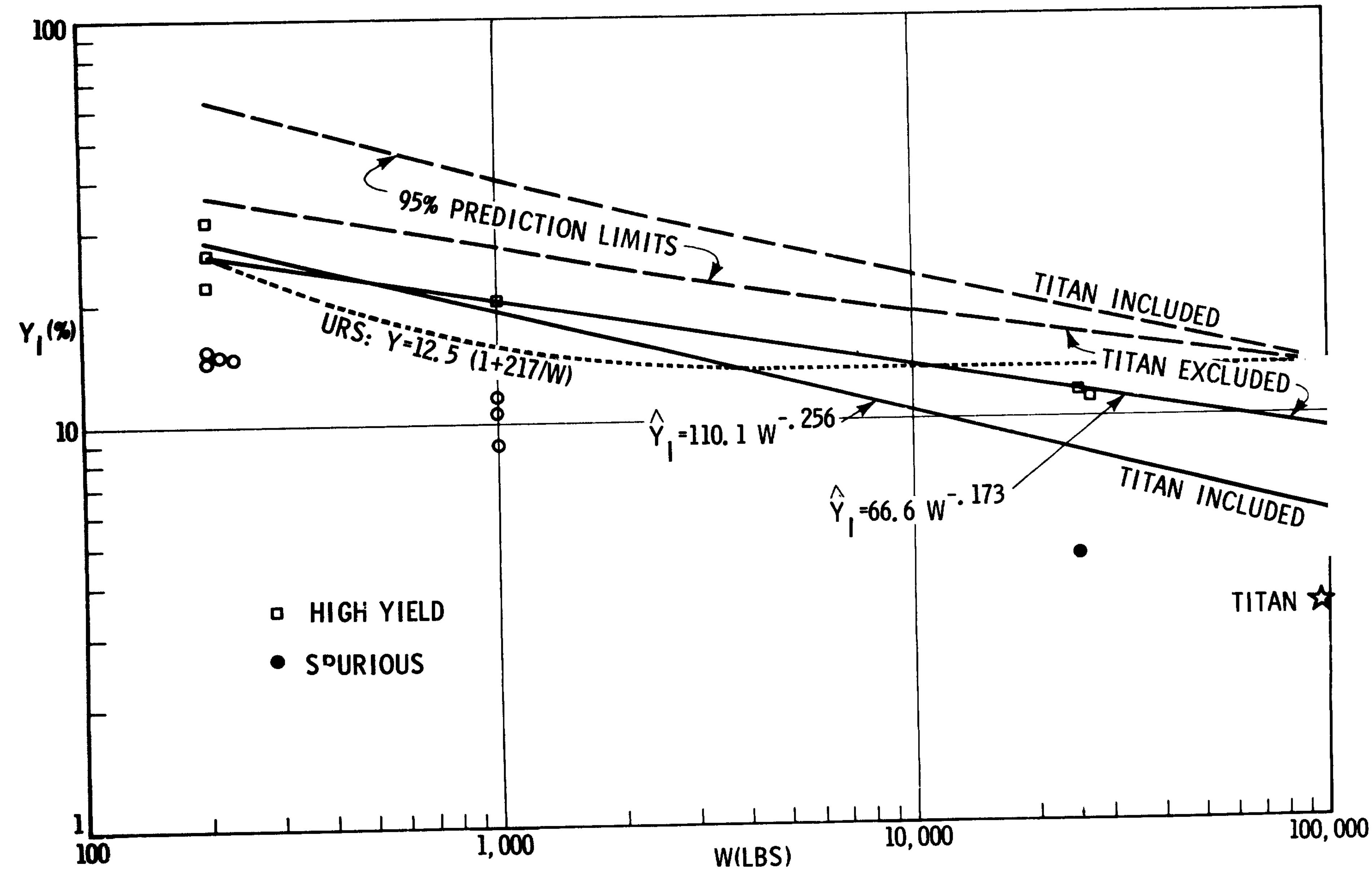

FIGURE 21 - CBM, LO_2/RP-1, REGRESSION OF IMPULSE YIELD ON WEIGHT--HIGH YIELD TESTS ONLY

1.2 Geometry

The effect of geometry variables was investigated by conducting three 200 lb. tests at each of the non-nominal conditions: L/D=.45, D_o/D_t=1.0; L/D=5.0, D_o/D_t=.45; and L/D=5.0, D_o/D_t=1.0. The results are shown in the lower portions of Figures 1P and 1I, which plot Y vs. t. The average impulse-to-pressure yield ratio of .85 is slightly less than for nominal geometry (200 lbs.).

The data appears to be fairly good. In all cases Y increases with t, although very slowly for L/D=5.0 and D_o/D_t=1.0. Also, t_{max}, the time to reach tank burst pressure, was longer than for the nominal case. In general, the maximum yield depends upon the mixing function (inferred from the Y vs. t plots) and on pressure buildup time (t_{max}). (For L/D=5.0 and D_o/D_t=1.0, mixing apparently was leveling off at 120 msec.) However, the data is not sufficient to infer quantitatively the physical characteristics of these processes. It seems appropriate, therefore, to restrict the analysis merely to maximum yields.

The yields are plotted in Figures 3P and 3I with maximum yields represented by stars. These are tabulated below:

D_o/D_t \ L/D		1.8	5.0
.45	P	31.5	25.3
	I	26.9	22.8
1.0	P	48.7	12.7
	I	45.3	9.8

Y clearly decreases as L/D increases. When L/D=1.8, Y increases markedly as D_o/D_t goes from .45 to 1.0; while for L/D=5.0, Y decreases markedly with D_o/D_t. Statistically, such a situation implies a large interaction effect. Straightforward analysis of the maximum yields in the preceding table leads to the following equation (equivalent to ordinary two way interpolation):

$$Y_P = 7.3 + 5.6\ L/D + 61.6\ D_o/D_t - 16.9\ L/D \cdot D_o/D_t$$

$$Y_I = -.3 + 6.8\ L/D + 65.6\ D_o/D_t - 17.8\ L/D \cdot D_o/D_t$$

Use of this equation for interpolation or extrapolation is not advisable without further data on the dynamics of mixing and pressure build-up. Indeed, for the Titan geometry conditions ($L/D=4$, $D_o/D_t=.1375$), the interaction term in the above equations just about offsets the other effects.

The two tests conducted at an ullage volume $V_u=40\%$ (tanks 2/3 full) increased the yield by 70 - 80% over that obtained at nominal 10% ullage (tanks full) (Figure 5). Assuming a log log linear relation, the specific yield (i.e., equivalent TNT weight relative to actual propellant weight) can be expressed approximately by the following equations:

$$Y_{S,P} = 161.5V_u^{.710}$$

$$Y_{S,I} = 130.7V_u^{.686}$$

To determine non-specific yield (i.e., relative to propellant weight for full tanks) the above equations are multiplied by the factor $(1-V_u)/.9$. When self-ignition occurs prior to bursting of the tank, apparently the ullage correction does not apply (see Appendix C.I.4).

URS' prediction equation for variable geometry and ignition time t, with W=200 lbs. and $V_u=10\%$, is

$$Y_{URS} = .373t(.87-.092\ L/D-.28\ D_o/D_t)$$

A detailed discussion of this equation, as well as URS' k-curves for variable ullage volume and tank burst pressure to be used with maximum ignition time, is presented in Appendix C.

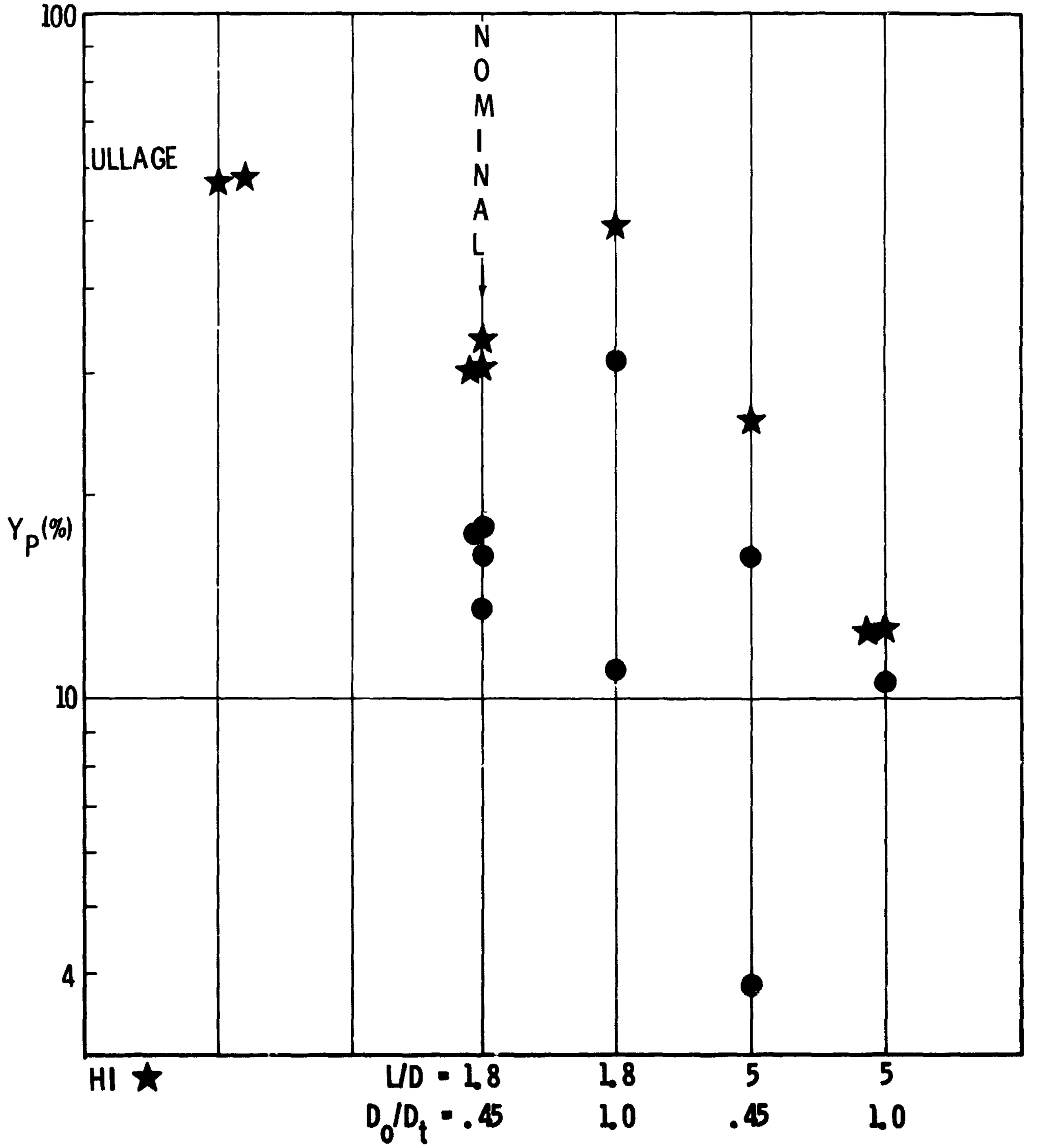

FIGURE 3P - CBM, LO_2/RP-1, 200 LBS

OVERPRESSURE YIELD vs. L/D, D_o/D_t AND ULLAGE

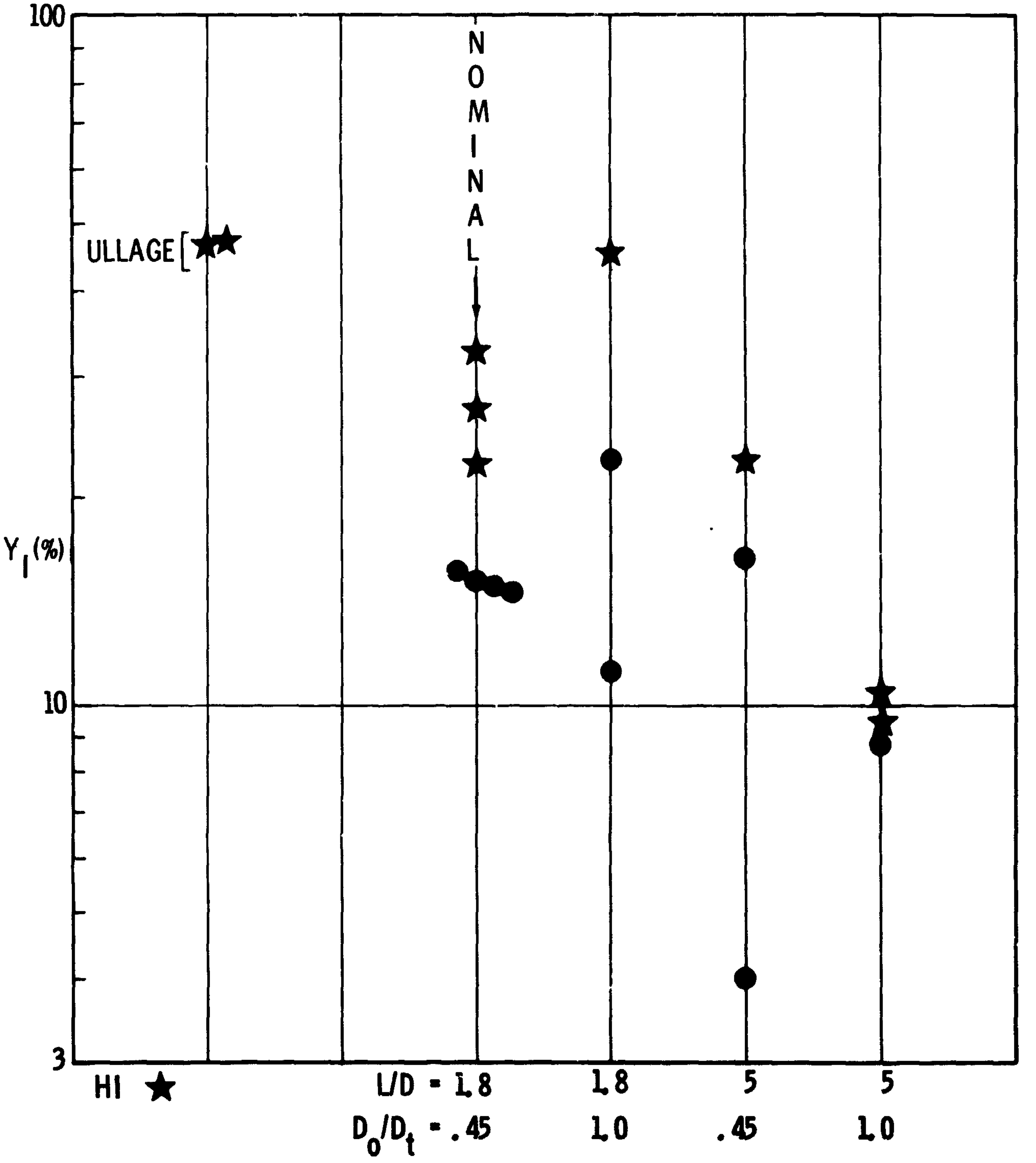

FIGURE 3I - CBM, LO_2/RP-1, 200 LBS

IMPULSE YIELD vs. L/D, D_o/D_t AND ULLAGE

2. CBM, LO_2/LH_2

2.1 Weight

Under nominal geometry conditions ($L/D=1.8$, $D_o/D_t=.45$), six tests were conducted at 200 lbs. and four tests at 1000 lbs. The three 25,000 lb. tests all gave extremely low yields and are not included. An SIV test, with $L/D=2$, $D_o/D_t=.083$, and W=91,000 lbs. was also conducted.

Figures 4P and 4I plot Y versus t. The principal characteristics of the data are as follows:

1. The data is highly erratic with essentially no reproducibility. The yields appear to be quite random.

2. Impulse yield is considerably greater than pressure yield, the average ratio being about 2.3:1 and with large scatter about the average. Apparently the pressure-time response was quite different from TNT, as well as from LO_2/RP-1.

3. Ignition time shows no consistent effect. At 200 lbs. several of the tests with very low (even zero) ignition time gave moderately high yields.

Yield versus W is plotted in Figures 5P and 5I. Yields for 200 lbs. do not differ significantly from 1000 lbs. although maximum pressure yield at 1000 lbs. was somewhat larger. The SIV pressure yield of 3.3% was the same as Titan, but impulse yield was 5.7%. SIV ignition time was only 183 msec. compared with 842 msec. for Titan.

With the SIV included, yield plausibly decreases with weight. However, because of the large scatter, the prediction limits are so large as to cast doubt on the meaningfulness of the statistically derived equations. The formal regression lines, using all non-spurious tests and including the SIV, are given by:

$$\hat{Y}_P = 17.6W^{-.148}$$

$$\hat{Y}_I = 61.4W^{-.190}$$

The 95% prediction limits are (for large W) eventually increasing. Asymptotically,

$$Y^A_{P,.95} \sim 2.8W^{.129}$$

$$Y^A_{I,.95} \sim 17.1W^{.002}$$

The expected yields are less than for LO_2/RP-1 but the prediction limits are larger. Calculated values for SIV and SII are shown in the table below.

		Actual	Expected	90% Prediction Limit	95% Prediction Limit
SIV (91,000)	P	3.26	3.3	15.2	26.2
	I	5.69	7.0	20.5	29.8
SII (930,000)	P	--	1.8	15.6	33.0
	I	--	3.3	14.8	24.8

The URS prediction equation is

$$Y_{URS} = \begin{cases} 2.8 + .82t/W^{1/3} & \text{for } t < 21.1W^{1/3} \\ 20 & \text{for } t \geq 21.1W^{1/3} \end{cases}$$

For unknown t, Y is 20% independent of W; the 90% prediction limit is 33%.

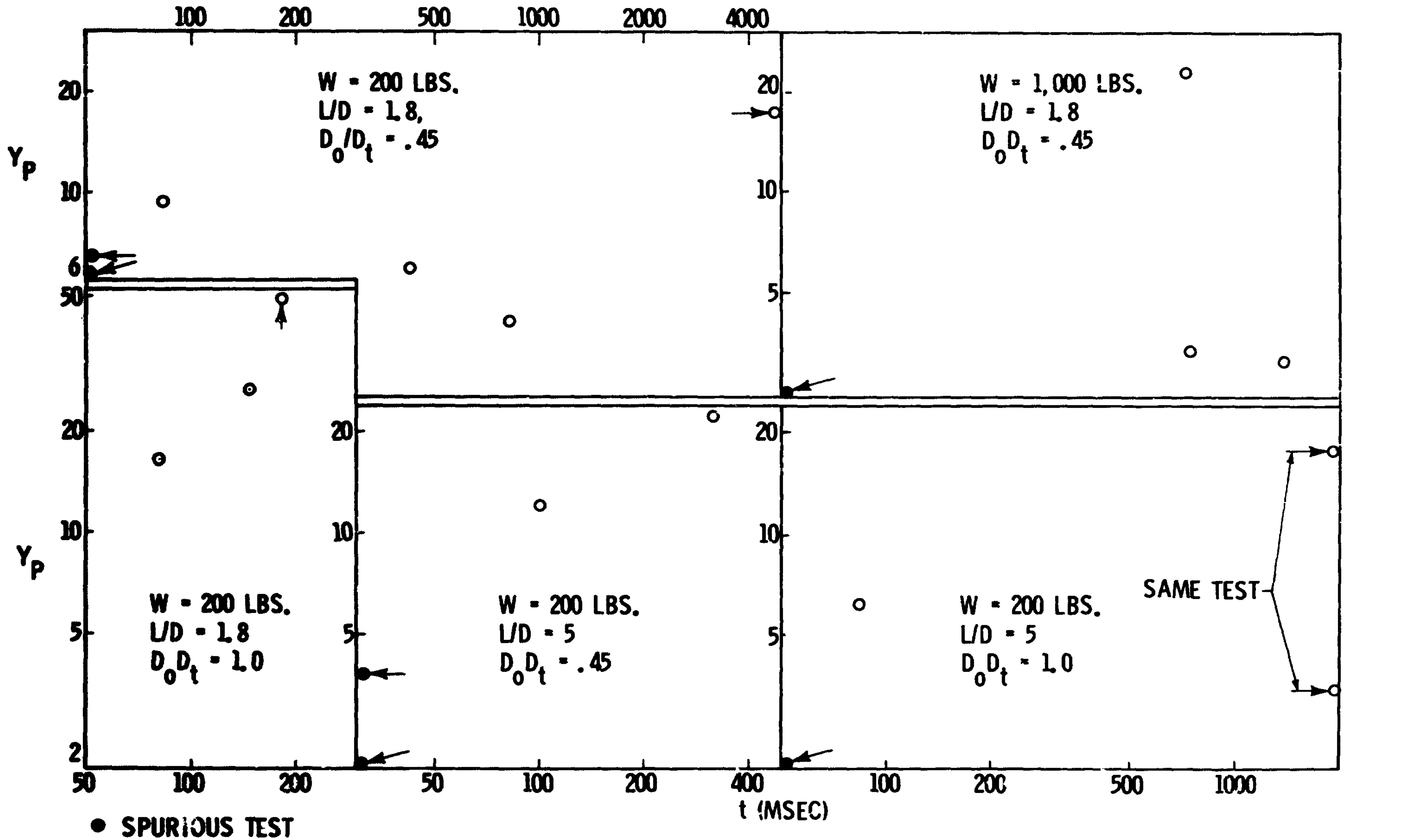

FIGURE 4P - CBM, LO_2/LH_2
OVERPRESSURE YIELD vs. IGNITION TIME

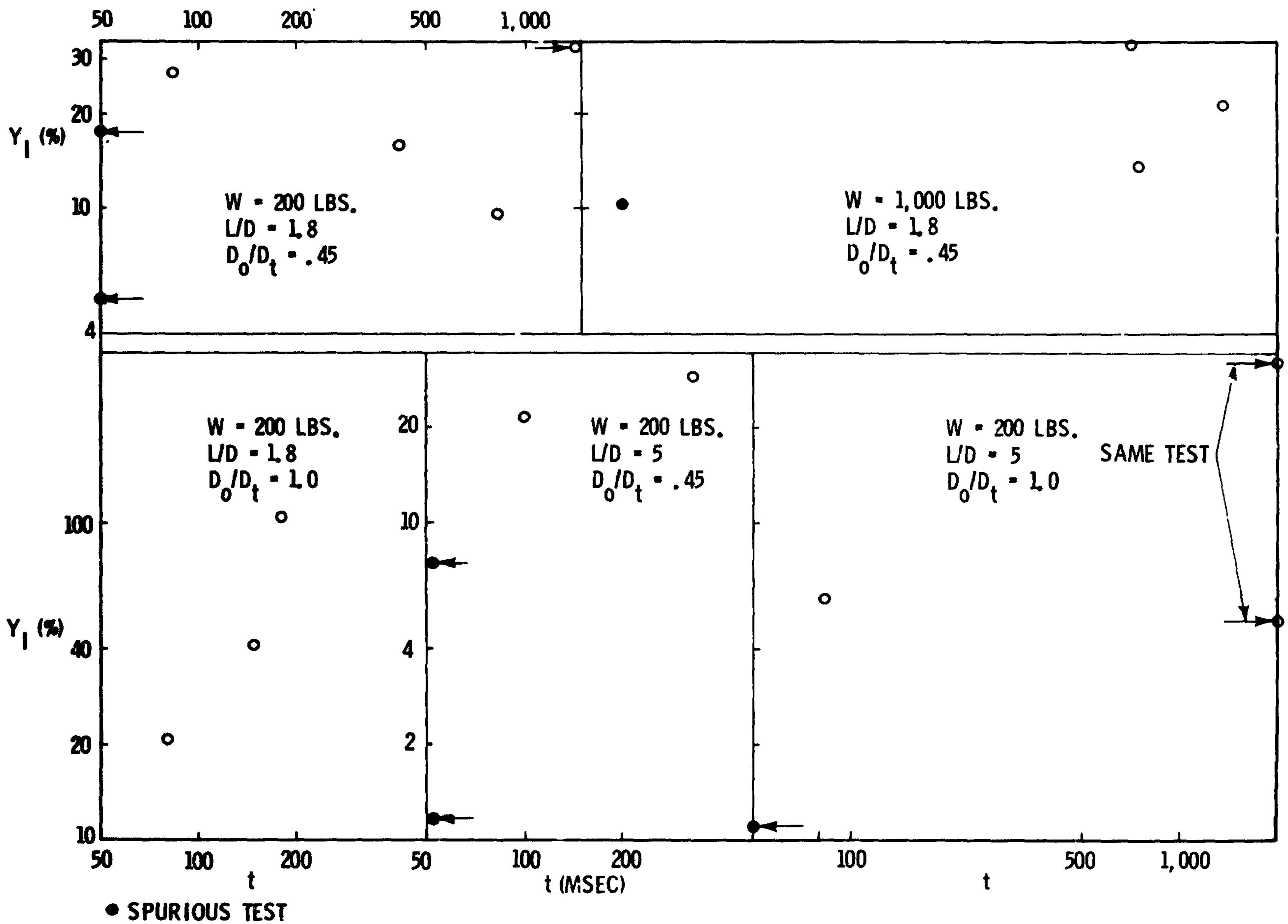

FIGURE 41 - CBM, LO_2/LH_2

IMPULSE YIELD vs. IGNITION TIME

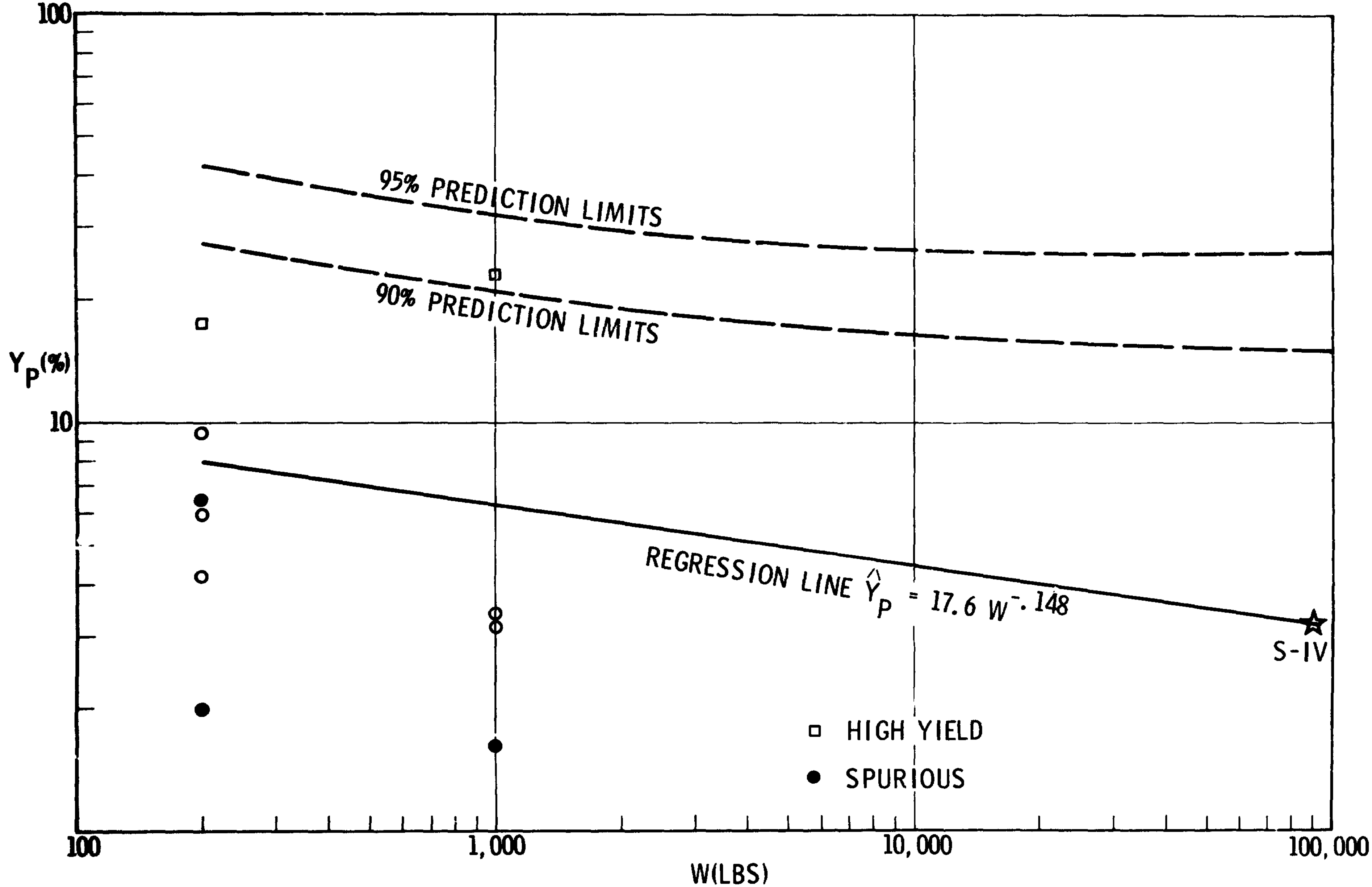

FIGURE 5P - CBM, LO_2/LH_2. REGRESSION OF PRESSURE YIELD ON WEIGHT -- ALL NON-SPURIOUS TESTS

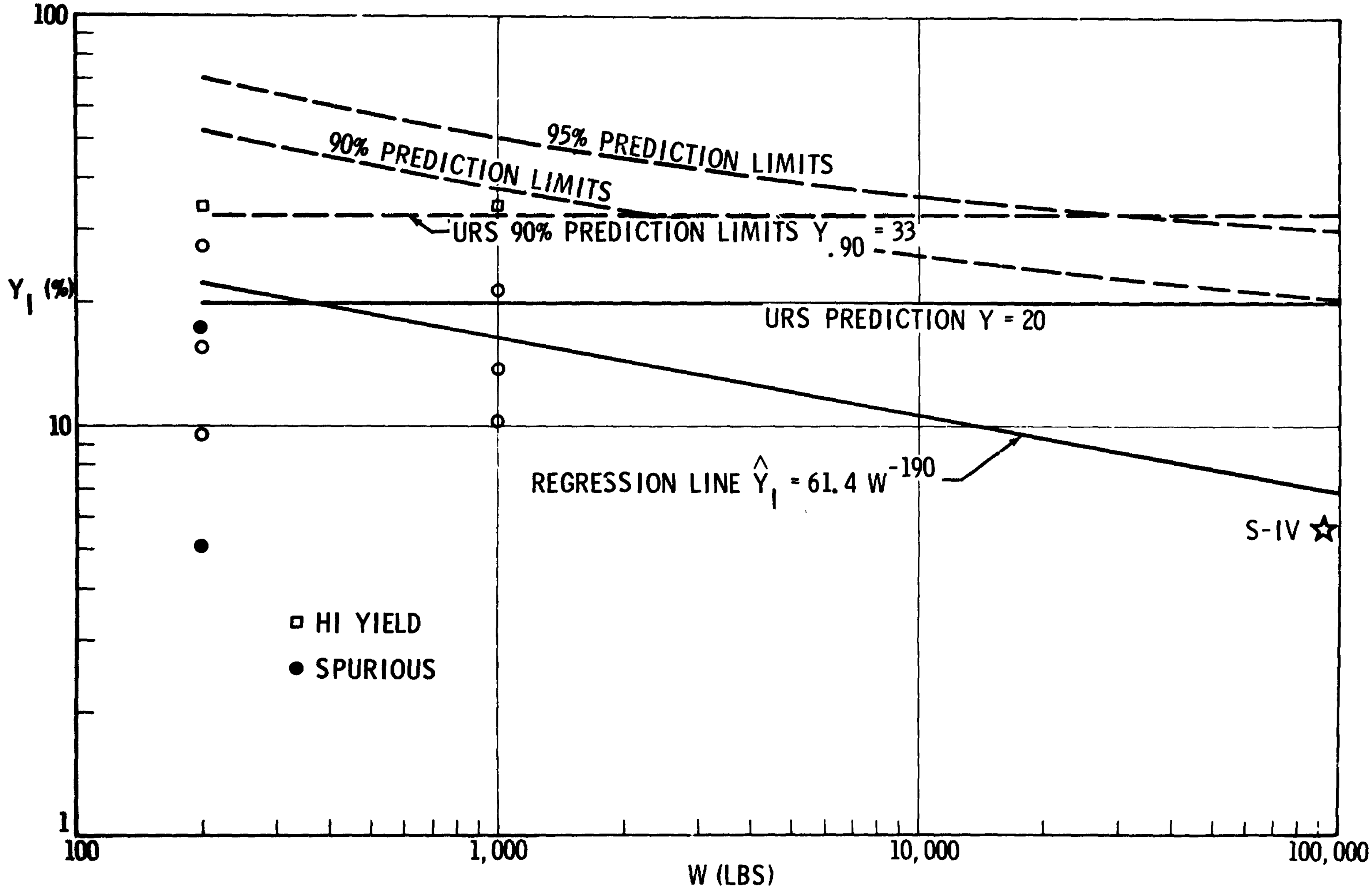

FIGURE 51 - CBM, LO_2/LH_2. REGRESSION OF IMPULSE YIELD ON WEIGHT--ALL NON-SPURIOUS TESTS

2.2 Geometry

The same geometry variations were investigated as for LO_2/RP-1, again with 200 lb. tests. The results are plotted versus ignition time in the lower portions of Figures 4P and 4I, and also in Figures 6P and 6I. Impulse yield averaged 1.6 times the pressure yield compared with 2.5 for nominal geometry (200 lbs.). The main characteristics are:

1. The data is extremely variable with about 4 of the 10 tests suspect. Three tests had very low ignition times and yields, and one test gave two explosions--the second resulted from LH_2 interaction with air and the yield was about five times greater.

2. Yield tends to increase with ignition time, more so than for nominal geometry. Consequently, comparison of the maximum yields, which are tabulated below, appears to be most appropriate.

D_o/D_t \ L/D		1.8	5.0
.45	P	17.5	21.8
	I	34.3	28.7
1.0	P	79.3	17.7
	I	104.5	32.7

3. Qualitative effects of the variables, when compared to yield for nominal geometry, are:

   a. For L/D=1.8, yield increases markedly for D_o/D_t=1. This is similar to LO_2/RP-1, but the effect is even greater. Two tests at D_o/D_t=.083 (scaled SIV) gave a 40-50% decrease (Y_P=9%, Y_I=20%).

   b. For D_o/D_t=.45, increasing L/D to 5 results in a small increase in pressure yield and a small decrease in impulse yield.

   c. When D_o/D_t=1 and L/D=5 the effect is uncertain. The yield is unchanged if the largest value of the double explosion is used; otherwise, the yield decreases greatly. In any case, because of (a) and (b), there is a large interaction effect similar to LO_2/RP-1.

d. Ullage volume of 40%, with Y_P=30% and Y_I=36.6%, shows a substantial (71%) increase in pressure yield but essentially no increase in impulse yield. However, in terms of specific yield, which is 50% greater, even impulse yield increases significantly.

Because of the many large uncertainties, even more than for LO_2/RP-1, an overall quantitative regression equation (Appendix C.II.3.2) does not appear meaningful.

URS concluded that, except when L/D=1.8 and D_o/D_t=1.0, the geometry variables had no significant effect.

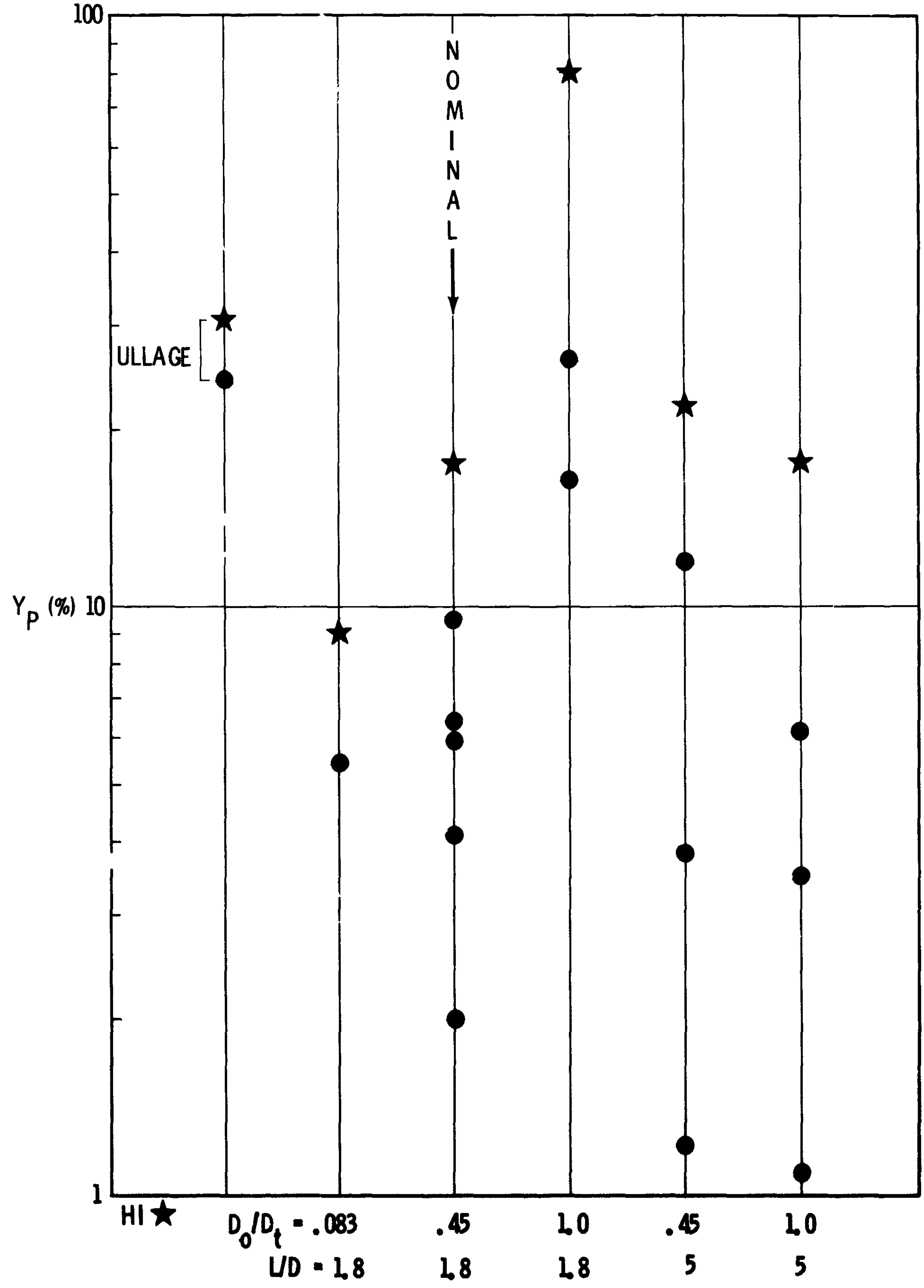

FIGURE 6P - CBM, LO_2/LH_2, 200 LBS

OVERPRESSURE YIELD vs. L/D, D_o/D_t AND ULLAGE

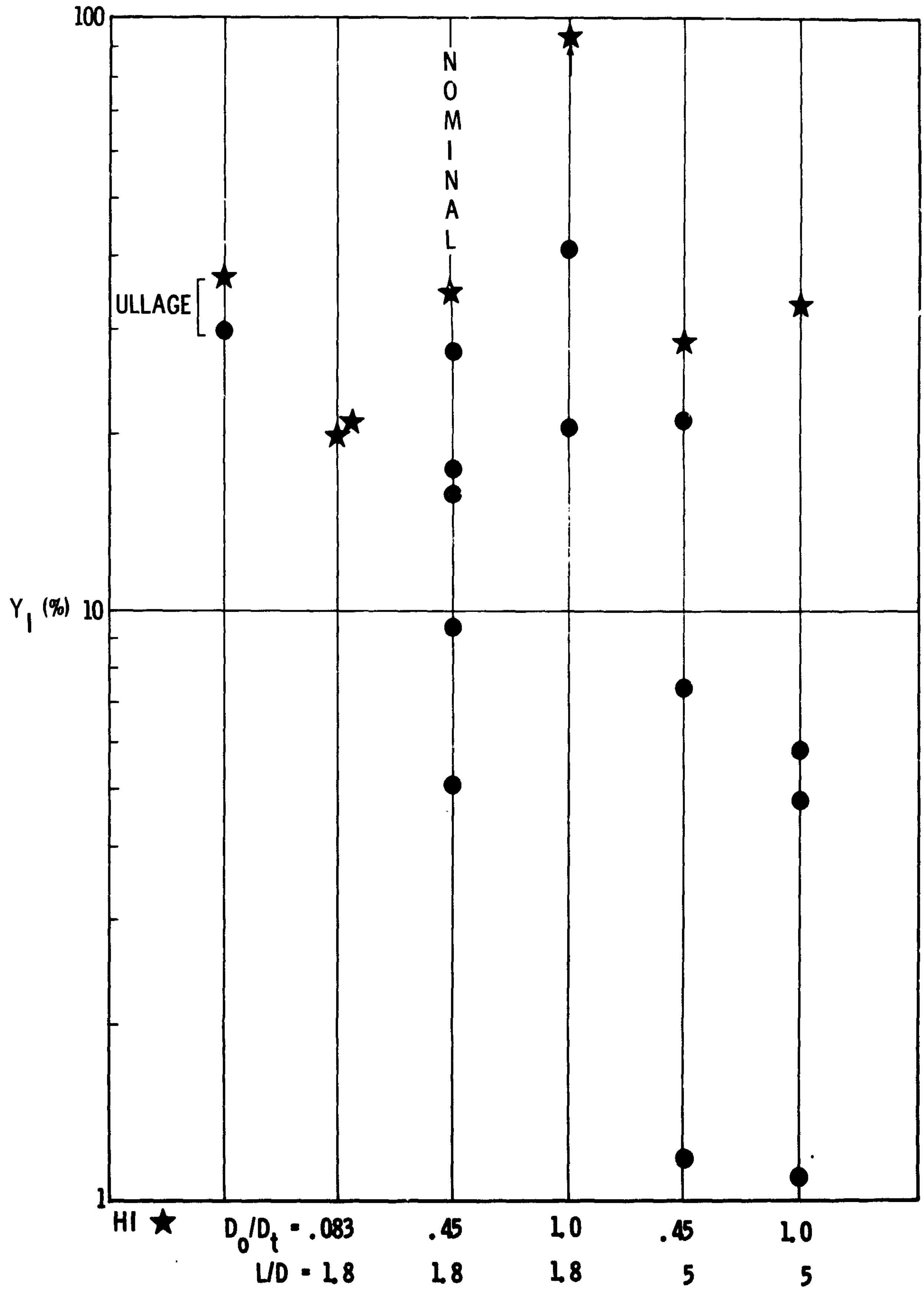

FIGURE 61 - CBM, LO_2/LH_2, 200 LBS
IMPULSE YIELD vs. L/D, D_o/D_t AND ULLAGE

3. CBGS, LO_2/LH_2

3.1 Weight, V=44 ft/sec.

A total of 17 tests were conducted with this propellant in normal orientation and L/D=1.8. These consisted of nine tests at 200 lbs., five tests at 1000 lbs. and three at 25,000 lbs. There is a moderate discrepancy between the pressure and impulse yields. The average Y_I/Y_P ratio is about 1.5 (Table A3).

Only one of the 200 lb. tests was spurious; on the other hand, three of the five 1000 lb. and two of the three 25,000 lb. tests were listed as spurious. The remaining 11 tests form the primary data used to study the relationship between yield and weight when V=44 ft/sec.

The eight nonspurious 200 lb. tests show essentially no correlation between yield and ignition time (cf. Figures 7P and 7I). Further, the ignition times can be grouped into four distinct sets at approximately 300, 500, 800 and 1400 msec. At these times the observed yields oscillate from high to low to highest to medium with a considerable amount of scatter at each time. Referring to Figure 7P it is seen that without the two low yield tests at 500 msec the remaining tests appear to indicate that the "asymptotic maximum yield" (cf. Section II) has been reached, even though its value is masked by the erratic nature of the data. However, there are no physical grounds for dropping these two low yield tests from the analysis; hence, the yields associated with 200 lb. tests may be low. It is clear from Figures 8P and 8I that higher yields than were actually observed at 200 lbs. would only result in a more rapid decrease of yield with increasing weight.

At 1000 lbs. the relative difference between the impulse yields of the two nonspurious tests is about 11%. Since the ignition times of the two tests are 900 and 1490 msec, it is probable that we are in the stable yield-pool diameter region of the LO_2/LH_2 Y,t-curve. The fact that the lowest yield occurs at 1490 msec could be due simply to scatter or to the eventual decay arising from evaporation of LH_2 (cf. Y,t prototype curve in Section II). This behavior is not as apparent in the pressure data because of test No. 217, but the pressure data for this is erratic and the scatter about the Kingery curve* is large.

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*Kingery fits are discussed in Appendix A.

Finally, the single 25,000 lb. test is self-ignited with t=365 msec.

The regressions of yield against weight for the nonspurious tests are given in Figures 8P and 8I. The expected yield for pressure and impulse falls off with weight according to $W^{-.172}$ and $W^{-.157}$, respectively:

$$(3.1) \qquad \hat{Y}_P = 73.8\ W^{-.172} \quad \text{and} \quad \hat{Y}_I = 72.1\ W^{-.157} \ .$$

Because of the large scatter at 200 lbs. the upper 90% prediction bounds are large and decrease slowly for large W (cf. Figures 8P, 8I and Table C15).

Although this group of 11 tests appears to be the most meaningful statistically, two other groupings were considered and regressions calculated (Appendix C). Both showed expected yield and upper 95%, as well as 90%, prediction limits strongly decreasing with increasing weight (cf. Figures C7, C8, C9, C10 and Table C15).

The following table shows numerical values of the expected impulse and pressure yields from equation (3.1) and their upper 95% (90%) prediction limits (cf. Figures 8P and 8I for a comparison with the observed values).

W (lbs.)	200	1000	25,000	SII (930,000)
$\hat{Y}_I$	31.4	24.4	14.8	8.4
$\hat{Y}_P$	29.7	22.5	12.9	6.9
$Y_{I,.95}(Y_{I,.90})$	68.6 (56.6)	53.3 (44.1)	40.1 (31.4)	34.8 (24.5)
$Y_{P,.95}(Y_{P,.90})$	68.2 (55.6)	42.3 (51.9)	37.6 (28.9)	31.8 (21.9)

The upper 90% prediction limit asymptotes are $Y^A_{P,.90}=(34.7)\ W^{-.047}$ and $Y^A_{I,.90}=(35.6)\ W^{-.039}$. The value of the pressure (impulse) asymptote at W=930,000 lbs. is 18.0% (20.1%).

The URS prediction equation when ignition time is unknown is a function of velocity only and is given by

$$Y_{URS} = 18.4 + 0.016\ V^2 \quad .$$

This equation gives an expected yield of 49.4% at V=44 fps independent of weight. The corresponding upper 90% prediction limit is approximately 75%.

The URS prediction equation in the case when ignition time is known is given in Section III.4 of Appendix C together with a table, C18a, comparing the yields observed by URS with their predicted yields.

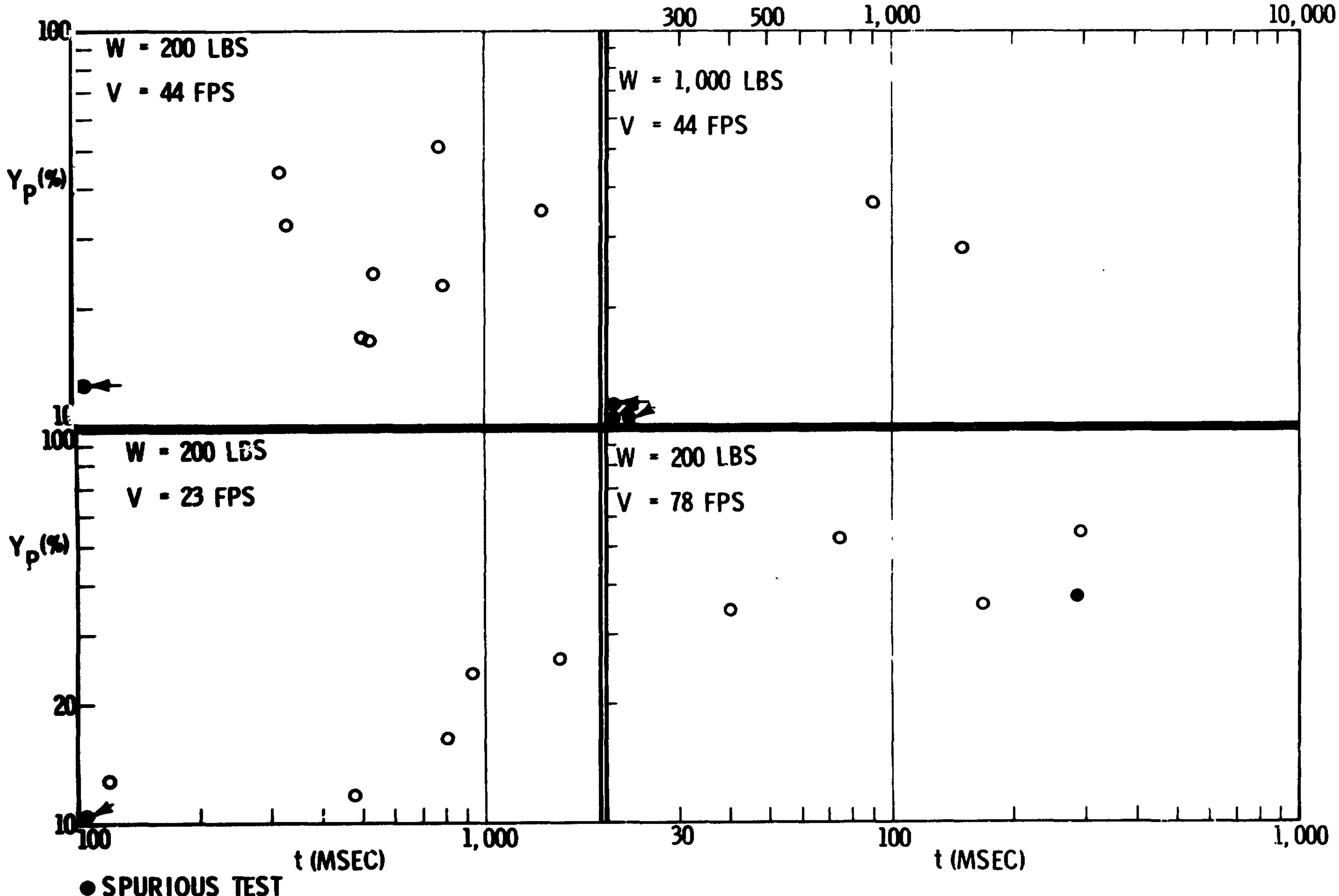
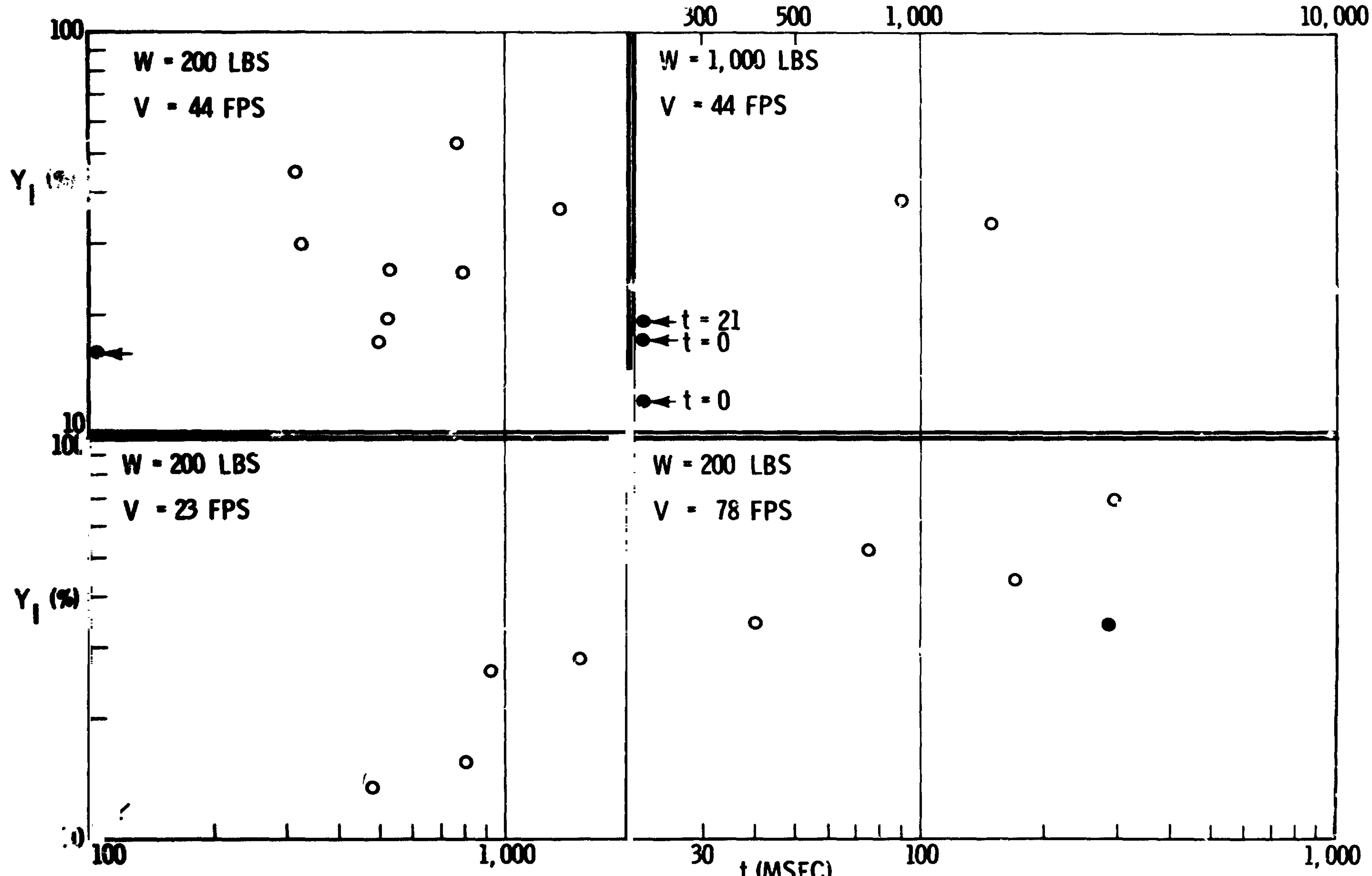


FIGURE 7P - CBGS, LO_2/LH_2, PRESSURE YIELD VS. IGNITION TIME



● SPURIOUS TEST

FIGURE 71 - CBGS, LO_2/LH_2, IMPULSE YIELD VS. IGNITION TIME

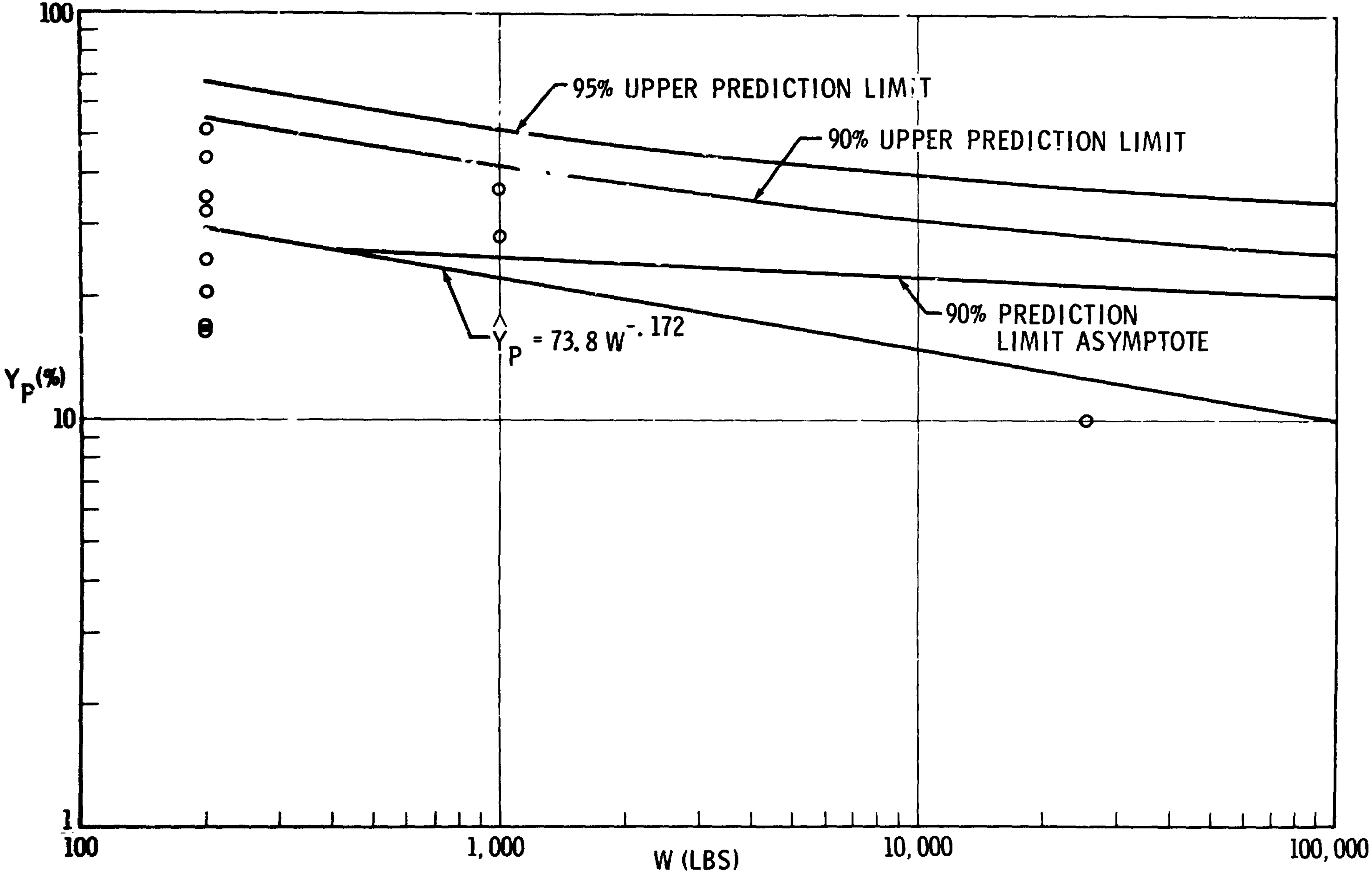


FIGURE 8P - CBGS, LO_2/LH_2, REGRESSION OF PRESSURE YIELD ON WEIGHT

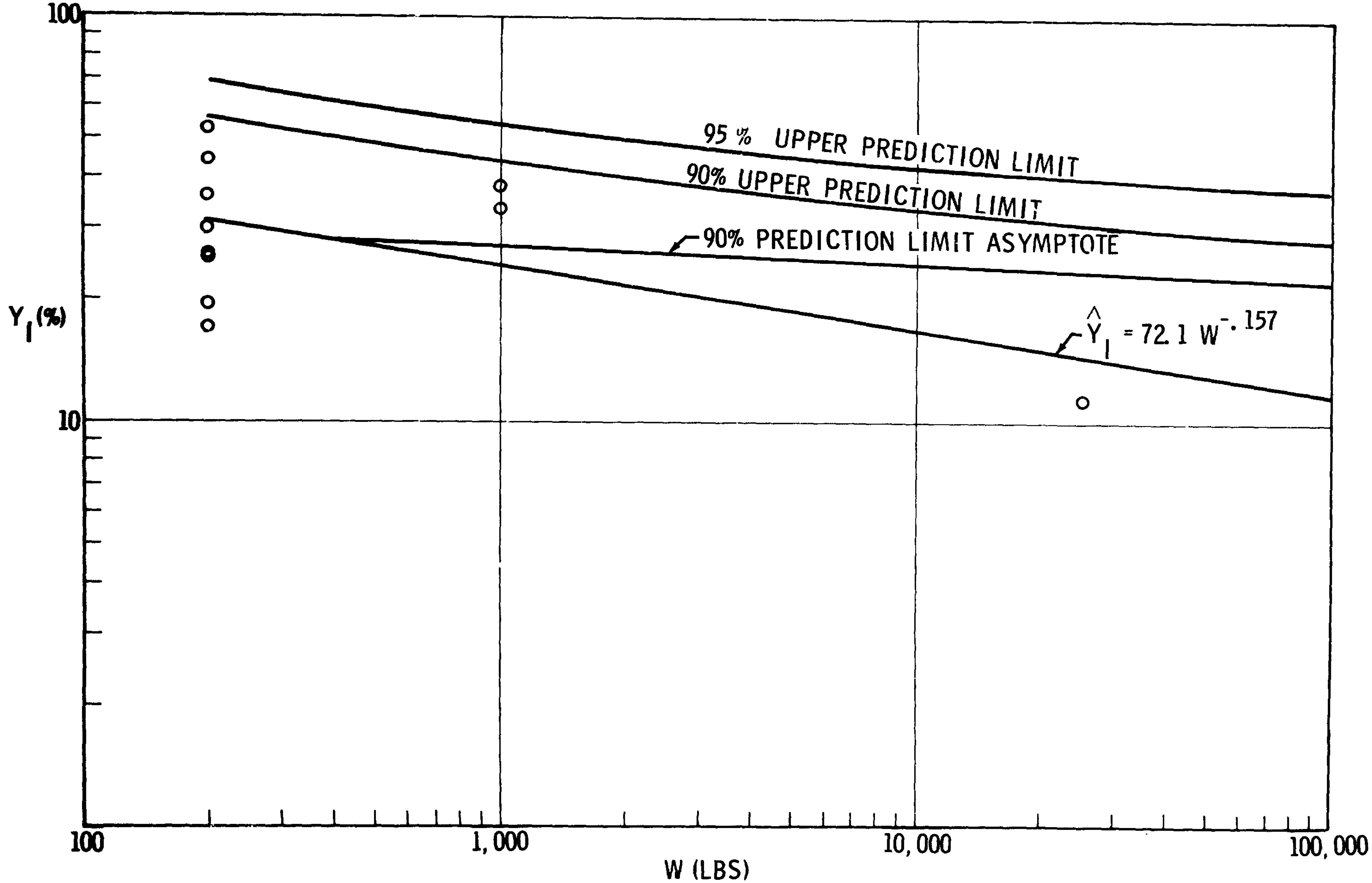


FIGURE 81 - CBGS, LO_2/LH_2, REGRESSION OF IMPULSE YIELD ON WEIGHT

3.2 Velocity with W=200 lb.

There are 17 nonspurious (drop) tests at 200 lb.: five at 23 ft/sec., eight at 44 ft/sec. and four at 78 ft/sec. In fact, test data at weights other than 200 lbs. is available only at 44 ft/sec. (Two 1000 lb. tests at 78 fps are both spurious.) Correlation between yield and ignition time is very good at 23 ft/sec.; this can be observed in Figures 7P and 7I. The correlation at 78 ft/sec. is not as strong and the confidence intervals are large, especially for the overpressure. That the correlation between Y_I and t is somewhat better than for Y_P and t is due primarily to test No. 151.

From Figures 9P and 9I it can be seen that there is a considerable amount of scatter at each of the velocities. The regression lines given there have

(3.2) $$\hat{Y}_P = 2.1\ V^{.680} \quad \text{and} \quad \hat{Y}_I = 2.0\ V^{.721} ,$$

with the upper 90% prediction limit asymptotes having exponents .963 and .995, respectively.

The following table gives several evaluations of equations (3.2) and the URS prediction equation for unknown ignition time, together with the corresponding values of the upper 90% prediction limits.

V (fps)	23	44	78
Y_{URS}	26.9	49.4	115.7
$Y_{URS,.90}$	47.0	75.0	—
$\hat{Y}_I$	19.2	30.7	46.4
$\hat{Y}_P$	18.3	28.4	42.0
$Y_{I,.90}$	32.2	50.1	77.7
$Y_{P,.90}$	31.3	47.1	71.5

Finally, although complete knowledge about ignition sources is lacking, there is some evidence that ignition time (or, at least, the ignition time for optimal yield) decreases with increasing velocity. This presents the possibility that the zero ignition high velocity impact (HVI) tests may be considered as a limiting case of the tower drop tests and suggests that the two types of tests be combined. (URS combines the HVI tests with the zero ignition tests regardless of weight.)

However, the principal justification for combining at least the flat wall HVI test with the drop tests is that the resulting prediction equations agree well with the drop test equations over the appropriate range of velocities and at the same time more accurately reflect the observed behavior of the flat wall HVI test. The resulting regression equations are

$$\hat{Y}_{P,HVI} = 3.9\ V^{.519} \quad \text{and} \quad \hat{Y}_{I,HVI} = 4.2\ V^{.524} ,$$

with upper 90% confidence limits on the slope of .677 and .679, respectively. The comparison of the extrapolated drop test with drop test plus flat wall prediction equations mentioned above is indicated in the following table for overpressure yield (Figures 10P, 10I show the observed values):

V (fps)	23	44	78	160	597
$\hat{Y}_P = 2.1\ V^{.680}$	18.3	28.4	42.0	68.4	167.4
$Y_{P,HVI} = 3.9\ V^{.519}$	20.0	28.0	37.3	54.8	108.5

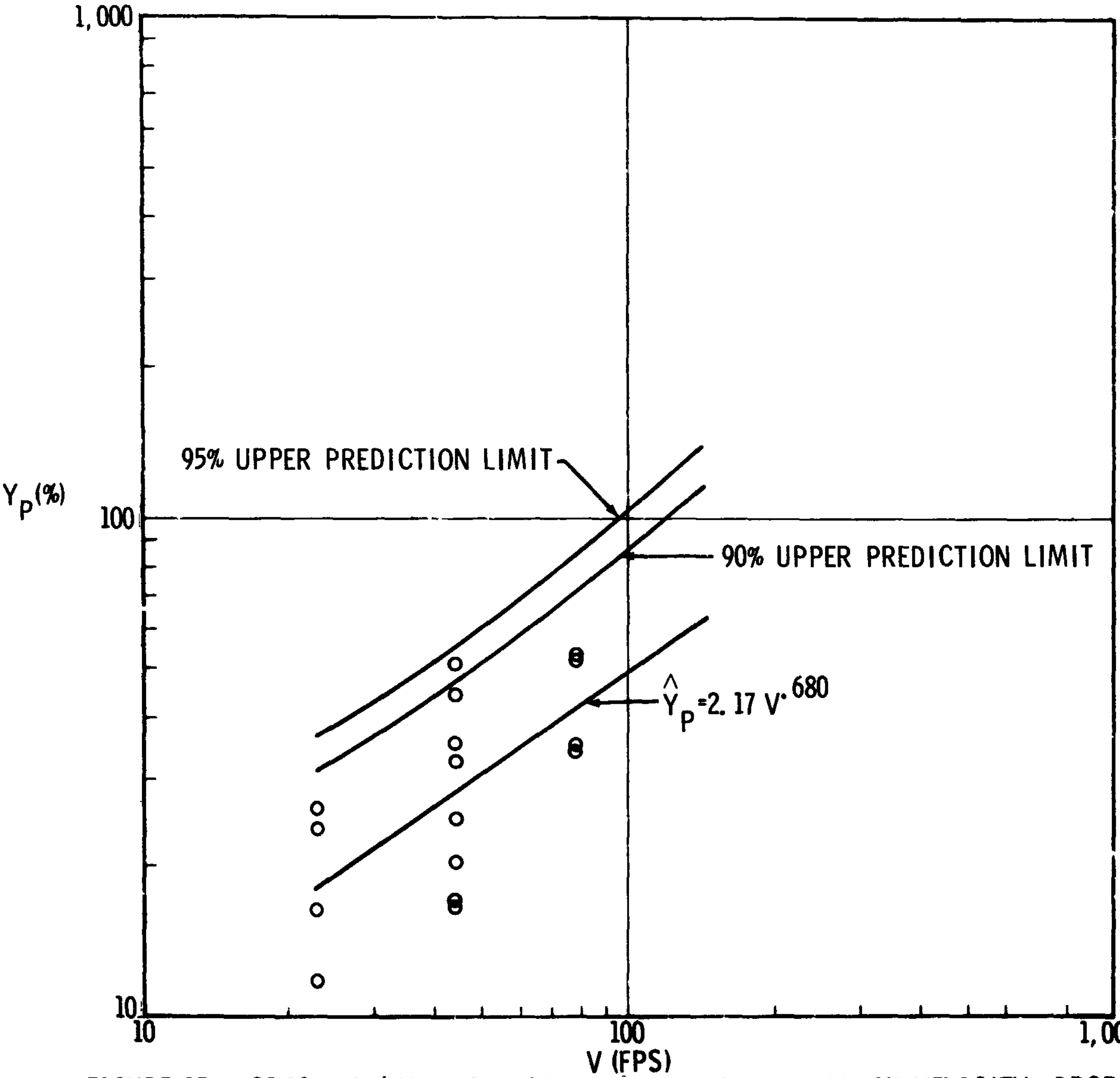


FIGURE 9P - CBGS, LO_2/LH_2, REGRESSION OF PRESSURE YIELD ON VELOCITY, DROP TESTS. (W = 200 LBS)

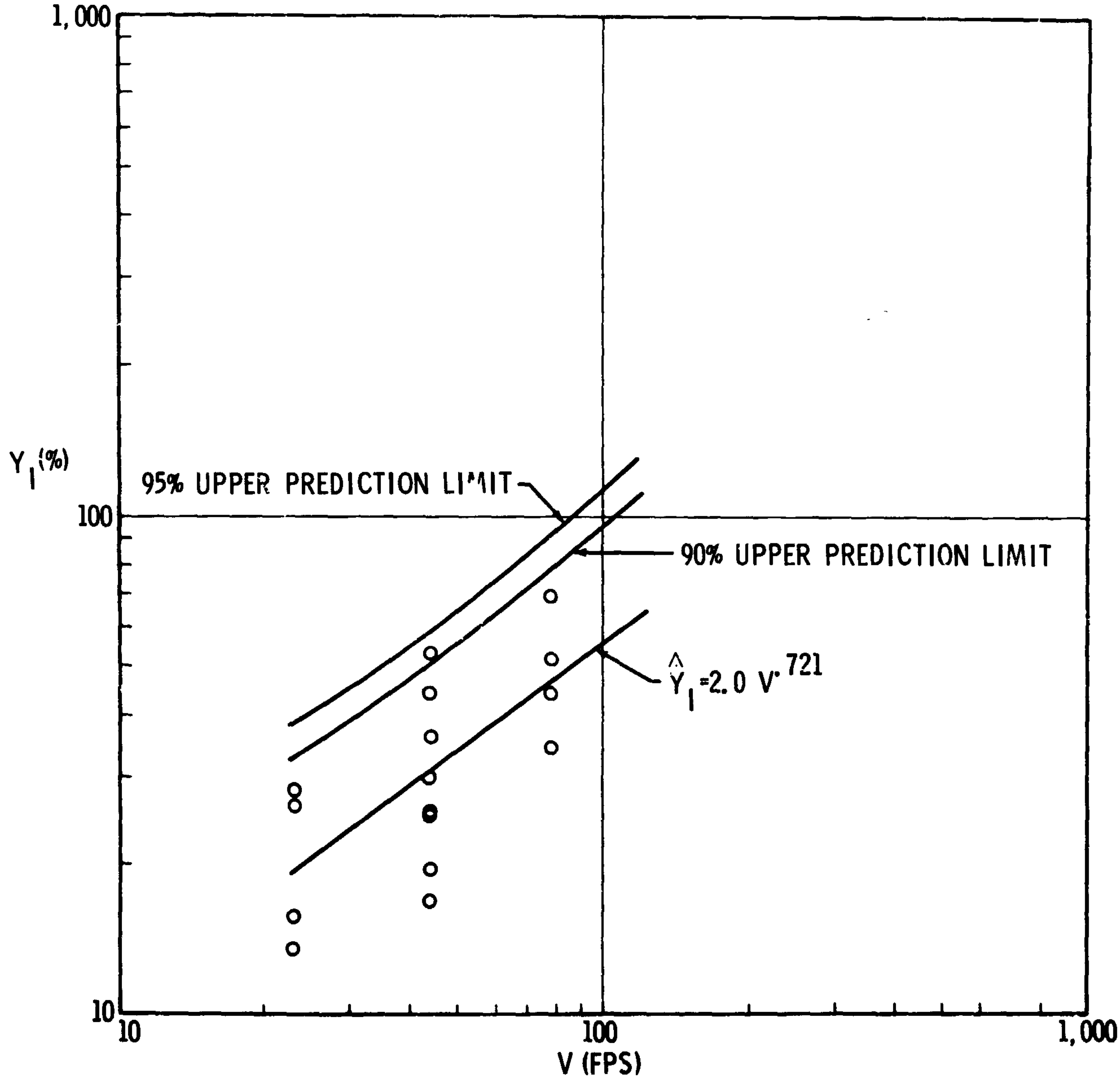


FIGURE 91 - CBGS, LO_2/LH_2, REGRESSION OF IMPULSE YIELD ON VELOCITY, DROP TESTS. (W = 200 LBS)

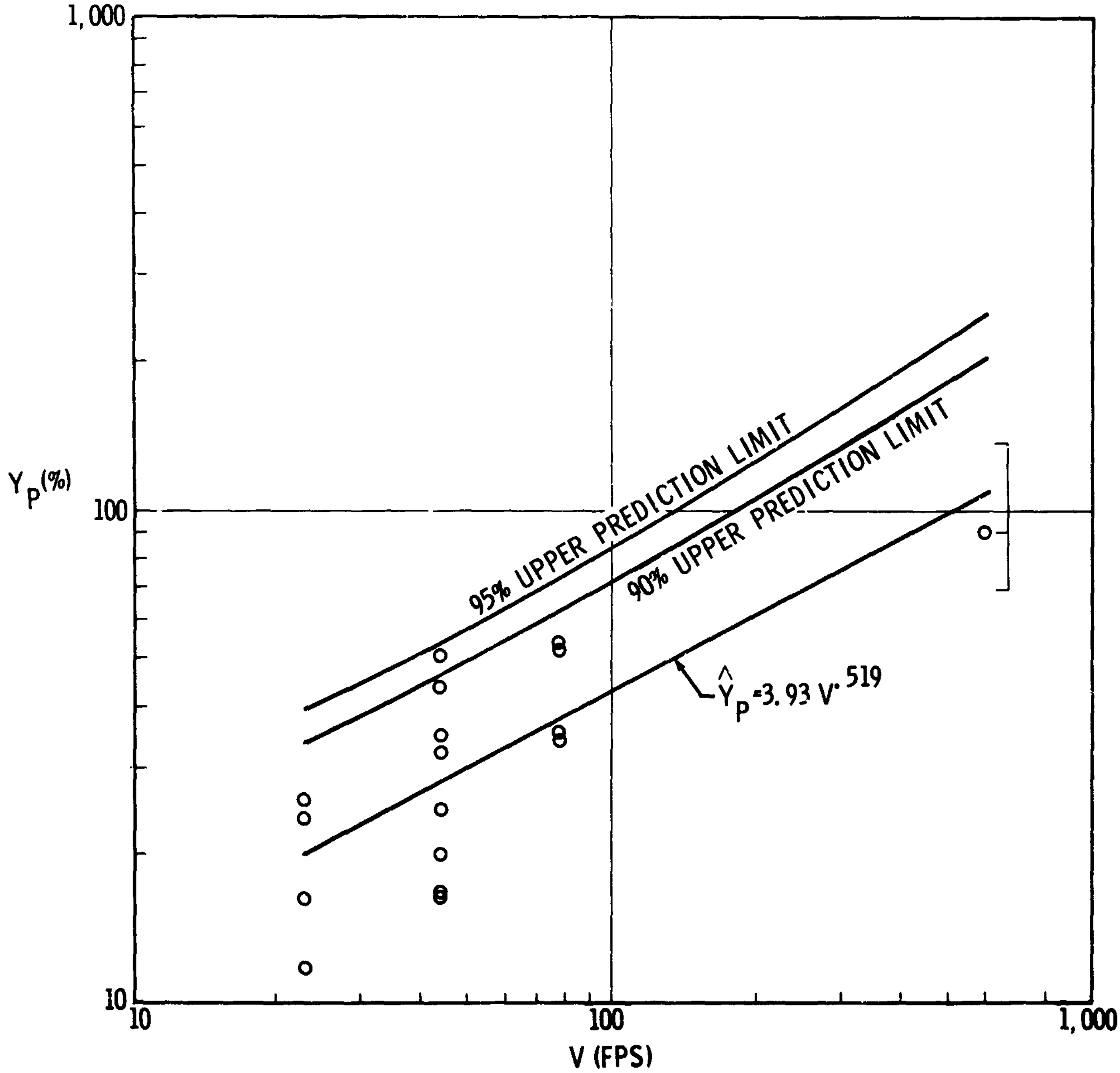


FIGURE 10P - CBGS, LO_2/LH_2, REGRESSION OF PRESSURE YIELD ON VELOCITY (DROP TESTS PLUS FLAT WALL HVI TEST)

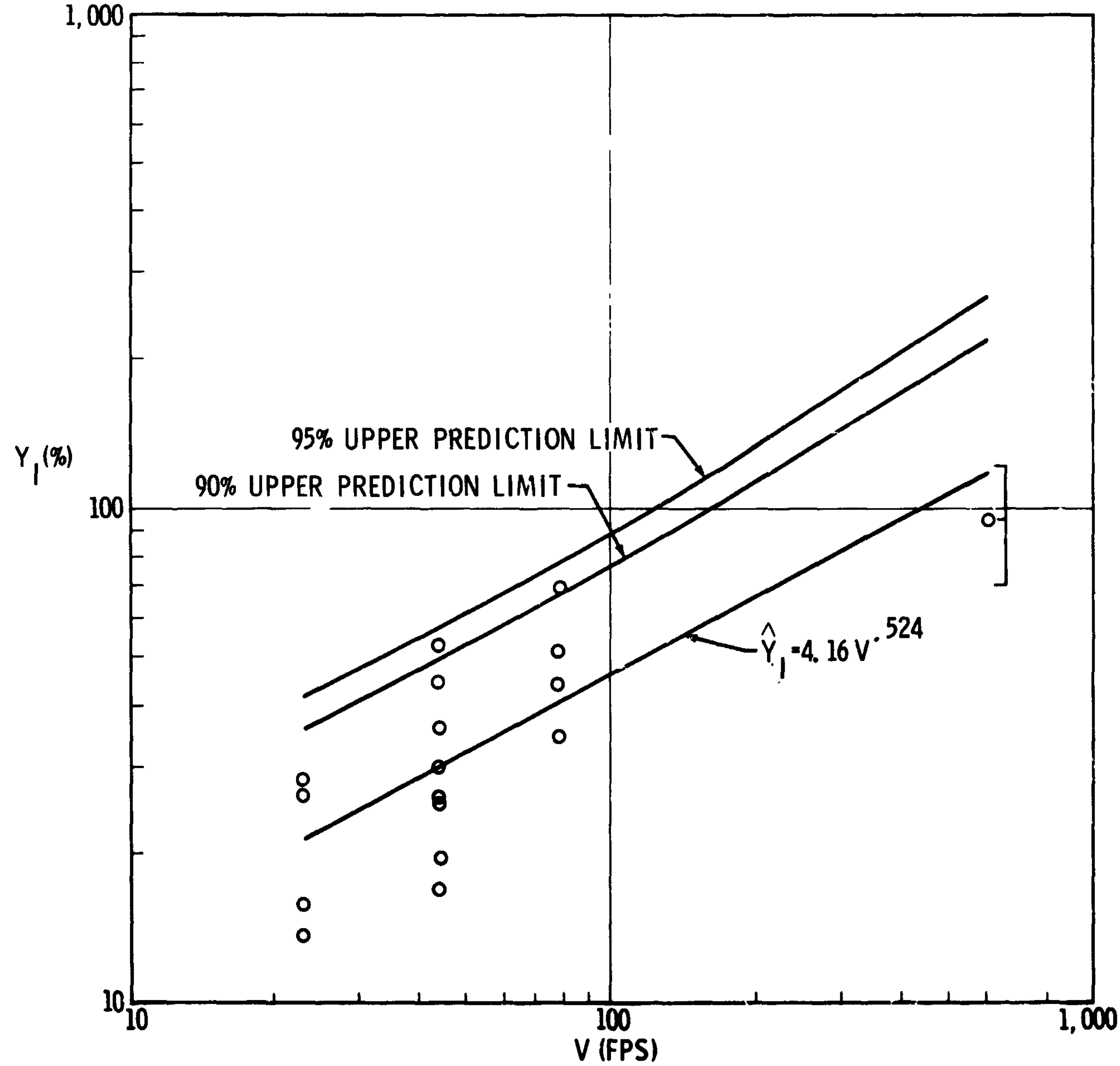


FIGURE 101 - CBGS, LO_2/LH_2, REGRESSION OF IMPULSE YIELD ON VELOCITY (DROP TESTS PLUS FLAT WALL HVI TEST)

3.3 Weight and Velocity

If one makes the assumption of no interaction effect* between weight and velocity, then the following joint regression is obtained using the nonspurious drop test data at 200, 1000 and 25,000 lbs. (cf. Appendix C, section III 3.2):

$$(3.3) \quad \begin{cases} \hat{Y}_P = (5.4)\ W^{-.169}\ V^{.683} \\ \hat{Y}_I = (4.8)\ W^{-.159}\ V^{.724} \end{cases}$$

The table given at the end of this section lists the values of these equations at 12 pairs of points together with the corresponding 90% upper prediction limits. The 200 lb. data is not listed since there is good agreement between the drop test equations of section 3.2 and the above multiple regression when W=200 lbs. Although equations (3.3) and the corresponding prediction limits were obtained from a multiple regression program, the results of Appendix D (section 3) show how equations (3.3) may be derived from the individual simple regressions on weight and velocity.

Multiple regressions based on both the drop tests and the flat wall HVI test are given in Appendix C, part 3.2 of Section III.

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*See Section IV, 3, (ii), p. 33.

W (lbs) \ V (fps)		23	44	78	160
1000	$\hat{Y}_P$	14.4	22.4[1]	33.1	54.0
	$(Y_{P,.90})$	(25.1)	(37.8)	(57.4)	(103.1)
	$\hat{Y}_I$	15.3	24.6	37.2	62.5
	$(Y_{I,.90})$	(26.4)	(40.9)	(63.6)	(117.2)
25,000	$\hat{Y}_P$	8.3	13.0[2]	19.2	31.4
	$(Y_{P,.90})$	(16.8)	(25.0)	(38.5)	(67.8)
	$\hat{Y}_I$	9.2	14.7	22.3	37.5
	$(Y_{I,.90})$	(18.2)	(28.3)	(43.7)	(79.2)
SII (930,000)	$\hat{Y}_P$	4.5	7.1	10.4	17.0
	$(Y_{P,.90})$	(12.0)	(18.3)	(27.3)	(47.2)
	$\hat{Y}_I$	5.2	8.3	12.5	21.1
	$(Y_{I,.90})$	(13.3)	(20.8)	(31.9)	(56.6)

(1) The average of the observed yields at 1000 lbs. and 44 fps is 32.6% for pressure and 37.9% for impulse; the corresponding maximum observed yields are 36.9% and 38.2%, respectively.

(2) The observed values for the pressure and impulse yields at 25,000 lbs. and 44 fps are 10.2% and 11.4%, respectively.

4. CBGS, LO_2/RP-1

4.1 Weight, V=44 ft/sec.

There are nine nonspurious tests available at this velocity: four at 200, four at 1000 and one at 25,000 lbs.

There is a strong negative correlation between Y and t for 200 lbs.: -.99 for the pressure yield and -.82 for the impulse yield. The formal correlations for the nonspurious 1000 lb. tests are similar. However, the relationship between Y and t is more accurately reflected in Figures 11P and 11I. As pointed out in section II, the RP-1 pool eventually stops spreading and is then overlapped at some instant by the continuously spreading LO_2 (the time of maximum yield for this failure mode), after which the potential yield tends to decrease with the increasing overlap. This behavior is particularly noticeable in the above mentioned plots of impulse yield against ignition time for both 200 and 1000 lbs. and explains the numerical correlations. In the case of the overpressure yield at 200 lbs. this phenomenon is not as distinct because of the low pressure yield associated with test No. 208 at t=460 msec. In this case, however, the Kingery curve provides only a marginal fit to the overpressure data. Relying more heavily on the impulse data, it is probable that the maximum yield-ignition time relationship has been attained for both 200 and 1000 lb. tests. On the other hand, if the yield-ignition time relationship for the 200 lb. data is more like that shown for the overpressure yields in Figure 11P, then the maximum yield may have been missed, resulting in lower than optimal yields at 200 lbs.

It can also be seen that while the average yield at 1000 lbs. was close to the average yield at 200 lbs. for the LO_2/LH_2 propellant, here the ranges of yields are essentially nonoverlapping, the yields at 1000 lbs. being the larger (Cf. Figure 12P). To this must be added a 25,000 lb. test with yield approximately equal to the average of the 200 lb. yields. It is then clear that a linear regression using the data at all of the three weights is somewhat forced (Figures 12P, I); further, since there are only three weights the data cannot realistically support a quadratic regression. Hence, without further information, the relationship between weight and yield is anomalous. For example, if the yields at 200 lbs. are low (recall the last paragraph, then primarily on the basis of the 1000 and 25,000 lb. tests it appears that yield is decreasing with increasing weight; however, no statistical significance can be attached to an extrapolation based on this data alone.

The URS prediction equation for t unknown assumes that ignition occurs at the peak of the Y vs. t curve and, since ignition time is scaled geometrically, is independent of weight: Y_{URS} = 95%, when V=44 fps.

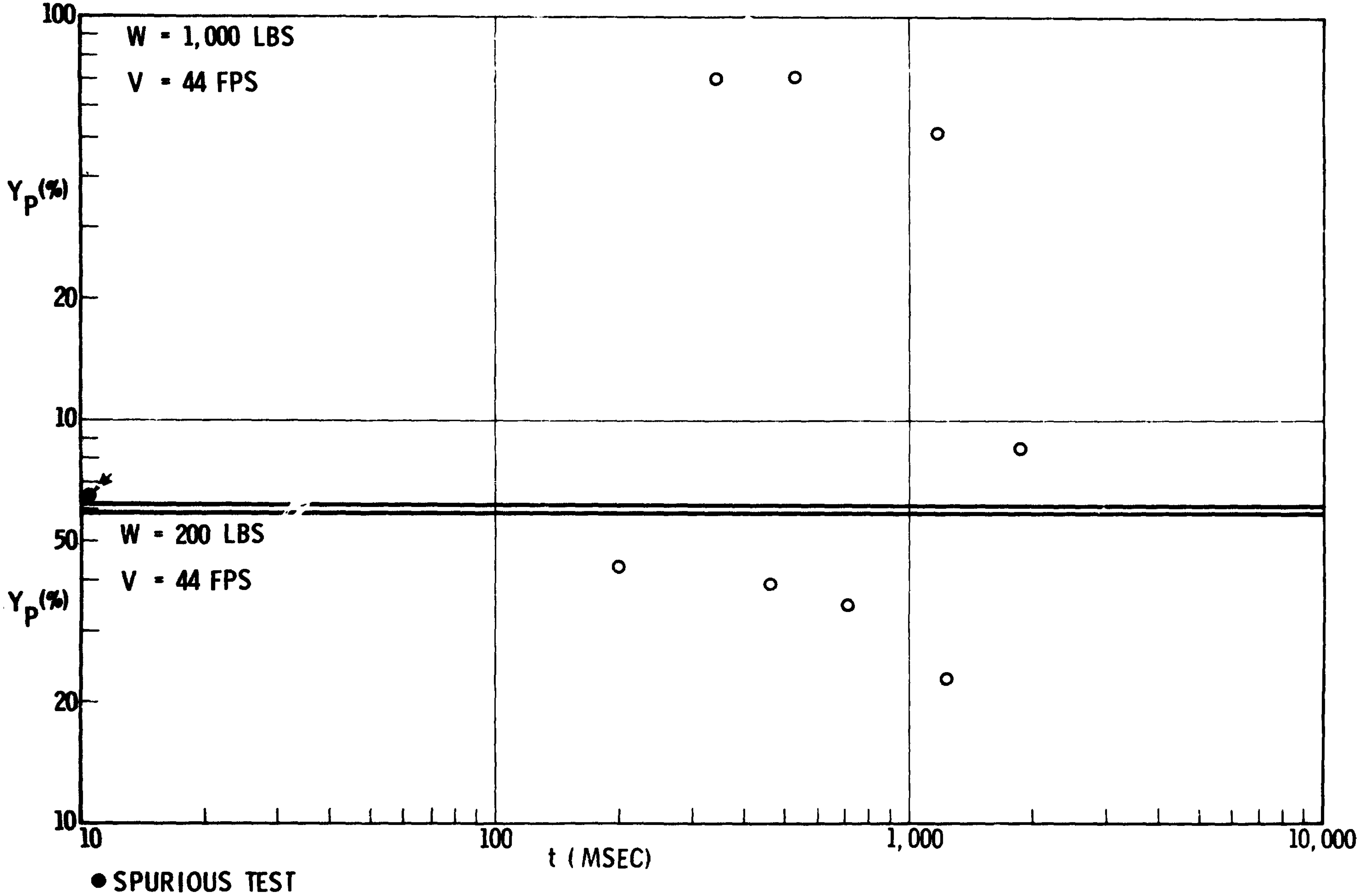

FIGURE 11P - CBGS, LO_2/RP-1, PRESSURE YIELD VS. IGNITION TIME

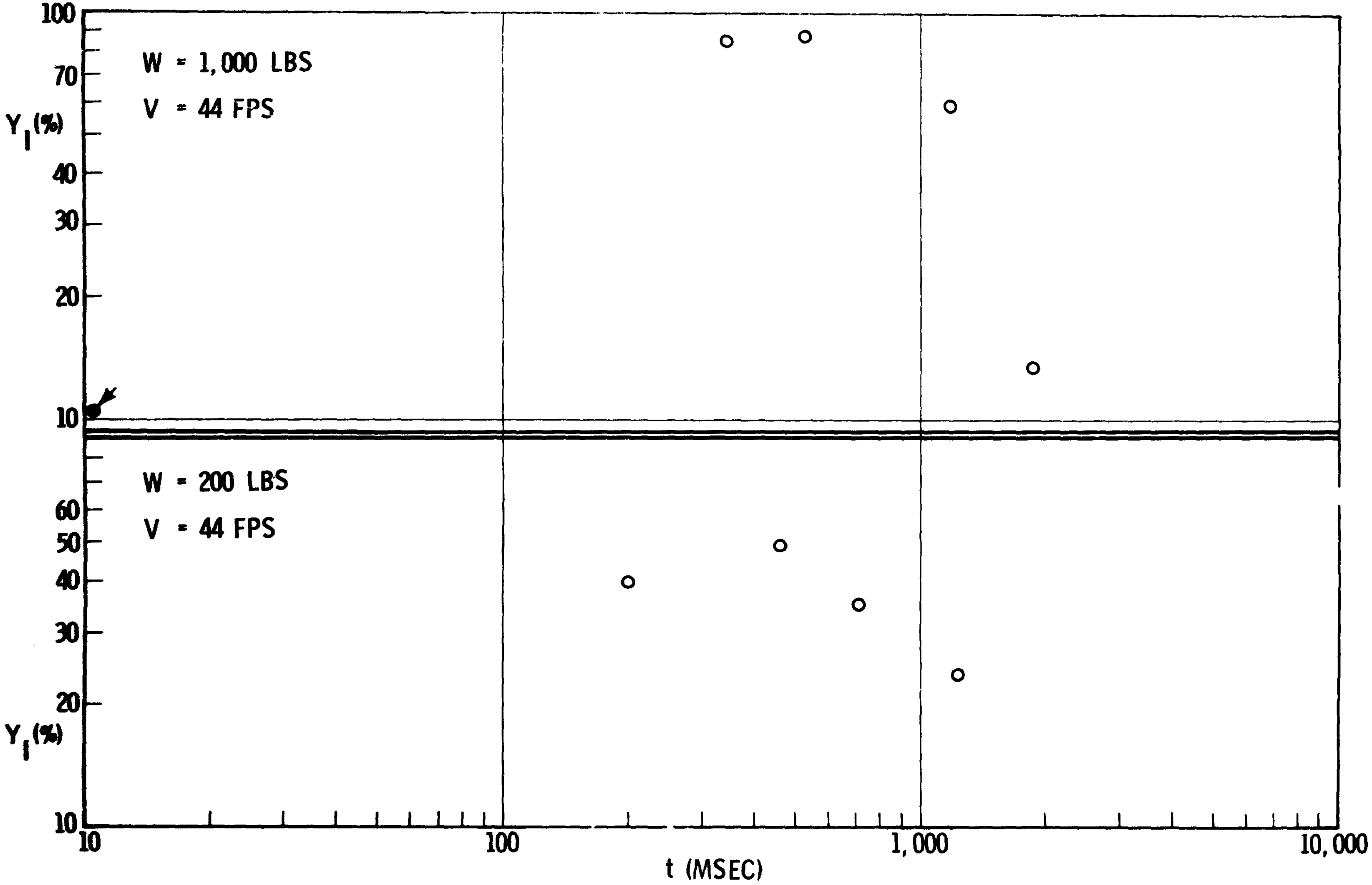

FIGURE 111 - CBGS, LO_2/RP-1, IMPULSE YIELD VS. IGNITION TIME

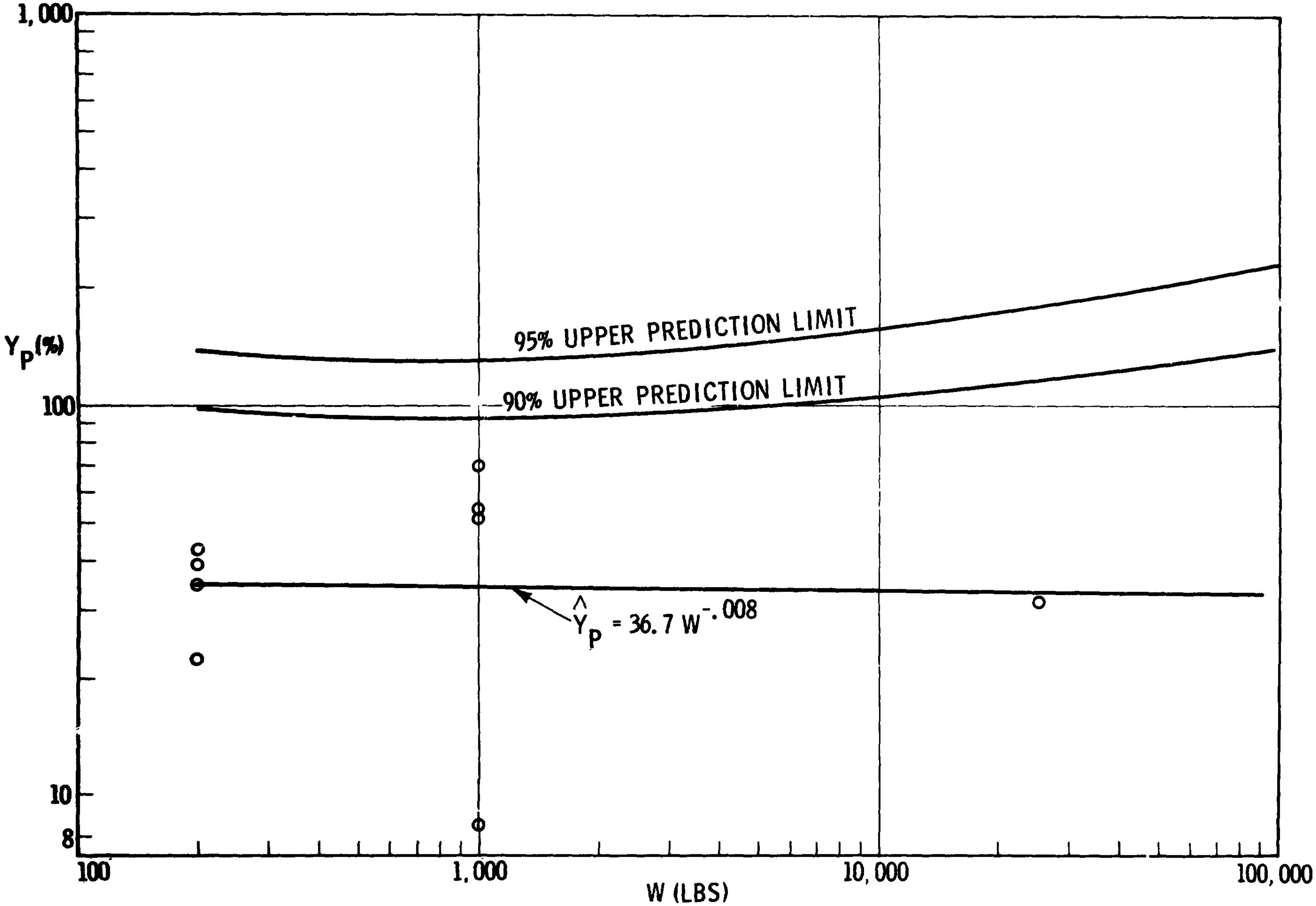

FIGURE 12P - CBGS, LO_2/RP-1, REGRESSION OF PRESSURE YIELD ON WEIGHT

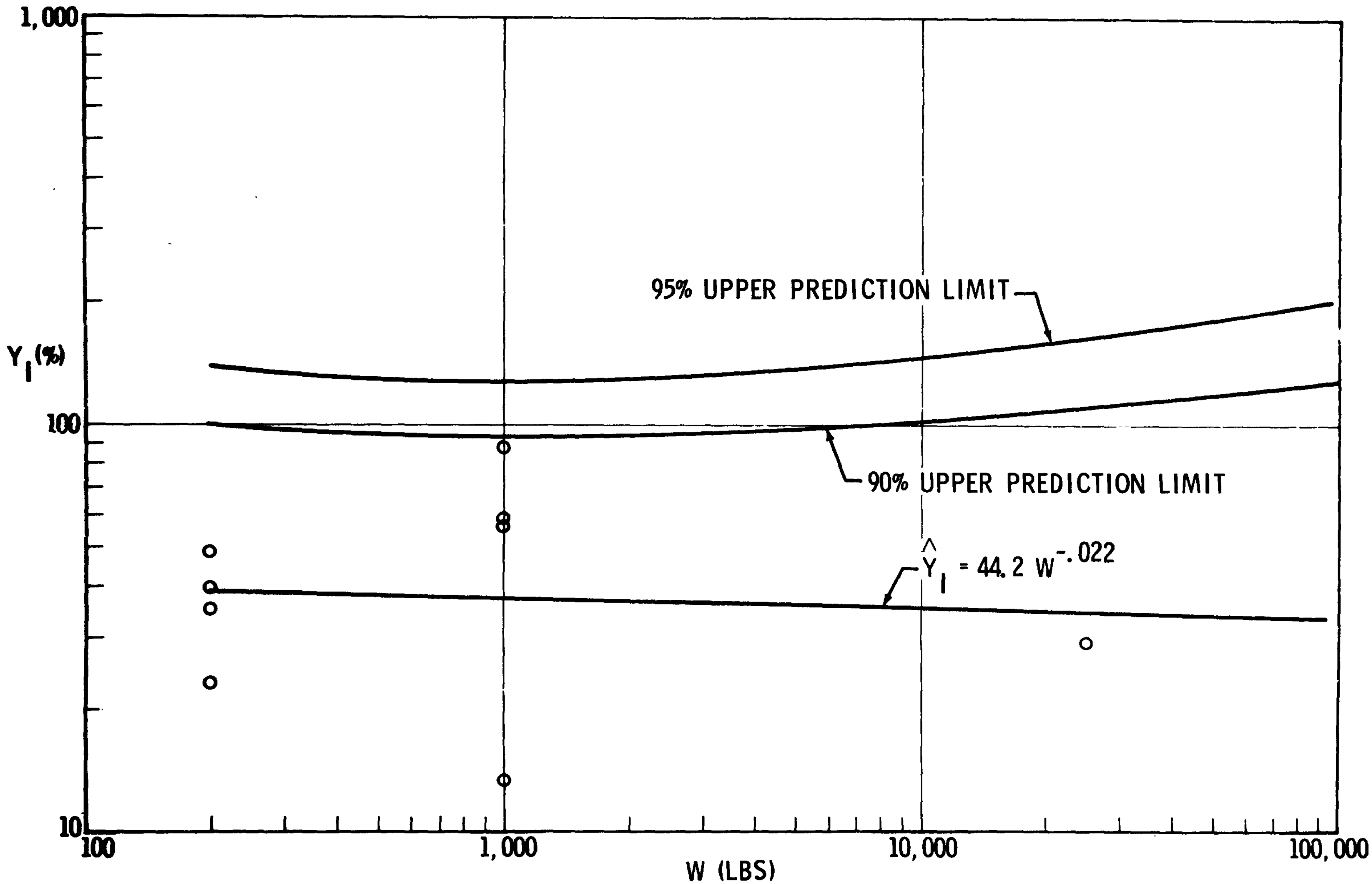

FIGURE 121 - CBGS, LO_2/RP-1, REGRESSION OF IMPULSE YIELD ON WEIGHT

4.2 Velocity with W=200 lbs.

The drop tests are those with velocities 23, 44 and 78 ft/sec.; there are 11 such tests using this propellant, four at 23 fps, four at 44 fps, and three at 78 fps. In addition, there are four high velocity impact tests. These four tests will not be considered here but they are indicated along with the drop tests in Figures 15P, I.

There is considerable variation in the magnitudes of the yields at each drop velocity. This is probably due to the critical manner in which yield appears to respond to the stage of overlap of the RP-1 propellant pool by the LO_2 pool. As mentioned in Section II, maximum yield should occur at those ignition delay times which coincide with the initial overlap of the RP-1 by the LO_2. This optimal ignition time (pool diameter) appears to be approximated in both the 44 and the 78 fps sequence of tests, but probably not at 23 fps (cf. Figures 11P, I and 13P, I). However, there are only four data points at 44 fps and three at 78 fps; this, combined with the anomalous nature of the 200 and the 1000 lb. yields at 44 fps, introduces a great deal of uncertainty concerning the realization of optimal ignition.

The following regression equations are formal in the sense that they assume that all the yields are the result of optimal pool diameter ignitions and the observed variation is random:

$$(4.2) \qquad \hat{Y}_P = 44\ V^{.527} \quad , \quad \hat{Y}_I = 2.0\ V^{.761} \ .$$

The exponents in these equations have upper 90% confidence limits of .90 and 1.12, respectively. These regressions provide a reasonably good fit to the averages at V=23, 44 and 78 fps (cf. Figures 14P, I).

On the other hand, if one takes the more realistic view that the low yield (nonspurious) tests do not reflect random variation at optimal pool diameter, but rather the sensitivity of yield to achieving ignition at times only "slightly" different from those corresponding to optimal pool diameter, then certain low yield tests must be discarded to obtain a consistent population.

Unfortunately, there is no objective way of doing this, but the following tests are suggested by the yield-ignition time graphs: at 23 fps, tests 096, 248, 144; at 44 fps test 232; at 78 fps, tests 110, 236. The remaining tests are then considered as optimal yield tests. The plots of these tests against velocity (W=200 lbs.) is extremely nonlinear (cf. Figures 14P, I); the availability of only three drop velocities precludes the use of any more complicated model.

The URS prediction equation, $Y_{URS} = 3.19\ V^{0.9}$, is compared with (4.2) in the following table. (The URS 90% upper prediction limit exceeds 115% at 44 fps and is not indicated here.)

V	23	44	78
Y_{URS}	53.6	95.7	160
$\hat{Y}_P$	23.1	32.5	43.9
$\hat{Y}_I$	21.1	34.6	53.5
$\hat{Y}_{P,.90}$	44.7	60.8	85.9
$\hat{Y}_{I,.90}$	40.1	63.5	102.6

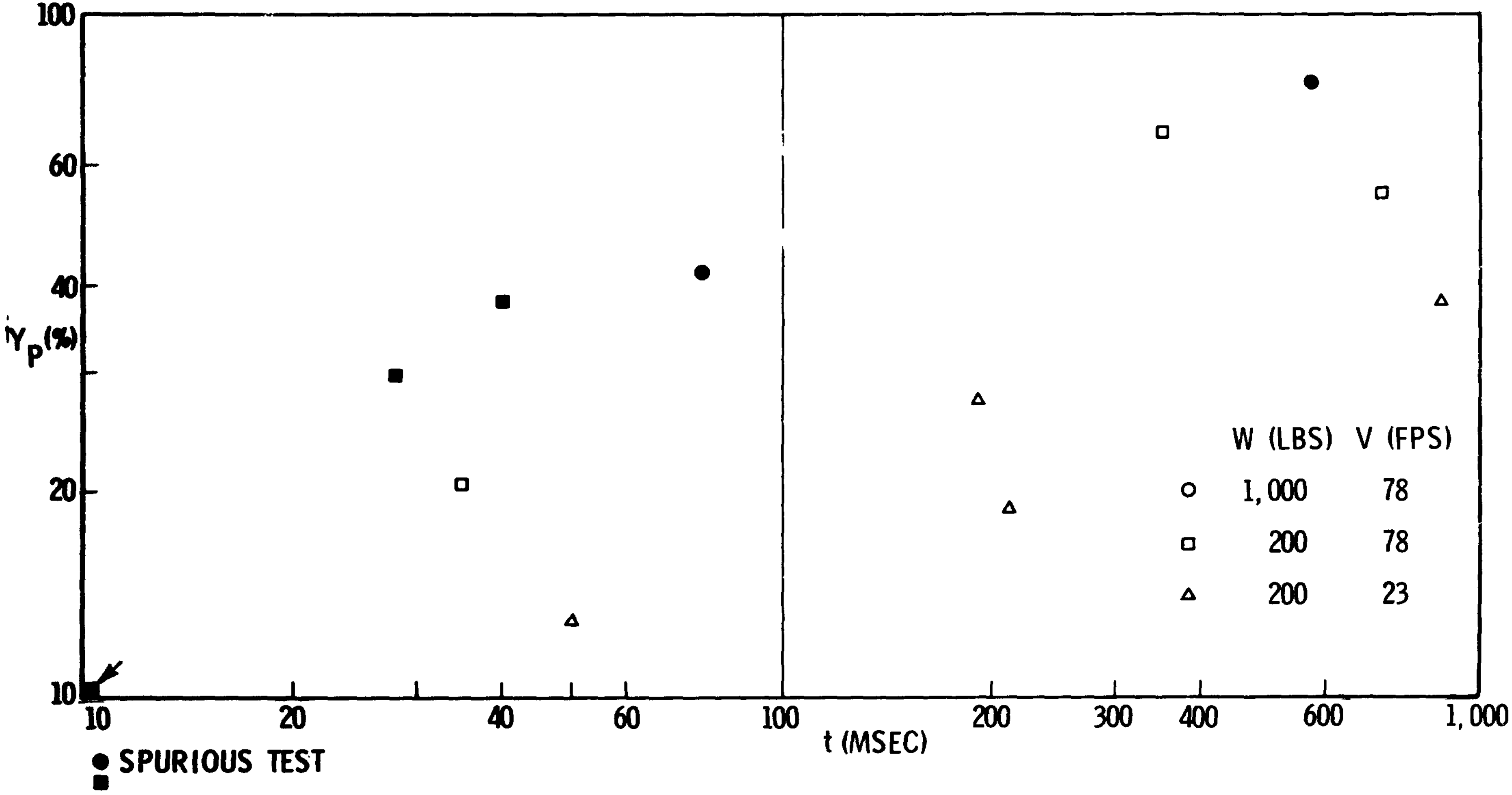

FIGURE 13P - CBGS, LO_2/RP-1, PRESSURE YIELD VS. IGNITION TIME

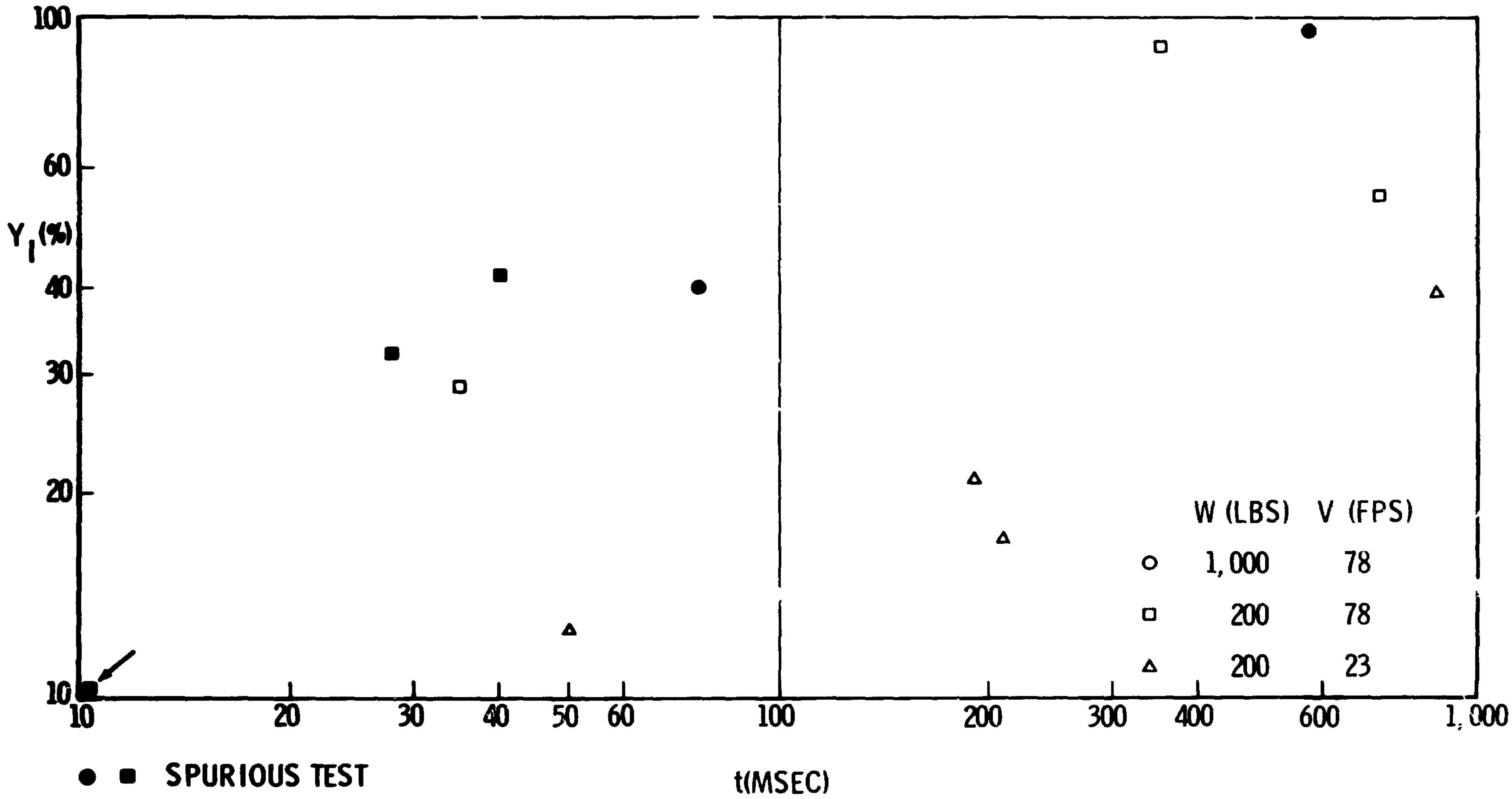

FIGURE 131 - CBGS, LO_2/RP-1, IMPULSE YIELD VS. IGNITION TIME

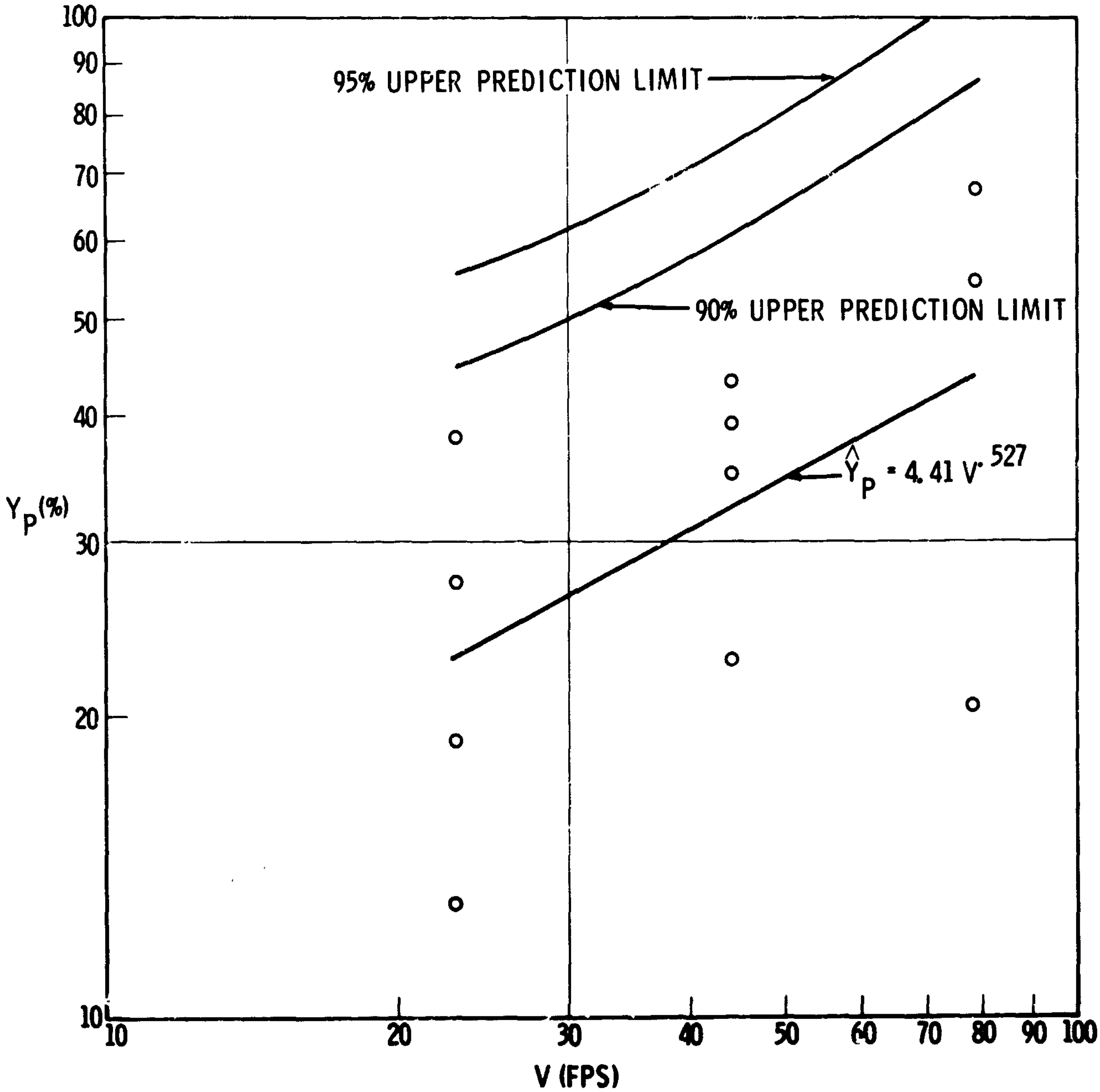

FIGURE 14P - CBGS, LO_2/RP-1, REGRESSION OF PRESSURE YIELD ON VELOCITY (DROP TESTS)

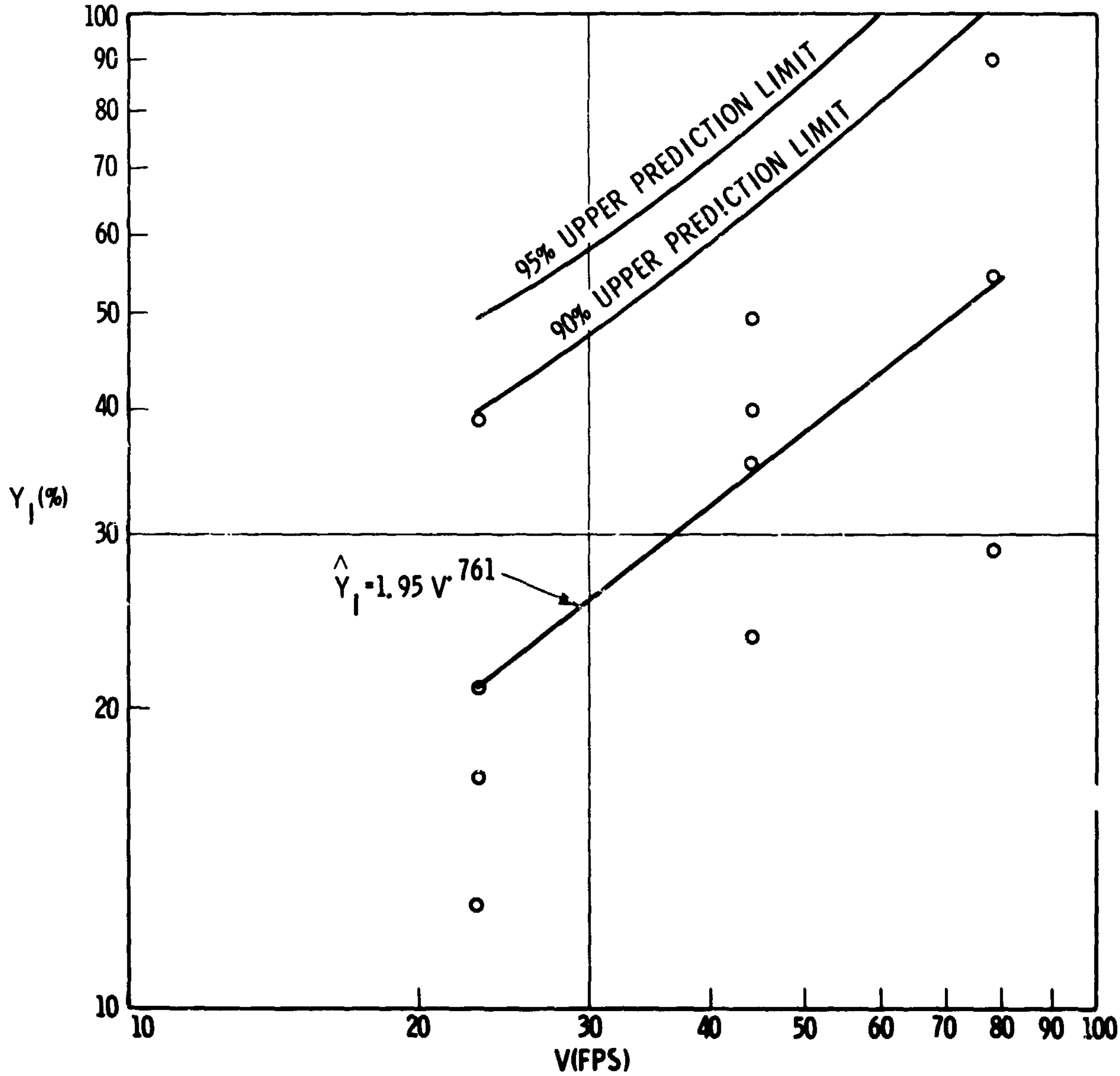

FIGURE 141 - CBGS, LO_2/RP-1, REGRESSION OF IMPULSE YIELD ON VELOCITY (DROP TESTS)

4.3 Velocity and Weight

There are two additional drop tests available at 1000 lbs. and 78 fps. Although both are listed as spurious, neither test is low yield and one of the tests, No. 190, has t=570 msec., a reasonably long ignition time. Hence these tests are probably comparable to the nonspurious drop tests. Furthermore, both are used by URS in their multiple regression.

Comparing these two tests with the 1000 lb., 44 fps data one can observe an increase of 7% (8%) for the maximum overpressure (impulse) yield. Since there are only 2 data points at 78 fps as opposed to 5 at 44 fps this would appear to indicate that yield increases with velocity as is the case for the 200 lb. drop tests. However, the 200 lb. data shows a 55% increase in maximum yield in the transition from 44 fps to 78 fps; hence, there seems to be an unexplained interaction between weight and velocity in the case of LO_2/RP-1. This, combined with obvious nonlinearities in both weight and velocity (Figures 14P, I), implies that a joint regression on weight and velocity would require 6 parameters. However, since there are only six velocity-weight combinations, such a regression would not have any statistical validity.

If one chooses to compare the 200 lb. drop tests with the high velocity impact tests (cf. Figures 15P, I), it can be seen that there is no increase in yield at 520 fps for the deep hole tests relative to the drop tests at 78 fps; the flat wall tests indicate a substantial decrease in yield.

Since the HVI tests show strong directional effects, the yield in each of the directions is indicated in Figures 15P and 15I for one of the flat wall and one of the deep hole tests.

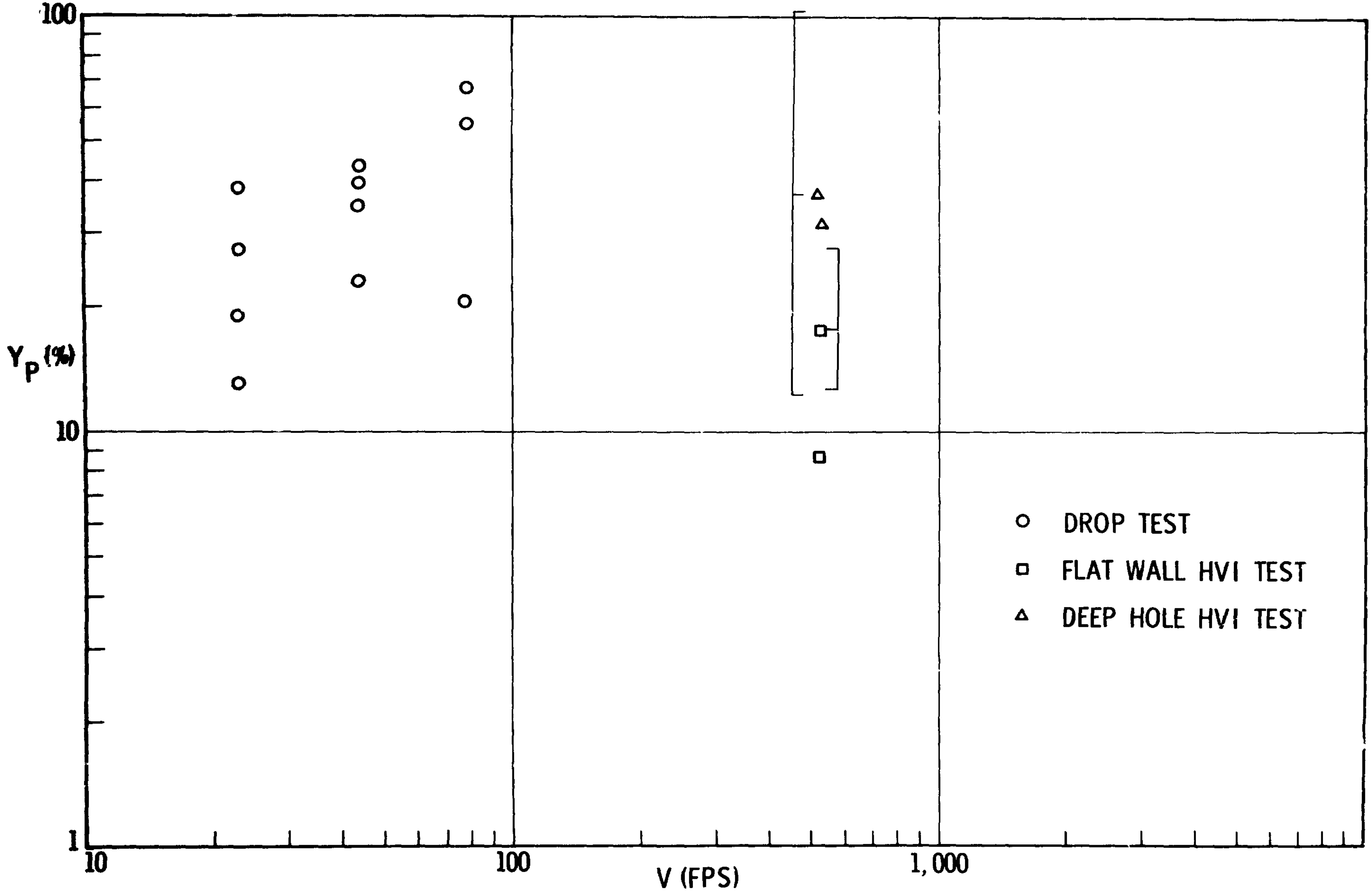

FIGURE 15P - CBGS, LO_2/RP-1, Y_P VS. V, DROP AND SLED TESTS

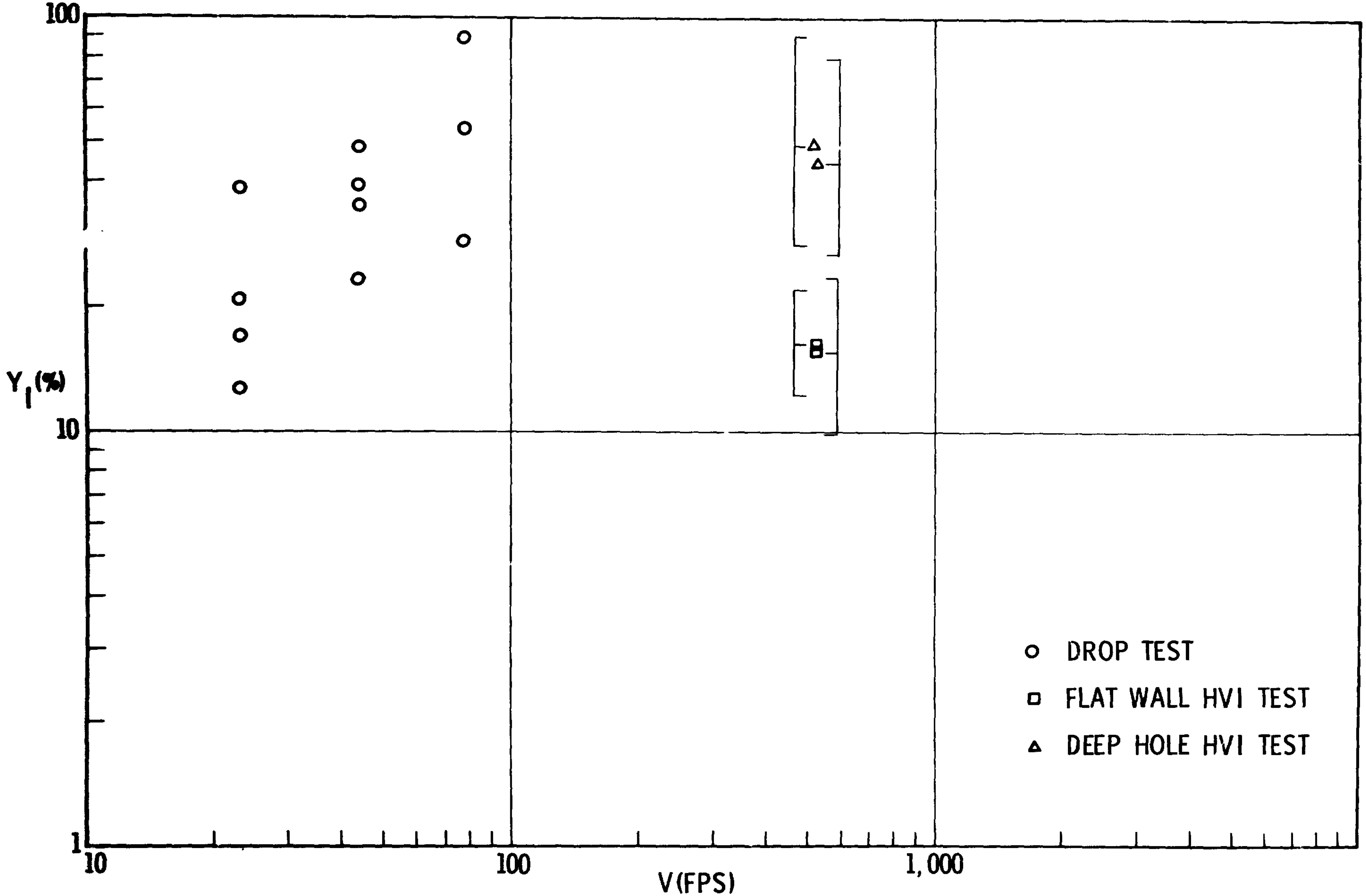

FIGURE 151 - CBGS, LO_2/RP-1, Y_1 VS. V, DROP AND SLED TESTS

IV. CONCLUDING REMARKS

The principal uncertainty in applying the Project Pyro tests to large weights involves the interpretation of the controlled small scale tests in an environment of self-ignition. In particular, it is not clear whether or not orifice size, ullage, or impact velocity--variables that are important in the small scale tests--are also consequential for self-ignited explosions, and, if so, in what manner. Such questions were not included within the scope of investigation for Project Pyro.

In the absence of even partial answers to these questions, the possibility of employing empirical regression estimates has been investigated in the previous sections. To aid in judging the meaningfulness of this analysis for purpose of prediction, the main conclusions pertaining to the following areas, for each of the four propellant-failure mode combinations, are presented. Overall general conclusions are also noted.

1. Characteristics of the small scale tests--very high yield is possible under proper conditions.

2. Principal areas of uncertainty regarding 1--too few tests combined with large scatter resulted in a number of gaps.

3. Characteristics of the large scale tests--low yield with one exception.

4. Applicability of regression analysis to prediction for large weights--mixed; prediction limits tend to be large because of large scatter.

To provide a convenient reference, the nonspurious large scale test results are listed in Table 1, and also results for a select number of "significant" small scale tests--principally those with highest yields (recalling, however, that for LO_2/LH_2 the average is more appropriate statistically than the maximum).

1. CBM, LO_2/RP-1

   (i) Small scale. Extremely high yields are obtained for 40% ullage, and for orifice ratio $D_o/D_t=1$. 1000 lb. tests give comparatively low yield.

TABLE 1

SUMMARY OF SIGNIFICANT TESTS

	LARGE SCALE TESTS					SMALL SCALE TESTS				
	W	GEOMETRY/ VELOCITY	t	Y_P	Y_I	W	GEOMETRY/ VELOCITY	t	Y_P	Y_I
CBM LO_2/RP-1	25,000	NOMINAL	540	11.3	11.4	200	NOMINAL	145	33.0	31.8
	25,000	NOMINAL	530	11.1	11.7	133	2/3 FULL†	150	85.0	69.8
	94,000 (TITAN)	L/D=4, D_o/D_t=.1375 V_u=50% ΔP_r=35	842	3.3	3.4	200	D_o/D_t=1.0	290	48.7	45.3
						1,000	NOMINAL	222	18.5	20.1
CBM LO_2/LH_2	91,000 (S-IV)	L/D=2, D_o/D_t=.083	183	3.3	5.7	200	NOMINAL	35 SEC	17.5	34.3
						200	NOMINAL	82	9.5	27.5
						133	2/3 FULL†	770	45.0	54.9
						200	D_o/D_t=1.0	180	79.3	104.5
						1,000	NOMINAL	810	23.3	34.5
CBGS LO_2/LH_2	25,000	44 FPS	365	10.2	11.4	200	23 FPS	1,524	26.0	28.2
						200	44 FPS	775	51.2	53.1
						200	78 FPS	292	53.6*	69.5
						200	569 FPS+	—	93.2	119.8
						1,000	44 FPS	900	36.9	38.2
CBGS LO_2/RP-1	25,000	44 FPS	465	31.9	29.4	200	23 FPS	870	38.0	39.1
						200	44 FPS	200	43.3	40.0
						200	44 FPS	460	39.3	49.5
						200	78 FPS	350	67.3	90.1
						200	526 FPS+	—	18.9	21.1
						1,000	44 FPS	525	71.0	87.7
						1,000	78 FPS	570	75.9	94.6

† SPECIFIC YIELD

+ SLED TEST, FLAT WALL (LARGE DIRECTIONAL VARIATION)

* 47% SCATTER IN PRESSURE DATA (ALL STATIONS INCLUDED)

(ii) Uncertainties. Mixing characteristics appear to change for L/D=5, D_o/D_t=1--a physical model is desirable. Ullage tests are too few and restrictive to extrapolate.

(iii) Large scale. This is the only case with more than one large scale nonspurious test. The two 25,000 lb. tests are reproducible. The Titan yield is quite low, with pressure and impulse response about the same as for the 25,000 lb. tests.

(iv) Regression. The statistical extrapolations, although conservative, appear to be the most reasonable of all the cases--expected values for 94,000 lbs. are 1.5-2.5 times observed Titan, 95% prediction limits are 3-4 times. Geometry and ullage variables should perhaps be ignored until uncertainties can be resolved. Differences between pressure and impulse yields are not sufficient to warrant separate predictions--pressure data seems better behaved.

2. CBM, LO_2/LH_2

(i) Small scale. Test results are highly erratic, and with large differences between pressure and impulse yields. Yield exceeding 100% is obtained for large orifice ratio (1.0); conversely, small orifice ratio (.083) gives low yield. 40% ullage increases yield, but less than for LO_2/RP-1. Maximum yield for 1000 lbs. is approximately the same as for 200 lbs., and greater than for LO_2/RP-1 at 1000 lbs.

(ii) Uncertainties. Mixing characteristics are even more uncertain than for LO_2/RP-1. Unusual behavior of pressure and impulse data requires further investigation.

(iii) Large scale. Unfortunately, no useful data was obtained from the 25,000 lb. tests. Whether the SIV ignition time of 183 msec is atypically low, and whether a longer ignition time would have increased yield, is difficult to judge. Pressure yield was about the same as Titan, impulse yield 70% greater.

(iv) Regression. Data is too erratic to place any confidence in the results, even though the expected regressions appear reasonable. One possible course is to use the LO_2/RP-1 regression for pressure yield, and 70% additional for impulse yield.

3. CBGS, LO_2/LH_2

(i) Small scale. (a) At 200 lbs. average yield shows a consistent and comparatively smooth increase with velocity; however the scatter is large. (b) At 44 fps, average yield for 1000 lbs. is approximately the same as for 200 lbs. (c) High velocity sled tests give extremely high yields. (d) Except for 200 lbs. at 44 fps (and high velocity sled tests), yields are less than for CBGS, LO_2/RP-1.

(ii) Uncertainties. Unfortunately, no useful data was obtained to determine the existence of any interaction between weight and velocity.

(iii) Large scale. The 25,000 lb. test appears representative. Yield is moderate, about the same as for CBM, LO_2/RP-1.

(iv) Regression. Although the regression on weight shows a definite decrease, the fall-off is rather shallow, primarily because average yields for 200 lbs. and 1000 lbs. were the same. Moreover, the large scatter at 200 lbs. leads to large prediction limits. The regression on velocity is satisfactory.

4. CBGS, LO_2/RP-1

(i) Small scale. For each combination of weight and impact velocity, there appears to be an optimum ignition time for critical pool diameter giving maximum yield. For 200 lbs. this maximum yield increases only slightly between 23 and 44 fps, but substantially between 44 and 78 fps. For 1000 lbs. the yield is very high at both 44 fps and 78 fps. The high velocity sled tests give comparatively low yields.

(ii) Uncertainties. It is not clear why maximum yield for 200 lbs. at 44 fps should be substantially less than for 1000 lbs. at 44 fps--the effect may be real or may result merely from failure to achieve maximum yield at 200 lbs. In general the nonlinear and interaction effects are pronounced and apparently unexplained. Finally, what significance, if any, should be attached to the low yield from the high velocity sled tests?

(iii) Large scale. The 25,000 lb. test gave very high yield, approximately 2.7 times that for CBM, LO_2/RP-1. The yield in fact exceeds the current 20% safety criterion for propellant weights less than 500,000 lbs. (DOD Instruction 4145.21, "Quantity-Distance Standards for Liquid Propellants.")

(iv) Regression. The relation between maximum yield and weight or velocity is too nonlinear for the limited data to provide meaningful regressions.

ACKNOWLEDGMENT

We owe special thanks to James H. Deese of the Future Studies Office, KSC, for his substantial technical and adminstrative assistance, and his advice on diverse aspects of this study. R. R. Singers, Bellcomm, patiently and expertly handled all computer programming, including a multitude of modifications. We wish also to thank Miss M. Harris, Bellcomm, for ably assisting in the painstaking task of drawing up the many figures and tables.

P. Gunther

G. R. Andersen

1033-$\frac{PG}{GRA}$-jf

Attachments
Appendices A, B, C, D
References

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APPENDIX A

ESTIMATION OF YIELDS AND DEPARTURES FROM KINGERY CURVES

1. TNT Kingery curves

Explosive effects of liquid propellants are customarily expressed in terms of equivalent weight of TNT. The yield is simply the ratio of equivalent TNT weight to total propellant weight. Figure A1 shows the variation in peak overpressure with scaled distance ($d/W^{1/3}$) for surface bursts of hemispherical TNT charges.[3] Similarly, Figure A2 plots scaled positive impulse* ($I/W^{1/3}$) vs. scaled distance.[4] (The above two curves are usually referred to as Kingery curves, after the principal author of the references cited.) Since all the observed impulse data were beyond the hump in Figure A2, the simple quadratic approximation (dashed curve) turned out to be satisfactory.

The TNT Kingery impulse curve represents a relatively recent revision. An earlier 1963 curve used by URS gave substantially higher equivalent TNT weights than the 1966 curve in Figure A2.

2. Terminal yield

Departures of liquid propellant explosions from the TNT Kingery curves have been reported in the literature[5,6]; at close in distances, peak overpressure is below the Kingery curve, while impulse is above. (According to Reference 5, impulse is below the Kingery curve when yield is less than 10%.) As distance increases, the observations tend to approach more closely the Kingery curves. This type of departure from TNT behavior has led to the notion of terminal yield, i.e., the yield for which the pressure and/or impulse would asymptotically approach their respective Kingery curves. URS, accordingly, for the most part estimated yields using only the data at the furthest distance.**

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*Positive impulse is the area under the overpressure-time trace, up to the time when the pressure becomes negative. For simplicity, the abbreviated terminology, pressure and impulse, is used.

**From conversations with URS, in some instances trends from the 2 or 3 furthest distances, and also shape of the traces, were qualitatively taken into account.

A preliminary examination of the data indicated that systematic departures from the Kingery curves would be difficult to establish because of the extremely large scatter. Apparently only very few traces conformed to the ideal TNT exponential pressure-time profile.* There were frequent occurrences of rounded peaks, double peaks, overshoots, etc. (Twelve different trace types are exhibited in Reference 1, volume 2.) Moreover, many of the measurements appeared to be discrepant.

Since the Project Pyro tests were greater in number than all previous liquid propellant tests, a valuable opportunity presented itself to study how fast terminal yield was approached, and, more generally, to investigate the nature of the departures from TNT. Because of the large scatter, it was considered necessary to employ objective quantitative criteria. Techniques were developed for making a least squares fit of all the data to the Kingery curves, followed by statistical tests for goodness of fit. If the close in points showed significant departure, a new least squares fit could then be made with these points omitted.

It was also decided not to combine the pressure yield and the impulse yield, since for so many tests these were exceedingly different. Although some form of average, as was used by URS, would have simplified the analysis, this was felt to be too coarse a procedure.

3. Description of computer analysis

A block diagram of the computer program is shown in Figure A3. For a particular run, the stations or distances to be externally omitted are specified. Then successive preliminary fits are made in order to identify and internally discard (with probability 95%) any extreme observations. A test for goodness of fit then compares the "within" variability (stations at the same distance) with the "between" variability about the Kingery curve. A second test is also performed to determine significance of direction (asymmetry of explosion). The principal output consists of the standard deviation of scatter (S_P%, S_I%) and the statistics of yield: the estimate (Y), standard deviation (S_Y%), and upper and lower 95% confidence limits (Y_U and Y_L). Detailed derivations, together with a description of the computer output, are given in Appendix B.

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*In reference to such departures, David Burgess of the Bureau of Mines, in a written communication, makes the following comment: "The measured blast is contaminated by some very messy function of the 90-odd percent of energy that is released after the "cutoff" time, i.e., the time when the transient enters the "suction" phase." (See also Reference 7.)

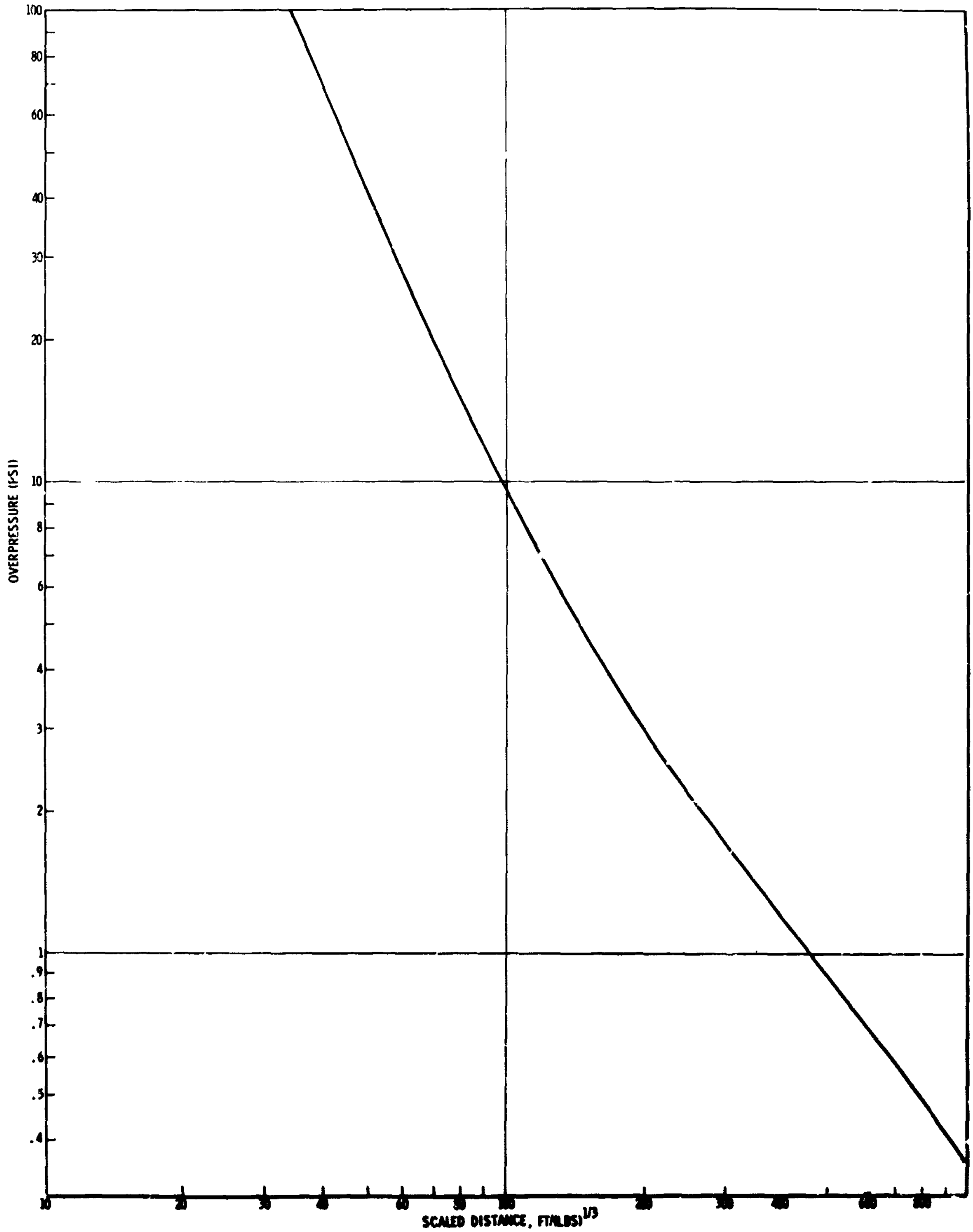

FIGURE A1 - PEAK OVERPRESSURE VS. SCALED DISTANCE FOR
TNT SURFACE BURSTS (REF. 3)

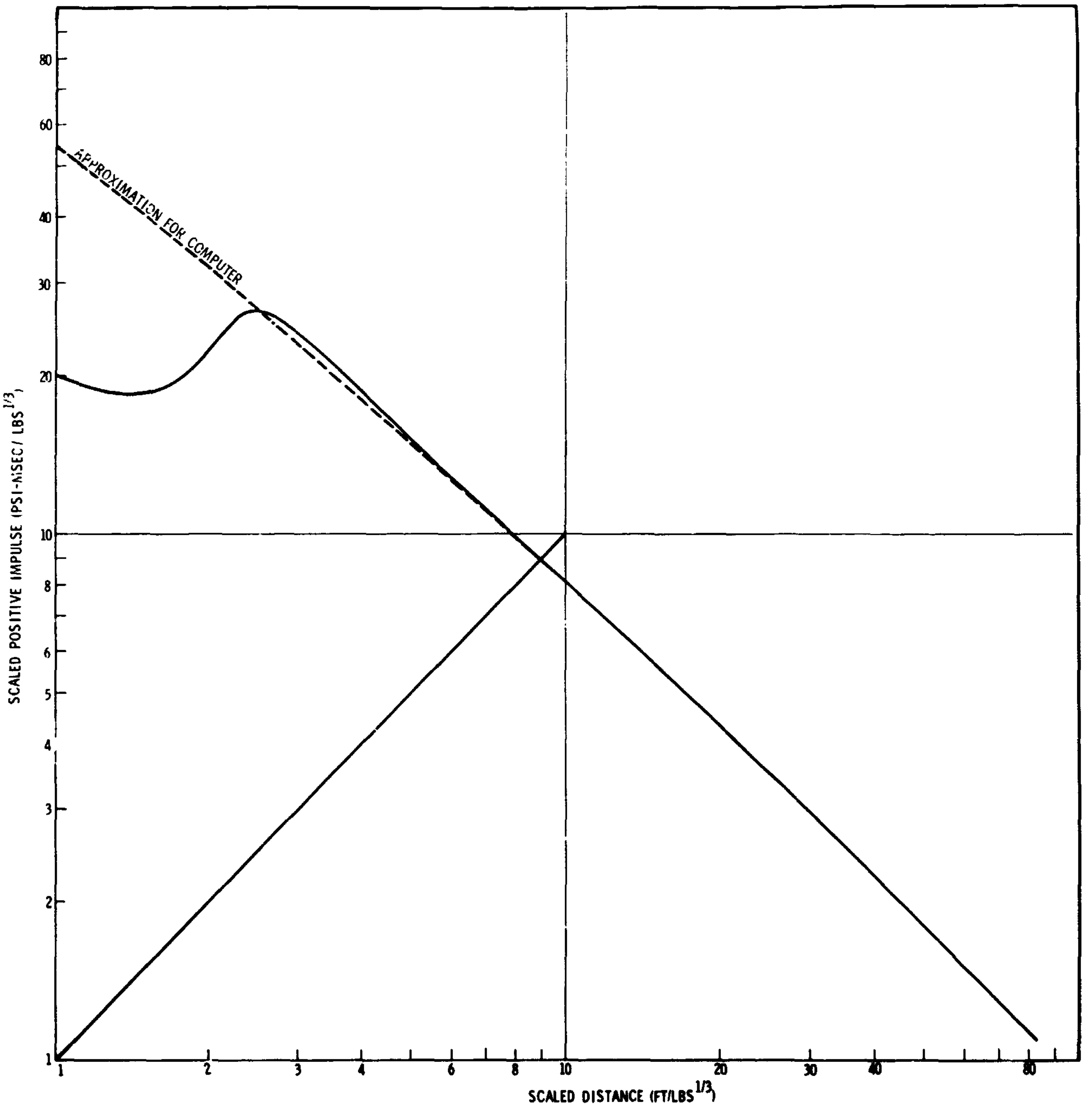

FIGURE A2 - SCALED IMPULSE VS SCALED DISTANCE FOR TNT SURFACE BURSTS (REF. 4)

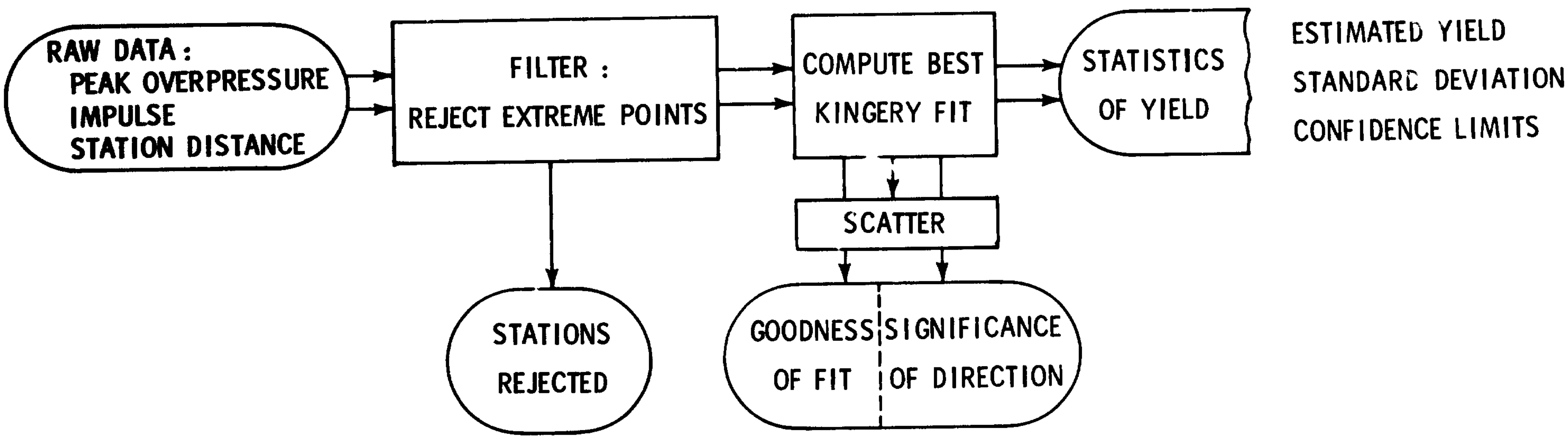

FIGURE A3- BLOCK DIAGRAM OF COMPUTER ANALYSIS

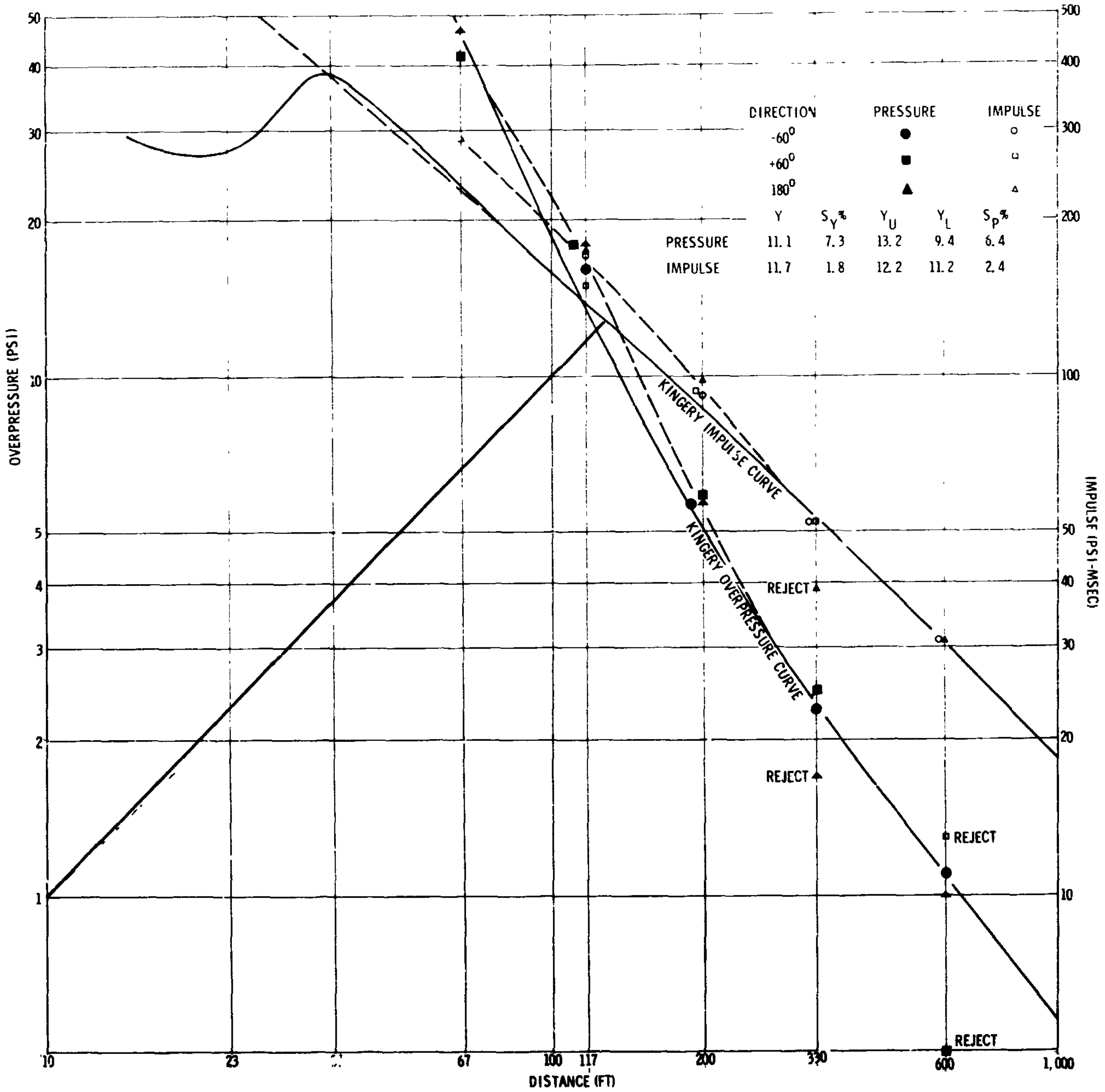

FIGURE A4- ILLUSTRATION OF LEAST SQUARES KINGERY FIT-TEST NO. 278

The least squares procedure is illustrated in Figure A4. To determine pressure yield, the scaled Kingery curve (Figure A1) is translated horizontally until, for the stations retained, the minimum sum of squares of deviations is obtained. For impulse yield the scaled Kingery impulse curve (Figure A2) is translated along the 45° diagonal.

4. Selection procedure for assigning yields

The procedure finally adopted for assigning yields, and characterizing departures from the Kingery curves, involved a certain amount of subjective judgment, but with the computer analysis supplying indispensable quantitative criteria. Initiallly successive fits were obtained: with no stations omitted (except those rejected as extreme), then omitting stations at the closest distance (d_1), and finally omitting the two closest distances (d_1 and d_2). For each run the individual deviations were examined along with changes in yield, scatter, and goodness of fit, all in conjunction with a graphical plot of the data. When a change in yield was accompanied by decreased scatter and an improved fit, one could usually determine whether d_1, or d_1 and d_2, departed significantly and should be omitted from the final fit. Complications arose when there was a significant directional effect, since this leads to large scatter and an insensitive measure for fit. In many cases additional runs were performed, sometimes with d_3 omitted, at other times with one or more marginally extreme points omitted. When data was available for only 3 distances the appropriate choice was often a difficult one to make.

The detailed results of the selection procedure are presented in Tables A1 - A4 with the most important extenuating factors briefly noted. To illustrate the process, a detailed discussion is given for test No. 278 (Figure A4). This was a 25,000 lb. CBM, LO_2/RP-1 test, and one of the more difficult (and interesting) tests analyzed.

Considering pressure first, although cursory examination of the data in Figure A4 suggested that the Kingery fit was adequate, the numerical evaluation indicated otherwise. The results of the successive runs are summarized below:

	All	d_1 omitted	d_1, d_2 omitted
Y_P	12.[illegible]	13.7	12.5
S_P%	11.5	9.2	7.4
Fit	<.01	<.05	<.10

Even after rejecting two extremely low values at stations 21 and 30 (cf. the computer output in Appendix B, Table B1P) the initial run resulted in a very bad fit with the d_1 stations having a negative deviation. As expected, the next run, with d_1 stations additionally omitted, increased the yield and reduced the scatter; however, the fit was still bad, with the d_2 stations having positive deviations. The third run, with d_1 and d_2 stations omitted (and also station 30 which was not internally rejected), decreased the yield and further decreased the scatter; but the fit was still bad, with d_3 positive and d_5 negative. A special run, with d_3 also omitted so that only two d_4 stations and two d_5 stations remained, gave a yield of 11.1 and a scatter of 6.4%, with a good fit. With this yield, the expected pressure at d_1 was 46.6, about the same as one of the observed d_1 values but somewhat higher than the other d_1 value of 42. Thus, it is uncertain whether a negative bias† exists at d_1.

It is of interest to note that although removal of the biased points did, as expected, reduce the scatter by almost one-half from the initial run, the standard deviation of the estimate of yield, $S_Y\%$, increased from 5.55% to 7.26% because the number of observations decreased from 12 to 4.* Further, with only 3 degrees of freedom remaining, the value of t for 95% confidence estimates is 3.18 compared with a t of 2.20 for the original 11 degrees of freedom. The final 95% confidence band of 9.4 - 13.2 compares with the original narrower band of 11.5 - 14.1. Although the final estimate of yield is more accurate, the precision is less.

The conclusion is that the departure from the Kingery curve consists of a bulge at d_2 and d_3. For this particular test, the variability in the data was small (after excluding the two

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*An additional factor is that the average distance for the remaining points has increased. Equations (7) and (8) in Appendix B can be roughly interpreted in terms of the formula $S_Y\% \simeq \frac{S_P\%}{K\sqrt{n}}$, where K depends on the slope of the Kingery curve, decreasing for increasing distance. Length of the confidence interval for Y is simply $t \cdot S_Y\%$.

†The term "bias", as used in this appendix, is synonomous with "deviation" or "departure" and does not imply statistical or measurement error.

extreme stations), so that the goodness of fit criterion was sufficiently sensitive to detect the departure. Fortunately, further corroboration is obtained from the second 25,000 lb. test, No. 282, which showed almost identical characteristics. Here the presence of a moderate directional effect served to desensitize the measure of fit. However, a similar bulge is clearly visible in the plot of the data (Figure A5P)--the contrast at d_2 with the Titan test is especially pronounced.

Concerning impulse departures for test 278, the results turned out similar to pressure but not quite as sharp. One can see from Figure A4 that the d_1-d_3 stations are considerably above the impulse curve. The successive runs, with the same stations (21 and 30) omitted as for pressure, gave the following results:

	All	d_1 omitted	d_1, d_2 omitted	d_1-d_3 omitted
Y_I	13.6	13.3	12.5	11.7
S_I%	9.8	8.8	5.9	2.4
Fit	∿.02	<.05	<.01	Indeterminate

Both the yield and the scatter continued to decrease, but the fit showed no improvement. For the final run, since the impulse values were unfortunately rounded by URS to the nearest integer, the error for d_4 and d_5 is zero and goodness of fit cannot be calculated. It is evident from the figure, however, that the fit is good. Hence a yield of 11.7 was assigned.

The companion test No. 282 differed somewhat (cf. Figure A5I). The values at d_3 were not biased high and these stations were retained. d_2 was definitely high. d_1, however, was much less than for test 278, with 2 directions having very low impulse values. The general conclusion, based upon both tests, is that the impulse departure is high at d_2, slightly high at d_3, and uncertain at d_1.

5. Summary of Departures from Kingery Curves[†]

Results of successive runs used in the selection procedure are presented in Tables A1-A4 for individual tests. Underlined quantities represent assigned yields--impulse-to-pressure yield ratios are shown in the last column. The "Bias" column lists those distances,* together with sign, where departures from the Kingery curves occurred. Plots of the data--pressure vs. distance and impulse vs. distance--for most tests are shown in Figures A5P, I-A23P, together with Kingery curves for the assigned yields and freehand estimates of the bias (dashed curves).

The individual departures were quite diverse, and often a simple satisfactory characterization was not readily available. This was especially so when there were marginally extreme points or when some distances showed large scatter. In some instances, particularly for CBM impulse, a bend was present, similar to that in the prototype (scaled) Kingery impulse curve, but shifted to the right, more gradual, and having a more pronounced bulge.

In general, except for several unusually "wild" tests, the data appeared to be in accord with, or at least not contradict, the concept of terminal yield. However, many tests recorded data at only three distances;** if close-in bias was present, determination of the terminal yield was somewhat ambiguous. (For example, using only the first three distances for test 239, CBM, LO_2/RP-1, would have given a very erroneous terminal impulse yield.) There were also tests where the slope of the furthest distances differed from the Kingery slope.

Since, as noted below, departures for the small scale tests appear difficult to extrapolate, similarities and differences between the Titan and SIV CBM tests are of special interest. The close-in negative overpressure bias for SIV is more severe than for Titan. Figures A5P and A7P show that at d_1 the overpressure is 50% less for SIV. At d_2, d_4 and d_5, the values are about the same, hence so also are the terminal pressure yields. The impulse departures (Figures A5I, A7I) are more dissimilar. Titan has a bulge at d_2, while SIV shows negative bias and a bend at d_1. Terminal impulse yield for SIV is about 65% greater than Titan.

[†]Pressure and impulse values were taken from Volume 2 of the URS preliminary report. A few minor changes (and some typographical errors) were made in the URS final report. These are listed at the end of this appendix.

*Recall that d_1=23 ft. for 200 and 1000 lbs., while d_1=67 ft. for 25,000 lbs. and higher.

**Number of distances and stations recording are listed in Tables A1-A4. Number omitted because of bias or extremeness are noted in the summary tables of Appendix C.

At the risk of oversimplification, Table A5 attempts to summarize the dominant trends of the departures;* these are based mainly on the bias characterizations in Tables A1-A4. The following remarks are intended both to highlight the salient features in the detailed Tables A1-A4, as well as to amplify the trend characterizations in summary Table A5.

1. CBM, LO_2/RP-1

   a. Pressure. Bias is negligible for 200 lbs., except for a positive bulge for high yield tests; 1000 lbs., including high yield test, is partly negative; 25,000 lbs. has positive bulge; and Titan is negative.

   b. Impulse. A positive bulge occurs for 25,000 lb. tests, Titan, and high yield 200 lb. tests. The biases for 1000 lbs. are very inconsistent, with the highest yield test having negative bias with a bend.

2. CBM, LO_2/LH_2

   a. Pressure. Negative bias occurs for most of the 200 lb. tests, all 1000 tests, and the SIV. In the latter case, the bias is quite large.

   b. Impulse. The departures show no consistent pattern, although there appears to be a slight propensity toward negative close-in bias (including SIV). The best fits are obtained from 200 lb. tests with limited data (3 distances). Impulse-to-pressure yield ratio varies greatly, the overall average being 2.3 (1.75 for SIV).

3. CBGS, LO_2/LH_2

   a. Pressure. The 200 lb. data is very erratic with, in general, negative bias; goodness of fit appears to decrease with increasing velocity--the high yield 78 fps test No. 195 has d_4 high. The 1000 lb. data has a small negative bias while the 25,000 lb. data indicates a small bulge at d_3. (Cf. Figures A20P, 21P).

   b. Impulse. The fits overall are good. The 200 lb., 44 fps data shows mixed behavior: the majority of tests are zero bias, but there are tests with negative bias, bends and bulges (cf. Figure A16I). The 200 lb., 23 fps data is in good agreement with the Kingery curve; the 78 fps data is similar, except for a small negative bias for test 114 and the strange value at d_4 in the case of the highest yield

*The terminology employed, although used loosely, is intended to be suggestive. E.g., "bulge" implies a positive bias at an intermediate distance, with smaller bias (or none) at adjacent distances.

test 195. (Cf. Figure A18I.) The high yield 25,000 lb., 44 fps data shows excellent agreement with the Kingery curve; the low yield test indicates a bulge at d_2 with a positive bias at d_1.

4. CBGS, LO_2/RP-1

   a. Pressure. There is a strong negative bias at 200 lbs. for all three velocities (23, 44, 78 fps), with a tendency to dip at 44 fps (i.e., a larger negative bias at d_2 than d_1). The 1000 lb., 44 fps data shows a negative bias, but has large scatter at the close-in distances. The single nonspurious 25,000 lb. test has a moderate negative bias and bend. In all weight and velocity combinations, the fit is generally good from d_3 on. (Cf. Figures A20P, A21P.)

   b. Impulse. There is a negative bias at 200 lbs. and 44 fps which decreases to zero in magnitude with decreasing yield; for 200 lbs., 23 fps there is a very small negative bias in the case of the two highest yield tests, with a slight bulge at d_2 for the lowest yield and large scatter, but zero bias for the remaining tests (cf. Figures A9I, A12I). With the exception of tests 141 and 236, the 200 lb., 78 fps data shows good agreement with the Kingery curve. In the case of the 1000 lb., 44 fps data, the fits are generally good at the last three distances. There is no data at d_2 for the maximum yield 1000 lb. test, but each of the next two highest yields seem to have low values at d_2 combined with high values at d_1 and d_3, i.e., a bend followed by a bulge. The lowest yield test provides a good fit to the Kingery curve, but with relatively large scatter. (Cf. Figure A10I.) The Kingery curve shows excellent agreement with the 25,000, 44 fps data (cf. Figure A11I), no deviation. Finally, there is a possible negative bias coupled with a dip at 1000 lbs. and 78 fps (cf. Figure A13I).

Main Conclusions

The overpressure exhibits a negative bias at close in distances except for positive bulges at 25,000 lbs. and for the high yield 200 lb. CBM LO_2/RP-1 tests.

The impulse bias depends strongly on propellant and failure mode. CBGS was relatively well-behaved with no bias for LO_2/LH_2 and a tendency to negative bias for low weights with LO_2/RP-1. CBM behaved erratically with a tendency to positive bulges for LO_2/RP-1 and to negative bias for LO_2/LH_2.

Impulse yields were significantly higher than pressure for CBM LO_2/LH_2 and for the high yield 200 lb. and 1000 lb. CBGS LO_2/RP-1 tests.

In general, departures from the Kingery curves do not follow any recognizable pattern for different weights and hence do not appear to be amenable to extrapolation. It is hoped, however, that the above detailed cataloging may prove useful in any future attempts to better understand the blast behavior of liquid propellants.

DISCREPANCIES IN URS FINAL REPORT

Test No.	Variable	Normal*	Appendix E** Discrepancy
047	Y	10	4 (Vol. II)
	t	120	515 (Vol. II)
267	t	1170	1770 (and Vol. II and Table 5-5)
058	Y	27	26
101	Y	35	25 (Vol. II)
	t	145	245
174	Y	52	51
	t	150	156
088	Y	4	8
208	Y	62	39
284	t	0	89
200	t	417	100
172	Y	35	32
	t	770	730
229	Y	53.	52
215	t	20	23
173	Y	13	15 (and Vol. II)

*Values given in Section 5, Appendix B, and Volume II unless otherwise indicated in discrepancy column.

**Unless otherwise indicated.

TABLE A1 CBM, LO_2/RP–1. SUMMARY OF ALTERNATIVE KINGERY FITS [1, 2]

				PRESSURE									IMPULSE									
W	L/D	Do/Dt	Test No.	Avail. Data d's (St's)	All d's Yp	All d's Sp%	d_1 Omitted Yp	d_1 Omitted Sp%	d_1, d_2 Omitted Yp	d_1, d_2 Omitted Sp%	Bias	Comments	Avail. Data d's (St's)	All d's Y_I	All d's S_I%	d_1 Omitted Y_I	d_1 Omitted S_I%	d_1, d_2 Omitted Y_I	d_1, d_2 Omitted S_I%	Bias	Comments	Y_I/Yp
200	1.8	.45	239	5(14)	33.5	9.5	31.0	7.9	31.6	6.7	$+d_1$		5(13)	25.7	12.2***	24.3	8.8**	23.6	7.5***	$+d_1, +d_2, +d_3$	d_1–d_3 omitted: Y=22.1, S=3.2	.71
			237	5(14)	30.6	12.4	30.7	12.3	31.1	6.4	0	Dir'l: 26–38	5(14)	26.8	8.9	26.4	9.2	26.1	8.6	0	Dir'l: 24–29	.88
			101	3(9)	31.0	12.5	33.0	12.7	35.5	9.8	$-d_1$	Dir'l: 29–42	3(9)	32.6	9.7	33.7	7.2	31.8	8.9	$+d_2$		.97
			044	3(8)	17.8	9.2	18.9	7.9	19.5	11.6	0		3(8)	17.4	10.3	17.1	10.7	15.5	5.0	$+d_1, +d_2$	Dir'l: 15–21	.87
			095A	3(9)	17.6	7.6	18.4	6.6	19.4	3.8	0		3(9)	14.7	6.7	14.8	6.8	14.1	8.4	0	d_2 slightly hi	.83
			238	5(13)	16.0	7.6	15.9	8.3	16.3	9.9	0	St. 29, 39(hi) omitted. Dec'l pt. wrong? [3]	5(13)	14.4	9.5	14.5	10.2	14.5	9.6	0		.90
			087A	3(9)	13.6	7.5	14.1	7.8	15.6	5.5	0		3(9)	15.1	6.3	14.9	5.2	14.9	2.4	0		1.11
																						(.90)
1,000	1.8	.45	193	4(10)	15.4	9.7***	16.0	9.7***	18.5	3.7	$-d_1, -d_2$		4(11)	17.3	12.9	18.3	9.0*	20.1	3.4	$-d_1, -d_2, +d_3$	St. 37(hi) omitted. Pronounced bend.	1.08
			270A	5(15)	10.7	9.4	10.7	7.1	10.8	7.5	0	St. 25 (hi), 37 (lo) omitted	5(15)	12.1	7.8**	12.4	7.6**	11.7	6.1	$+d_2$	d_3 slightly hi	1.10
			192	4(10)	8.9	9.4*	9.4	9.4*	10.9	5.6	$-d_1, -d_2$		4(10)	9.6	11.5**	10.4	9.0	11.2	6.8	$-d_1, -d_2$	d_1 and St. 26 (lo) omitted: Y=10.8, S=6.4	.99
			209	4(12)	9.5	5.7	9.6	6.3	10.3	5.8	0	St. 38 (hi) not omitted, scatter too low	4(12)	8.9	12.1	9.0	13.0	9.2	15.6	$+d_3$	Variable data. d_3 slightly hi	.93
																						(1.02)
25,000	1.8	.45	282	5(13)	12.3	10.4	12.7	9.6	12.1	10.0	$+d_2, +d_3$	d_1–d_3 omitted: Y=11.3, S=6.8 Dir'l: 9.5–12.9	5(15)	11.8	11.5**	12.1	10.0**	11.4	6.9	$+d_2$		1.01
			278	5(14)	12.7	11.5***	13.7	9.2**	12.5	7.4*	$+d_2, +d_3$	d_1–d_3 omitted: Y=11.1, S=6.4	5(13)	13.6	9.8**	13.3	8.8**	12.5	5.9***	$+d_1, +d_2, +d_3$	d_1–d_3 omitted: Y=11.7, S=2.4	1.06
			275	5(15)	1.3	9.5*	1.3	10.5*	1.2	10.3	$+d_2$		5(15)	4.6	15.7	4.6	15.7	4.7	14.0**	$+d_4$	Variable data. Dir'l: 3.7–5.3	2.56
94,000	4	.1375	Titan	5(13)	2.84	11.4**	2.94	10.8**	3.27	8.8	$-d_1, -d_2$		5(10)	3.49	11.2	3.55	11.3	3.42	7.5	$+d_2$	St. 19(lo) omitted. d_5 slightly hi	1.05
200 {	1.8	.45]	240	5(13)	65.5	13.1	62.2	12.9	55.8	5.7	$+d_1, +d_2$		5(15)	44.4	10.5*	46.4	10.0	46.5	10.8	$-d_1, -d_4$	Alternative: $+d_2, +d_3, +d_5$	.83
{	2/3 Full]		174	3(5)	56.7	4.5	57.8	5.6	– –	– –	0	Only d_1 and d_3 available	3(7)	44.6	5.8*	46.5	3.9	47.8	3.6	$-d_1$		.82
200	1.8	1.0	042	3(8)	49.8	17.9**	59.3	13.8	48.7	2.9	$-d_1, +d_2$	Difficult choice	3(8)	48.5	8.1*	50.8	7.8*	45.3	2.5	$+d_2$		.93
			058	3(8)	28.6	10.0	31.3	7.0	33.4	7.1	$-d_1$	d_2 slightly lo.	3(8)	22.8	9.1	22.2	9.7	21.3	18.5	0		.73
			086	3(8)	12.1	13.0	11.8	10.7	13.5	8.3	0		3(8)	11.3	10.0	11.6	8.6	11.4	1.4	0	St. 36 hi	.93
200	5	.45	100	3(9)	25.3	10.4	23.7	9.5	22.3	12.3	0	d_1 slightly hi.	3(9)	22.8	7.2	22.9	8.3	21.5	8.8	0	d_2 slightly hi	.90
			046	3(8)	16.1	6.1	16.3	6.3	17.1	7.5	0	Slightly dir'l: 15–18	3(8)	16.2	6.8	15.7	3.8	16.1	6.0	0		1.01
			088	3(8)	3.8	10.6	3.9	11.2	3.8	4.9	0	Dir'l: 3.1–4.6	3(8)	4.0	4.8	3.9	3.3	3.8	2.4	0	Slope slightly steep	1.06
200	5	1.0	085	3(9)	12.9	8.5	12.9	7.2	13.7	8.0	0		3(9)	9.3	8.2	9.5	8.4	9.1	3.5	0	d_2 slightly hi	.72
			049	3(8)	12.6	8.0	12.3	7.3	13.0	5.7	0	Slightly dir'l: 11.7–14.7	3(8)	10.4	10.1	10.6	10.0	11.1	15.2	0	Variable data	.83
			047	3(7)	10.7	14.0*	9.5	12.4	11.7	17.8	$-d_2$	Large scatter in d_3	3(7)	8.4	11.1	7.9	11.3	7.7	20.1	0	St. 37(lo) omitted: Y=8.8, S=7.4	.83
																						(.85)

1. Underlined quantities denote yields selected as most representative and used in the regression analysis.
2. Asterisks denote significantly poor Kingery fits relative to scatter.
3. Corrected in URS final report.

TABLE A2. CBM, LO_2/LH_2. SUMMARY OF ALTERNATIVE KINGERY FITS

				PRESSURE									IMPULSE									
			Test	Avail. Data d's	All d's		d_1 Omitted		d_1,d_2 Omitted				Avail. Data d's	All d's		d_1 Omitted		d_1,d_2 Omitted				
W	L/D	D_o/D_t	No.	(St's)	Yp	Sp%	Yp	Sp%	Yp	Sp%	Bias	Comments	(St's)	Y_I	S_I%	Y_I	S_I%	Y_I	S_I%	Bias	Comments	Y_I/Y_p
200	1.8	.45	090	3(7)	18.3	10.3	19.1	11.4	21.9	19.7	0	St. 37 (hi) omitted: Y=17.5, S=3.5	3(7)	25.5	18.7**	24.3	21.1***	34.3	8.3	$-d_2$	d_2 very lo	1.96
			118	3(9)	8.4	8.5***	9.5	3.0	9.5	2.7	$-d_1$		3(9)	27.5	7.6	28.7	5.9	27.3	2.5	$+d_2$	d_2 slightly hi	2.88
			200	4(10)	6.0	12.0	5.9	13.9	5.0	11.4	0	Variable data	4(10)	15.9	17.9*	15.4	20.1**	16.3	27.2*	$-d_3,+d_4$	St. 37(lo) and 28(hi) omitted: Y=15.7, S=11.2	2.62
			091	3(8)	6.5	8.6	6.9	10.3	6.6	19.2	0		3(8)	16.6	8.9	17.2	5.0	16.5	5.9	0	St. 25(lo) omitted: Y=17.3, S=4.9	2.65
			199	4(11)	4.2	4.9	4.3	4.9	4.4	6.1	0		4(10)	9.5	13.1	9.3	13.0*	8.5	9.4*	$-d_3$	St. 16(lo) omitted. Bend	2.26
			053	3(8)	1.7	20.5	2.1	5.6	2.2	6.2	$-d_1$(?)	St. 15 (lo) omitted: Y=2.0, S=6.0	3(6)	4.7	9.9	5.1	1.0	5.1	1.1	$-d_1$		2.54
																						(2.49)
1,000	1.8	.45	213	5(14)	19.8	14.8	20.7	12.9	23.3	9.5	$-d_1,-d_2$	Variable data	5(14)	28.2	18.8***	30.8	15.2**	34.5	8.4	$-d_1,-d_2$		1.48
			212	4(11)	2.2	15.8*	2.3	18.0**	3.2	12.1	$-d_1,-d_2$	d_4 omitted, has Y=24.3	4(10)	23.3	15.6	24.0	17.2	24.9	19.9	$+d_4$	d_4 (St. 28,38) omitted: Y=21.3, S=11.5. Dir'l	6.67
			265	5(15)	2.6	19.1*	2.9	15.4*	3.4	12.1	$-d_1,-d_2$		5(14)	11.8	33.3	13.7	19.8	13.8	7.5*	?	d_1 and d_2 variable. d_3 and d_4 slightly hi	4.03
			210	4(12)	1.4	16.9	1.5	11.2	1.6	8.6	$-d_1,-d_2$	Dir'l: 1.1–2.0	4(12)	10.2	10.9	10.6	10.8	10.5	12.7	0	d_3 slightly hi	6.27
																						(4.61)
25,000	1.8	.45	277	5(15)	.09	28.2	.08	26.4	.08	27.3	?	Dir'l: .05–.15	5(15)	.18	11.0	.17	9.7	.18	8.8	?		2.02
			279	2(6)	.05	5.7	– –	– –	– –	– –	– –	No d_1-d_3	2(6)	.15	24.0	– –	– –	– –	– –	– –	No d_1-d_3	3.15
			281	1(3)	.05	– –	– –	– –	– –	– –	– –	No d_1-d_4	1(3)	.13	17.1	– –	– –	– –	– –	– –	No d_1-d_4	2.49
																						(2.55)
91,000	2	.083	SIV	5(10)	2.44	17.2***	2.77	9.1**	2.90	9.7**	$-d_1,-d_2,-d_3$	d_1-d_3 omitted: Y=3.26, S=7.6	4(6)	5.25	14.9	5.69	7.6	5.76	8.6	$-d_1$	No d_3	1.75
200	1.8	.45	172	3(5)	25.3	12.1**	30.0	4.8	31.9	– –	$-d_1$		3(5)	36.6	4.5	37.5	3.1	36.8	2.5	0	d_1 slightly lo, d_2 slightly hi	1.22
	2/3 Full		167	3(6)	16.7	19.1***	21.8	8.5*	23.9	3.4	$-d_1,-d_2$		3(5)	29.2	5.6	28.7	6.8	27.9	5.1	0	d_1 and d_2 slightly hi	1.22
200	1.8	1.0	050	3(6)	49.2	27.8***	66.1	13.3	79.3	10.9	$-d_1,-d_2$(?)	Difficult choice	3(6)	95.9	13.0	104.5	7.6	106.4	13.0	$-d_1$		1.32
			093	3(7)	23.4	9.0**	26.1	3.8	25.8	4.0	$-d_1$	St. 36 (lo) omitted	3(7)	40.4	7.7	41.0	5.1	39.8	5.8	0		1.55
			051	3(7)	11.9	22.1**	16.2	10.6	20.3	– –	$-d_1$		3(7)	21.6	7.7	22.1	10.4	23.4	– –	0	*St. 16(hi) omitted: Y=20.7**, S=4.7*	1.28
200	5	.45	094	3(8)	21.8	12.1	20.0	8.1	19.4	5.0	0	Dir'l: 19-26	3(8)	28.7	4.9	28.2	5.7	27.6	6.0	0	d_3 slightly lo	1.32
			138	3(6)	12.0	6.1	12.6	6.4	11.4	3.0	0		3(8)	21.3	5.0	20.9	5.2	20.0	4.9	0		1.78
			054	3(6)	2.7	17.2***	3.2	6.5**	3.8	4.9	$-d_1,-d_2$		3(6)	7.4	4.6	7.5	4.5	7.6	7.1	0		1.95
			055	3(8)	.9	21.2**	1.2	9.0	1.4	7.5	$-d_1$	Dir'l: .9–1.5	3(8)	1.1	13.1	1.2	10.8	1.2	17.8	$-d_1$	Dir'l: 1.0–1.4	.98
200	5	1.0	092b	3(9)	13.1	25.7***	18.6	10.4	20.1	10.7	$-d_1$	d_1 and St. 37 omitted: Y=17.7, S=7.5	3(9)	30.4	10.8	32.7	8.4	33.6	9.6	$-d_1$		1.85
			092a	3(9)	3.5	7.7	3.5	6.4	3.5	7.2	0		3(9)	5.8	11.3**	5.5	11.2*	4.9	5.0	$+d_1,+d_2$		1.39
			052	3(8)	4.7	17.7**	6.1	4.3	5.9	4.0	$-d_1$		3(8)	5.8	9.0	6.1	7.7	6.6	9.7	0	d_1 slightly lo	.94
			057	3(8)	.7	19.3*	.95	12.1	1.1	8.3	$-d_1$		3(8)	1.1	13.0*	1.2	2.9	1.2	2.9	$-d_1$		1.32
200	1.8	.083	169	3(6)	9.0	6.0	8.6	5.4	9.4	3.3	0		3(6)	19.7	3.1	19.6	3.2*	19.0	2.5	0		2.19
			173	3(7)	5.4	7.1	5.5	7.5	6.0	8.8	0		3(7)	19.9	7.0	20.8	4.6	20.9	6.4	$-d_1$		3.83
																						(1.61)

TABLE A3. CBGS, LO_2/LH_2, SUMMARY OF ALTERNATIVE KINGERY FITS

			PRESSURE									IMPULSE									
W(lbs)	V (fps)	Test No.	Avail. Data d's (stn's)	All Dist's (0)		d_1 Omitted (1)		d_1 & d_2 Omitted (2)		Bias	Comments	Avail. Data d's (stn's)	All Dist's (0)		d_1 Omitted (1)		d_1 & d_2 Omitted (2)		Bias	Comments	Final Y_I/Y_p
				Y_p	S_p%	Y_p	S_p%	Y_p	S_p%				Y_I	S_I%	Y_I	S_I%	Y_I	S_I%			
200	44	230(SP)	5(15)	13.7	13.5	13.0	13.5	12.89	12.5	$-d_2$	Directional effect (2)	5(15)	16.09	3.3	16.3	5.3	17.2	9.3	0		1.248
		197	4(11)	13.7	20.1	14.3	17.4	16.95	6.2	$-d_2-d_3$		4(11)	17.07	6.9	17.2	7.6	17.3	6.4	0		1.007
		231	5(14)	17.5	10.4	16.65	8.7	17.2	9.4	$-d_1-d_2+d_3-d_4$	Directional effect (2)	5(14)	19.66	6.8	19.9	6.8	19.1	12.8	0		1.181
		254	5(14)	20.34**	11.0	21.8**	9.8	24.5	7.9	$-d_1$	Poor fit at (2) also	5(14)	24.4*	8.6	25.9	4.9	25.81	3.3	$-d_1$		1.053
		203	4(11)	22.94	16.6	24.1	14.1	25.4	18.1	0	Large scatter at each d.	4(11)	25.51	2.9	26.7	8.0	27.4	9.7	0		1.112
		229	5(14)	24.5***	24.3	26.0***	24.0	35.25*	13.4	$-d_1-d_2$		5(14)	35.2	14.5	34.0	14.5	36.35	12.2	$+d_1-d_2-d_3$	d_5 is high; data erratic – convex	1.031
		252	5(14)	27.7*	10.5	29.3	10.3	32.58	8.1	$-d_1-d_2$	Tail ragged	5(14)	30.3	8.9	31.3	7.3	30.04	5.1	$-d_1+d_2$		.922
		204	4(11)	35.8***	10.5	38.2***	9.5	44.07	4.4	$-d_1-d_2$		4(11)	44.66	11.0	45.3**	7.3	44.9**	9.2	0	d_4 low; directional effects	1.013
		251	5(12)	38.5	21.7	36.8	19.6	44.08	17.0	$-d_1-d_2-d_3$	Sp.Y_p = 51.16, S_p = 7.7%	5(13)	50.6	6.7	51.65	5.3	52.0	6.3	$-d_1$	Sp: Y_I = 53.07, S_p = 5.8%; direct. eff'ts	1.037
																				AVERAGE	1.067
1000	44	211(SP)	4(12)	4.94	13.3	5.2	18.7	6.1	19.0	$-d_2$	$+d_4$	4(12)	12.36	12.7	12.5	10.6	12.8*	12.1	0	d_4 high	2.502
		266(SP)	5(14)	6.7	5.4	6.7	2.5	6.76	2.4	$-d_1-d_2$		5(14)	14.4*	20.5	16.0	11.7	17.39	7.3	$-d_1-d_2$		2.574
		264(SP)	5(14)	11.36	17.9	11.9	16.4	12.9*	11.1	0	Large scatter	5(14)	18.1	17.5	19.26	15.8	20.3	17.0	$-d_1$		1.695
		217	5(11)	17.9	52.1	15.73	39.0	15.1	41.4	$-d_1$	Sp. Y_p = 28.22, S_p = 14.8%	5(10)	38.5	28.6	33.67	5.0	33.7	5.0		Few stat's and large scatter at d_1 & d_2	1.193
		262	5(15)	28.7	25.7	29.2	18.3	36.86	6.3	$-d_1-d_2-d_3$		5(15)	37.8	13.2	35.6	10.5	38.15	8.2	$+d_1-d_2$		1.035
																					1.80
25,000	44	289(SP)	5(13)	2.67	4.1	2.72	4.3	2.65	4.3	0 $+d_2$		5(13)	3.87	3.5	3.9	3.7	3.8	2.9	0	Extremely variable at d_1	1.449
		290(SP)	5(14)	2.27	13.7	2.33	14.3	2.15	11.0	$-d_1+d_2+d_3$	Bulge at d_2 and d_3 d_5 low(.85 avi. eff)	5(14)	4.83	13.2	4.78	12.5	4.46	10.1	$+d_1+d_2$		2.074
		288C	5(14)	10.11**	12.0	11.14	9.9	11.37*	10.1	$-d_1+d_2+d_3$	Sp. Y_p = 10.17, S_p = 7.5%	5(14)	11.42	6.6	11.55	7.0	11.67	6.9	0		1.123
											Bulge at d_3										1.549
200	23	105(SP)	3(7)	6.04	5.6	6.0	7.3	5.9	4.0	0		3(7)	6.75	5.8	7.3	7.7	6.8	.95	$+d_2$		1.119
		152	3(9)	11.0	12.7	11.88	11.5	13.3	8.4	$-d_1-d_2$	Directional effect	3(9)	13.67	6.3	13.5	7.3	13.4	10.0	0	Directional effect	1.151
		153	3(9)	12.78	13.4	13.2	13.8	13.0	6.8	0	Variable		—	—	—	—	—	—	—	No data	1.135
		184	3(6)	15.0	11.9	16.30	9.0	17.2	8.3	$-d_1$		3(7)	15.79	6.7	15.8	7.6	15.0	2.6	0		.969
		225	5(15)	15.6***	25.0	19.0***	19.3	23.8***	11.5	$-d_1-d_2$	The fit is good at (2)	5(15)	22.9*	9.2	24.8*	11.1	26.31	9.0	$-d_1-d_2$	Variable; d_5 high	1.104
		201	4(11)	26.0	14.4	25.0	13.5	22.1	8.4	0		4(11)	28.22	11.7	26.7	11.3	26.9	8.8	0		1.0854
																					1.053
200	78	226(SP)	5(14)	26.2	36.0	24.8	32.9	37.04	12.5	$-d_1-d_2$		5(14)	34.41	10.7	35.1	10.6	35.3	9.7	0		.929
		150	3(9)	27.8**	8.6	28.6**	10.1	34.18	3.7	$-d_1-d_2$		3(9)	34.77	4.0	35.1**	2.6	34.0	2.1	0		1.017
		151	3(7)	28.2***	16.7	35.18**	6.0	41.7	---	$-d_1-d_2$		3(7)	44.27	3.5	45.3*	2.1	44.4	1.4	0		1.259
		114	3(9)	41.4***	20.3	52.01***	7.1	58.5	2.8	$-d_1-$ 0 $+d_3$		3(9)	49.6*	5.7	51.84	4.5	50.4	5.6	$-d_1$		.997
		195	4(11)	74.1***	47.0	84.7***	53.5	176.0***	23.3	0 $-d_2+d_4$	Sp. Y_p = 53.60. S_p = 24.9%	4(11)	67.1***	14.6	71.6***	13.3	75.8***	14.9	0 d_4 high	Sp. Y_I = 69.46, S_p = 17.1%, (used);	1.296
																				Sp. Y_I = 62.40, S_p = 10.1%. (pref'ble)	1.10
1000	78	215(SP)	5(15)	12.13	9.2	11.6	8.6	12.2	8.4	0		5(15)	22.9**	7.0	22.9**	7.0	24.05	4.5	0	Significant directional effects	1.983
1000	78	216(SP)	5(15)	5.48	9.6	5.1	13.2	5.4	10.1	0		5(14)	9.3**	19.0	10.8	10.2	11.35	8.5	0		2.071
200	597	079	4(11)	91.2	22.4	91.2	22.4	90.36	18.9	$+d_2$(?)		4(11)	98.2	18.8	98.2	18.8	94.59	16.8	$+d_2$(?)		1.047
200	569	080	4(11)	101.7	52.8	101.2	52.8	93.25	45.2	$+d_2$(?)		4(11)	127.5	36.9	127.5	36.9	119.8	34.3	$+d_2$(?)		1.285
																					1.597

TABLE A4. CBGS, LO_2/RP-1 SUMMARY OF ALTERNATIVE KINGERY FITS

			PRESSURE									IMPULSE									
			Avail. Data d's	All Dist's (0)		d_1 Omitted (1)		d_1 & d_2 Omitted (2)				Avail. Data d's	All Dist's (0)		d_1 Omitted (1)		d_1 & d_2 Omitted (2)				
W(lbs)	V(fps)	Test No.	(stn's)	Y_p	S_p%	Y_p	S_p%	Y_p	S_p%	Bias	Comments	(stn's)	Y_I	S_I%	Y_I	S_I%	Y_I	S_I%	Bias	Comments	Final Y_I/Y_p
200	44	232	5(10)	17.0**	20.3	15.1***	20.3	22.9	5.1	$-d_1-d_2$		5(10)	23.7	7.8	23.6	8.5	23.7	4.4	0	Scatter large; 1 data point at d_3 and d_5	1.034
		208	4(9)	21.8*	37.8	28.5*	30.8	39.3*	22.2	$-d_1-d_2-d_3$	d_4 high	4(10)	43.7*	20.7	49.2	14.2	49.0	10.0	$-d_1$	Directional effect at (0) and (1)	1.261
		249	5(14)	24.8**	20.2	31.0*	12.9	35.0*	7.4	$-d_1-d_2$	Large negative bias	5(15)	34.1	8.5	34.4	8.4	35.3	7.3	$-d_1-d_2$	Directional effect at (2)	1.009
		250	5(15)	35.3*	17.5	32.4**	25.4	43.3	14.3	$-d_1-d_2$	Variable, d_5 high	5(15)	35.8	16.1	36.1	14.7	40.0	6.7	$-d_1-d_2$		1.081
																				AVERAGE	1.096
1000	44	218(SP)	5(14)	3.1	14.9	2.9*	13.4	2.6	11.3	$+d_2$	d_4 is low	5(12)	4.4	12.5	4.3	11.8	4.2	10.2	0 ?		1.410
		219	5(14)	6.9	18.5	7.0	19.6	8.5	13.0	$-d_1-d_2-d_3$		5(15)	12.7	10.6	12.9	9.8	13.5	9.4	$-d_1-d_2$	Direct. effect at (1), (2)	1.588
		267	5(15)	37.6	54.5	43.5	42.4	51.9	12.3	$-d_1-d_2$		5(14)	58.7	9.7	61.3	13.5	65.9	12.0	$-d_2$		1.132
		268	5(14)	47.7*	17.0	48.5*	17.7	54.5*	16.8	$-d_1-d_2-d_3$	d_4 is high	5(14)	55.9	7.2	54.7	6.2	58.6	9.2	$-d_2$		1.026
		220	4(10)	71.0	14.8	71.4	15.7	71.4	15.7	0	No data at d_2	4(11)	87.7	7.8	88.7	7.9	88.7	7.9	0	No data at d_2; Direct. effect at (1), (2)	1.235
																					1.278
25,000	44	284(SP)	5(15)	1.3	9.4	1.2	7.5	1.2*	6.6	$+d_1$	Directional effect	5(14)	1.4	3.5	1.4	3.2	1.4	3.1	0		1.168
		285	5(13)	25.7*	16.7	26.1**	16.9	31.93	6.8	$-d_1-d_2$	good fit at last 3 dist's	5(13)	29.4	3.7	28.2	6.8	28.9	5.5	0		.922
																					1.045
200	23	096	3(9)	13.0	5.4	13.5	5.2	14.1	6.8	0		3(9)	12.8*	8.4	13.2*	9.4	12.0	4.2	0 $+d_2$	Bulge at d_2	1.0146
		248	5(15)	19.0	12.4	20.3	10.4	21.7	10.0	0		5(15)	17.2	9.7	17.5	9.1	17.7	10.4	0	Variable data; directional effect (0), (1), (2)	.903
		144	3(7)	21.5**	11.4	22.6**	10.7	27.3	5.6	$-d_1-d_2$		3(9)	20.6*	3.6	21.0	3.2	20.5	2.7	$-d_1$	d_2 slightly high	.771
		202	4(12)	30.5	20.1	33.0	15.7	38.0	9.4	$-d_1-d_2$	Variable	4(11)	36.7**	10.0	39.1	7.3	40.6*	5.9	$-d_1$		1.030
																					.930
200	78	141(SP)	3(7)	4.4	6.5	4.4	7.0	4.8	8.9	0	Directional effect	3(7)	5.2	12.6	4.7	9.2	4.3	7.7	$+d_1$	Directional effect at (1)	1.064
		110	3(9)	18.3***	9.8	20.6*	3.4	21.7	0.0	$-d_1$		4(9)	28.9	6.7	28.3	7.1	28.7	8.8	0	Directional effect at (0)	1.406
		207(SP)	4(11)	29.7	4.9	29.9	5.7	32.0	5.1	0		4(11)	32.2	2.6	34.6	6.4	34.5	7.0	0		1.085
		205(SP)	4(11)	38.0**	8.1	39.8**	8.2	44.2	6.8	$-d_1-d_2$		4(11)	42.0	6.9	42.1	6.0	41.7*	6.8	0	d_4 is high	.950
		236	5(15)	32.3***	33.2	43.5*	23.0	54.8	19.0	$-d_1-d_2-d_3$	Directional effect (2)	5(14)	49.6*	10.1	54.5	10.8	57.1	8.9	$-d_1-d_2$	Variable at d_5; d_5 high	.995
		206	3(8)	67.3	8.7	67.3	8.7	67.5	10.6	0		3(8)	90.1	4.3	--	--	88.9	4.3	0	Directional effect at (2)	1.339
																					1.140
1000	78	190(SP)	4(11)	58.2*	23.7	63.6*	20.2	75.9	10.2	$-d_1-d_2$		4(10)	86.7**	30.0	79.6**	22.0	94.6	16.7	$-d_1-d_2$		1.246
		269A(SP)	5(14)	27.6***	25.6	32.8***	20.2	41.92***	8.3	$-d_1-d_2$		5(14)	38.1	9.2	39.0	5.0	40.06	3.7	$-d_1-d_2$	Almost zero bias at d_1	.956
																					1.101
200	526	075	4(11)	18.9	21.1	--	--	17.8	16.9	$+d_2$		4(11)	17.6	29.2	--	--	15.7	25.1	$+d_2$	Significant directional effects	.882
	523	077	4(11)	8.8	23.9	--	--	8.7	23.5	$+d_2$		4(11)	17.1	19.7	--	--	16.4	17.1	$+d_2$		1.885
	523	076	4(11)	38.9	56.2	--	--	31.4	46.4	$+d_2$		4(11)	47.9	33.4	--	--	44.6	31.0	$+d_2$		1.420
	518	078	4(11)	39.1	48.9	--	--	37.3	35.3	$+d_2$		4(11)	52.5	35.7	--	--	49.3	34.0	$+d_2$		1.322
																					1.377

TABLE A5 - OVERALL SUMMARY OF DEPARTURES FROM KINGERY CURVES

			PRESSURE			IMPULSE		
	W	GEOMETRY/ VELOCITY	GENERAL TREND	QUALITATIVE DESCRIPTION	FIGURE	GENERAL TREND	QUALITATIVE DESCRIPTION	FIGURE
CBM, LO_2/RP-1	200	NOMINAL	0		A5P	{ 0 $+d_1+d_2$	HIGH YIELD TESTS HAVE POSITIVE BIAS	A5I
	200	NON-NOMINAL	{ 0 $+d_2$	POSITIVE BULGE FOR HIGH YIELD	A6P	0	HIGH YIELD TESTS HAVE BULGES AND BENDS	A6I
	1,000	NOMINAL	{ 0 $-d_1-d_2$	NEGATIVE BIAS FOR HIGH YIELD	A5P	?	BAD FITS--MIXED BIAS – NEGATIVE BIAS PLUS BEND FOR HIGHEST YIELD	A5I
	25,000 (NON SP)	NOMINAL	$+d_2,+d_3$	POSITIVE BULGE	A5P	$+d_2$	POSITIVE BULGE	A5I
	TITAN		$-d_1,-d_2$	NEGATIVE BIAS	A5P	$+d_2$	SMALL POSITIVE BULGE	A5I
CBM, LO_2/LH_2	200	NOMINAL	0	SMALL NEGATIVE BIAS ON SOME TESTS	A7P	?	BAD FITS--MIXED BIAS. $-d_2$ DIP FOR HIGHEST YLD	A7I
	200	NON-NOMINAL	$-d_1,-d_2$ (?)	LARGEST NEGATIVE BIAS FOR HIGHEST YIELD	A8P	{ 0 $-d_1$	5 OF 15 TESTS WITH NEGATIVE BIAS	A8I
	1,000	NOMINAL	$-d_1,-d_2$	NEGATIVE BIAS	A7P	?	MOSTLY BAD FITS--NEGATIVE BIAS FOR HIGHEST YIELD TEST	A7I
	SIV		$-d_1-d_2-d_3$	LARGE NEGATIVE BIAS	A7P	$-d_1$	NEGATIVE BIAS	A7I
CBGS, LO_2/LH_2	200	44	$-d_1,-d_2$	NEGATIVE BIAS; $-d_3$ FOR HIGHEST YIELD	—	0	BIASES SLIGHT AND MIXED	A161
	1,000	44	{ 0 $-d_1,-d_2,-d_3$	NEGATIVE BIAS FOR HIGHEST YIELD	A22P	0	BAD FITS--MIXED BIASES	A11I
	25,000	44	$-d_1,+d_2?,+d_3$	POSITIVE BULGE	A22P	0	BULGE FOR LOWEST YIELD	A15I
	200	23	{ 0 $-d_1,-d_2$(?)	MIXED	A23P	0		A18I
	200	78	$-d_1,-d_2$	LARGE NEGATIVE BIAS	A23P	0	UNCERTAIN BIAS FOR HIGHEST YIELD	A18I
	200	HVI	$+d_2$(?)		—	$+d_2$(?)		A19I
CBGS, LO_2/RP-1	200	44	$-d_1,-d_2$	NEGATIVE BIAS, WITH LARGE DIP AT d_2	A20P	$-d_1,-d_2$	NEGATIVE BIAS	A9I
	1,000	44	$-d_1,-d_2,-d_3$(?)	LARGE NEGATIVE BIAS	A20P	0	POSSIBLE DIP AT d_2	A10I
	25,000	44	$-d_1,-d_2$	NEGATIVE BIAS	A20P	0		A11I
	200	23	{ $-d_1,-d_2$ 0	NEGATIVE BIAS EXCEPT FOR LOW YIELDS	A21P	0	MIXED-NEGATIVE BIAS FOR HIGHEST YIELD	A12I
	200	78	{ $-d_1,-d_2$ 0	NEGATIVE BIAS AT INTERMEDIATE YIELDS	A21P	0	POSSIBLE NEGATIVE BIAS FOR INTERMEDIATE YIELDS	—
	1,000	78	$-d_1,-d_2$	LARGE NEGATIVE BIAS	—	$-d_2$	LARGE DIP FOR HIGHEST YIELD	A13I
	200	HVI	$+d_2$	SLIGHT POSITIVE BIAS (NO DATA AT d_1)	—	$+d_2$	SLIGHT POSITIVE BIAS (NO DATA AT d_1)	A14I

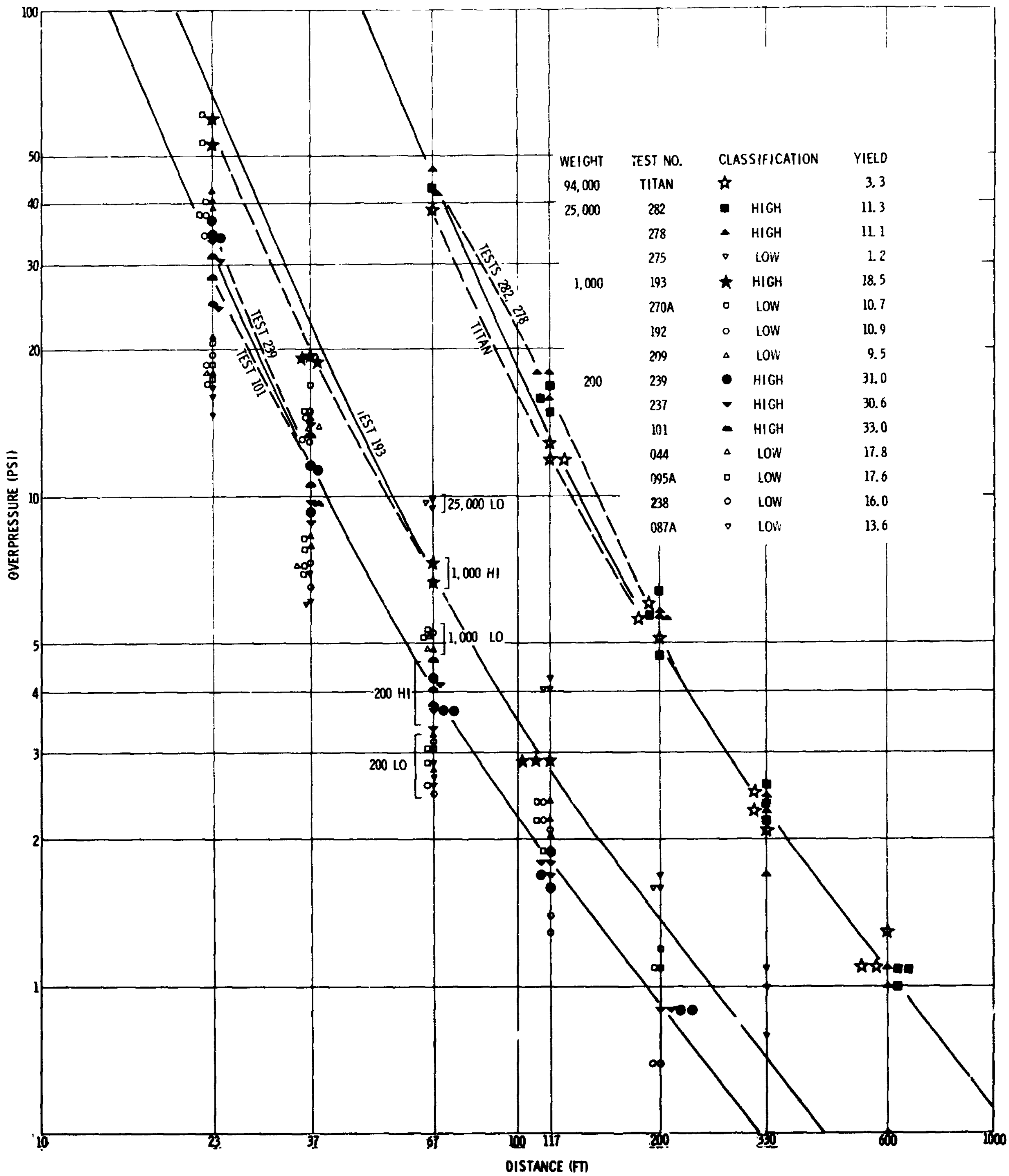

FIGURE A5P - CBM, LO_2/RP-1, NOMINAL GEOMETRY, PEAK OVERPRESSURE VS DISTANCE

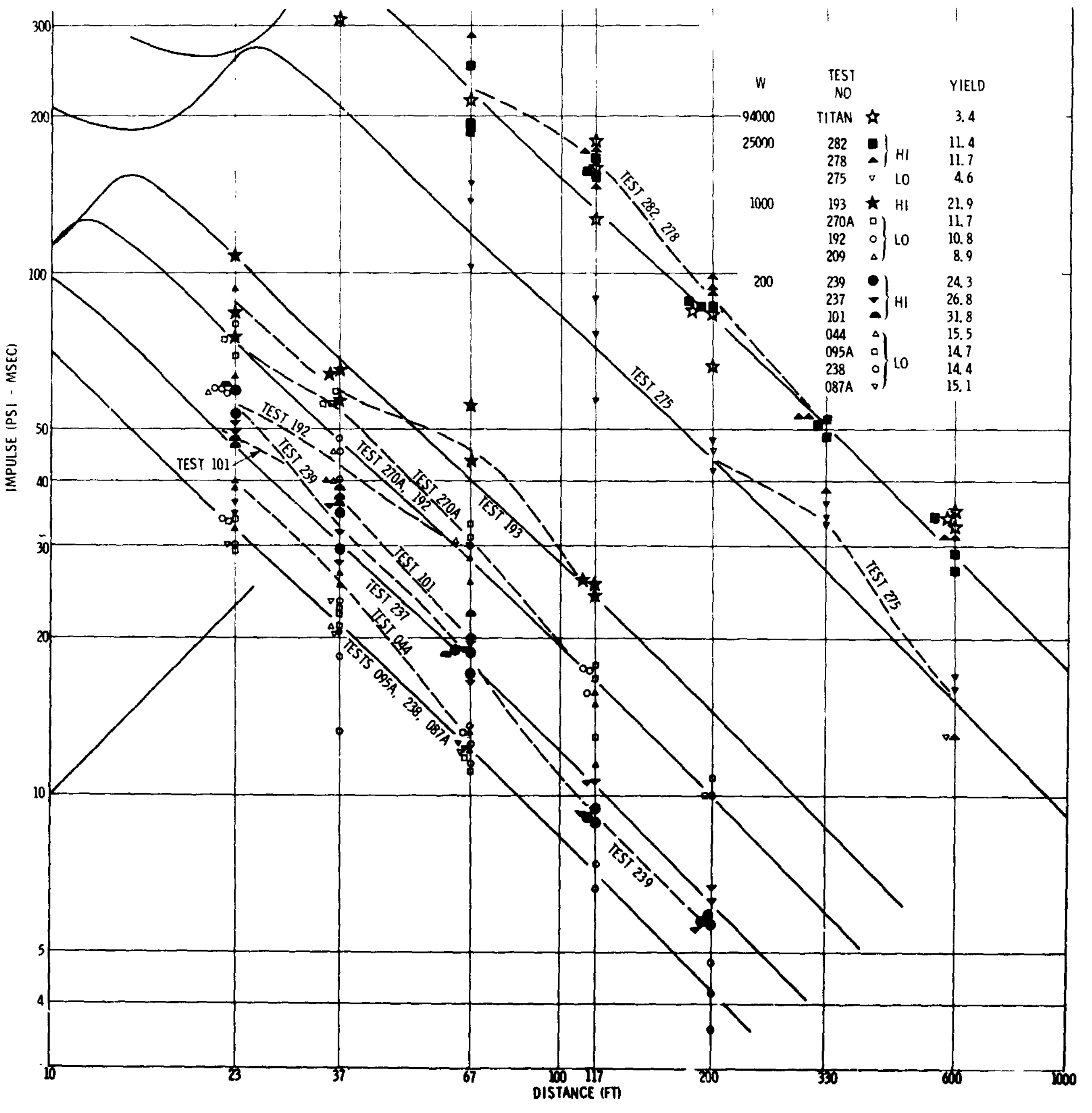

FIGURE A51 - CBM, LO_2/RP-1, NOMINAL GEOMETRY IMPULSE VS. DISTANCE

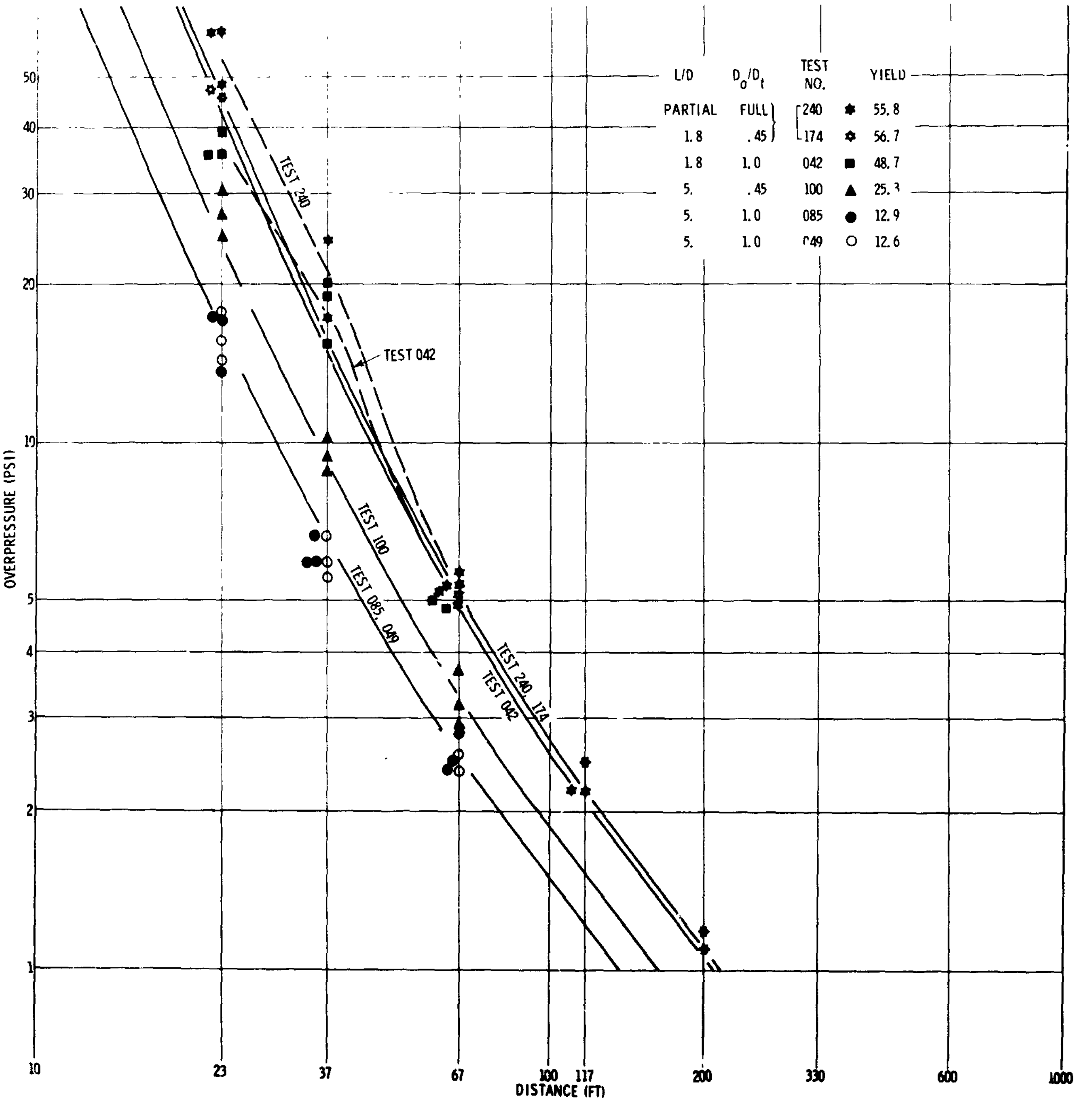

FIGURE A6P- CBM, LO_2/RP-1, 200 LBS, PEAK OVERPRESSURE VS. DISTANCE

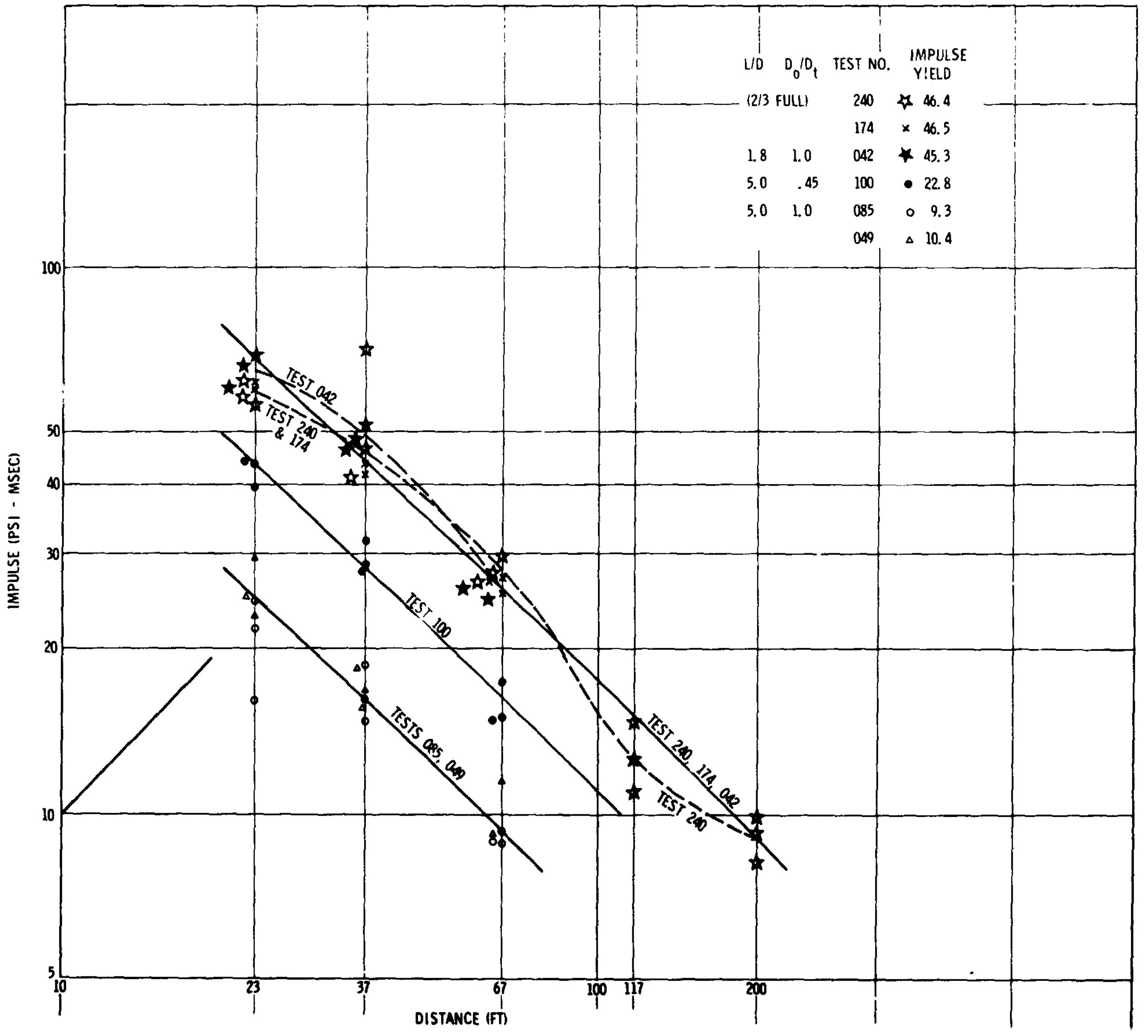

FIGURE A61 - CBM, LO_2/RP-1, 200 LBS., HIGH YIELD TESTS. IMPULSE VS. DISTANCE

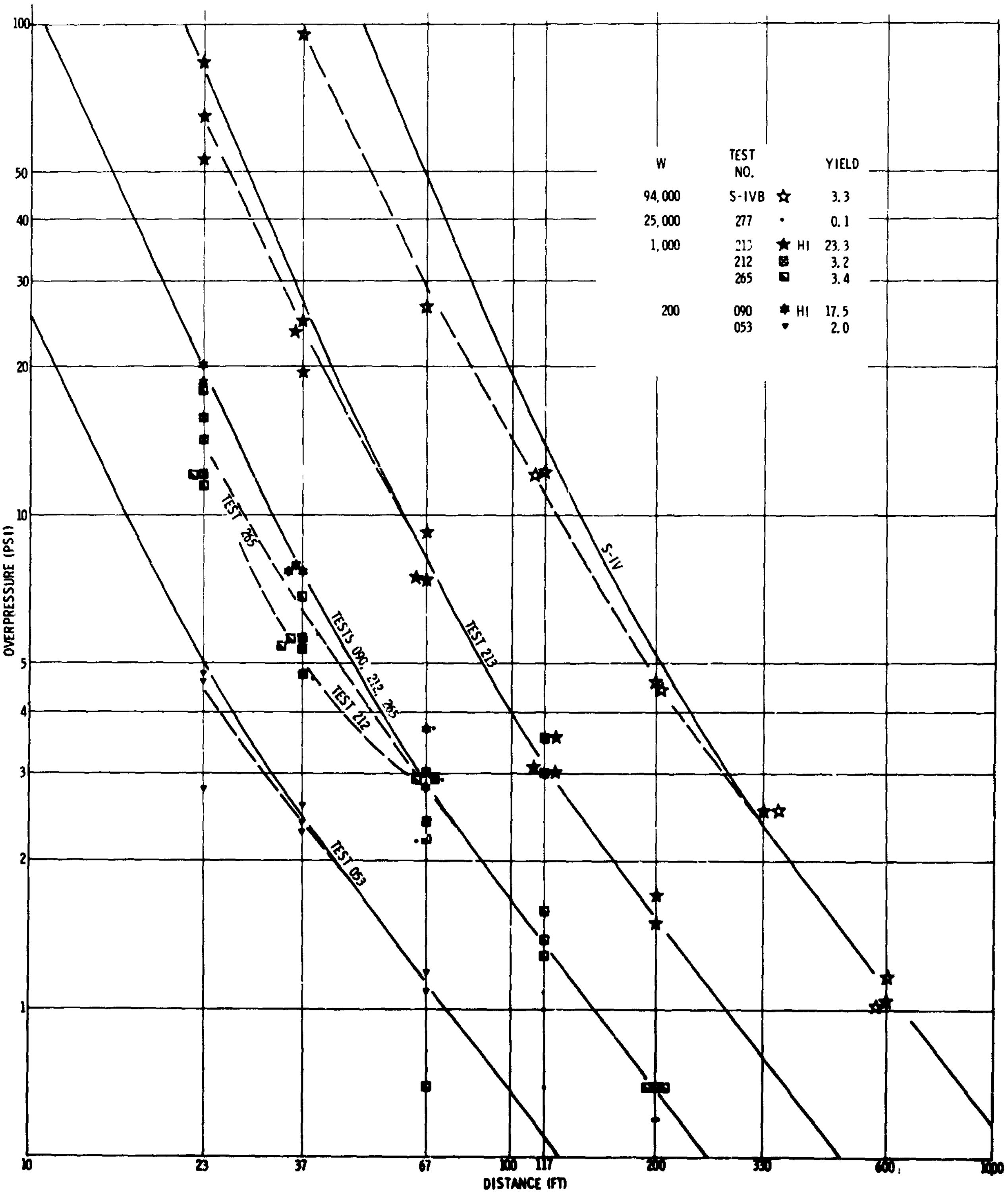

FIGURE A7P - CBM, LO_2/LH_2, NOMINAL GEOMETRY, PEAK OVERPRESSURE VS. DISTANCE

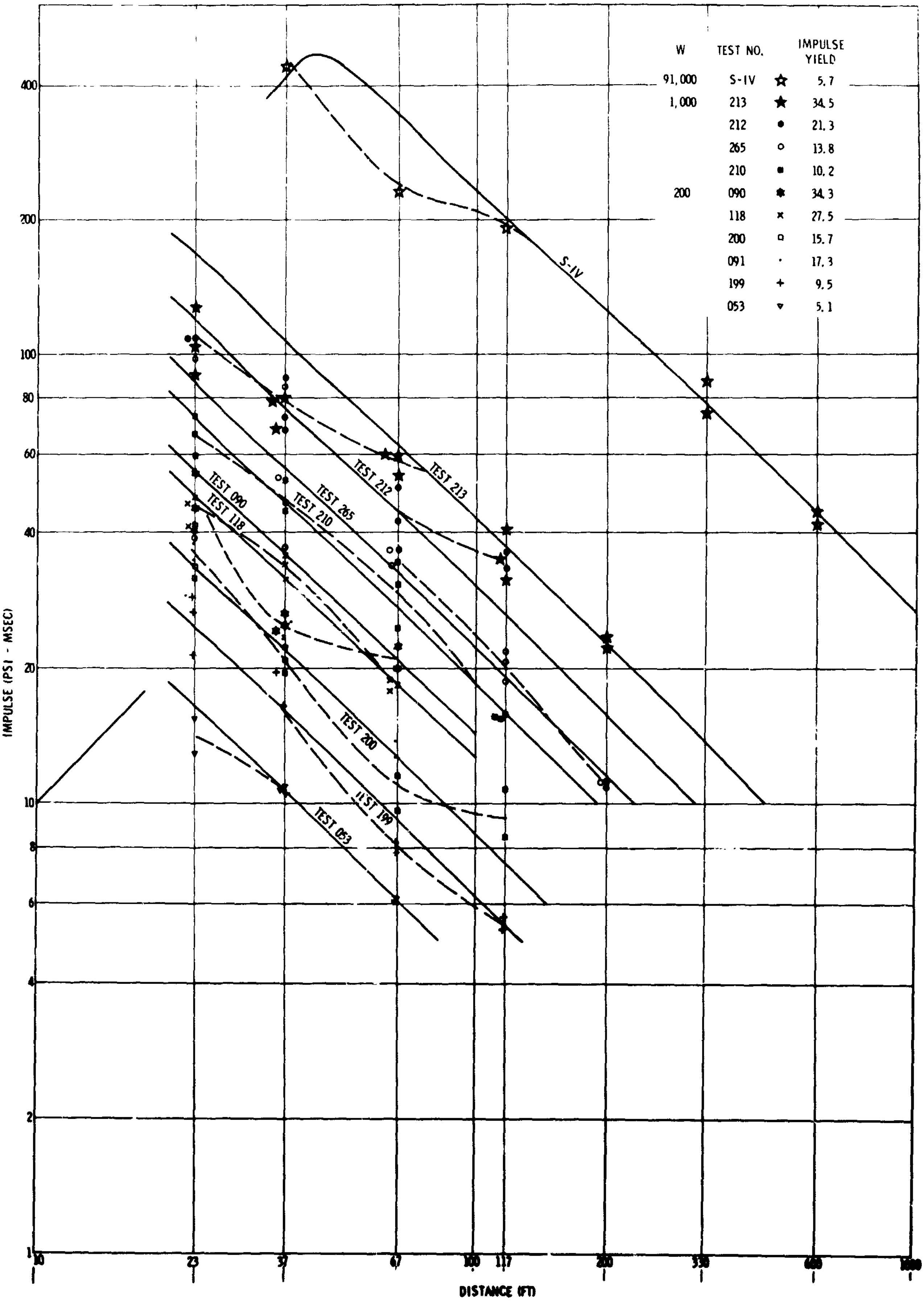

FIGURE A71 - CBM, LO_2/LH_2, NOMINAL GEOMETRY, IMPULSE VS. DISTANCE

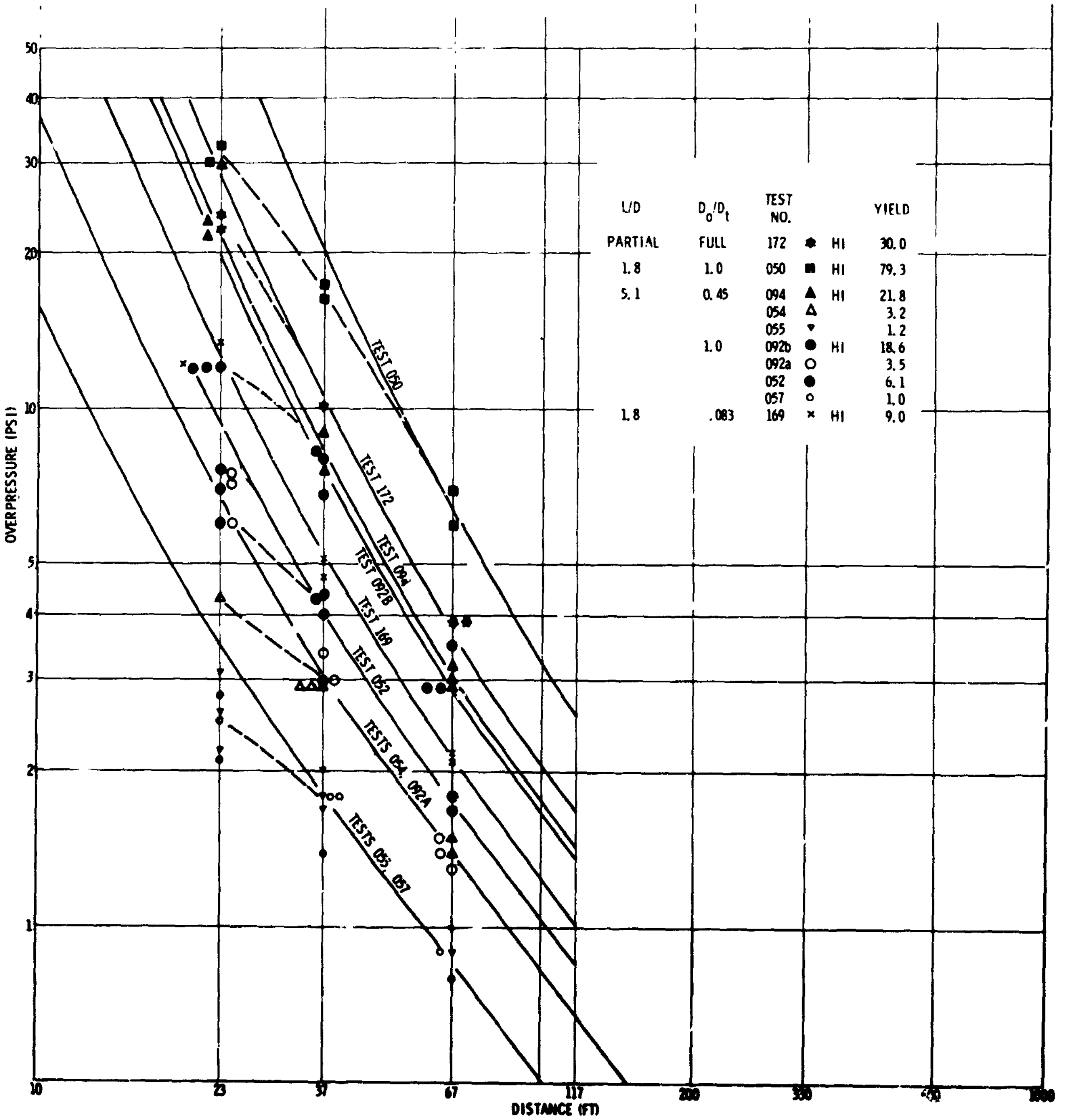

FIGURE ABP - CBM, LO_2/LH_2 200 LBS, PEAK OVERPRESSURE VS. DISTANCE

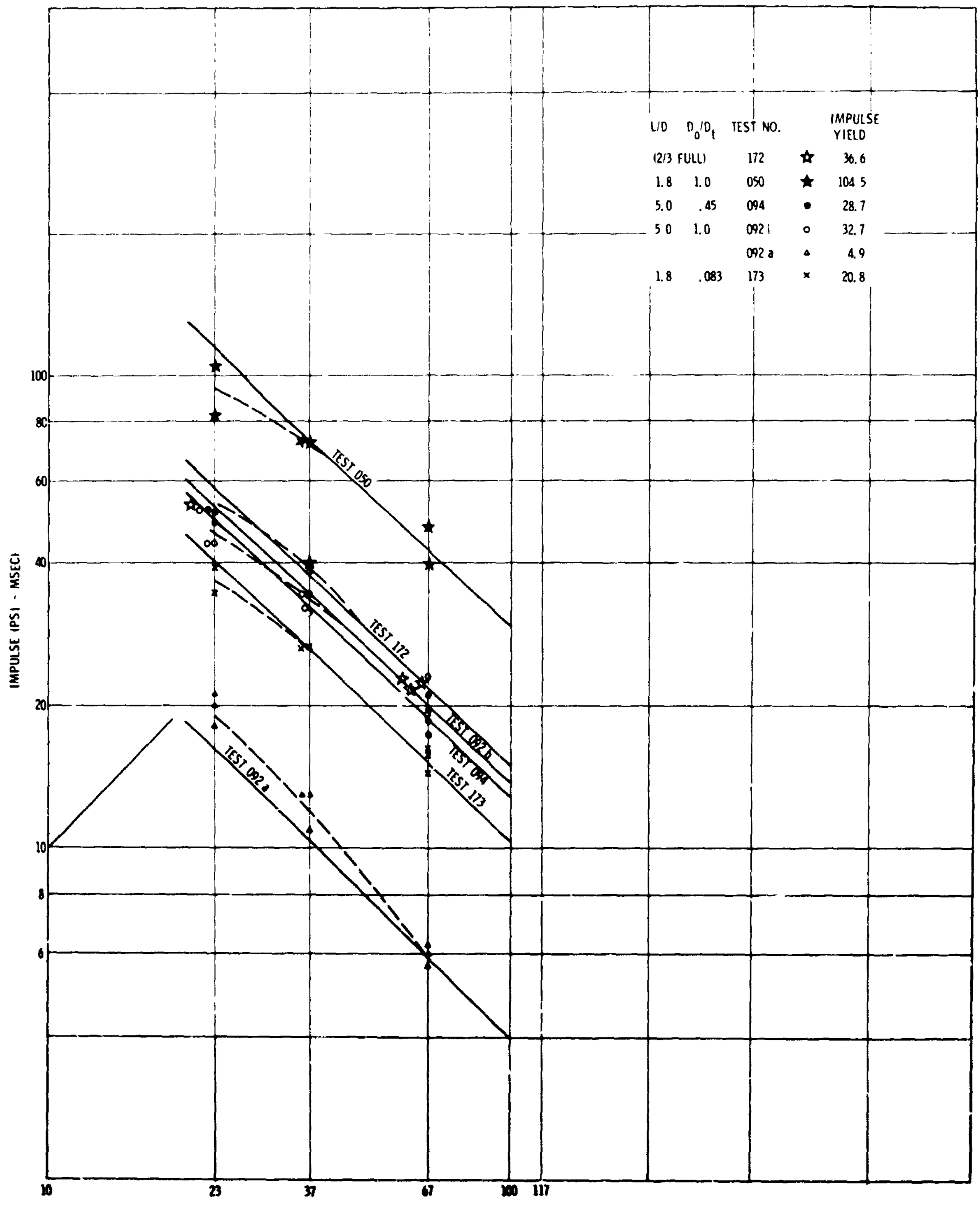

FIGURE A81 - CBM, LO_2/LH_2, 200 LBS., HIGH YIELD TESTS IMPULSE VS. DISTANCE

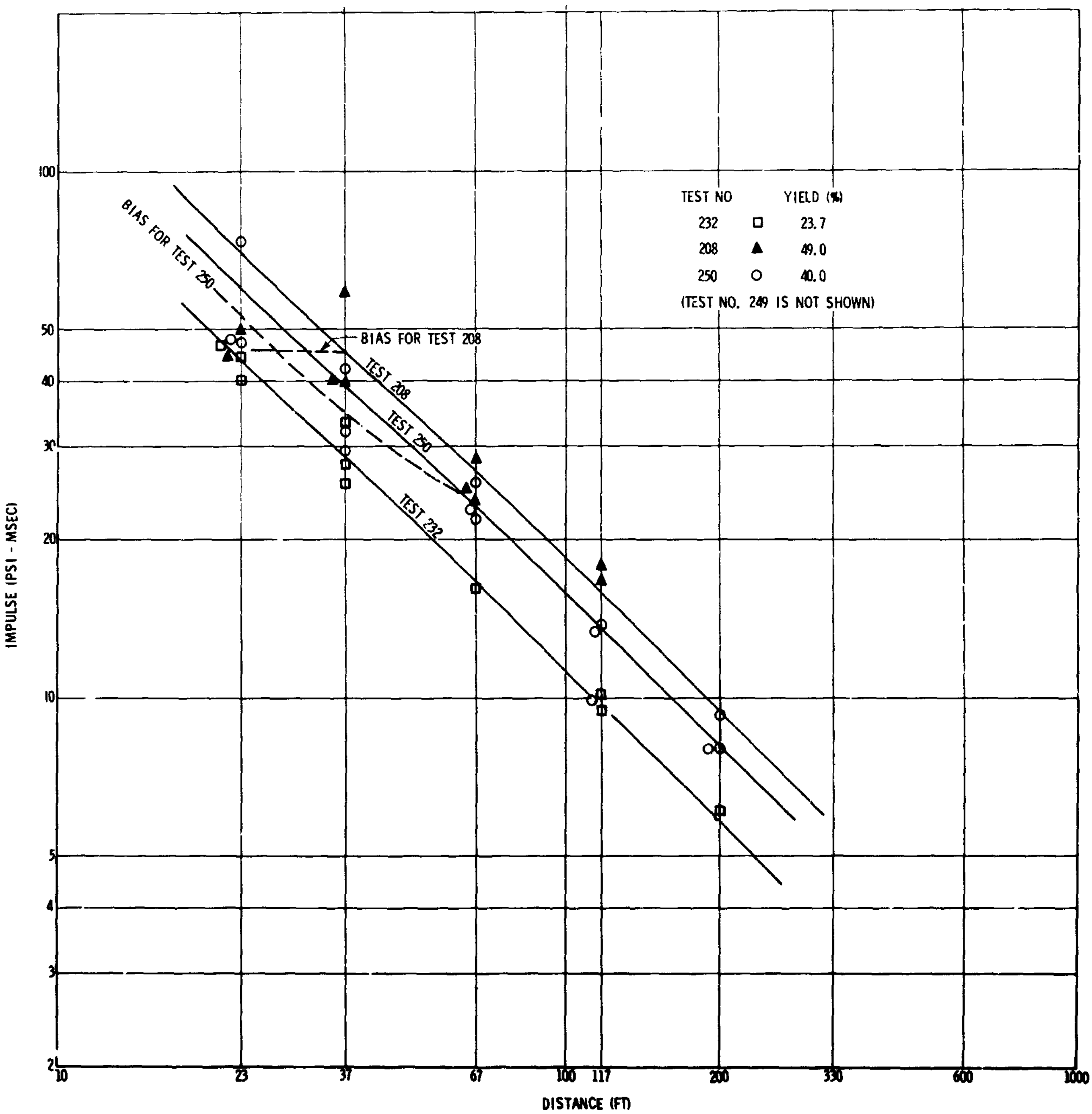

FIGURE A91 - CBGS, LO_2/RP-1, V = 44 FPS, W = 200 LBS, IMPULSE VS DISTANCE

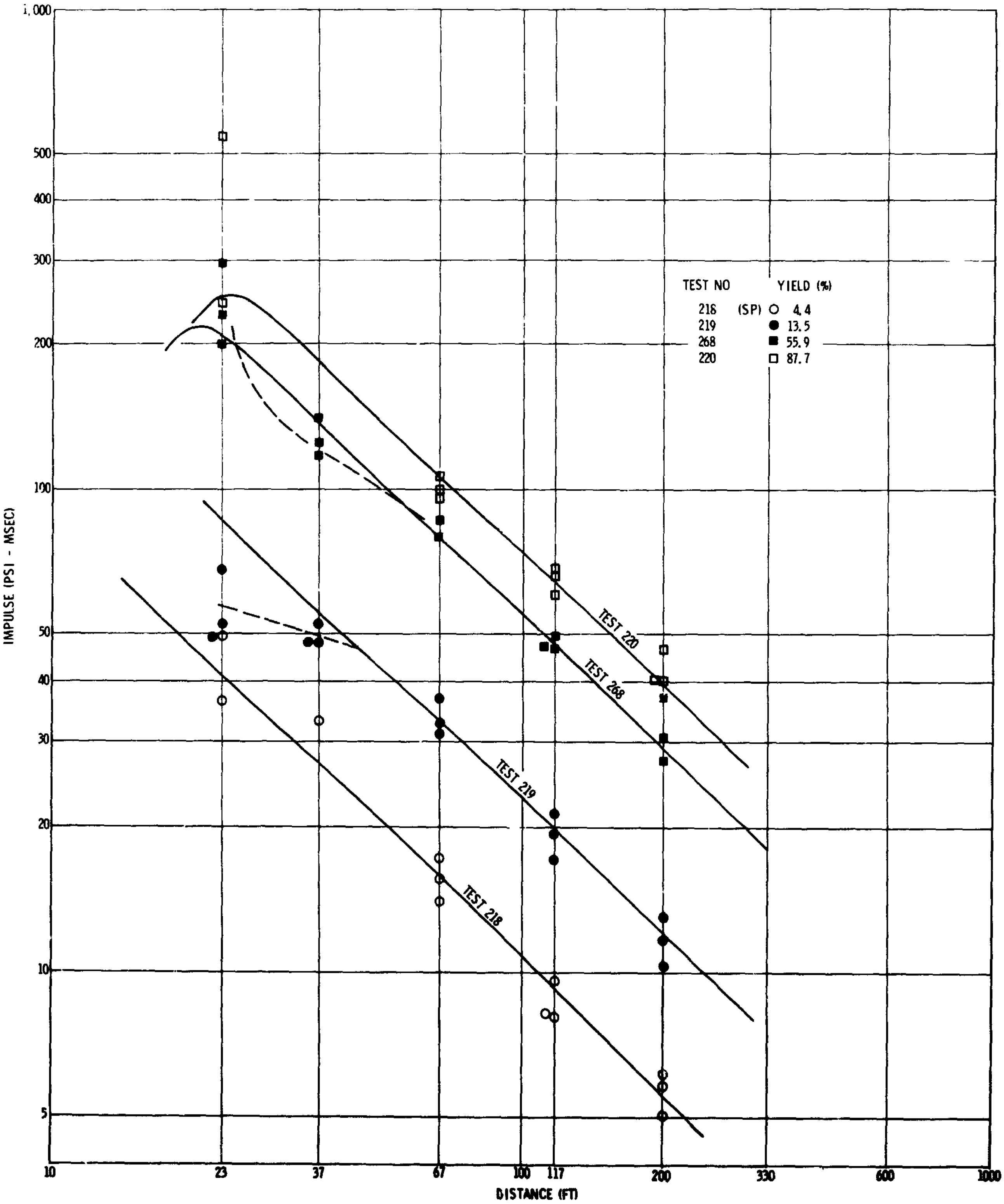

FIGURE A101 - CBGS, LO_2/RP-1, V = 44 FPS, W = 1000 LBS, IMPULSE VS DISTANCE

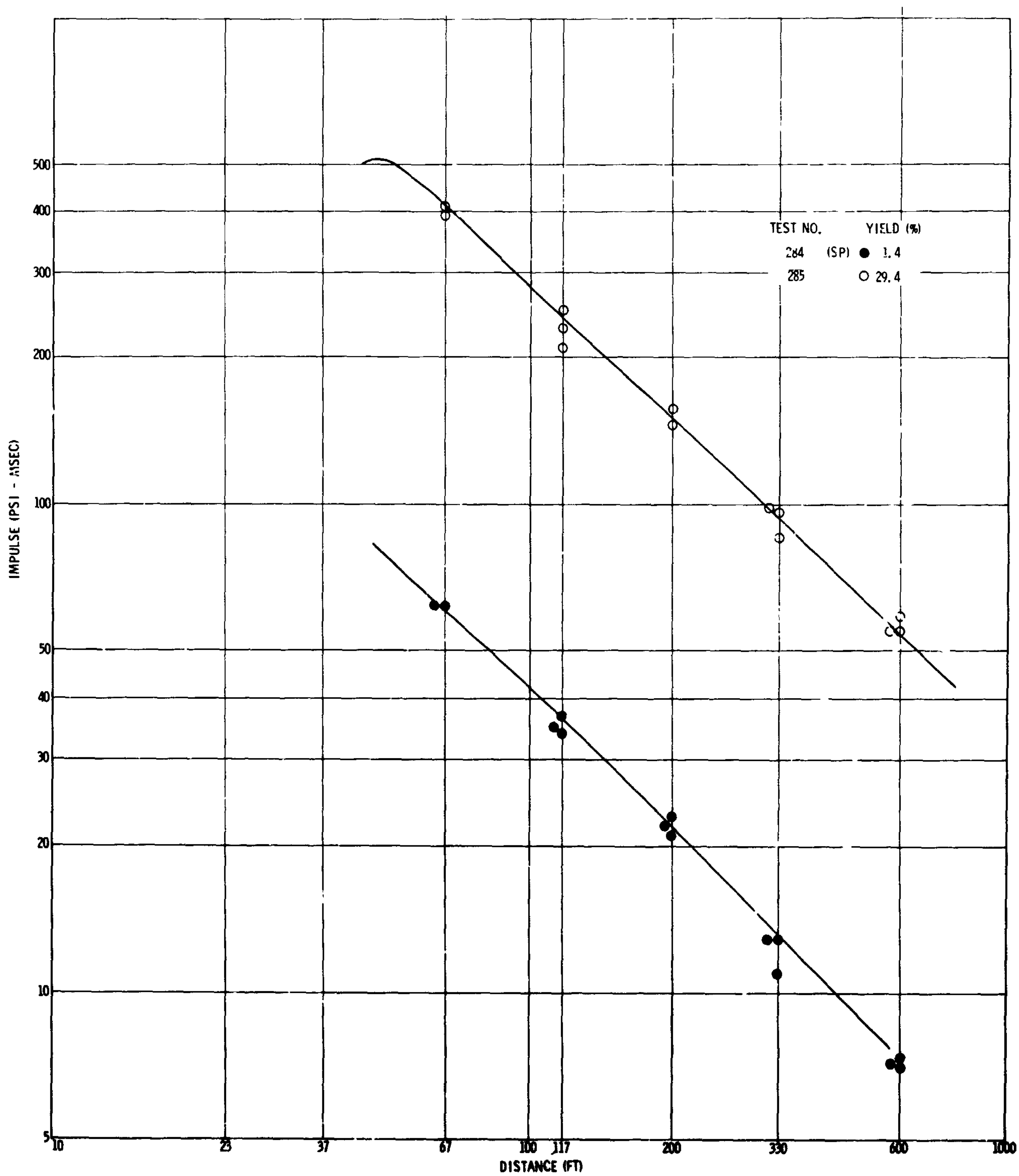

FIGURE A111 - CBGS, LO_2/RP-1, V = 44 FPS, W = 25,000 LBS, IMPULSE VS DISTANCE

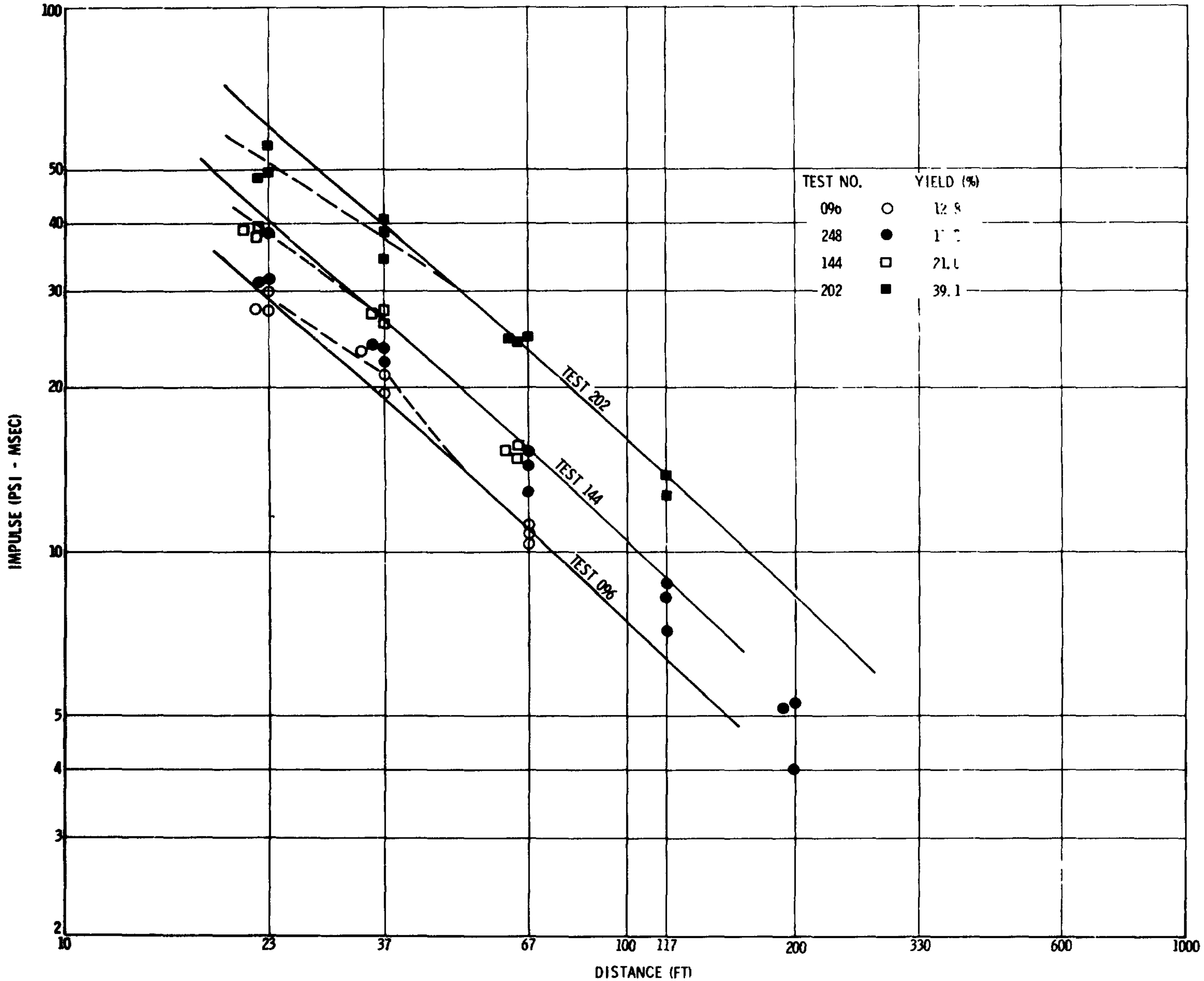

FIGURE A121 - CBGS, LO_2/RP-1, V = 23 FPS, W = 200 LBS, IMPULSE VS DISTANCE

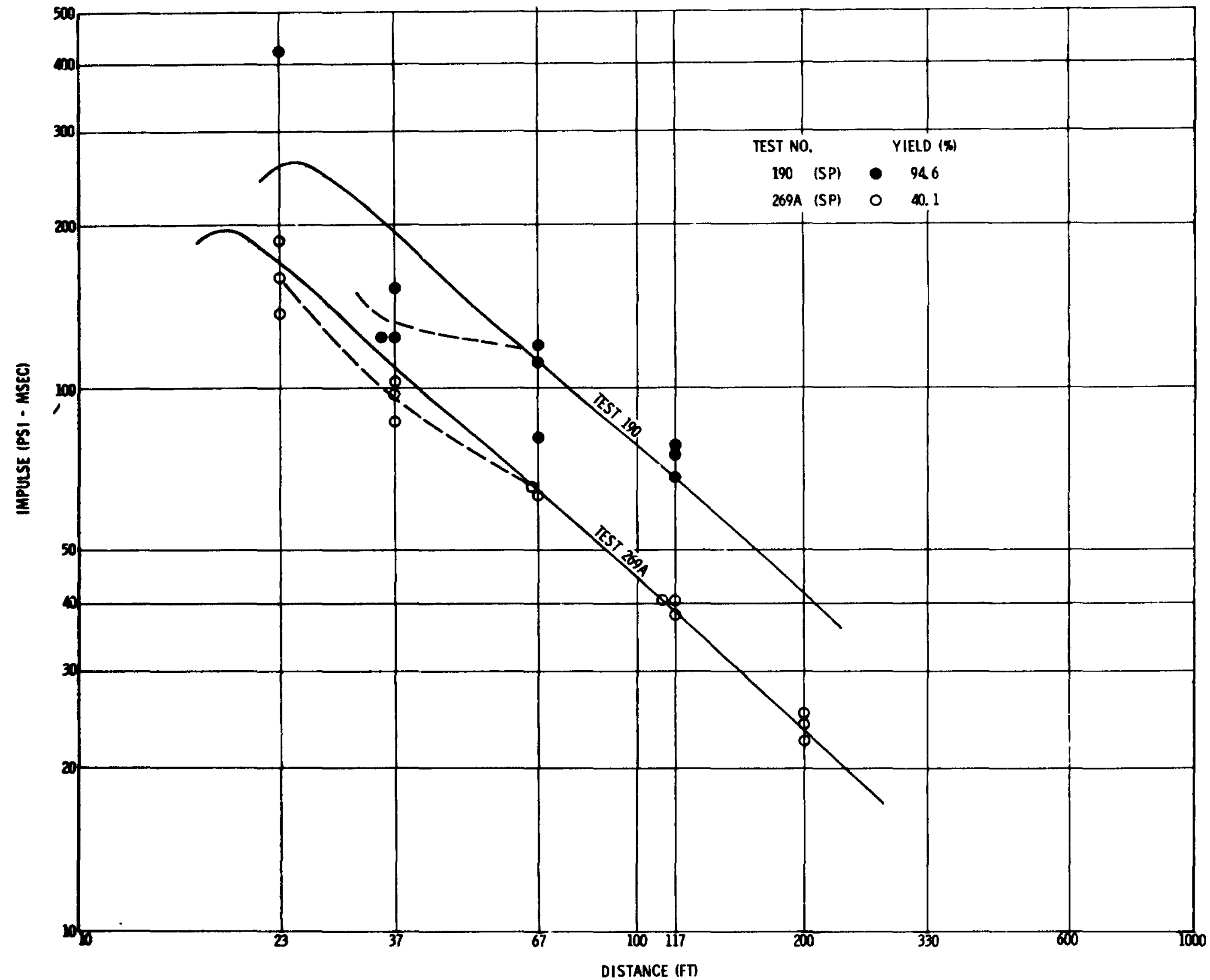

FIGURE A131 - CBGS, LO_2/RP-1, V = 78 FPS, W = 1000 LBS, IMPULSE VS DISTANCE

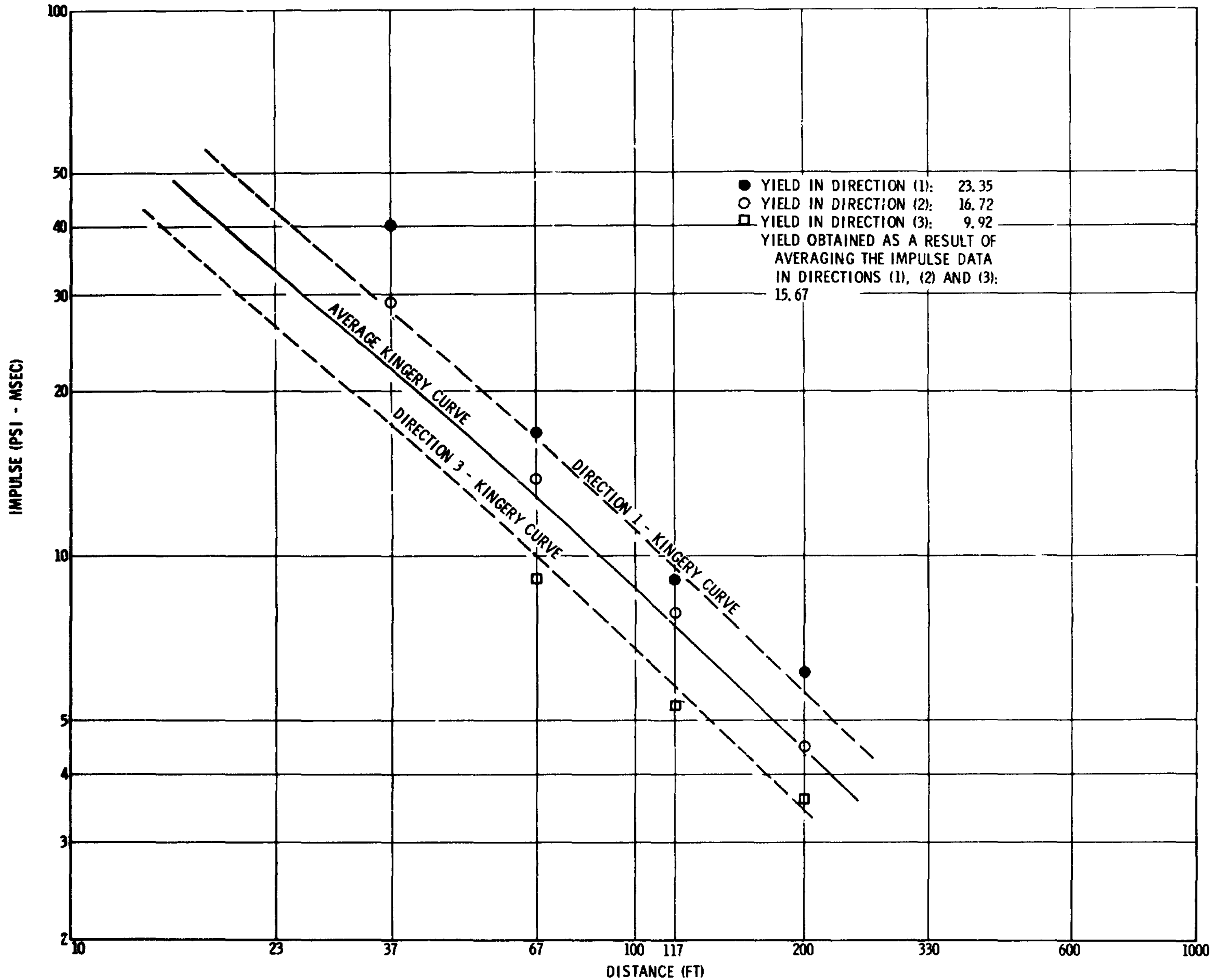

FIGURE A141 - CBGS, LO_2/RP-1, FLAT WALL HIGH VELOCITY IMPACT TEST NUMBER 075, IMPULSE VS DISTANCE

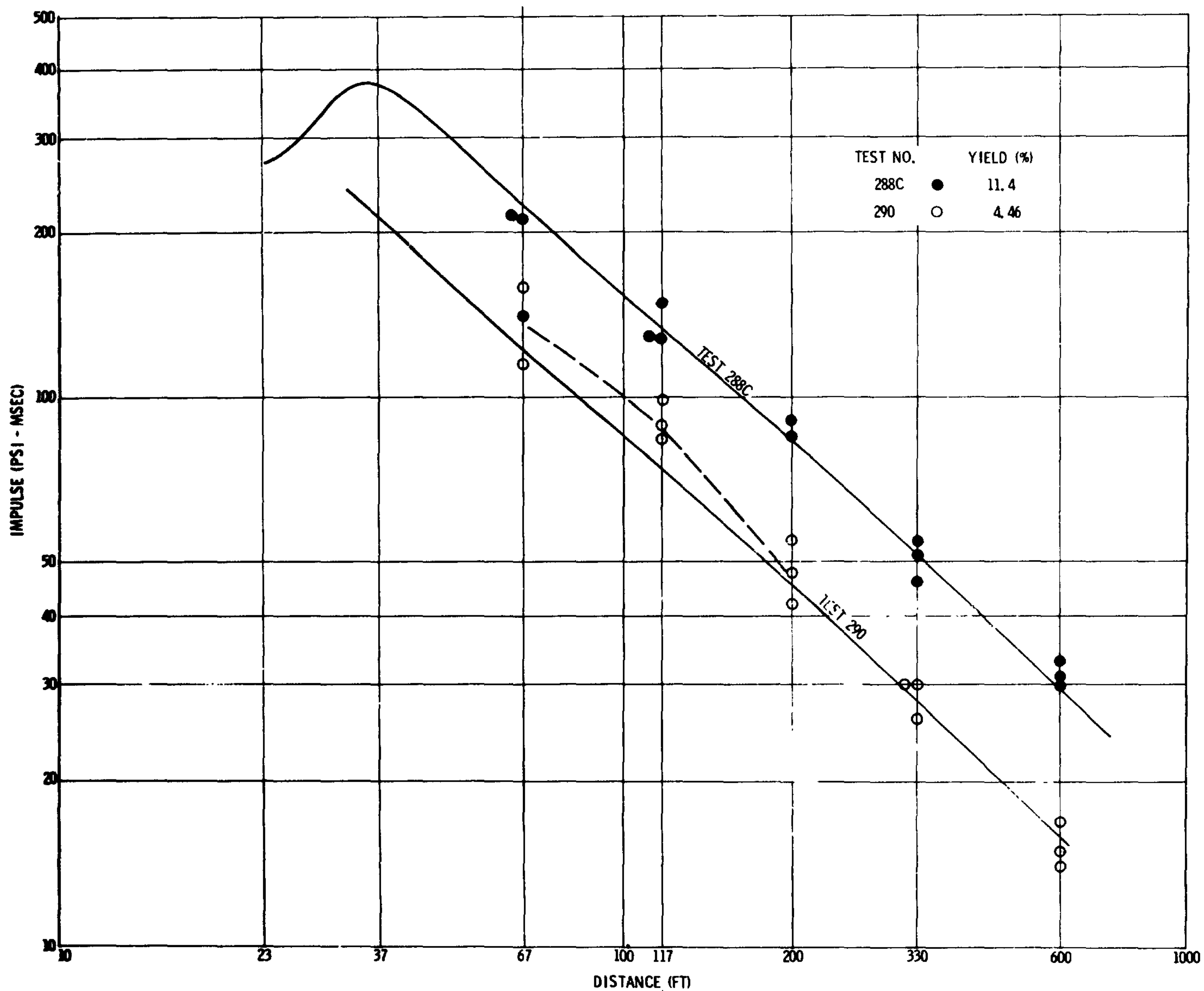

FIGURE A151 - CBGS, LO_2/LH_2, V = 44 FPS, W = 25,000 LBS, IMPULSE VS DISTANCE

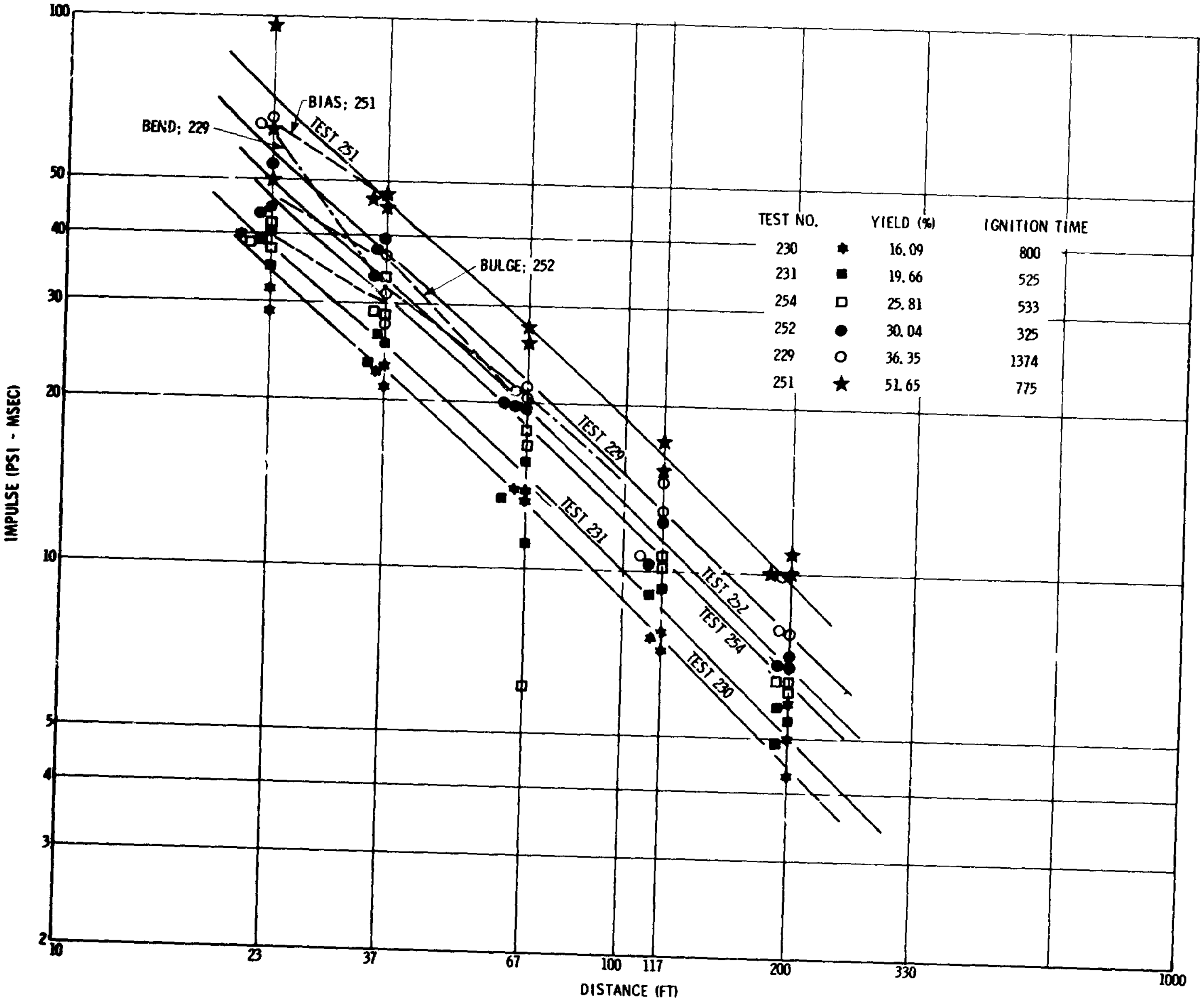

FIGURE A161 - CBGS, LO_2/LH_2, V = 44 FPS, W = 20C !BS, IMPULSE VS DISTANCE

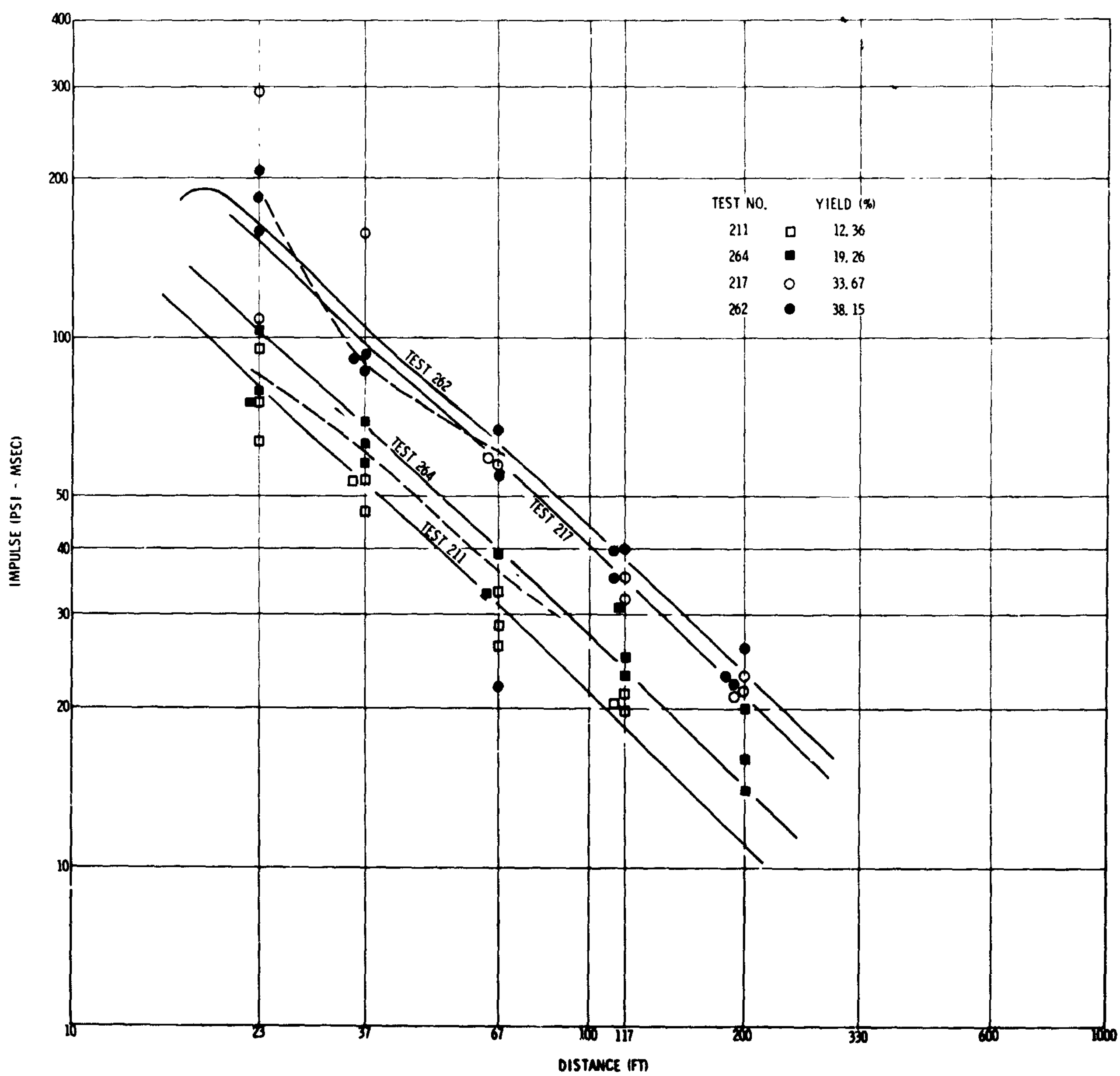

FIGURE A171 - CBGS, LO_2/LH_2, V = 44 FPS, W = 1000 LBS, IMPULSE VS. DISTANCE

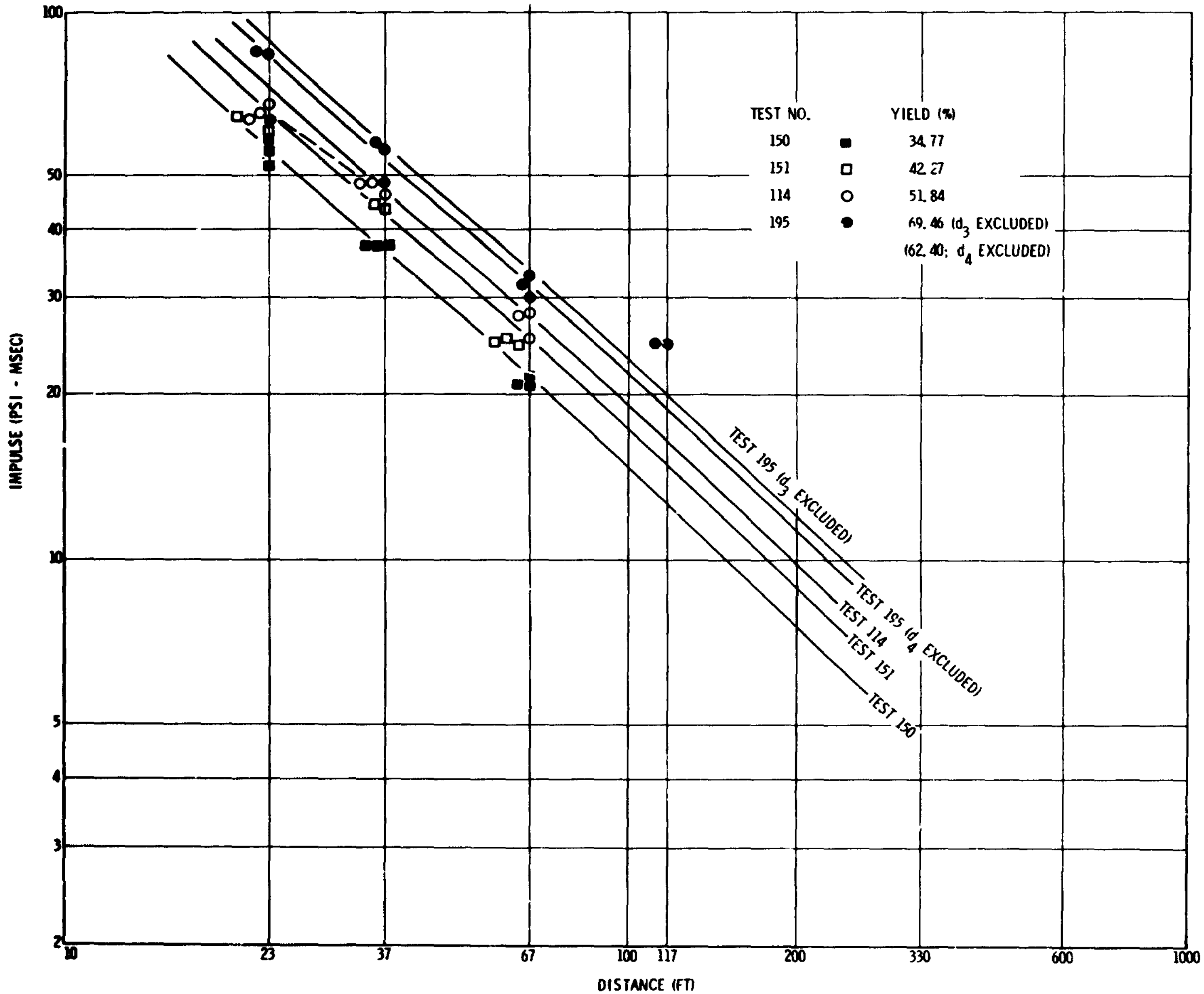

FIGURE A181 - CBGS, LO_2/LH_2, V = 78 FPS, W = 200 LBS, IMPULSE VS DISTANCE

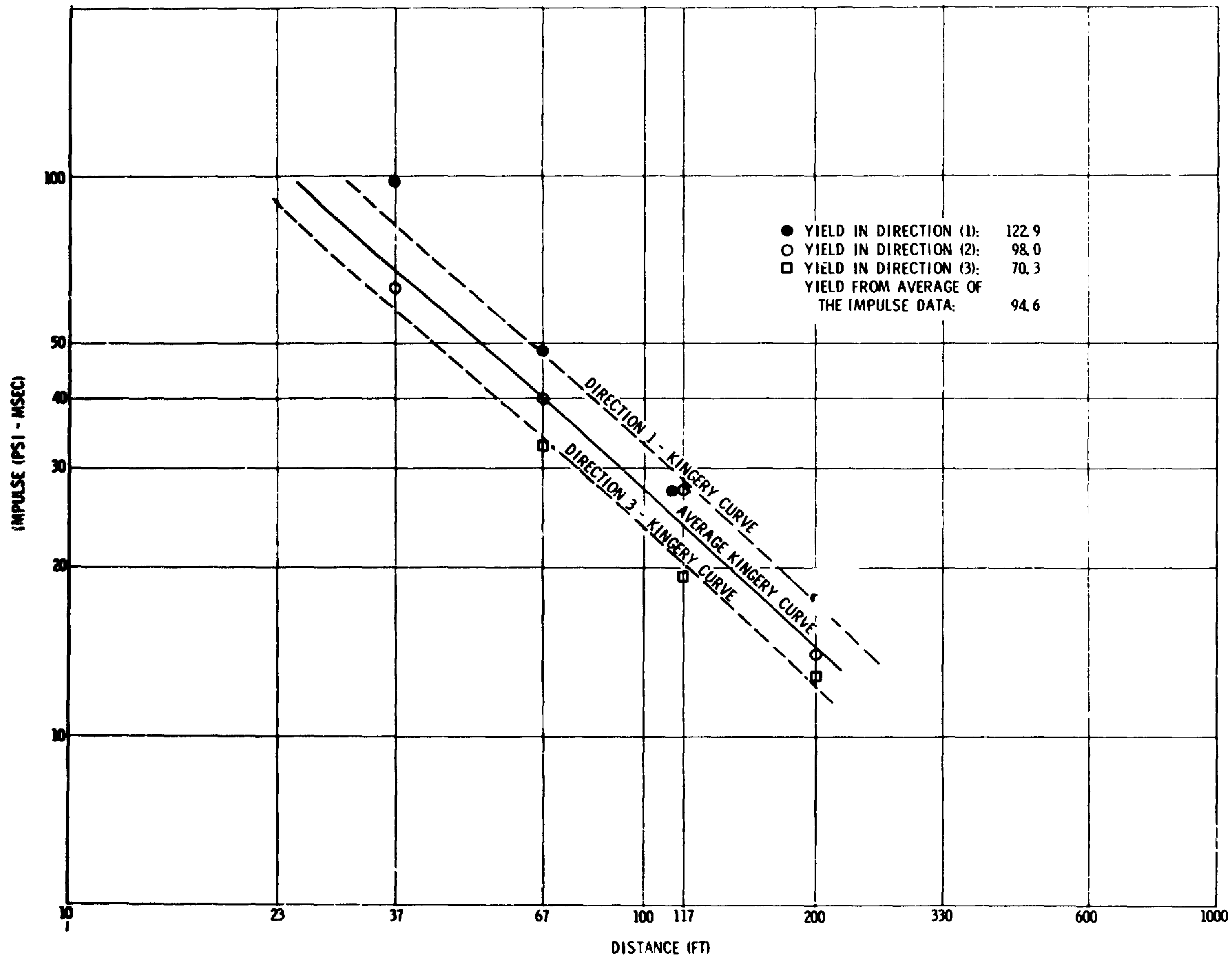

FIGURE A191 - CBGS, LO_2/LH_2, FLAT WALL HIGH VELOCITY IMPACT TEST, W = 200 LBS, V = 597 FPS, IMPULSE VS DISTANCE
TEST 079

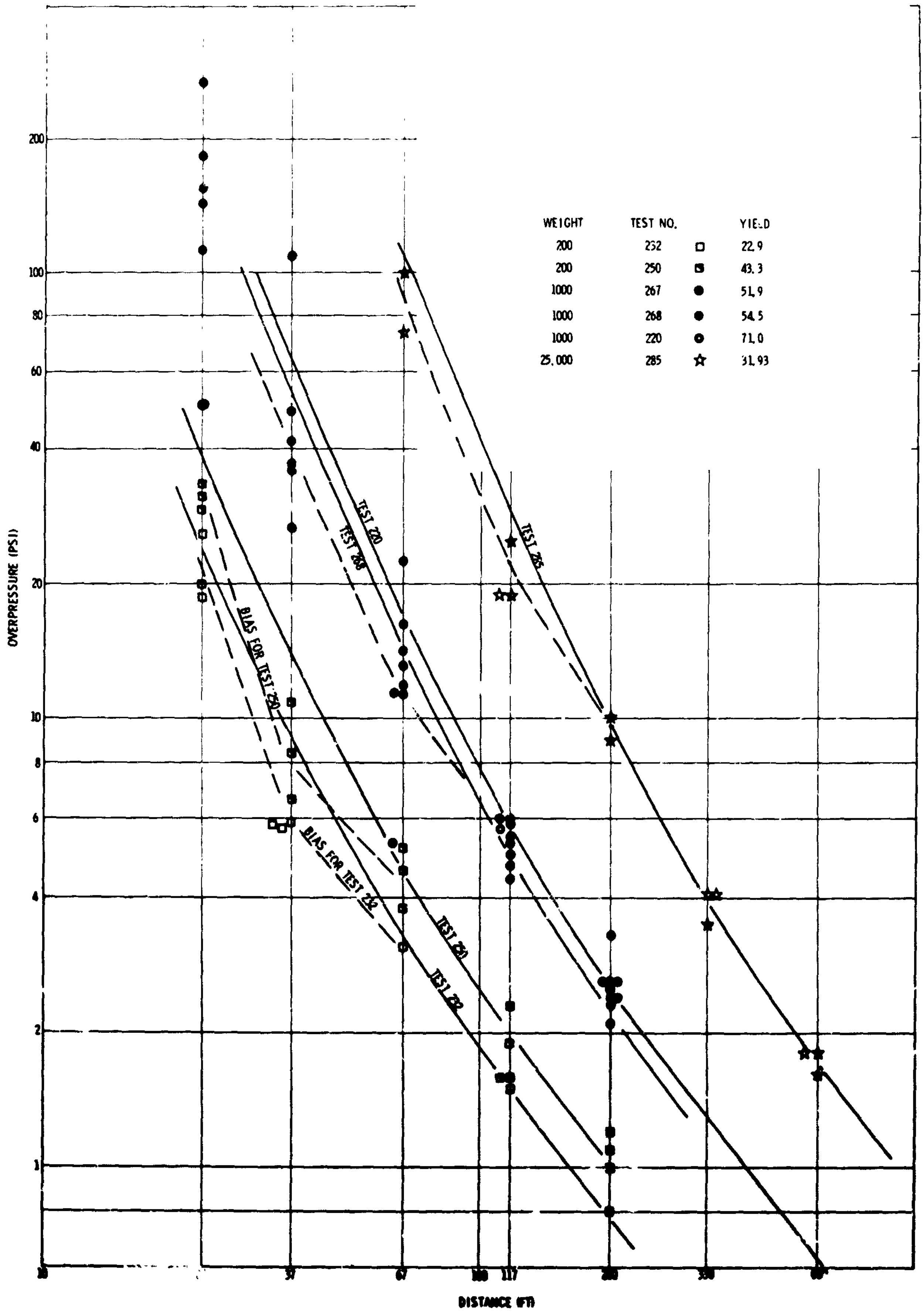

FIGURE A[illegible] - CBGS, LO_2/RP-1, V = 44 FPS, OVERPRESSURE VS. DISTANCE

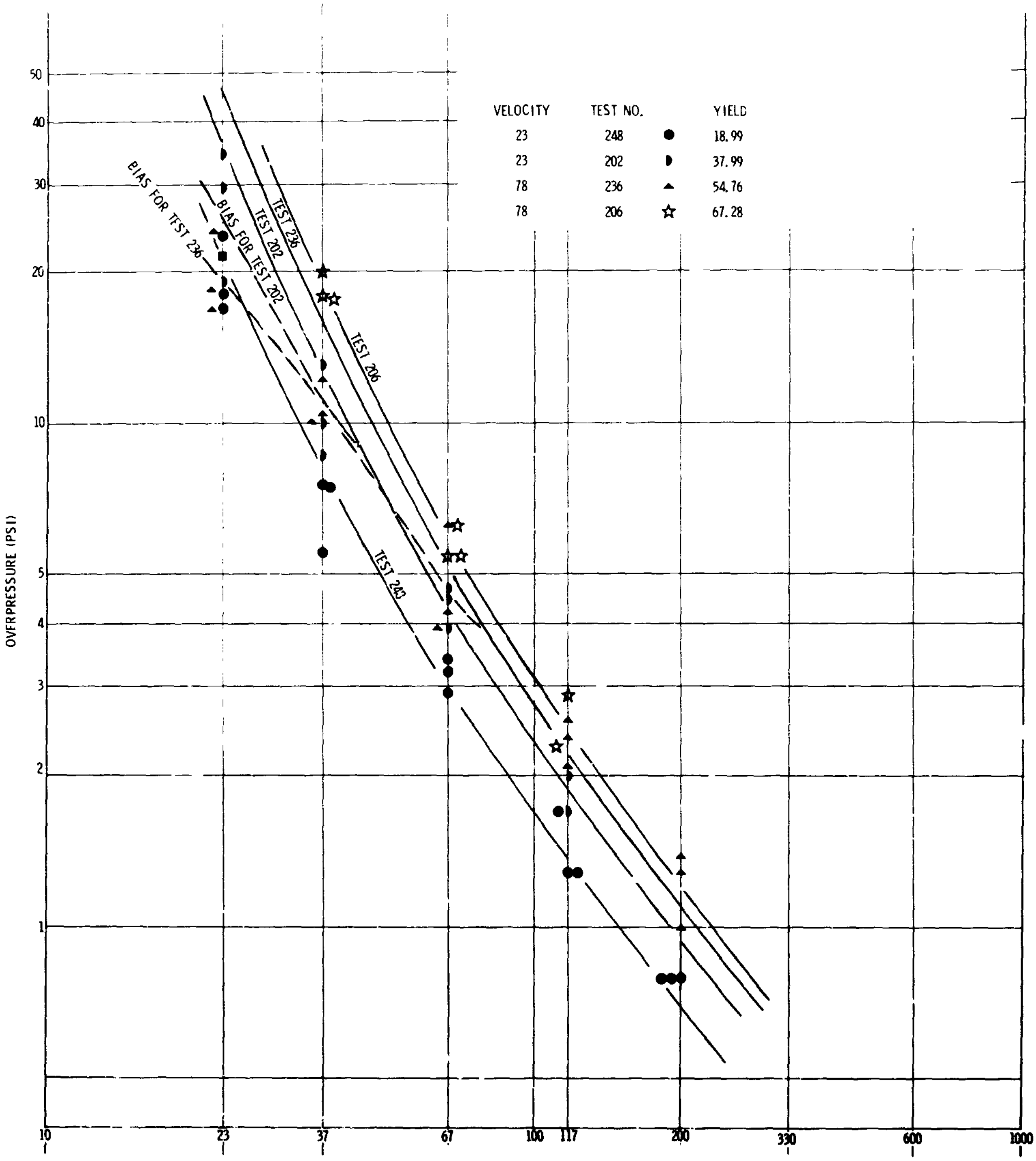

FIGURE A21P - CBGS, LO_2/RP-1, W = 200 LBS, OVERPRESSURE VS. DISTANCE

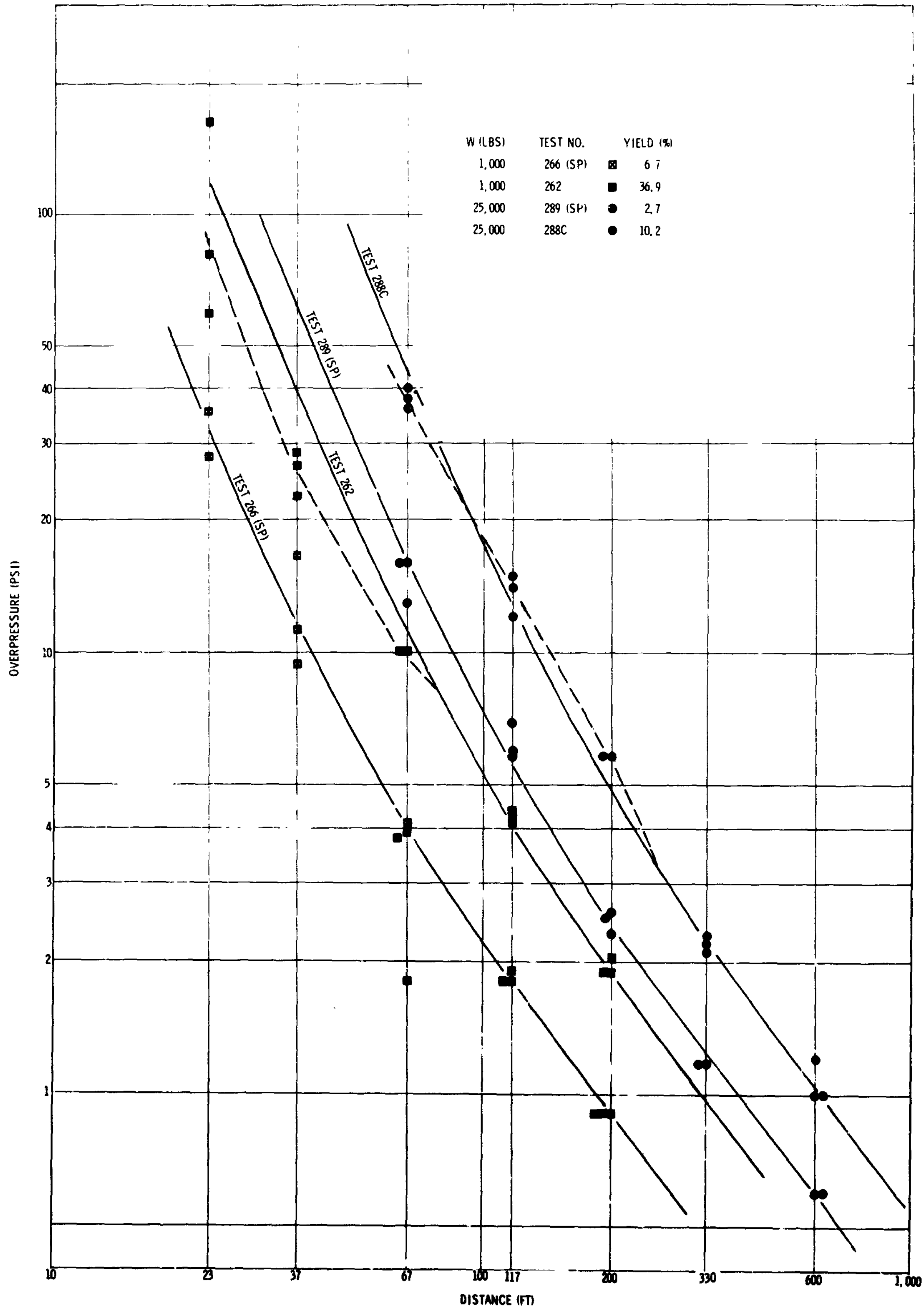

FIGURE A22P - CBGS, LO_2/LH_2, V = 44 FPS, OVERPRESSURE VS. DISTANCE

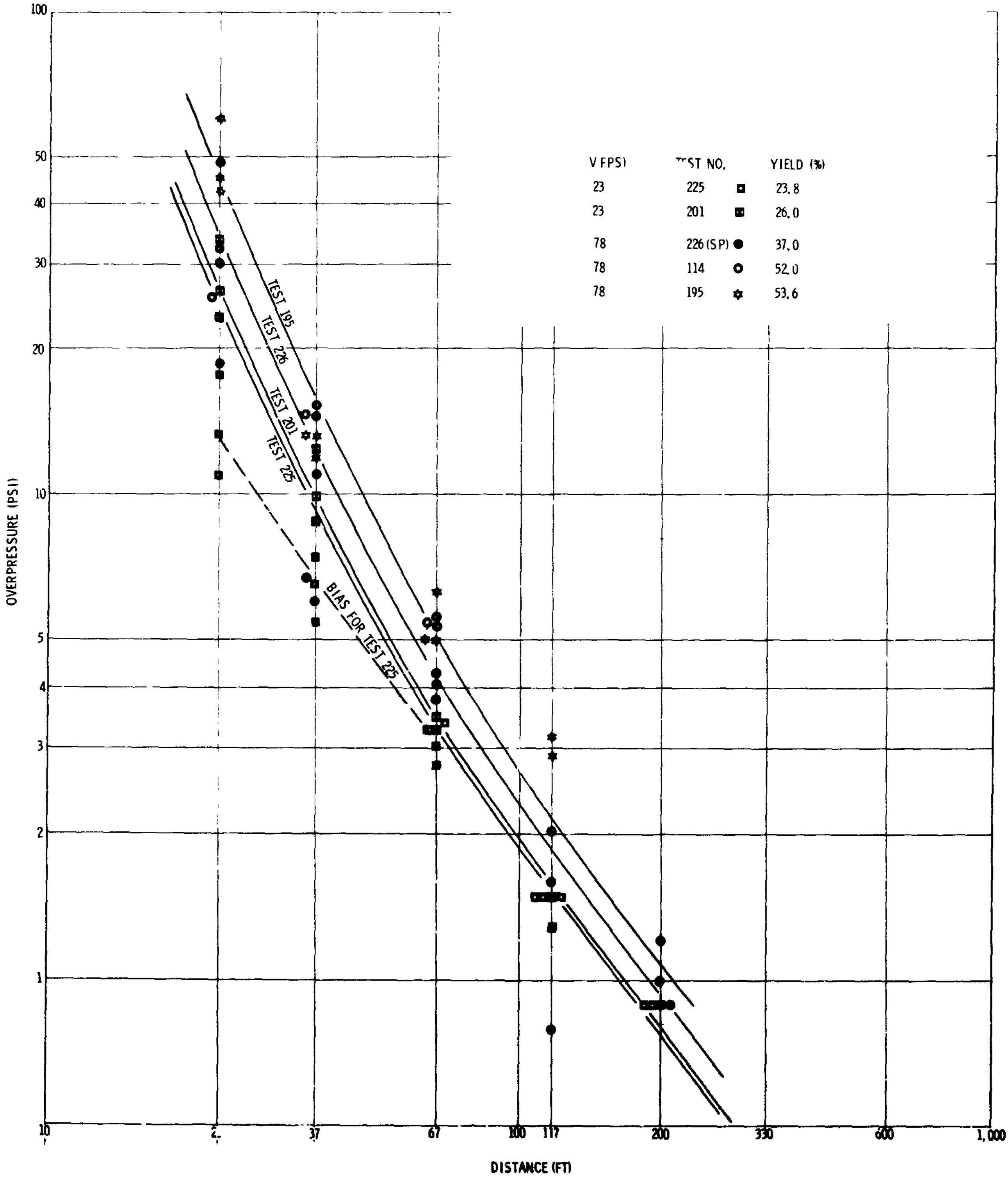

FIGURE A23P- CBGS, LO_2/LH_2, W= 200 LBS, OVERPRESSURE VS. DISTANCE

APPENDIX B

LEAST SQUARES KINGERY FIT DERIVATIONS AND COMPUTER OUTPUT

Let the Kingery overpressure curve be represented by

$$P = K_P\left(\frac{d}{W_T^{1/3}}\right) \tag{1}$$

where

P = peak overpressure

d = distance

W_T = equivalent weight of TNT

(1) can be rewritten in terms of (natural) logs:

$$y = K(x+a) \tag{2}$$

where

$$y = \log P$$
$$x = \log d \tag{3}$$
$$a = -1/3 \log W_T$$

The function K(z), tabulated in reference 3 and plotted in Figure A1, corresponds precisely (for $P \geq .5742$) to the following eighth degree polynomial.

$$K(z) = 7.045 - 1.628z - .274z^2 - 0.66z^3 + .0065z^4 + .048z^5$$
$$- .020z^6 + .003z^7 - .00016z^8$$

Only very few overpressure measurements less than .5742 were recorded for the liquid propellant tests.

Given the observations (x_i, y_i), i=1,..., N, the least squares method of fit minimizes with respect to a the quantity*

$$\sum_{i=1}^{N} [y_i - K(x_i+a)]^2 \qquad (4)$$

Differentiating and then equating to zero yields the following equation to be solved for $\hat{a}$, e.g., by Newton's iteration method.

$$\sum_{i=1}^{N} K'(x_i+\hat{a})[y_i-K(x_i+\hat{a}] = 0 \qquad (5)$$

Substituting $\hat{a}$ into (4) and dividing by N-1 (to allow for the loss of one degree of freedom) gives the following estimate for the variance of y about the Kingery curve:

$$S_y^2 = \frac{1}{N-1} \sum_{i=1}^{N} [y_i-K(x_i+\hat{a})]^2 \qquad (6)$$

Since dy = dP/P, or $\sigma_y \simeq \sigma_P/EP$ (where EP designates expected value of P), one can interpret S_y as the per cent standard deviation in P, i.e., $S_y \simeq S_P\%$.

---

* Since, at different distances, relative errors are more comparable than absolute errors, deviations in the log of pressure, as implied by (4), are more appropriate than deviations in the actual pressure. Note also that if each observation is first converted to an equivalent yield (or to a) and the least squares fit then obtained, this procedure incorrectly minimizes the horizontal deviations.

If the random deviations from the Kingery curve are assumed to be (log) normally distributed, then $\hat{a}$ will be approximately normal. Using the first term in the Taylor's expansion of (6), the variance of $\hat{a}$ is given approximately by

$$S_{\hat{a}}^2 = \frac{S_y^2}{H} \tag{7}$$

where

$$H = \frac{\left\{\sum K'^2(x_i+\hat{a}) - \sum K''(x_i+\hat{a})[y_i-K(x_i+\hat{a})]\right\}^2}{\sum K'^2(x_i+\hat{a})} \tag{8}$$

The interpretation as per cent standard deviation is given by

$$S_Y\% = S_{W_T}\% \simeq 3S_{\hat{a}} \tag{9}$$

Approximate confidence limits for $\hat{a}$ can be obtained from (7) in the usual manner using the t-distribution with N-1 degrees of freedom.

A test of significance of the Kingery fit can be derived, again in the usual manner, based upon the analysis of variance. Let

n_i = number of stations at the i^{th} distance $(i=1,\ldots,k)$

$y_{i.}$ = mean value of y's at the i^{th} distance $(i=1,\ldots,k)$

Then the within error, σ_y^2, from stations at the same distance, is estimated by

$$S_{y,w}^2 = \frac{1}{N-k} \sum_{j=1}^{N} y_j^2 - \sum_{i=1}^{k} n_i y_{i.} \qquad (10)$$

where

$$N = \sum_{i=1}^{k} n_i$$

If the y_j are normal, $S_{y,w}^2/\sigma_y^2$ is exactly χ^2 distributed with N-k degrees of freedom. An estimate of the between error, due to the Kingery fit, is

$$S_{y,b}^2 = \frac{1}{k-1} \sum_{i=1}^{k} n_i [y_{i.} - K(x_i + \hat{a})]^2 \qquad (11)$$

This quantity is approximately independent of $S_{y,w}^2$; moreover, if the Kingery curve is appropriate (i.e., null hypothesis true), then $S_{y,b}^2/\sigma_y^2$ is approximately χ^2 distributed with k-1 degrees of freedom. It follows that the F-ratio

$$F(k-1, N-k) = \left(\frac{S_{y,b}}{S_{y,w}}\right)^2 \qquad (12)$$

will be an approximate test of goodness of fit.

Note that $S_{y,w}$ estimates the "true" random error, whereas S_y in equation (6) includes also the error in fit. Pooling these is approximately the same as the estimate in (6) of overall scatter:

$$S_y^2 \simeq \frac{1}{N-1} [(N-k) S_{y,w}^2 + (k-1) S_{y,b}^2]$$

The test for significance of direction is similar. Let $\hat{a}_\nu$ (ν=1,2,3) be the Kingery estimate for each of the three directions separately, H_ν the corresponding precision factor from equation (8), and $\bar{a}$ the weighted average

$$\bar{a} = \sum_{1}^{3} H_\nu a_\nu / \sum_{1}^{3} H_\nu \qquad (13)$$

If now S_{yp}^{2} represents the pooled scatter, i.e.,

$$S_{yp}^{2} = \frac{1}{N-3} \sum_{\nu=1}^{3} \sum_{i=1}^{N_\nu} [y_{i\nu} - K(x_{i\nu} + \hat{a}_\nu)]^2 \qquad (14)$$

then the appropriate F-test is given approximately by

$$F(2,N-3) = \frac{\frac{1}{2} \sum_{1}^{\nu} H_\nu (\hat{a}_\nu - \bar{a})^2}{S_{y,p}^{2}} \qquad (15)$$

The derivations for the least squares Kingery fit to the impulse data can be reduced to the overpressure case. Let the Kingery impulse curve be represented by

$$\frac{I}{W_T^{1/3}} = K_I \left(\frac{d}{W_T^{1/3}}\right)$$

or, taking (natural) logs,

$$y = J(x+a) - a$$

where now $y = \log I$. Then the previous formulas hold with $K(x+a)$ replaced by $J(x+a)-a$, K' by $J'-1$, and K'' by J''. The tabulation of $J(z)$ in reference 4 has been plotted in Figure A2. The following quadratic approximation (shown as the dashed curve in Figure A2) was used in the computations.

$$J(z) = 3.998049 - .7620427z - .0285779z^2$$

The approximation is very good except for the initial portion of the curve. However, very few observations were in this region.

Description of Computer Output*

Figures B1P (pressure) and B1I (impulse) illustrate the computer output for test number 278, the example used in Appendix A. Since five separate least squares fits are performed for both pressure and impulse, it is convenient to define the "standard output" (SO) as follows:

Y = estimated yield = $\frac{1}{W} e^{-3\hat{a}}$

S.D.P% = $S_P\%$ = S_y = per cent standard deviation of scatter about the Kingery curve (equation (6))

S.D.Y% = $S_Y\%$ = $3\ S_{\hat{a}}$ = per cent standard deviation of estimate of Y (equations (7) - (9))

YU = upper 95% confidence limit for Y
$= Ye^{t_{N-1,\ 2.5\%}\ (3\ S_{\hat{a}})}$**

YL = lower 95% confidence limit
$= Ye^{-t_{N-1,\ 2.5\%}(3\ S_{\hat{a}})}$

N = number of stations used to fit the Kingery curve

LN(TNT) = $\ln W_T = -3\hat{a}$

N-EQUIV[a] = H (equation (8))

---

*Copies of the computer output for all tests can be made available.

**If F(t) is the distribution function of the t-distribution with ν degrees of freedom, then $t_{\nu,P}$ is defined by $F(t_{\nu,P}) = 1-P$

[a]The N-EQUIV nomenclature arises from analogy with the well-known equation for the standard deviation of the mean of N observations. Compare footnote on page A4 of Appendix A.

The program contains the options of externally designating certain stations to be omitted, or of performing successive preliminary runs to internally omit significantly extreme observations, or both, or neither. The print-out for each preliminary run shows, besides the SO, the external and additional internal stations* that have been omitted, the pressures for each of the remaining stations, and the normalized deviations defined by

$$t_i = \frac{y_i - K(x_i + \hat{a})}{S_y}$$

Significant deviations (i.e., $|t_i| > t_{N-1,P}$) are asterisked. (*) is used for P = 2.5%, and (**) for P = 1%. If no observations show significant deviation, the next run is the final one.

The output for the final run consists of: new internal stations omitted, SO, individual stations and pressures, and normalized deviations. To aid in plotting the Kingery fit, the pressures P_K at various distances d, together with 90% upper and lower confidence limits, are given. These are calculated from

$$P_K = e^{K(\ln d + \hat{a})}$$

$$P_{KU} = e^{K(\ln d + \hat{a} + t_{N-1,5\%}S_{\hat{a}})}$$

$$P_{KL} = e^{K(\ln d + \hat{a} - t_{N-1,5\%}S_{\hat{a}})}$$

---

*A 2-digit station code, identical to URS', is used. The first digit refers to direction: 1 for -60°, 2 for +60°, 3 for 180°. The second digit refers to distance: 5 = 23 ft, 6 = 37 ft, 7 = 67 ft, 8 = 117 ft, 9 = 200 ft, 0 = 335 ft, 1 = 600 ft.

Next, the F-ratio value which tests goodness of the Kingery fit is given (equation (12)), together with the appropriate degrees of freedom. The square root of numerator (representing the fit error $S_{y,b}$) and denominator (representing pure error $S_{y,w}$) are also shown. Significance is determined from tabulated values for F.

Finally, the existence of any directional effect is ascertained. First are shown the SO for each direction and the average (using equations (13) and (14)), followed by the value of the F-ratio (equation (15)).

The output for the impulse analysis is identical to the pressure.

TABLE B1P - COMPUTER OUTPUT FOR TEST 278 - PRESSURE

```
                    STANDARD OUTPUT OF ESTIMATES:
   SO: (     Y     , S.D.P% , S.D.Y% ,  YU    ,   YL    , N , LN(TNT), M-EQUIV)

OVERALL ESTIMATE/PRELIMINARY RUN:

   SO: ( .11377+00 .26625-00 .12448+00 .1418-00 .9126-01  14 .7953+01 .4117+02)

   (STATION,PRESS. ,NORMALIZED DEVIATION)
   (27 42.0-.4610-00  )(37 47.0-.3851-01  )(18 16.0 .4840-00  )(28 18.0 .9263-00  )
   (38 18.0 .9263-00  )(19  5.6 .3062-00  )(29  5.9 .5022-00  )(39  5.7 .3727-00  )
   (10  2.3-.8059-01  )(20  2.5 .2326-00  )(30  1.7-.1216+01  )(11  1.1 .1308-01  )
   (21   .5-.2948+01**)(31  1.0-.3449-00  )(

OVERALL ESTIMATE/PRELIMINARY RUN:

   SO: ( .12254+00 .15294-00 .72073-01 .1393-00 .1078+00  13 .8027+01 .4053+02)

   (STATION,PRESS. ,NORMALIZED DEVIATION)
   (27 42.0-.1171+01  )(37 47.0-.4353-00  )(18 16.0 .5099-00  )(28 18.0 .1280+01  )
   (38 18.0 .1280+01  )(19  5.6 .2591-00  )(29  5.9 .6003-00  )(39  5.7 .3748-00  )
   (10  2.3-.3659-00  )(20  2.5 .1793-00  )(30  1.7-.2342+01* )(11  1.1-.1825-00  )
   (31  1.0-.8057-00  )(

OVERALL ESTIMATE/FINAL RUN:


   INTERNAL STATIONS OMITTED -      21 30


   SO: ( .12735-00 .11518+00 .55531-01 .1407-00 .1153+00  12 .8066+01 .3872+02)

   (STATION,PRESS. ,NORMALIZED DEVIATION)
   (27 42.0-.1809+01  )(37 47.0-.8323-00  )(18 16.0 .4464-00  )(28 18.0 .1469+01  )
   (38 18.0 .1469+01  )(19  5.6 .1537-00  )(29  5.9 .6067-00  )(39  5.7 .3073-00  )
   (10  2.3-.6424-00  )(20  2.5 .8152-01  )(11  1.1-.3840-00  )(31  1.0-.1211+01  )
   (

   KINGERY ESTIMATES,UPPER,LOWER
          D           PK             PKL             PKU
        67.0    .51728930+02    .50037599+02    .53477430+02
       117.0    .15198108+02    .14701190+02    .15711822+02
       200.0    .55017622+01    .53218764+01    .56877283+01
       335.0    .24766343+01    .23956582+01    .25603475+01
       600.0    .11497420+01    .11121500+01    .11886047+01

   GOODNESS OF KINGERY FIT: F(  4,  7)=( .17388-00/ .59762-01)**2= .84656+01
                                                                  (P= .0098***)
DIRECTIONAL ESTIMATES (SO):

   -60 ( .12654-00 .58700-01 .53691-01 .1436-00 .1115+00   4 .8059+01 .1076+02)
    60 ( .12757-00 .16021-00 .12782-00 .1723-00 .9443-01   4 .8068+01 .1414+02)
   180 ( .12777-00 .13967-00 .11273+00 .1666-00 .9800-01   4 .8069+01 .1382+02)
   AVE ( .12736-00 .12731-00 .61382-01 .1425-00 .1138+00  12 .8066+01 .3872+02)

   SIGNIFICANCE OF DIRECTION: F(  2,  9)= .22360-02     (P>.5)
```

TABLE B11 - COMPUTER OUTPUT FOR TEST 278 – IMPULSE

```
                    STANDARD OUTPUT OF ESTIMATES:
   SO: (      Y     , S.D.P% , S.D.Y% ,  YU    ,   YL    , N , LN(TNT), N-EQUIV)

OVERALL ESTIMATE/PRELIMINARY RUN:

   SO: ( .115[illegible]2+00 .28939-00 .12521-00 .1438-00 .9202-01  13 .7964+01 .4808+02)

   (STATION,IMPLSE ,NORMALIZED DEVIATION)
   (37287.0 .8867-00  )(18170.0 .7464-00  )(28149.0 .2908-00  )(38174.0 .8268-00  )
   (19 92.0 .2881-00  )(29 94.0 .3624-00  )(39 98.0 .5064-00  )(10 52.0-.2961-01  )
   (20 52.0-.2961-01  )(30 39.0-.1024+01  )(11 31.0 .1149+00  )(21 13.0-.2888+01**)
   (31 31.0 .1149+00  )(

OVERALL ESTIMATE/PRELIMINARY RUN:

   SO: ( .12869-00 .14908-00 .67371-01 .1452-00 .1140+00  12 .8076+01 .4407+02)

   (STATION,IMPLSE ,NORMALIZED DEVIATION)
   (37287.0 .1257+01  )(18170.0 .9767-00  )(28149.0 .9224-01  )(38174.0 .1133+01  )
   (19 92.0 .7920-01  )(29 94.0 .2235-00  )(39 98.0 .5030-00  )(10 52.0-.5448-00  )
   (20 52.0-.5448-00  )(30 39.0-.2475+01* )(11 31.0-.2727-00  )(31 31.0-.2727-00  )
   (

OVERALL ESTIMATE/FINAL RUN:


   INTERNAL STATIONS OMITTED -      21 30


   SO: ( .13574-00 .97788-01 .46234-01 .1476-00 .1248+00  11 .8130+01 .4026+02)

   (STATION,IMPLSE ,NORMALIZED DEVIATION)
   (37287.0 .1580+01  )(18170.0 .1147+01  )(28149.0-.2010-00  )(38174.0 .1385+01  )
   (19 92.0-.2265-00  )(29 94.0-.6548-02  )(39 98.0 .4196-00  )(10 52.0-.1183+01  )
   (20 52.0-.1183+01  )(11 31.0-.7745-00  )(31 31.0-.7745-00  )(

   KINGERY ESTIMATES,UPPER,LOWER
          D          PK             PKL             PKU
        67.0    .24590384+03    .23913011+03    .25286944+03
       117.0    .15195803+03    .14777216+03    .15626248+03
       200.0    .94060210+02    .91469204+02    .96724610+02
       335.0    .58378515+02    .56770406+02    .60032176+02
       600.0    .33438881+02    .32517765+02    .34386089+02

   GOODNESS OF KINGERY FIT: F(  4,  6)=( .14104-00/ .51730-01)**2= .74332+01
                                                                        (P= .02**)
DIRECTIONAL ESTIMATES (SO):

   -60 ( .13022-00 .98929-01 .77026-01 .1561-00 .1086+00   4 .8088+01 .1485+02)
    60 ( .12631-00 .61128-01 .55400-01 .1485-00 .1074+00   3 .8058+01 .1096+02)
   180 ( .14959-00 .10718+00 .84562-01 .1825-00 .1226+00   4 .8227+01 .1446+02)
   AVE ( .13574-00 .94403-01 .44634-01 .1475-00 .1249+00  11 .8130+01 .4026+02)

   SIGNIFICANCE OF DIRECTION: F(  2,  8)= .13650+01     (P= .32)
```

APPENDIX C

DETAILED ANALYSIS OF THE DATA

This appendix amplifies the summary analysis in Section III of the report. For each of the four propellant-failure mode combinations, the following topics are discussed:

1. Summary of data for individual tests
2. Correlation of yield and ignition time
3. Regression analysis
4. Comparison with URS

Derivations of specialized statistical formulas used in 2 and 3 are given in Appendix D.

I. CBM, LO_2/RP-1

1. Summary of data

1.1 Weight

Using the yields obtained from the selection procedure in Appendix A, Table C1 presents summary statistics from the computer output for the 200 lb., 1000 lb., and 25,000 lb. tests at nominal geometry (L/D=1.8, D_o/D_t=.45), and also the Titan test. For each test the standard output is given, i.e., the least squares estimate of yield for stations not omitted, standard deviation of the scatter in percent, standard deviation of the estimated yield in percent, upper and lower 95% confidence limits, total data available (distances and stations), number of distances and stations omitted (number of additional "extreme" stations omitted shown in parentheses), and URS' estimate of yield.

Regarding type of ignition, X refers to an external ignition source, either cap or squib. Ignition from other sources, as described in Appendix E of the URS report[1], are explicitly noted. Regarding Titan, there is some uncertainty whether ignition occurred before or after tank rupture (see Section III, p. 10 footnote). The symbol SP indicates a spurious test. As noted in Section III, such tests encountered unusual conditions which led to premature self-ignition. For 200 and 1000 lbs. none were externally ignited. For 25,000 lbs. conditions such as diaphragm rupture, possibly small orifice (test 275), etc., occurred.

Ignition times and URS yields are taken from the URS report.[1] However, the values in the various tables did not always agree. The principal discrepancies are noted at the end of this appendix.

For each test in Table C1 the first line refers to pressure and the second to impulse. Plots of pressure and impulse were given in Figures A5P and A5I, along with the least squares Kingery fits and departures therefrom. Of special significance is the fact that the magnitude of the Titan response was about the same as for the two (non-spurious) 25,000 lb. tests.

Figure C1 presents a scatter diagram of impulse yield vs. pressure yield. For the 200 lb. tests, including non-nominal geometry tests, the average ratio Y_I/Y_P was .87. For larger weights, the ratio is approximately unity.

1.2 <u>Geometry</u>

The statistics of yield for the 200 lb. non-nominal geometry tests are presented in Table C2, and distance plots in Figures A6P and A6I. Except for test No. 240 (two-thirds full), data was available at only three distances.

Exceptionally high yields of 46 - 56% were obtained for the two partially full tests and for test No. 042, with L/D=1.8 and D_o/D_t=1.0. These tests also generally showed severe departures from the Kingery curves (see Appendix A).

2. <u>Correlation with Ignition Time</u>

The dependence of yield Y upon ignition time t, as discussed in Section II of the report, was a principal area investigated by URS--in fact, the URS prediction equation (paragraph 4 below) assumes that ignition time scales geometrically. The computed correlations r and regression slopes b are shown in Table C3 both for logs of yield and time, as well as without taking logs. In the former case departure of b from unity indicates roughly the extent to which mixing is nonlinear.

The correlations in Table C3 should be viewed with caution. Because of the small number of tests in each category, slight departures from linearity may lead to very large confidence limits.* Conversely, spuriously high correlations are not unusual. For these reasons, it is necessary to rely also on visual interpretation of the Y vs. t plots in Figures 1P and 1I.

---

*The 90% confidence limits, r_U and r_L, assume ignition time to be nonrandom (see Appendix D2).

2.1 Weight

The 200 lb. tests show good correlation, especially for pressure yields, and with a log slope not too different from unity. However, there is considerable variation at intermediate ignition times (120-127 msec.) with the three largest yields being rather close to one another. These together are treated below as representative of maximum yield.

The scatter is large at 1000 lbs. with resulting low correlation, while at 25,000 lbs. t was not varied. Hence for these weights one cannot meaningfully estimate the dependence of Y on t. However, if yield is assumed to be proportional to t, then a plot of Y/t vs. W should give some idea of scaling with W. Using the high yield tests, the results are shown in Figures C2P and C2I. The fall-off is seen to be considerably faster than $W^{1/3}$, even with Titan excluded.

2.2 Geometry

Correlation with ignition time was good for all three non-nominal cases. (For 2/3 full t was not varied.) However, the mixing rate was not constant for D_o/D_t=1, as evidenced by the departure from unity of the log log slope b. The values of b from Table C3 are summarized below.

D_o/D_t \ L/D	1.8	5.0
.45	1.08 .74	1.05 .80
1.0	1.31 1.27	.17 .08

Maximum ignition times, tabulated below, were greater than for nominal geometry in all cases except 2/3 full.

D_o/D_t \ L/D	1.8	5.0
.45	156, 145	220
1.0	290	380, 316

For this table the interaction term (see 3.2) turns out to be very small.

3. Regression

The statistical regression analysis for weight is performed separately from geometry (or velocity in the case of CBGS, LO_2/LH_2). Although this greatly simplifies the analysis and interpretation of results, a slight inconsistency in the regression value for the nominal condition may occur. Formulas are derived in Appendix D3 for the corrections to the single variable regressions which yield the joint regression on both variables.

The analysis for the geometry variables turns out to require merely the determination of main effects and interaction without performing a least squares analysis.

Logs of the variables are used in the regression analysis since relative errors in the estimated yields are more comparable than absolute errors. In addition, logs arise naturally when attempting to determine scaling over a weight range of several orders of magnitude.

Prediction limits for the regression line are also determined. The distinction between confidence limits and prediction limits is that the former pertains only to errors in the regression line (the mean and the slope), whereas the latter includes in addition the test-to-test variation, as estimated by the scatter about the regression line. Both prediction and confidence limits are asymptotic to the line through the center of gravity of the observations and using the upper confidence estimate of slope (Appendix D1). Thus the latter determines how fast the prediction limit for yield falls off for large weight; this depends of course upon the probability level selected.

3.1 Weight

Several alternative regressions have been performed--using high yield tests, or all tests (except the spurious 25,000 lb. test No. 275), each with and without the Titan. Results are summarized in Table C4, which lists the scatter (σ_Y%), the regression constants (A,B), 90% and 95% prediction asymptotes ($A^a_{.90}$, $B_{.90}$, etc.), and expected yields and prediction limits for 94,000 lbs. and 4.6×10^6 lbs. The most relevant regressions have been plotted in Figures 2P and 2I.

Using high yield tests only (three at 200 lbs., one at 1000 lbs.*, two at 25,000 lbs.) and excluding Titan, the scatter is quite small and the 95% confidence slope of .12-.18 is still appreciable. Note that the observed yield for 1000 lbs. is considerably below the regression line. The expected yield for 94,000 lbs. is 8-9% compared with 3.3% observed for the Titan test. The fact that observed impulse yields at 200 lbs. are less than pressure yields leads to a shallower regression slope and a larger expected yield for large weights. Moreover, since the regression scatter for impulse is greater than for pressure, the difference between the prediction limits is widened further.

With Titan included in the regression,** the slope becomes much steeper. Although the scatter increases appreciably the 95% prediction limits remain lower than when Titan is omitted.

If Farber's theory of critical volume for self-ignition is assumed to be correct, then in Figure 2 the intersection point of the line (with slope unity) joining the self-ignited 25,000 and 94,000 lb. points, and the line through the non-self-ignited 200 lb. and 1000 lb. points, should provide a rough estimate of critical weight for self-ignition. This turns out to be approximately 12,000-20,000 lbs., which compares with Farber's estimate of 2,800 lbs.

The regression line using all tests shows very large scatter and with essentially flat 95% confidence slope. This regression is probably not too meaningful, since it depends upon the number of controlled low ignition time tests performed.

3.2 Geometry

The yields for non-nominal geometry conditions were previously plotted in Figures 3P and 3I. Since the data is not sufficient to determine the dependence of the mixing function

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*In principle, the high yield tests for each weight should be statistically comparable. At maximum ignition time, poor reproducibility occurs only at 1000 lb. In this case, selection of the largest of the three yields seems to offer the most meaningful analysis.

**Because of the large uncertainties (see Section 3.2 and 4), no adjustment is made for the different Titan geometry and ullage conditions. Also, actual propellant weight for the Titan test of 94,000 lbs. is used rather than full weight of 170,000 lbs.

upon geometry, a direct analysis of the high yield tests seems to be the best course. These are tabulated below.

D_o/D_t \ L/D	1.8	5.0	
.45	31.5 26.9	25.3 22.8	28.4 24.9
1.0	48.7 45.3	12.7 9.8	30.7 27.6
	40.1 36.1	19.0 16.3	29.6 26.2

The following qualitative trends are apparent:

1. When the orifice increases to D_o/D_t=1, yield increases substantially, by over 50%.

2. When tank geometry increases to L/D=5, yield decreases moderately, by about 15-20%.

3. When both L/D=5 and D_o/D_t=1, the yield is very low, less than half the nominal.

The statistical analysis of the above 2 x 2 factorial design is straightforward* and can be expressed in terms of the main effect L of L/D, the main effect D of D_o/D_t, and the interaction term LD. These main effects are calculated from the row and column averages shown in the table. For example, for pressure yields one gets

$$L = \frac{1}{2}(19.0 - 40.1) = -10.5$$

$$D = \frac{1}{2}(30.7 - 28.4) = 1.2$$

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*For the nominal case the average of the three high yield tests has been used. The joint regression of yield on both weight and geometry requires that the nominal value be replaced by that derived from the simple regression of yield on weight (see Appendix D). When the weight regression uses only high yields, the nominal yield decreases by one when Titan is excluded, and increases by one when included.

The interaction is obtained from

$$LD = \frac{1}{2}\left[\frac{1}{2}(31.5 - 25.3) - \frac{1}{2}(48.7 - 12.7)\right] = -7.4$$

These quantities can be incorporated into a regression equation, which is actually equivalent to the usual two way interpolation formula.

$$Y_P = 29.6 - 10.5\left(\frac{L/D-3.4}{1.6}\right) + 1.2\left(\frac{D_o/D_t-.725}{.275}\right) - 7.4\left(\frac{L/D-3.4}{1.6}\right)\left(\frac{D_o/D_t-.725}{.275}\right)$$

$$= 7.3 + 5.6\ L/D + 61.6\ D_o/D_t - 16.9\ L/D \cdot D_o/D_t \qquad (*)$$

$$Y_I = 26.2 - 9.9\left(\frac{L/D-3.4}{1.6}\right) + 1.4\left(\frac{D_o/D_t-.725}{.275}\right) - 7.9\left(\frac{L/D-3.4}{1.6}\right)\left(\frac{D_o/D_t-.725}{.275}\right)$$

$$= -.3 + 6.8\ L/D + 65.6\ D_o/D_t - 17.8\ L/D \cdot D_o/D_t \qquad (*)$$

The interpolation also could be performed using logs of the variables or any other transformation, for example, $(D_o/D_t)^2$, the orifice area ratio.

Since repeated tests for maximum yield were not conducted for all conditions, it is difficult to estimate the statistical error in the above equations. Perhaps the best one can do is to assume that the scatter determined from the three high yield nominal tests and the two tests at L/D=5, D_o/D_t=1 applies generally.

The large interaction term in (*) can be considered to result from an "unusual" yield for some one of the four geometry conditions--probably L/D=5, D_o/D_t=1--and suggests a change in mixing dynamics. URS states (reference 1, p. 5-57): "The experimental results suggest that some slowdown in mixing may have occurred for the L/D of 5 case but that mixing is not basically limited by ... freezing (of the RP-1), but rather by the pressure buildup in the tank ... to a value greater than the bursting pressure ..."

In any event, the change may not occur in the smooth manner implied by the preceding equation (*), and may be especially unsatisfactory for extrapolation outside the tested range.+ In fact, for small orifices (D_o/D_t<.45) one might prefer to ignore the response at L/D=5, D_o/D_t=1 and use merely the linear interpolation formulas:

$$Y_P = 31.5 - 6.2\left(\frac{L/D-1.8}{3.2}\right) + 17.2\left(\frac{D_o/D_t-.45}{.55}\right)$$

$$= 21.0 - 1.95\ L/D + 31.2\ D_o/D_t$$

(**)

$$Y_I = 26.9 - 4.1\left(\frac{L/D-1.8}{3.2}\right) + 18.4\left(\frac{D_o/D_t-.45}{.55}\right)$$

$$= 14.2 - 1.3\ L/D + 33.5\ D_o/D_t$$

For the Titan geometry (L/D=4, D_o/D_t=.1375) but with W=200 lbs., one gets

Y_P = 30.1 , Y_I = 25.9 , equation (*)

Y_P = 17.5 , Y_I = 13.7 , equation (**)

Depending upon whether or not the interaction term is included, the decrease in yield with respect to nominal geometry is thus either negligible or substantial.

Of the three geometry variables investigated, ullage volume V_u gave the largest increase in yield. The effect of V_u can be obtained by interpolating between the observed yields for V_u=10% and V_u=40%. Figure C3 plots specific yields (i.e., relative to actual propellant weight) and also the limiting yield of 120%, assuming perfect mixing for V_u=100%. The line for impulse exceeds only slightly the limiting yield, while that for pressure is considerably above; URS' freehand curve is intermediate over almost the entire range. The equations of the lines are

$$Y_{S,P} = 161.5\ V_u^{.710}$$

$$Y_{S,I} = 130.7\ V_u^{.686}$$

+Farber's unpublished data from model studies may be relevant here; unfortunately, the data do not agree with the Pyro yields.

If URS' procedure is adopted, V_u can be replaced by the effective ullage volume V_{u-eff} (see paragraph 4 below). The conversion from specific yield to nonspecific yield (i.e., relative to tanks full) is given simply by

$$Y = Y_S \cdot (1-V_u)/.9$$

Also, if normalized to unity for nominal conditions (V_u=V_{u-eff}=.1), one gets the so-called k-factors.

$$k_P = 5.69\ V_{u-eff}^{\ .710}(1-V_u)$$

$$k_I = 5.39\ V_{u-eff}^{\ .686}(1-V_u)$$

URS' k curves are intermediate to these equations for $V_{u-eff} < .75$.

4. Comparison with URS

URS' prediction equation is given by

$$Y = t^* \cdot (1 + \frac{217}{W}) \cdot (.869 - .092\ L/D - .276\ D_o/D_t), \text{ for } t < k \cdot t_{max} \quad (1a)$$

$$Y_{max} = t^*_{max} \cdot (1 + \frac{217}{W}) \cdot (.869 - .092\ L/D - 276\ D_o/D_t) \cdot k(V_u, \Delta p_r) \quad (1b)$$

for $t = t_{max}$ or t unknown

Yield is thus the product of four factors, each of which depends upon appropriate variables:

1. **Scaled ignition time $t^* = t/W^{1/3}$ (or $t^*_{max} = t_{max}/W^{1/3}$)**
2. **Weight W**
3. **Geometry L/D, D_o/D_t**
4. **Ullage volume V_u and tank differential burst pressure Δp_r (k=1 for V_u=10% and Δp_r=85 psia)**

t_{max}, the time required for tank pressure to reach the burst pressure, depends upon geometry.

Consider first the effect of geometry and ignition time in relation to the 200 lb. data. For W=200 equation (1a) becomes

$$Y = ct \tag{2}$$

where

$$c = .357(.87 - .092\ L/D - .28\ D_o/D_t) \tag{3}$$

The numerical values of c are:

(4)

D_o/D_t \ L/D	1.8	5.0
.45	.207	.102
1.0	.152	.0476

Although (2) assumes yield proportional to t, Table C3 indicates considerable deviation from proportionality, especially when L/D=5 and D_o/D_t=1.0. This shows up in Table C5, which compares the observed yields (both URS and least squares) with the yields calculated from equation (2) using the observed ignition times and the values of c in (4).

With respect to equation (1b), under nominal 10% ullage, URS uses the following t_{max} values.

(5)

D_o/D_t \ L/D	1.8	5.0
.45	126	194
1.0	252	333

These appear to be about 30 msec less than the observed maxima (cf. 2.2 and Table C1 and C2). The corresponding maximum yields, obtained by multiplying the tabular values in (4) and (5), are as follows. (URS estimated yields are shown in parentheses; Section III of the report or Tables C1 and C2 give the least squares yields.)

D_o/D_t \ L/D	1.8	5.0
.45	26.0 (33)	19.7 (23)
1.0	38.4 (48)	15.8 (12)

(6)

The agreement is not too good, especially for D_o/D_t=1. Most likely this reflects the absence of an interaction term in the geometry factor (3), since only negligible interaction appears in the t_{max} values in (5).

It should be noted that URS recommends (Volume 3 of the URS report) the use of D_o/D_t=.45 in equations (1a) and (1b) when $D_o/D_t \leq .45$, and the use of D_o/D_t=1 when $D_o/D_t > .45$, with linear interpolation for L/D in (5) to determine t_{max}. This procedure in effect bypasses the anomalous behavior of the data noted above.

The t_{max} values in (5) are extrapolated by URS (under nominal ullage conditions) to weights other than 200 lbs. through geometric scaling, i.e., multiplying by the factor $(W/200)^{1/3}$.

Consider next the effect of W. Since, with one exception, t was not varied for the 1000 and 25,000 lb. tests, the data apply essentially to equation (1b). Substituting the nominal geometry values (L/D=1.8, D_o/D_t=.45, k=1) gives

$$Y_{max} = .58\, t^{*}_{max} (1 + 217/W) \qquad (7)$$

URS uses the estimate (cf. entry in (5) for nominal geometry)

$$t^{*}_{max} = 21.5 \qquad (8)$$

or

$$t_{max} = 21.5\ W^{1/3} \qquad (9)$$

Actually, if one plots the observed t_{max} values (Figure C4), these appear to be better approximated by

$$t_{max} = 33.9\ W^{.275} \qquad (10)$$

Substituting (8) into (7) gives the hyperbola

$$Y_{max} = 12.5\ (1 + \frac{217}{W}) \qquad (11)$$

This equation has been previously plotted in Figure 2I along with the log log regressions for high yield tests. Comparison of the observed and predicted maximum yields is shown below.

W	Predicted			Observed		
		log-log*			Least Squares	
	Y_{URS}	Y_P	Y_I	Y_{URS}	Y_P	Y_I
200	26.1	30.5	26.6	32	31.0	22.1
				32	30.6	26.2
				35	33.0	31.8
1000	15.2	21.6	20.1	20	18.6	20.1
				13	10.7†	11.7†
				14	10.9†	10.8†
25,000	12.6	10.9	11.5	13	11.3	11.4
				13	11.1	11.7

*Titan excluded

†Not used in log-log regression

Making due allowance for the differing estimates of yield, it would appear that the log log regression equation gives a slightly better fit. URS' estimate of 12.5 for the hyperbolic parameter in (7) depends primarily on the five high yield 1000 and 25,000 lb. tests shown in the table.** The essential difference is in the model assumed. URS' hyperbolic equation implies that Y is constant for large W (for W≥10,000, 1+217/W≈1), whereas the power law assumes constant scaling with weight.

Finally, consider the effect of ullage volume V_u. URS' treatment is similar to that outlined in paragraph 3.2. First, from the two 40% ullage tests at 200 lb. and the limiting yield of 120%, a curve of specific yield vs. V_u was estimated (Figure C3). To allow for variable tank differential burst pressure, Δp_r (i.e., burst pressure minus average initial pressure), URS assumed that specific yield depends upon the number of moles of evaporated gas required for tank pressure to reach burst pressure, so that Δp_r has the same effect as V_u. Specifically,

**URS used the nonmaximum ignition time and non-nominal geometry 200 lb. data as well. But because of the nonlinear behavior of these variables (see 3.2) this procedure decreases statistical precision in predicting <u>maximum</u> yields.

the effective ullage volume is defined by

$$V_{u-eff} = \frac{\Delta p_r}{85} V_u \qquad (12)$$

since for the test tanks Δp_r=85 psia (LO_2 tank pressure 35-40 psia, RP-1 tank pressure 15 psia, burst pressure 115 psia). V_{u-eff} is then substituted for V_u in determining specific yield.* Converting to nonspecific yield and normalizing at $V_u=V_{u-eff}$=10% gives curves for the k factor as a function of Δp_r and V_u.

In general, for large weights (i.e., $1 + 217/W \simeq 1$), maximum yield is constant and related to the maximum yield for 200 lbs. and nominal ullage (tabled in (6)) by

$$Y_{max} = \frac{k}{2.085} \cdot Y_{max(W=200,\ k=1)} \qquad (13)$$

When t is known, no ullage correction is applied. However, in order to obtain the same limiting value as given by equation (1b), URS permits t to be (only) as large as $k \cdot t_{max}$.

It is of interest to apply the URS prediction equations to the conditions for the Titan test, namely W = 170,000 lbs., L/D=4, D_o/D_t=.1375, V_u=50%,** Δp_r=35 psia. Consider first the Y_{max} equation (1b). From (12) one gets V_{u-eff}=20.6%. URS' estimate of k=.9 compares with the values calculated from the equations in paragraph 3 of k_P=.93 and k_I=.86. Replacing the actual D_o/D_t of .1375 by .45, and then interpolating in (5) for L/D=4 gives t_{max}=173 for W=200 lbs., or t^*_{max}=29.6 (and also t_{max}=29.6 $(170,000)^{1/3}$ = 1640 msec which compares with the observed t=842 msec).

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*Note that the value of limiting yield, for perfect mixing, will then depend upon Δp_r.

**RP-1 tank was two-thirds full (35,000 lbs.) and LO_2 tank one-half full (59,000 lbs.); overall this is equivalent to 55% full.

If ignition occurred after tank rupture, using equation (1b) gives Y_{max} = 29.6 (1) (.377) (.9) = 10.0.* In case self-ignition occurred before tank rupture, then the observed t of 842 msec would be used with equation (1a). One gets Y = 15.2 (.377) = 5.7. These predicted yields for Titan are actually nonspecific yields, whereas the yields shown in Table C1 (pressure yield 3.27, impulse yield 3.42, URS yield 4) are specific yields relative to 94,000 lbs. propellant--for nonspecific yield one should multiply by .55. Thus the predicted yields are three to five times the observed yield. Even if W=94,000 is used in (1a), one gets a specific yield of 7.0 which is still more than twice the observed yield.

Finally, some mention should be made of URS' application of impulse correction factors to their equivalent TNT weights, ranging from 1.3 for far distances (1.4 for LO_2/LH_2) to 2.0 for near distances. This procedure does not appear to agree with the results obtained from the detailed study in Appendix A of departures from the TNT Kingery curves.

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*Alternatively, one can use (14); however, linear interpolation in the table of yields in (6) is only approximate, giving $Y_{max} = (\frac{.9}{2.085}) \cdot 21.7 = 9.3$.

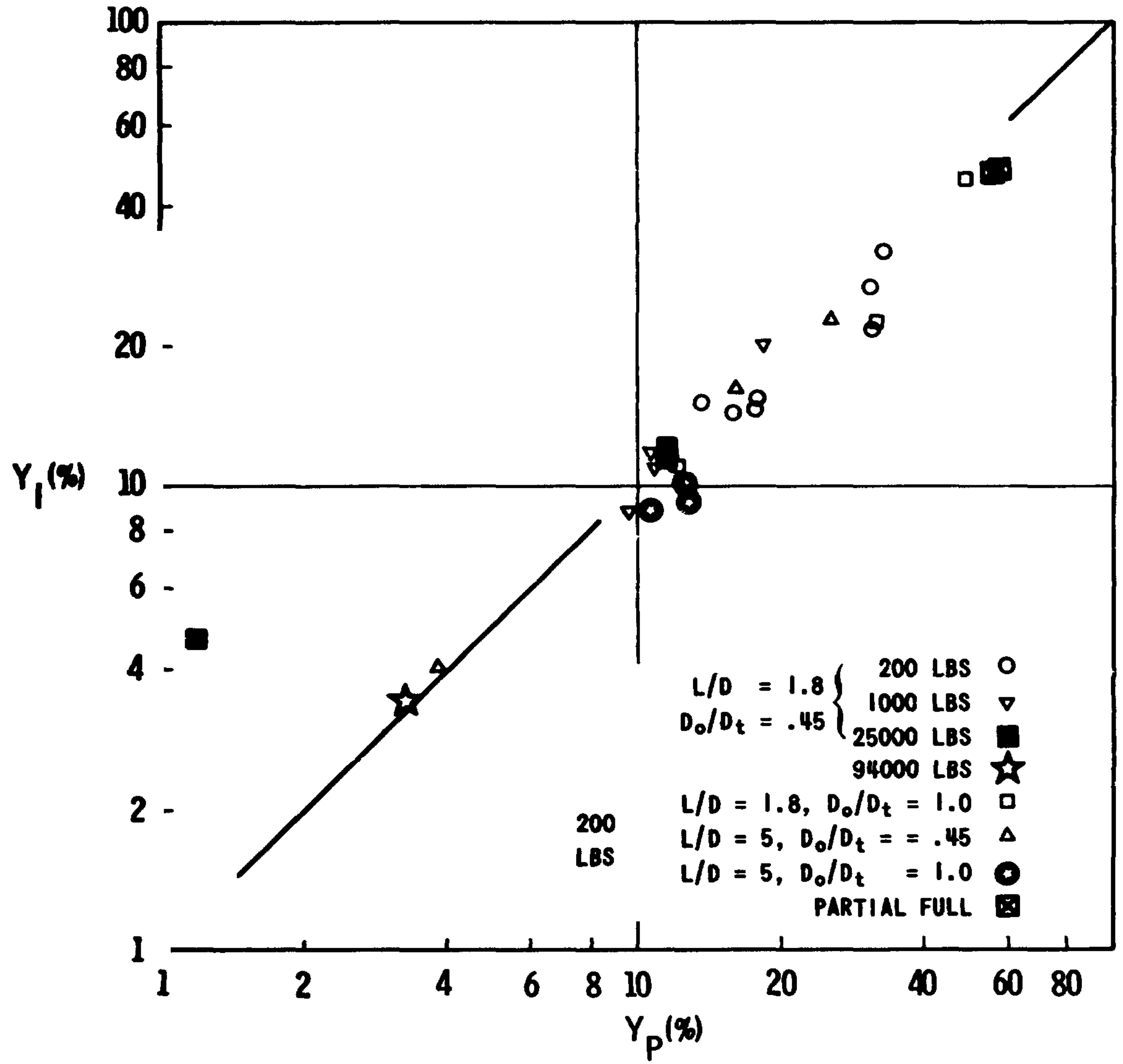

FIGURE C1 - CBM, LO_2/ RP-1, COMPARISON OF OVERPRESSURE AND IMPULSE YIELDS

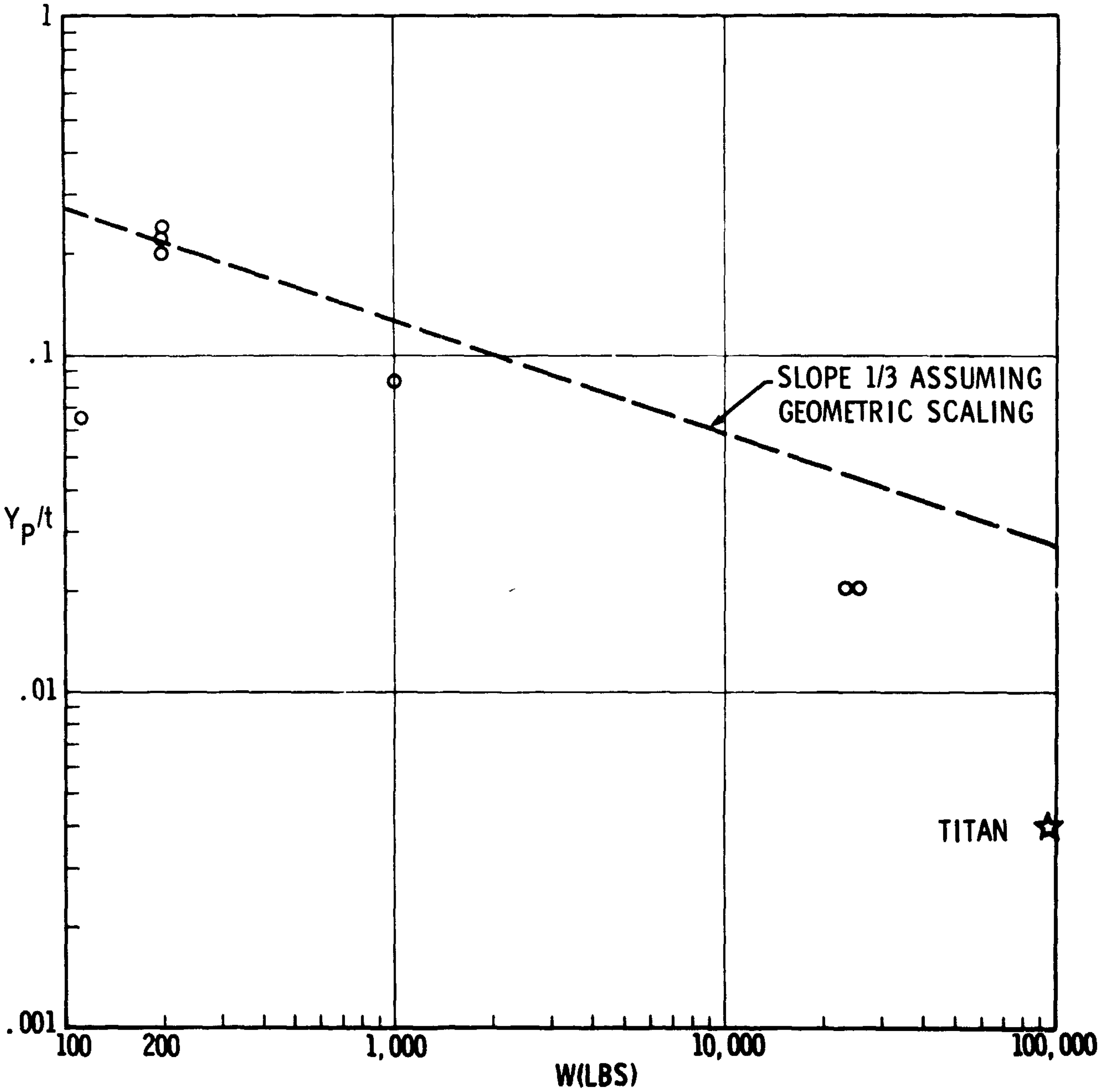

FIGURE C2P - CBM, LO_2/RP-1, TIME SCALING OF .IGH YIELD TESTS WITH WEIGHT--PRESSURE

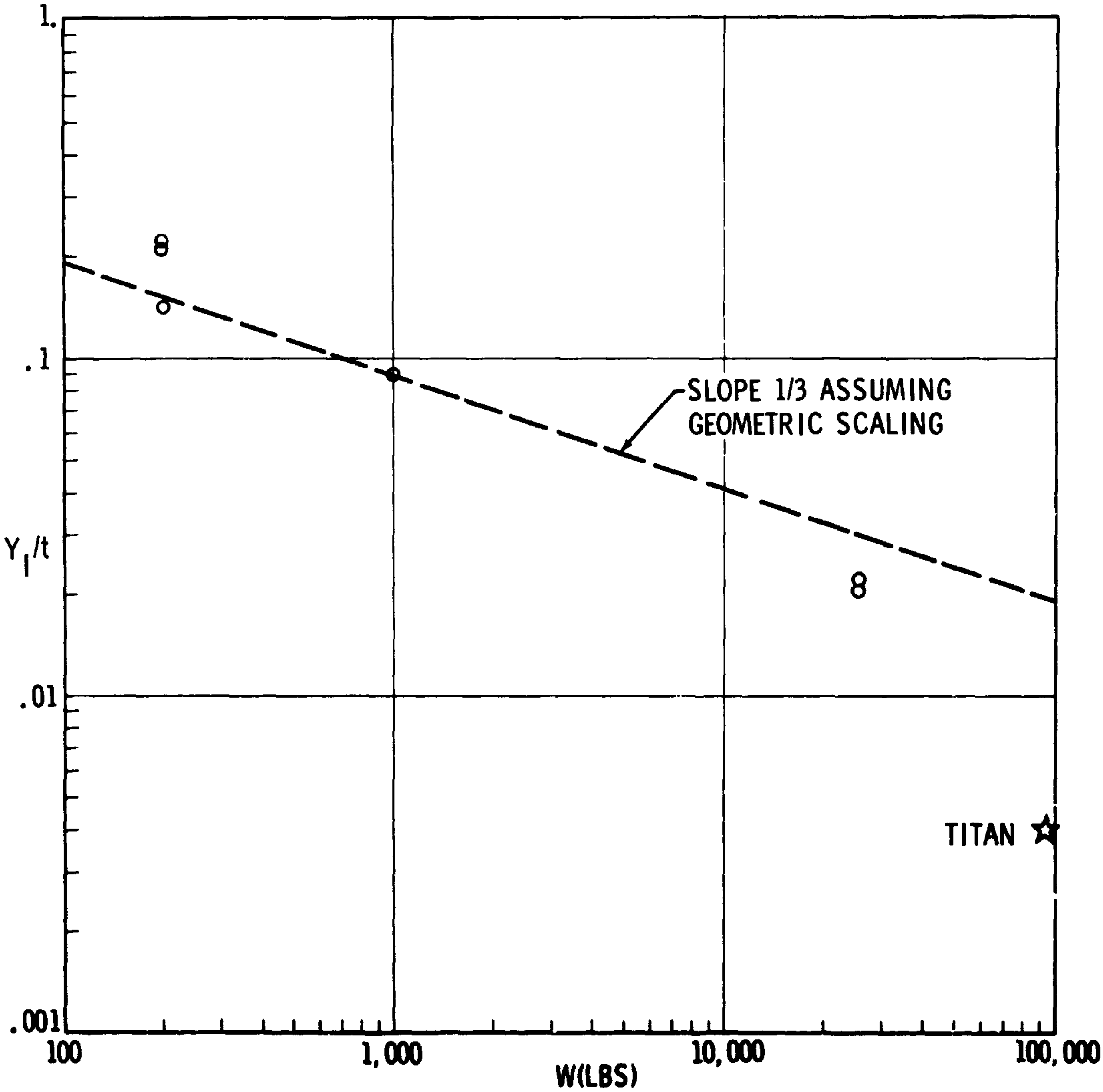

FIGURE C21 - CBM, LO_2/RP-1, TIME SCALING OF HIGH YIELD TESTS WITH WEIGHT - IMPULSE

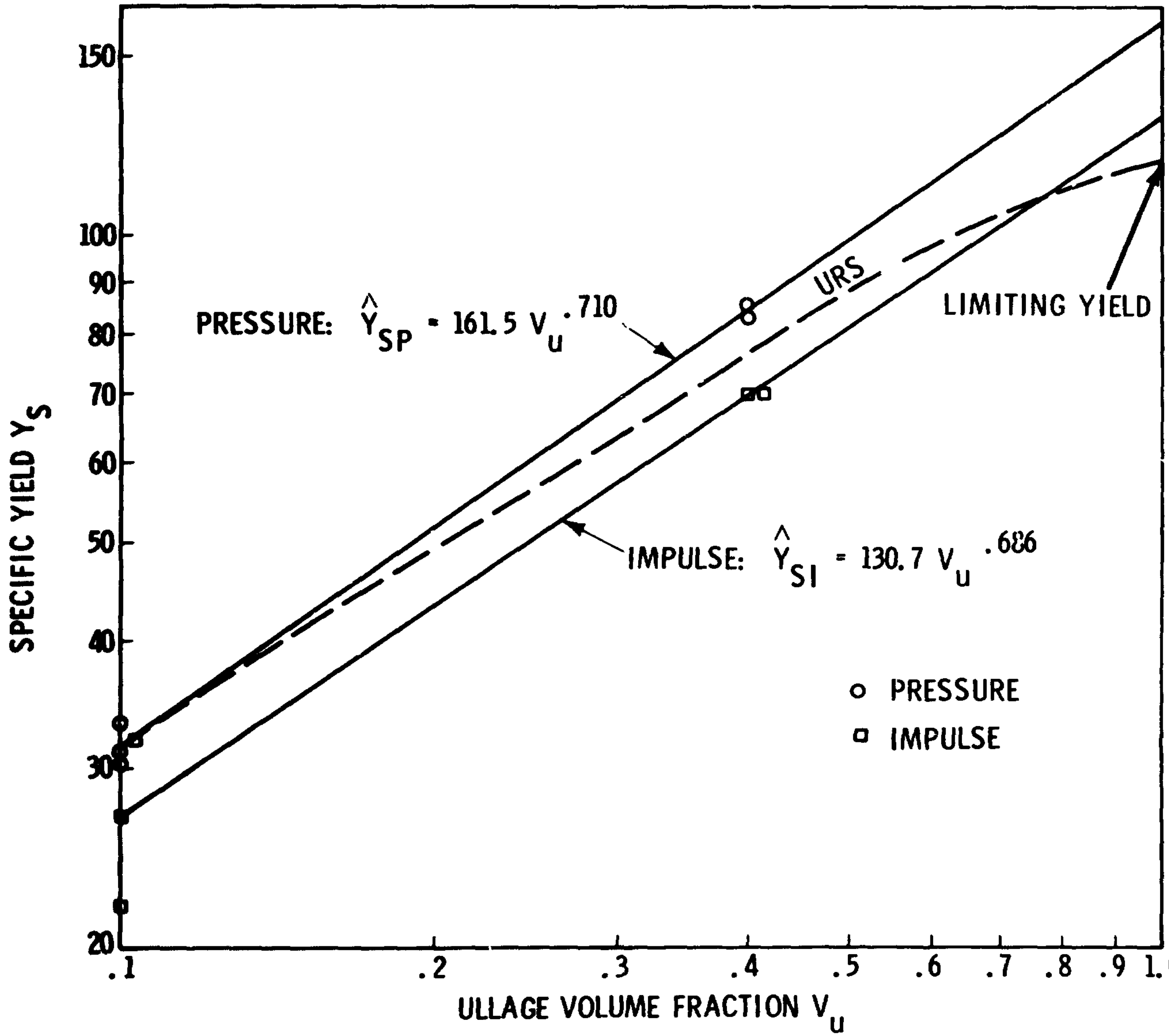

FIGURE C3 - CBM, LO_2/RP-1, 200 LBS., SPECIFIC YIELD VS. ULLAGE VOLUME

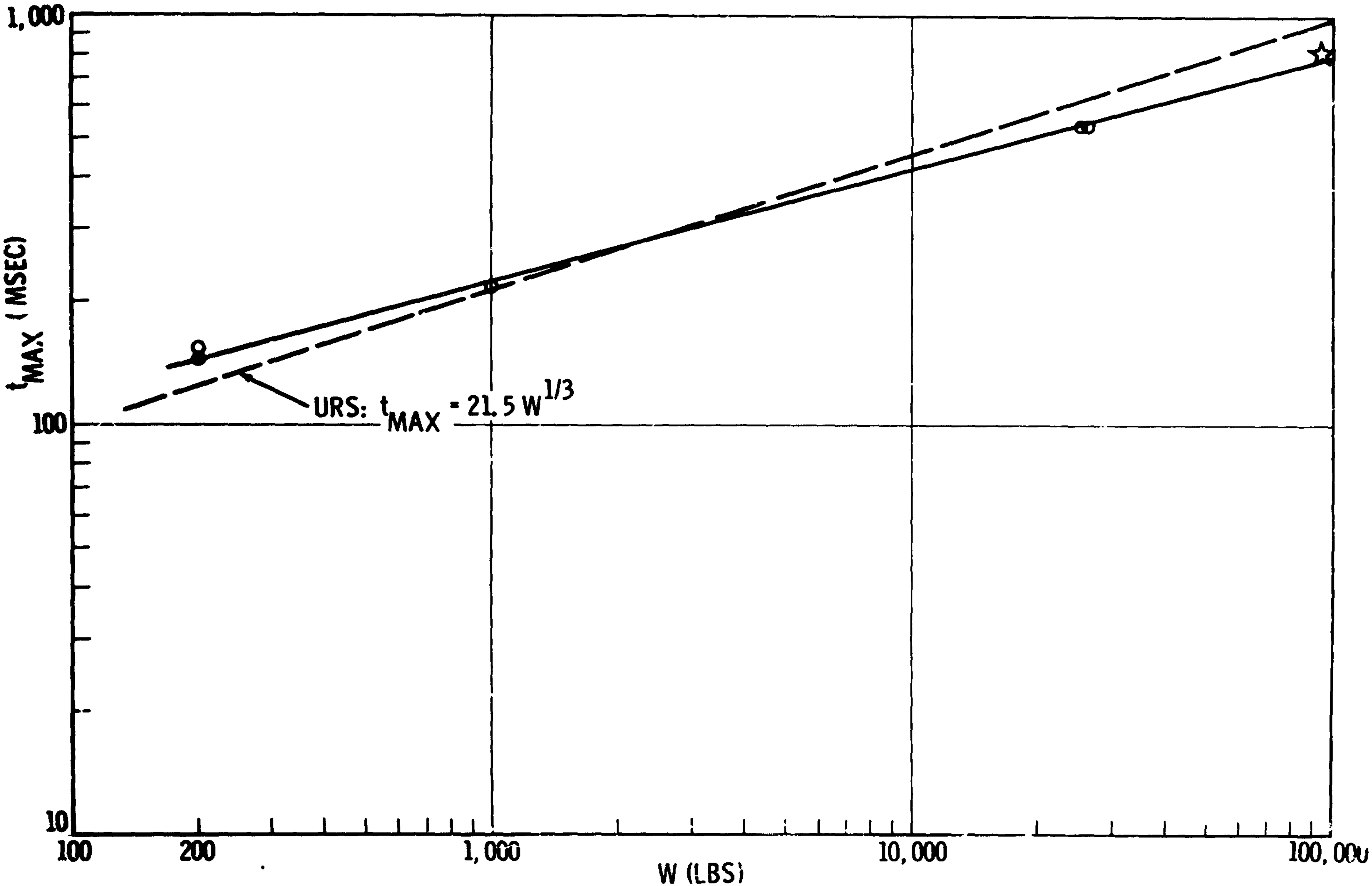

FIGURE C4 - CBM, LO_2/RP-1. SCALING OF t_{MAX} WITH WEIGHT

TABLE C1 - CBM, LO_2/RP-1, L/D = 1.8, D_o/D_t = .45, SUMMARY OF TEST DATA

W	TEST NO.	IGNITION TYPE (1)	TIME	Y	$S_{P(1)}$%	S_Y%	Y_U	Y_L	AVAILABLE DATA d's, ST'NS	DATA OMITTED d's, ST'NS	Y_{URS}
200	239	X or TR	156	31.03	7.9	4.5	33.7	28.6	5d,14	1d,3	32
				22.13	3.2	2.0	23.0	21.3	5d,13	3d,7	
	237	TR	127	30.58	12.4	5.7	33.8	27.6	5d,14	0	32
				26.81	8.9	3.7	28.6	25.1	5d,14	0	
	101	X	145[d]	32.98	12.7	8.7	39.3	27.7	3d,9	1d,3	35
				31.84	8.9	8.0	40.2	25.2	3d,9	2d,6	
	044	TR	120	17.82	9.0	5.1	19.6	16.2	3d,8	0	18
				15.54	5.0	5.5	22.0	11.0	3d,8	2d,6	
	095A	X	120	17.61	7.6	4.1	19.0	16.3	3d,9	0	17
				14.66	6.7	3.5	15.6	13.7	3d,9	0	
	238	X	85	15.99	7.6	4.0	17.2	14.9	5d.13	(2)	19
				14.43	9.5	4.2	15.6	13.4	5d,13	(1)	
	087A	X	70	13.56	7.5	4.2	14.7	12.5	3d,9	0	16
				15.08	6.3	3.3	16.0	14.2	3d,9	0	
1000	193	X(2)	222	18.55	3.7	3.1	19.8	17.4	4d,10	2d 5	20
				20.10	3.4	2.6	21.4	18.9	4d,11	2d,6(1)	
	270A	X	225	10.68	9.4	4.5	11.6	9.9	5d,15	(2)	13
				11.74	6.1	3.6	12.6	11.0	5d,15	2d,6(2)	
	192	X(2)	216	10.94	5.6	5.8	12.5	9.6	4d,10	2d,6	14
				10.82	6.4	4.1	11.8	10.0	4d,10	1d,3(1)	
	209	X	121	9.51	5.7	2.7	10.0	9.1	4d,12	0	10
				8.86	12.1	5.8	9.8	8.0	4d,12	(1)	
25000	282	TR	540	11.28	6.8	6.3	12.8	9.9	5d,13	3d,7	13
				11.37	6.9	3.6	12.1	10.6	5d,15	2d,6	
	278	TR	530	11.11	6.4	7.3	13.2	9.4	5d,14	3d,8(2)	13
				11.73	2.4	1.8	12.2	11.2	5d,13	3d,7(2)	
	275	SP(3)	515	1.19	10.3	7.8	1.4	1.0	5d,15	2d,6	4
				4.64	15.7	6.3	5.2	4.2	5d,15	0	
94000	301	(4)	842	3.27	8.8	6.0	3.7	2.9	5d,13	2d,4	4
				3.42	7.5	5.2	3.8	3.1	5d,10	2d,4(1)	

(1) X = IGNITION BY CAP OR SQUIB

TR = SELF-IGNITION AT OR AFTER TANK RUPTURE

(2) PROBABLE CAP IGNITION

(3) SUSPECT ORIFICE TOO SMALL

(4) SOURCE OF IGNITION UNKNOWN

d MAJOR DISCREPANCY IN VALUES IN URS APPENDIX E.

TABLE C2 - CBM, LO_2/RP-1, W = 200. SUMMARY OF TEST DATA

L/D	D_o/D_t	TEST NO.	IGNITION TYPE	IGNITION TIME	Y	$S_{P(I)}$%	S_Y%	Y_U	Y_L	AVAILABLE DATA d's, STN'S	DATA OMITTED d's, STN'S	Y_{URS}
1.8	.45	240	X	156	55.77	5.71	4.1	60.3	51.6	5d,13	2d,5	60
2/3	FULL				46.44	10.0	4.9	50.8	42.5	5d,15	1d,3(2)	
		174	TR	150[d]	56.65	4.5	3.1	60.5	53.0	3d,5	0	52
					46.53	3.9	2.7	49.3	43.9	3d,7	1d,2	
1.8	1.0	042	TR	290	48.69	2.9	3.7	61.4	38.6	3d,8	2d,6	48
					45.34	2.5	2.8	54.0	38.0	3d,8	2d,6	
		058	X	200	31.33	7.0	5.1	35.0	28.1	3d,8	1d,3	27
					22.79	9.1	5.1	25.1	20.7	3d,8	0	
		086	X	100	12.14	13.0	7.6	14.0	10.5	3d,8	0	14
					11.27	10.0	5.6	12.5	10.1	3d,8	0	
5.	.45	100	X	220	25.29	10.4	5.4	28.0	22.9	3d,9	0	23
					22.84	7.2	3.8	24.5	21.3	3d,9	0	
		046	X	143	16.11	6.1	3.5	17.2	15.1	3d,8	0	17
					16.20	6.8	3.8	17.4	15.1	3d,8	0	
		088	X	60	3.81	10.6	7.1	4.4	3.3	3d,8	0	4[d]
					4.02	4.8	2.7	4.2	3.8	3d,8	0	
5.	1.0	085	X	380	12.93	8.5	4.8	14.1	11.8	3d,9	0	12
					9.27	8.2	4.5	10.1	8.5	3d,9	(1)	
		049	X	316	12.56	8.0	4.7	13.7	11.5	3d,8	0	12
					10.40	10.1	5.6	11.6	9.3	3d,8	0	
		047	X	120	10.68	14.0	9.1	12.8	9.0	3d,7	0	10
					8.83	7.4	4.7	9.7	8.0	3d,7	(1)	

(1) X = IGNITION BY CAP OR SQUIB; TR = SELF-IGNITION AT OR AFTER TANK RUPTURE

TABLE C3 - CBM, LO_2/RP-1. CORRELATIONS BETWEEN YIELD AND IGNITION TIME.*

W	L/D	D_o/D_t	Tests	Direct r	Direct r_u	Direct r_l	Direct b	Logs r	Logs r_u	Logs r_l	Logs b
200	1.8	.45	All (7)	.85 .67	92 .84	.70 .23	.23 .15	.86 .67	.92 .84	.72 .23	1.10 .76
1000	1.8	.45	All (4)	.48 .56	.88 .89	-.62 -.55	.04 .06	.52 .63	.89 .91	-.58 -.46	.52 .73
25000	1.8	.45	All (3)	.92 .90	.98 .98	-.56 -.72	.42 .29	.92 .91	.98 .98	-.57 -.67	50.0 20.0
			Non SP (2)	– –	– –	– –	.02 -.04	– –	 –	– –	.81 -1.7
200	1.8	1.0	All (3)	1.0 .98	1.0 .99	1.0 .83	.19 .18	1.0 .99	1.0 .99	1.0 .95	1.31 1.27
200	5.0	.45	All (3)	1.0 .99	1.0 .99	1.0 .97	.13 .12	.99 .99	1.0 1.0	.99 .97	1.49 1.38
200	5.0	1.0	All (3)	1.0 .53	1.0 .97	.99 -.93	.01 .32	1.0 .62	1.0 .97	1.0 -.92	.17 .08
200	2/3	Full	All (2)	– –	– –	– –	-.15 -.02	– –	– –	– –	-.40 -.05

*r_u and r_l represent 90% upper and lower confidence limits for the correlation coefficient r;b is the slope.

TABLE C4 - CBM, LO_2/RP-1. REGRESSION SUMMARY ($Y = AW^{-B}$)

Tests	No.	$\sigma Y\%$	A	B	$A_{.90}$[a]	$B_{.90}$	$A_{.95}$[a]	$B_{.95}$	W = 94,000* $\hat{Y}$	$Y_{.90}$	$Y_{.95}$	W = 4.6 x 10^6 $\hat{Y}$	$Y_{.90}$	$Y_{.95}$
Highs, Titan out	6	3.9	93.9	.213	78.1	.187	72.6	.177	8.23	9.90	10.64	3.60	4.66	5.15
		5.6	66.6	.173	50.9	.136	45.9	.121	9.18	11.99	13.31	4.63	6.80	7.86
Highs, Titan in	7	13.8	150.4	.290	86.8	.219	71.0	.194	5.43	9.54	11.71	1.76	3.65	4.77
		15.1	110.1	.256	60.2	.178	48.3	.150	5.89	10.91	13.67	2.18	4.85	6.50
All except No. 275, Titan out	13	15.8	43.3	.150	25.5	.069	21.5	.043	7.77	14.90	18.32	4.34	10.47	13.84
		14.5	32.9	.116	20.2	.041	17.3	.017	8.75	15.89	19.20	5.58	12.52	16.18
All except No. 275, Titan in	14	17.2	64.3	.216	39.8	.148	34.2	.125	5.40	10.24	12.52	2.32	5.23	6.74
		16.5	50.5	.188	31.9	.121	27.6	.100	5.89	10.88	13.19	2.84	6.17	7.87

*The observed yields for Titan are Y_P = 3.27% and Y_I = 3.42%

TABLE C5 - COMPARISON BETWEEN URS PREDICTED AND OBSERVED YIELDS
CBM, LO_2/RP-1. W = 200 LBS.

TEST NO.	L/D	D_o/D_t	t	URS PREDICTED	ESTIMATED YIELD Y_{URS}	Y_P	Y_I	% DEVIATION[1] Y_{URS}	Y_P	Y_I
239	1.8	.45	156	32.2	32	31.0	22.1	+ 0.6	+ 3.9	+45.7
237[S]			127	26.2	32	30.6	26.8	-18.1	-14.2	- 2.1
101			145	30.0	35	33.0	31.8	-14.3	- 9.2	- 5.9
044			120	24.8	18	17.8	15.5	+37.8	+39.1	+59.6
095A[S]			120	24.8	17	17.6	14.7	+45.9	+40.8	+69.1
238			85	17.5	19	16.0	14.4	-7.9	+ 9.8	+21.7
087A			70	14.5	16	13.6	15.1	-9.4	+ 6.7	- 4.1
042	1.8	1.0	290	44.2	48	48.7	45.3	-7.9	- 9.2	- 2.5
058			200	30.5	27	31.3	22.8	+13.	- 2.7	+33.8
086			100	15.2	14	12.1	11.3	+8.6	+25.6	+35.3
100	5.0	.45	220	22.3	23	25.3	22.8	- 2.6	-12.5	- 2.0
046			143	14.5	17	16.1	16.2	-14.4	- 9.7	-10.2
088[S]			60	6.1	2	3.8	4.0	+52.7	+60.2	+51.9
085		1.0	380	18.1	12	12.9	9.3	+50.8	+39.9	+95.1
049			316	15.0	12	12.6	10.4	+25.	+19.7	+44.6
047			120	5.7	10	10.7	8.8	-42.9	-46.5	-35.3

[1]Computed from 100 $(Y_{pred} - Y)/Y$
[S]Designates spurious test

II. CBM, LO_2/LH_2

1. Summary of Data

1.1 Weight (Table C6, Figures A7P and A7I)

All three of the 25,000 lb. tests were early ignition, self-ignited, spurious, and with low yields--these have been excluded from the analysis. Much of the data is extremely variable or discrepant, e.g., for test No. 212 the two overpressure measurements at the furthest distance (117 ft.) correspond to a yield of 24%, whereas the measurements at the nine closest stations give a least squares yield of 3%.

There was substantial difference between pressure and impulse yields, with the 1000 lb. tests ranging as high as 1:6, (Figure C5 and Table A2).

Maximum pressure yield for 200 lbs. was much less than for LO_2/RP-1, maximum impulse yield slightly greater. For 1000 lbs. both pressure and impulse yields were greater than for LO_2/RP-1.

The geometry for the S-IV test was L/D=2, D_o/D_t=.083. The test self-ignited at 183 msec giving pressure yield of 3.3%, the same as Titan, and impulse yield of 5.7%.

1.2 Geometry (Table C7 and Figures A8P and A8I)

Non-nominal geometry yields (all at 200 lbs.) appear better behaved than the nominal, except possibly for L/D=5, D_o/D_t=1. Here test No. 092, with ignition time of three minutes gave two explosions. The first was quite small while the second, due to interaction of LH_2 with air, gave substantial yield.

Pressure and impulse data were available at only three distances.

2. Correlation with Ignition Time (Table C8 and Figures 4P and 4I)

2.1 Weight

Correlation between yield and ignition time was very poor for the 200 lb. and the 1000 lb. tests at nominal geometry.

2.2 Geometry

For all non-nominal cases, except 2/3 full, yield increased monotonically with ignition time. For L/D=5, D_o/D_t=.45, the numerical correlation was excellent and the log slope was about .5; for D_o/D_t=1, L/D=.45, the response was nonlinear (either with logs or direct) with an average log slope of 1.7. In the remaining two cases, the data is either too few or too uncertain to make an estimate.

3. Regression

3.1 Weight (Table C9, Figures 5P and 5I)

The scatter is extremely large for the eight regressions summarized in Table C9. Those with maximum yields only, or with the SIV test excluded, give meaningless regressions and/or prediction limits. Even the first three regressions in Table C9, which use all, or almost all, tests with SIV included, have prediction limits that appear impractically high.

3.2 Geometry (Figures 6P and 6I)

Maximum yields, using test No. 092b, are shown in the table below.

L/D → D_o/D_t ↓	1.8	5.0	
.45	17.5 34.3	21.8 28.7	19.6 31.5
1.0	79.3 104.5	17.7 32.7	48.5 68.6
	48.4 69.4	19.7 30.7	34.1 50.1

Maximum yield for D_o/D_t=.083, L/D=.45 was Y_P=9.0%, Y_I=20.8%; and for 2/3 full Y_P=30%, Y_I=36.6%. Most of the maximum yield tests exhibited negative pressure and/or impulse bias close-in (see Table A2).

Qualitatively, the individual entries are perhaps not too dissimilar from the corresponding ones for LO_2/RP-1, especially if one makes allowance for the low confidence in the nominal value and for L/D=5, D_o/D_t=1.0 (3 minutes ignition time with double explosion). In particular, D_o/D_t=1, L/D=1.8 resulted in a substantial increase in yield, even more than for LO_2/RP-1. Of equal significance is the fact that D_o/D_t=.083, L/D=1.8 gave also a moderately large decrease in yield (Y_P=9%, Y_I=20%).

Ullage volume of 40% appears to have increased pressure yield by more than 2/3, although impulse yield increased only slightly. (Impulse response at 37 ft. for the nominal test was very low and not used in estimating yield.) In terms of specific yield, which is 50% greater, the overall increase is substantial, although not as much as for LO_2/RP-1.

Formal analysis of the preceding table leads to the following main effects and interactions (impulse values shown in parentheses):

$$L = -14.3 \ (-19.3)$$

$$D = 14.4 \ (18.5)$$

$$LD = -16.5 \ (-16.6)$$

The corresponding regression equations are:

$$Y_P = 25.6 - 8.1\left(\frac{L/D-3.4}{1.6}\right) + 5.5\left(\frac{D_o/D_t-.725}{.275}\right) - 10.0\left(\frac{L/D-3.4}{1.6}\right)\left(\frac{D_o/D_t-.725}{.275}\right)$$

$$Y_I = 45.6 - 16.1\left(\frac{L/D-3.4}{1.6}\right) + 17.5\left(\frac{D_o/D_t-.725}{.275}\right) - 16.7\left(\frac{L/D-3.4}{1.6}\right)\left(\frac{D_o/D_t-.725}{.275}\right)$$

Because of the large uncertainties, even greater than for LO_2/RP-1, these equations do not appear to be too meaningful--nor would the extension of the analysis to include D_o/D_t=.083.

4. URS Prediction Equation

URS concluded that the only situations affecting yield were very low ignition time and the geometry configuration $D_o/D_t=1$, L/D=1.8. Specifically

$$Y = \begin{cases} 2.8 + .82\ t/W^{1/3} & , \text{ for } t < 21.1\ W^{1/3} \\ 20 & , \text{ for } t \geq 21.1\ W^{1/3} \end{cases} \qquad (*)$$

For $D_o/D_t=1$ and L/D=1.8, a free hand curve through the three yields, similar to those shown in Figure 4, is used by URS with t* substituted for t for weights larger than 200 lbs. A comparison of the observed URS yields with those predicted by the above equation is shown in Table C9a.

For the SIV, with a propellant weight of 91,000 lbs., the ignition time required to reach maximum yield becomes 948 msec. Using the actual time of 183 msec the above equation gives Y=6.1.

Because of the erratic nature of the data, the formula in (*) for t_{max} (time to reach "stabilized" maximum yield) is difficult to verify. For W=200 lbs. the equation gives t_{max}=123 msec. Examining the most pertinent tests in Table C9a, for nominal geometry the two highest yields occurred for ignition times of 82 msec and 35,000 msec. For L/D=5, D_o/D_t=.45, yield appears not to have stabilized at the largest observed ignition time (329 msec); while for D_o/D_t=.083, yield is approximately the same for 56 msec and 318 msec. Likewise, for W=1000 lbs. with calculated t_{max} of 211 msec, the observ maximum yield occurs at 708 msec.

Clearly the data itself does not enable inferences to be drawn about scaling of ignition time with weight, and for the SIV test in particular. (Farber's model simulation of the SIV test (Reference 2, p. II-35) suggests that ignition occurred during the initial rise of the mixing function.) Whether 183 msec constitutes a representative time for self-ignition cannot of course be judged.

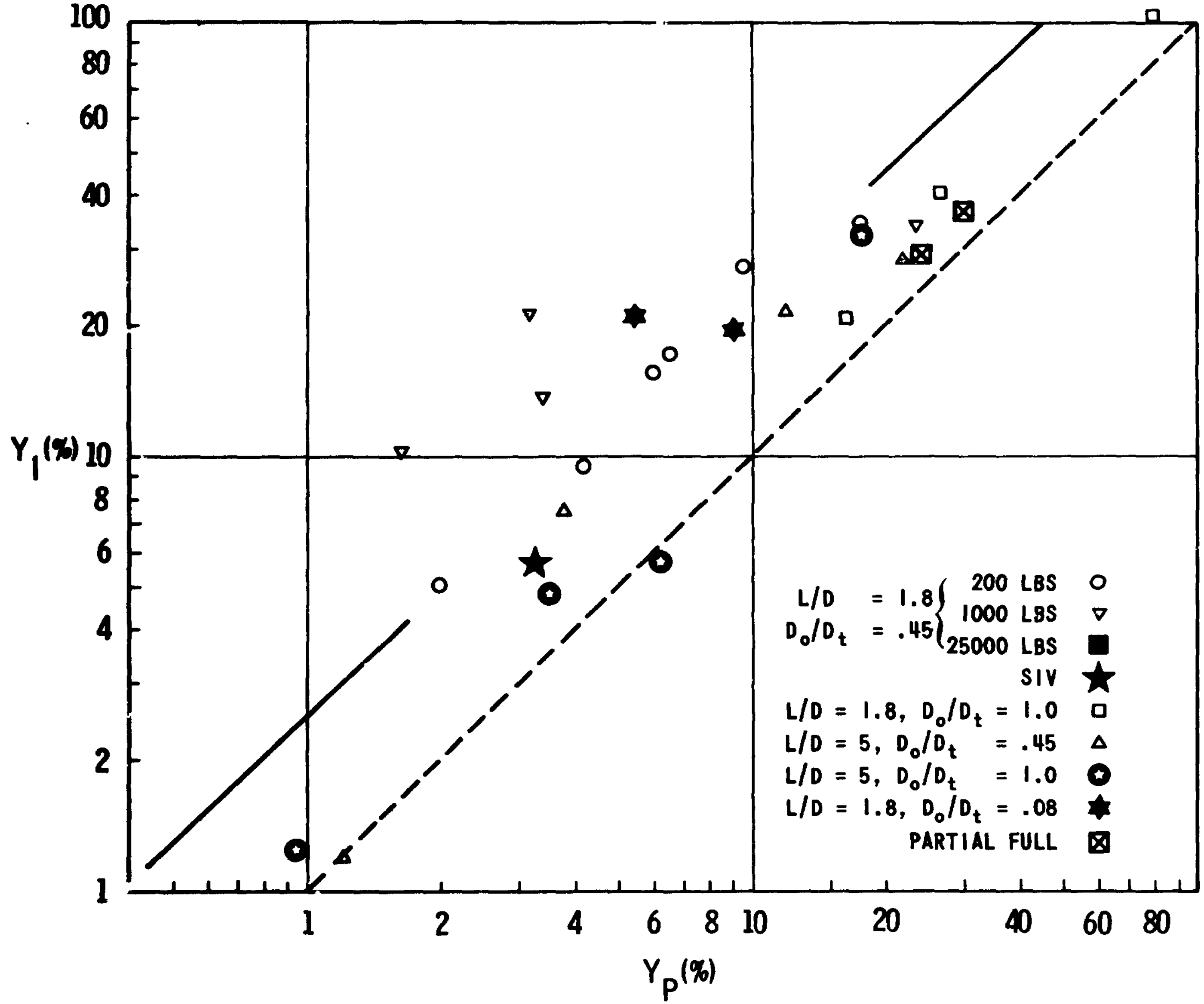

FIGURE C5 - CBM, LO_2/LH_2, COMPARISON OF OVERPRESSURE AND IMPULSE YIELDS

TABLE C6 - CBM, LO_2/LH_2, L/D = 1.8, D_o/D_t = .45, SUMMARY OF TEST DATA

W	TEST NO.	IGNITION TYPE	IGNITION TIME	Y	$S_{P(1)}$%	S_Y%	Y_U	Y_L	AVAILABLE DATA d's, ST'NS	DATA OMITTED d's, ST'NS	Y_{URS}
200	090	X(1)	35 SEC	17.48	3.5	2.3	18.3	16.7	3d,7	(1)	29
				34.32	8.3	9.2	61.3	19.2	3d,7	2d,5	
	118	X(1)	82	9.53	3.0	2.4	10.0	9.1	3d,9	1d,3	20
				27.45	7.6	4.0	29.6	25.5	3d,9	0	
	200	X	417 d	5.98	12.0	7.2	6.8	5.2	4d,10	0	17
				15.66	11.2	6.2	17.6	13.9	4d,10	(2)	
	091	SP(2)	0	6.53	8.6	5.4	7.2	5.9	3d,8	0	13
				17.31	4.9	2.9	18.3	16.4	3d,8	(1)	
	199	X	816	4.21	4.9	3.0	4.4	4.0	4d,11	(1)	8
				9.52	13.1	6.8	10.8	8.4	4d,10	(1)	
	053	SP(2)	1	2.00	6.0	4.7	2.2	1.8	3d,8	(1)	4
				5.08	1.0	0.8	5.2	5.0	3d,6	1d,2	
1000	213	X	708	23.29	9.5	6.3	26.2	20.7	5d,14	2d,6	35
				34.54	8.4	4.6	37.7	31.6	5d,14	2d,6	
	212	X	1366	3.19	12.1	14.6	4.9	2.1	4d,11	2d,6(2)	27
				21.29	11.5	6.5	24.1	18.8	4d,10	(2)	
	265	X	750	3.42	12.1	9.6	4.1	2.9	5d,15	2d,6	10
				13.79	7.5	4.1	14.9	12.8	5d,14	2d,6	
	210	SP(3)	20	1.63	8.6	8.0	1.9	1.4	4d,12	2d,6	7
				10.22	10.9	5.0	11.2	9.3	4d,12	0	
25000	277	SP(2)	31	.088	28.2	17.7	.12	.06	5d,15	(2)	.2
				.178	11.0	4.6	.19	.16	5d,15	(2)	
	279	SP(2)	33	.048	5.7	4.9	.05	.04	2d,6	0	.2
				.151	24.0	14.4	.20	.11	2d,6	0	
	281	SP(2)	–	.053	–	–	–	–	1d,3	0	.1
				.132	17.1	14.3	.20	.09	1d,3	0	
91000	S IV	(4)	183	3.26	7.6	7.8	3.8	2.8	5d,10	3d,5	5
				5.69	7.6	5.3	6.4	5.1	4d,6	1d,1	

(1) CAP IGNITION AT 100 PSI

(2) DIAPHRAGM RUPTURE. TEST 091 NOT USED IN URS ANALYSIS

(3) DIAPHRAGM OR TANK RUPTURE

(4) SELF-IGNITION SOURCE UNKNOWN

TABLE C7 - CBM, LO_2/LH_2, W = 200. SUMMARY OF TEST DATA

L/D	D_o/D_t	TEST NO.	IGNITION TYPE	IGNITION TIME	Y	$S_{P(1)}$%	S_Y%	Y_U	Y_L	AVAILABLE DATA d's,ST'NS	DATA OMITTED d's,ST'NS	Y_{URS}
1.8	.45	172	SP(1)	770d	30.00	4.8	4.9	34.6	26.0	3d,5	1d,2	35
2/3	FULL				36.62	4.5	3.2	39.2	34.2	3d,5	0	
		167	SP(2)	8740	23.94	3.4	3.9	26.8	21.4	3d,6	0	24
					29.20	5.6	3.9	31.7	26.9	3d,5	0	
1.8	1.0	050	TR(3)	180	79.34	10.9	13.0	180.5	34.9	3d,6	2d,4	86
					104.50	7.6	6.1	120.5	90.6	3d,6	1d,2	
		093	X	147	26.11	3.8	3.2	28.2	24.2	3d,7	1d,2(1)	34
					40.42	7.7	4.6	44.2	36.9	3d,7	0	
		051	X(4)	80	16.24	10.65	9.2	20.2	13.1	3d,7	1d,3	22
					20.74	4.7	3.0	22.0	19.5	3d,7	(1)	
5.0	.45	094	(5)	329	21.80	12.1	6.9	24.8	19.1	3d,8	0	25
					28.73	4.9	2.8	30.3	27.3	3d,8	0	
		138	X	100	11.97	6.1	4.3	13.0	11.0	3d,6	0	17
					21.29	5.0	2.8	22.4	20.2	[illegible]	0	
		054	SP(6)	17	3.82	4.9	8.1	6.4	2.3	3d,6	2d,4	6
					7.43	4.6	2.9	7.9	7.0	3d,6	0	
		055	SP(6)	1	1.21	9.0	9.2	1.5	0.99	3d,8	1d,3	1
					1.18	10.8	7.4	1.4	1.0	3d,8	1d,3	
5.0	1.0	092b	X(5)	3 MIN	17.69	7.5	5.9	20.1	15.6	3d,9	1d,3(1)	26
					32.71	8.4	5.4	36.5	29.3	3d,9	1d,3	
		092a			3.50	7.7	5.0	3.8	3.2	3d,9	0	
					4.87	5.0	4.4	5.5	4.3	3d,9	2d,6	
		052	X	83	6.15	4.3	3.9	6.7	5.7	3d,8	1d,3	7
					5.79	9.0	5.0	6.4	5.3	3d,8	0	
		057	SP(6)	12	0.95	12.1	12.5	1.2	0.73	3d,8	1d,3	1
					1.25	2.9	2.0	1.3	12.0	3d,8	1d,3	
1.8	.083	169	X	318	9.00	6.0	4.3	9.8	8.3	3d,6	0	15
					19.71	3.1	2.0	20.5	18.9	3d,6	0	
		173	(1)	56	5.43	7.1	5.1	6.0	4.9	3d,7	0	13d
					20.78	4.6	3.2	22.2	19.4	3d,7	1d,2	

(1) FIRE
(2) COLD FLOW; IGNITION CAUSED BY POOR VENTING
(3) TANK RUPTURE IGNITION
(4) CAP IGNITION AT TANK RUPTURE
(5) IGNITION AT 100 PSI
(6) DIAPHRAGM RUPTURE

TABLE C8 - CBM, LO_2/LH_2, CORRELATIONS BETWEEN YIELD AND IGNITION TIME.

W	L/D	D_o/D_t	Tests	Direct				Logs			
				r	r_u	r_l	b	r	r_u	r_l	b
200	1.8	.45	All[1] (6)	.89 .72	.94 .87	.75 .25	0 0	.67 .57	.86 .83	.14 -.70	.12 .10
			Non SP (4)	.92 .74	.97 .92	.71 -.24	0 0	.58 .34	.90 .86	-.53 -.70	.14 .08
1000	1.8	.45	All (4)	.17 .52	.83 .89	-.76 -.59	0 .01	.55 .72	.89 .92	-.56 -.28	.32 .20
			Non SP (3)	-.35 .01	.94 .95	-.96 -.95	-.01 0	-.31 .20	.94 .96	-.96 -.94	-.98 .26
200	1.8	1.0	All (3)	.84 .88	.98 .98	-.84 -.77	.56 .76	.87 .93	.98 .98	-.79 -.51	1.68 1.79
200	5.0	.45	All (4)	.97 .90	.98 .96	.95 .61	.06 .08	.99 .99	1.0 .99	.99 .99	.51 .57
			Non SP (2)	– –	– –	– –	.04 .03	– –	– –	– –	.50 .25
200	5.0	1.0	All[2] (2)	– –	– –	– –	.07 .06	– –	– –	– –	.97 .79
200	1.8	.083	All (2)	– –	– –	– –	.01 0	– –	– –	– –	.29 -.03
200	2/3	Full	All (2)	– –	– –	– –	0 0	– –	– –	– –	-.09 -.09

[1]Test No. 091 with t = 0 replaced by t = 1 for log correlation.
[2]Test No. 092 with t = 3 min. omitted.

TABLE C9 - CBM, LO_2/LH_2. REGRESSION SUMMARY ($Y = AW^{-B}$)

Tests	No.	σ_Y%	A	B	$A^a_{.90}$	$B_{.90}$	$A^a_{.95}$	$B_{.95}$	W = 91,000* $\hat{Y}$	$Y_{.90}$	$Y_{.95}$	W = 930,000 $\hat{Y}$	$Y_{.90}$	$Y_{.95}$
All non-spurious, 200, 1000, SIV	8	34.0	17.6	.148	4.48	-.057	2.78	-.129	3.24	15.24	26.17	1.82	15.58	32.98
		23.6	61.4	.190	23.8	.048	17.1	-.002	7.03	20.50	29.79	4.52	16.67	26.29
All 200, 1000, SIV	11	37.4	10.7	.112	2.84	-.094	1.85	-.161	2.97	14.88	25.15	2.29	16.10	30.38
		28.0	35.7	.137	13.2	-.017	9.58	-.067	7.47	24.94	36.92	5.43	23.38	37.60
All except 053 and 199	9	36.83	19.4	.178	4.51	-.040	2.75	-.114	2.55	13.05	22.68	1.69	12.37	24.29
		18.74	76.5	.221	36.4	.110	28.3	.072	6.15	14.11	18.70	3.69	10.14	14.30
Max 200, 1000, and SIV	3	24.9	125.6	.309	5.66	-.084	.22	-.498	3.67	43.42	583.	1.79	38.95	994.
		16.38	228.7	.317	29.8	.579	3.50	-.214	6.15	31.21	172.	2.95	22.34	188.
All non-spurious, SIV out	7	37.3	17.7	.149	.48	-.452	.13	-.672	3.24	110.98	404.	2.29	294.	1732.
		23.1	13.4	-.069	1.43	-.443	.63	-.579	29.59	265.70	592.	34.77	709.	2131.
All except SIV	10	39.5	17.2	.194	.83	-.316	.30	-.485	1.88	41.48	116.	1.20	79.4	319.
		27.9	8.8	-.103	1.04	-.462	.51	-.582	28.54	252.97	521.	36.26	698.	1860.
All except 053, 199, and SIV	8	38.2	86.6	.428	2.90	-.129	.89	-.323	.655	16.86	52.5	.24	20 9	99.
		19.9	47.2	.140	8.04	-.150	4.33	-.251	9.54	51.89	93.8	6.89	70.4	159.
Max 200 and 1000	2	–	6.80	-.178	–	–	–	–	52.1	–	–	78.8	–	–
		–	33.6	-.004	–	–	–	–	35.2	–	–	35.5	–	–

*The observed yields for SIV are Y_P = 3.26% and Y_I = 5.69%

TABLE C 9a - CBM, LO_2/LH_2, COMPARISON BETWEEN URS AND OBSERVED PREDICTED YIELDS

TEST NO.	W	L/D	D_o/D_t	t	URS PREDICTED	OBSERVED Y_{URS}	OBSERVED Y_P	OBSERVED Y_I	% DEVIATION*
090	200	1.8	.45	35 SEC	20	29	17.5	34.3	- 31
118				82	14.3	20	9.5	27.4	- 28
200				417	20	17	6.0	15.7	+ 18
091[S]				0	2.8	13	6.5	17.3	- 78
199				816	20	8	4.2	9.5	+150
053[S]				1	2.9	4	2.0	5.1	- 28
213	1,000			708	20	35	23.3	34.5	- 43
212				1366	20	27	3.2	21.3	- 26
265				750	20	10	3.4	13.8	+100
210[S]				20	4.4	7	1.6	10.2	- 37
SIV	91,000	2	.083	183	6.14	5	3.3	5.7	+ 23
172[S]	200	1.8	.45	770	20	35	30.0	36.6	- 43
167[S]		2/3	Full	8740	20	24	23.9	29.2	- 17
094		5.0	.45	329	20	25	21.8	28.7	- 20
138				100	16.8	17	12.0	21.3	- 1
054[S]				17	5.2	6	3.8	7.4	- 13
055[S]				1	2.9	1	1.2	1.2	+190
092b							17.7	32.7	
092a		5.0	1.0	3 MIN	20	26	3.5	4.9	- 23
052				83	14.4	7	6.1	5.8	+106
057[S]				12	4.5	1	0.9	1.2	+350
169		1.8	.083	318	20	15	9.0	19.7	+ 33
173				56	10.7	13[d]	5.4	20.8	- 18

* $\left[(Y_{PRED} - Y_{URS})/Y_{URS}\right] \cdot 100$

S DESIGNATES SPURIOUS TEST

III. CBGS, LO_2/LH_2

1. Summary of Data

1.1 Weight, V=44 fps (Tables C10 and C11, Figures C6, A15I, A16I, A17I, A22P)

The average Y_I/Y_P ratio is approximately 1.1 for the 200 lb. tests, 1.8 for the 1000 lb. tests and 1.6 for the 25,000 lb. tests. The combined average ratio, Y_I/Y_P, is approximately 1.5, with a moderate amount of scatter about this value.

The one spurious test (No. 230) at 200 lbs. has ignition time t=24 msec. and Y_P=12.9% (Y_I=16.1%). This is comparable to the lowest yield 200 lb. nonspurious test (No. 197). The non-spurious tests have ignition times between 317 and 1374 msec. and Y_P approximately uniformly distributed between 12.9 and 44.1%. (Y_I is between 16.1 and 53.1%). Hence, the scatter at 200 lbs. is large.

There are three tests at 25,000 lbs.; two of these are spurious. They are the only spurious tests in the LH_2, 44 fps group which have ignition times greater than 100 msec. Further, the only nonspurious test at this weight, No. 288C, also has a relatively low ignition time, t=365 msec. All three tests are self-ignited. The two spurious tests are clearly low yield tests, while the yield of test No. 288C is approximately 11%. If self-ignition invariably occurs at large weights, this test is probably representative of what is achievable at 25,000 lbs. Alternately, consider the relation between ignition time and "asymptotic maximum" yield in the case of the 200 and 1000 lb. tests: there are four tests with t=0, 0, 21 and 24 msec. which are low yield, two with t=317 and 325 msec. which are high yield; the maximum 200 lb. test is at t=775 msec. and the maximum at 1000 lbs. is at t=900 msec. Therefore, on the basis of ignition time alone, the yield observed for test No. 288C could be anywhere from representative to high. We consider this yield to be representative of the type of yields that can be expected at 25,000 lbs.

1.2 Velocity, Weight=200 (Table C12, Figures A18I, A23P, 9P, 9I)

There are five nonspurious tests at 23 ft/sec. These group into three distinct sets: the test with the largest yield, a pair of tests with similar ignition times, but with the yield of one test showing a 46% increase over the other and two tests with similar yields and widely differing ignition times.

The data at 200 lbs. and 44 ft/sec. has been discussed in Section 1.1. Of the five tests at 78 ft/sec., one is listed as spurious and three are ignited by an external ignition source. Test 226 is listed as spurious because of accidental ignition by fire; it is neither a low yield nor an early ignition time test.

The Kingery curve for overpressure provides a poor fit to all of the data for each of the nonspurious 200 lb., 78 fps tests. The impulse fit for test No. 195 is also poor (cf. 3.a., p. A-7).

2. Correlation, LO_2/LH_2, (Tables C13 and C14, Figures 7P and 7I)

The 200 and 1000 lb. data at 44 fps is treated in sufficient detail in Section III of the report.

The 200 lb. data at 23 fps shows a strong increase in yield with ignition time. It is impossible to state with confidence that the "asymptotic maximum" yield has been attained. This is apparent from Figures 7P and 7I. There the increasing nature of the data argues against the maximum being achieved, while an almost 600 msec. difference in ignition time combined with a 2.2% difference in the yield of the two highest tests argues for it being achieved.

There is a moderate correlation between Y and t, when W=200 lbs. and V=78 fps, but because of the large scatter the confidence intervals are very wide. The "asymptotic maximum yield" condition appears to have been reached at 78 fps; however, the scatter is such that it cannot be determined with high confidence.

There are two spurious, low yield, early ignition time tests at 1000 lbs. when V=78 fps. These are not included in either the Bellcomm of the URS analyses.

3. Regression

3.1 Regression on Weight (Tables C15 and C7P, C7I and C8P, C8I)

The regression equations for the eleven nonspurious tests are

$$(1^\circ) \quad \begin{cases} \hat{Y}_P = (73.8)\, W^{-.172}, \; Y^A_{P,90} = (34.7)\, W^{-.047} \\ \hat{Y}_I = (72.1)\, W^{-.157}, \; Y^A_{I,90} = (35.6)\, W^{-.039} \end{cases}$$

where $Y^A_{P,90}$ and $Y^A_{I,90}$ denote the asymptotes of the upper 90% prediction limits (Figures 8P and 8I). As indicated in Section III of the report, this regression was the result of those "asymptotic maximum" yields attainable from the given test configuration. The following is the list of the tests used in this regression: test numbers 197, 203, 204, 229, 231, 251, 252, 254 at W=200 lbs.; 217, 262 at W=1000 lbs.; 288C at W=25,000 lbs.

It is of some interest to determine how sensitive the regressions in (1°) are to certain variations in the population.

The following two cases were considered:

(i) Non-zero ignition time tests (Figures C7P, C7I). This amounts to adding the tests numbered 230 at 200 lbs., 264 at 1000 lbs. and 289, 290 at 25,000 lbs. to the set of nonspurious tests at 44 fps. The resulting regressions are

$$\hat{Y}_P = (212.0)\ W^{-.382}\ , \quad Y^A_{P,90} = (102.3)\ W^{-.271}\ ,$$

$$\hat{Y}_I = (159.1)\ W^{-.312}\ , \quad Y^A_{I,90} = (82.3)\ W^{-.221}$$

Because of the low yield nature of the tests added to the group in (1°), this regression might be considered as giving the expected yield when the full range of attainable yields is taken into account.

(ii) Maximum yields (Figures C8P, C8I)

$$Y_P = (274.9)\ W^{-.319}\ , \quad Y^A_{P,90} = (153.6)\ W^{-.235}$$

$$Y_I = (253.7)\ W^{-.300}\ , \quad Y^A_{I,90} = (144.6)\ W^{-.219}$$

These regressions are based on the maximum yield tests at each weight. Specifically, tests No. 204, No. 251 at 200 lbs., No. 262 at 1000 lbs., and No. 288C at 25,000 lbs.

These results suggest that the regression in (1°) is conservative; the expected yield in (1°) is essentially an upper 95% prediction limit for (i) and (ii). On the other hand, all three regressions are extremely sensitive to the yields at 200 lbs. and the scatter at this weight produces the large prediction limits in (1°) and (i). As a result, the relevance of the regression equations in extrapolation depends heavily on the relation, if any, between large scale tests (≧25,000 lbs.) and the 200 lb. tests. (Cf. Section II of the report.)

3.2 <u>Velocity</u> (Tables C16, C17 and C18; Figures 9P, 9I)

The regression equations are given in Section III.

The single flat wall high velocity test was included in the discussion in III. The data for this test showed a strong directional effect. The three pressure (impulse) yields based on measurements taken in each of the three directions separately are 136.8 (122.9), 76.6 (98.0), and 69.5 (70.3). The highest yield corresponds to readings taken on the impacted side of the wall; the next corresponds to values recorded parallel to the wall and the lowest to those taken on the side of the wall opposite to the target. The first and the last of these direction-dependent yields for the single flat wall HVI test are indicated in Figures 10P and 10I together with assigned yield. (Cf. Appendix A.)

The only nonspurious tests at velocities other than 44 fps are those conducted at 200 lbs.

The following is a list of the tests used in the multiple regression of Section III, 3.3.3: test numbers 152, 184, 201, 225 at W=200 lbs., V=23 fps; 197, 203, 204, 229, 231, 251, 252, 254 at W=200 lbs., V=44 fps; 217, 262 at W=1000 lbs., V=44 fps; 288C at W=25,000, V=44 fps; 114, 150, 151, 195 at W=200 lbs., V=78 fps.

If the results of the flat wall HVI test (No. 079) are added to the drop test data above, then we obtain the following multiple regression:

$$\hat{Y}_P = (9.96)\ W^{-.165}\ V^{.512}$$

$$\hat{Y}_I = (10.0)\ W^{-.154}\ V^{.517}$$

The following table applies these equations to the SII of 930,000 lbs. and gives the corresponding 90% upper prediction limit values:

	V (fps)	23	44	78	160	597
SII	$\hat{Y}_P, (Y_{P,.90})$	5.2 (13.3)	7.2 (18.5)	9.7 (25.0)	13.9 (36.9)	27.4 (77.9)
	$Y_I, (Y_{I,.90})$	6.1 (15.4)	8.5 (21.5)	11.4 (29.1)	16.5 (43.1)	32.6 (91.5)

4. Geometric Scaling and the URS Prediction Equations (Table C18a)

We have already observed (Figures 7P, 7I) the tenuous relationship that exists between yield and ignition delay time when Bellcomm yields are used. The URS counterpart of our Y vs t plots are given in Figure 5-44 of the URS final report. In this figure URS presents three plots of yield against scaled pool diameter; one for each of the three tower drop velocities. Since the velocity is constant in each plot and the different propellant weights are distinguishable, they may be considered as showing yield vs. ignition time. At V=78 fps and W=200 lbs. the high yield test is number 195, which has an ignition time of 292 msec and a URS yield of 104%.* This is approximately 100% larger than the yield determined by URS for the next highest yield test at this velocity and weight. This fact, combined with the extremely high and inconsistent pressure and impulse values at one distance for test 195 (cf. Appendix A), indicates that some caution should be exercised in the use of this test. Without this test there is no visible effect of ignition delay time on yield indicated in Figure 5-44. Including test 195, at the URS yield of 104%, indicates an effect, but the consequent scatter prevents any definite conclusion.

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*The Bellcomm overpressure (impulse) yield for this test is 54% (70%).

At V=44 fps, URS plots the spurious test, 230, together with the eight remaining 200 lb. tests, the two high yield 1000 lb. tests and the high yield 25,000 lb. tests. Any effect of ignition time on the yields of the 200 lb. tests is masked by the erratic nature of the data. (See Section III.) Adding the two 1000 lb. tests improves the situation, but only slightly; while the inclusion of the 25,000 lb. test begs the question of scaling. Finally, the 23 fps data suggests the weakest effect of ignition delay time on yield of the three velocities. On the other hand, the yields calculated by Bellcomm for this case show the strongest effect of ignition time (Figures 7P, 7I).

Now the URS prediction equation for CBGS, LO_2/LH_2 is given by

$$(1°) \qquad \hat{Y}_{URS} = 18.4 + 0.0031\, V^2 D^* - 0.00015\, V^2 (D^*)^2 ,$$

where $D^* = t^*V$, $t^* = t/\sqrt[3]{W}$, t is the ignition time in seconds, V is the velocity in fps and W is the propellant weight in pounds. Evaluations of equation (1°) are presented in Table 18a together with a comparison of these predictions with the corresponding observed yields as determined by URS. The yields calculated by Bellcomm are also listed in this table.

It is interesting to consider the significance (both statistically and physically) of the quadratic term in this equation. The percentage of the total variation explained by the quadratic term is 4%. Using the URS statistical calculations in Appendix B of their report, we find that the introduction of $V^2(D^*)^2$ into the regression does not significantly increase the regression sum of squares. That is, using the standard F-test we find that $F(1,17) = 354.66/206.77 = 1.717$ is not significant. Therefore, instead of the regression (1°), the following regression may be used (Appendix B, URS):

$$(2°) \qquad \hat{Y} = 19.85 + .0022\, V^2 D^* .$$

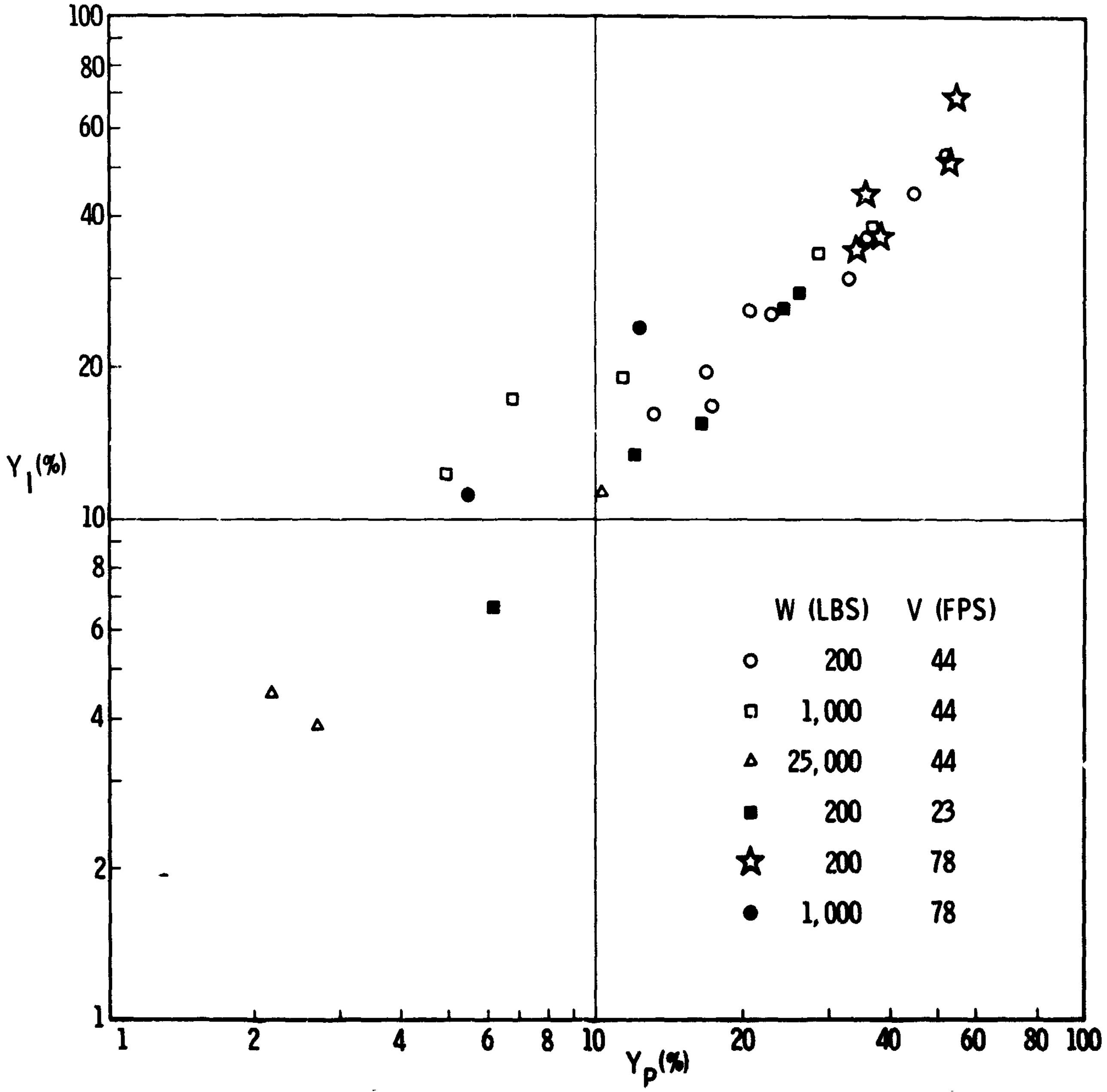

FIGURE C6 - CBGS, LO_2/LH_2, L/D = 1.8, Y_I VS Y_P, SCATTER DIAGRAM

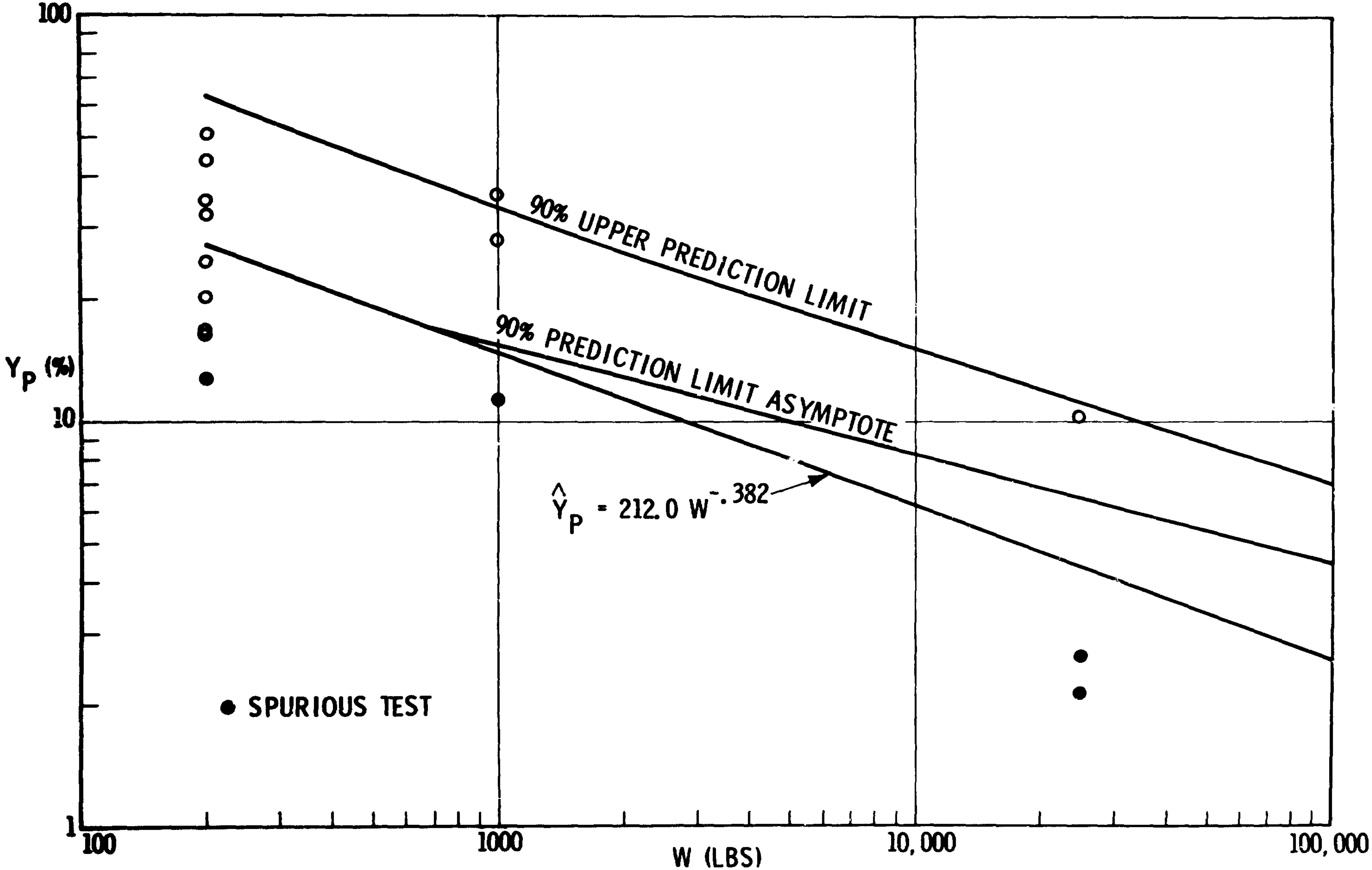

FIGURE C7P - CBGS, LO_2/LH_2, REGRESSION OF PRESSURE YIELD ON WEIGHT (NON-ZERO IGNITION TESTS)

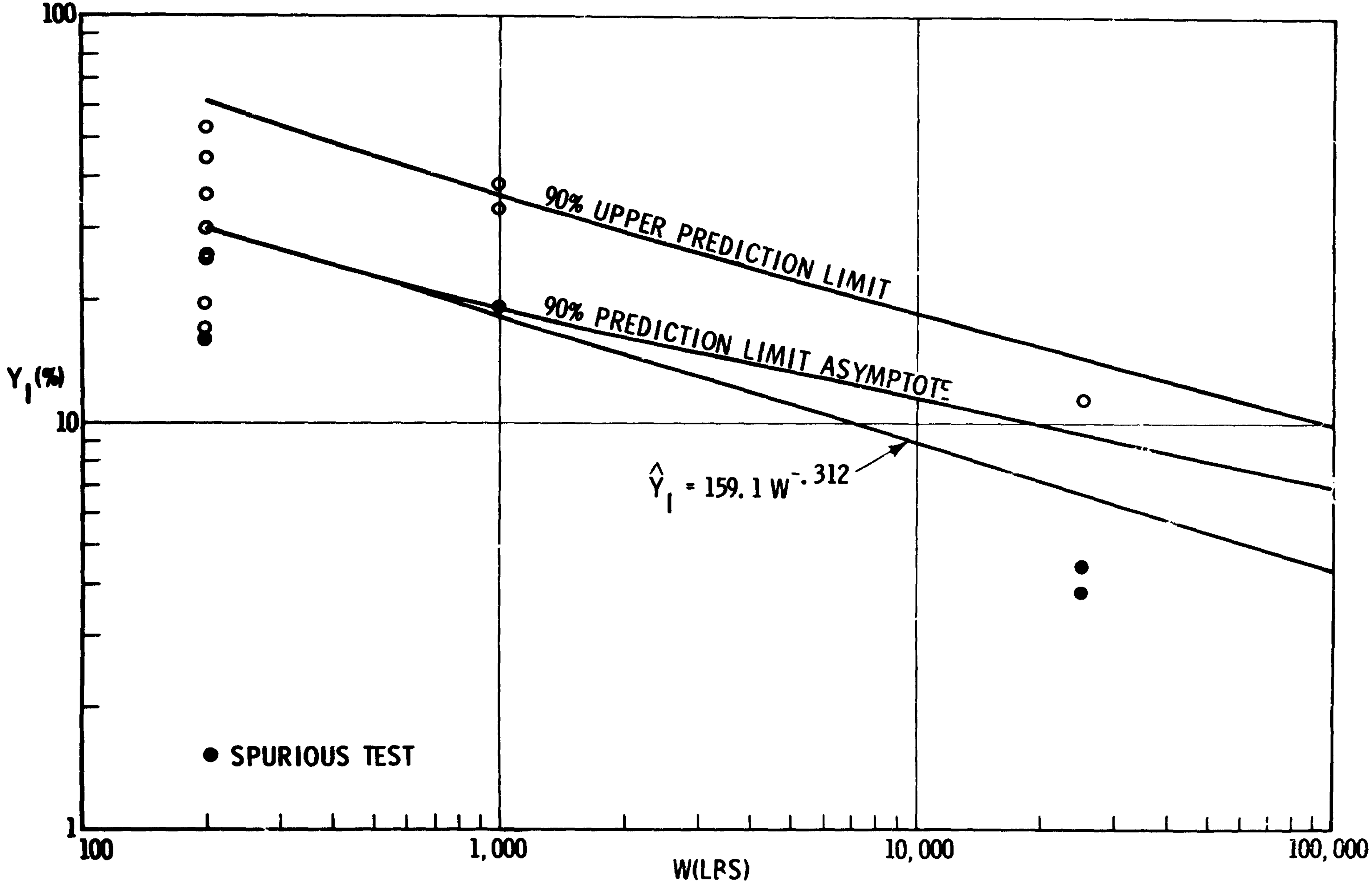

FIGURE C71 - CBGS, LO_2/LH_2 REGRESSION OF IMPULSE YIELD ON WEIGHT - (NON-ZERO IGNITION TESTS)

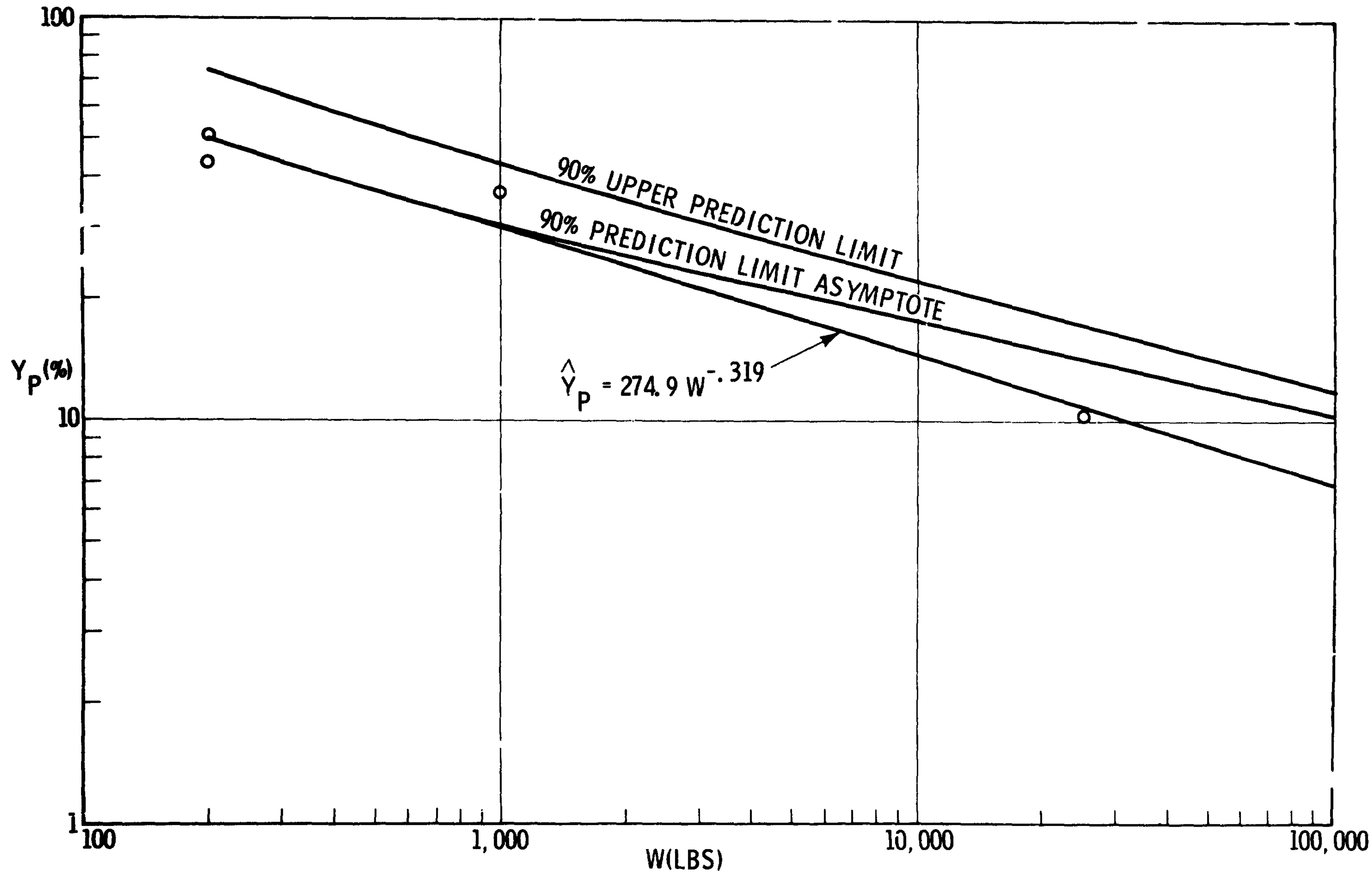

FIGURE C8P - CBGS, LO_2/LH_2, REGRESSION OF PRESSURE YIELD ON WEIGHT (HIGH YIELD TESTS)

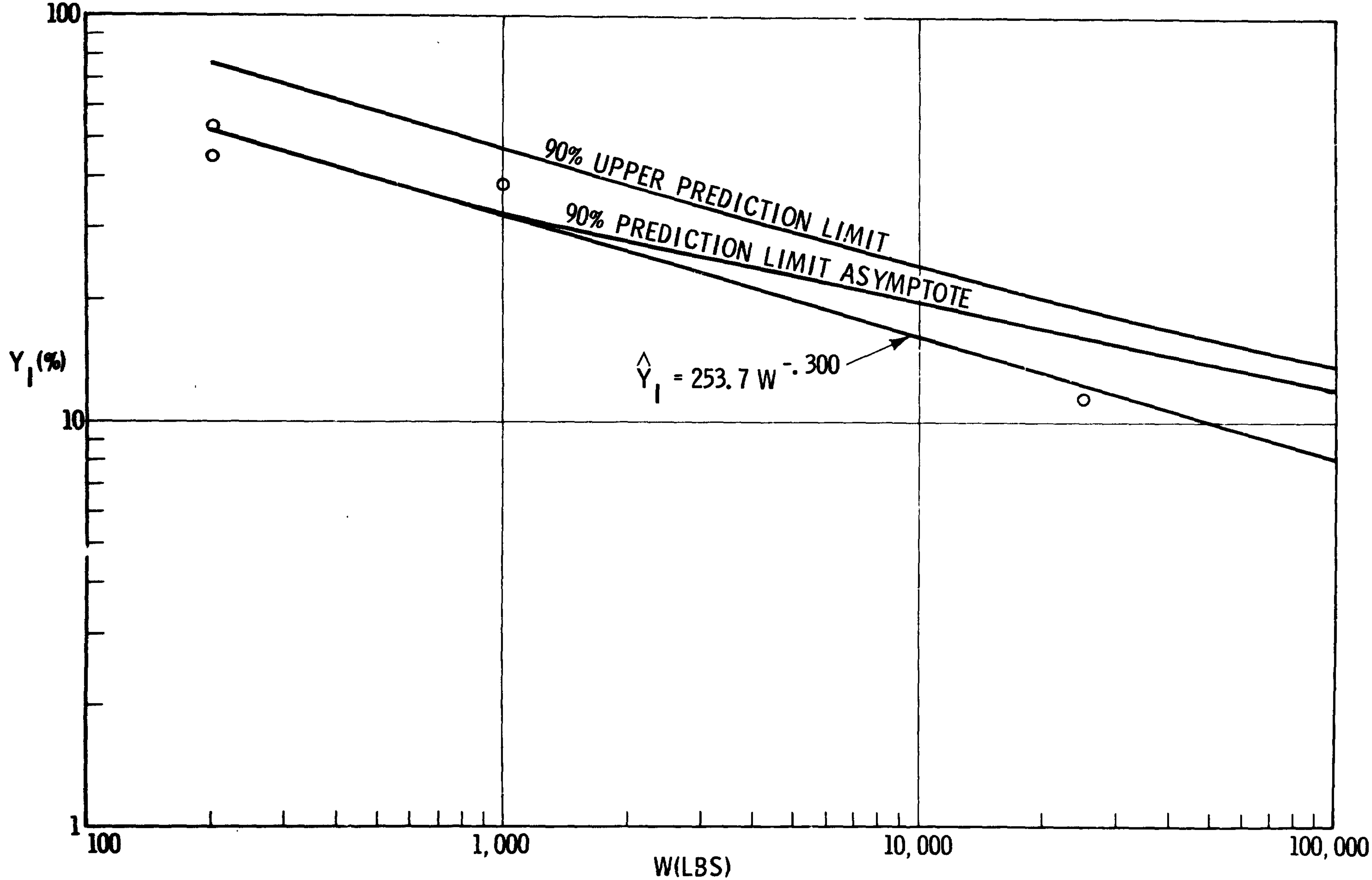

FIGURE C81 - CBGS, LO_2/LH_2, REGRESSION OF IMPULSE YIELD ON WEIGHT (HIGH YIELD TESTS)

TABLE C10 - CBGS, LO_2/LH_2, L/D = 1.8, SUMMARY OF TEST DATA

W	V	TEST NO.	IGNITION TYPE	IGNITION TIME	Y	$S_{P(I)}$%	S_Y%	Y_U	Y_L	AVAILABLE DATA d's, ST'NS	DATA OMITTED d's, ST'NS	Y_{URS}
200	44	230	SP (1)	24	12.89	12.5	9.3	15.3	10.8	5d,15	2d,6	21
					16.09	8.0	3.3	17.1	15.2	5d,15	(1)	
		197	X	500	16.95	6.2	6.0	19.3	14.9	4d,11	2d.6	19
					17.07	6.9	3.4	18.2	16.0	4d,11	(1)	
		231	X	525	16.65	8.7	5.6	18.5	15.0	5d,14	1d,3,(1)	24
					19.66	6.8	2.9	20.7	18.6	5d,14	(1)	
		254	X	533	24.49	7.9	6.6	27.8	21.6	5d,14	2d,6(1)	32
					25.81	3.3	1.9	26.8	24.9	5d,14	2d,6,(1)	
		203	X	800	22.94	16.6	8.4	26.7	19.7	4d,11	0	31
					25.51	2.9	1.5	26.2	24.8	4d,11	(2)	
		229	X	1374	35.25	13.4	9.9	42.5	29.2	5d,14	2d,6,(1)	53
					36.35	12.2	6.3	40.8	32.4	5d,14	2d,5	
		252	X	325	32.59	8.1	6.1	36.6	29.0	5d,14	2d,6	38
					30.04	5.1	2.8	31.7	28.5	5d,14	2d,6	
		204	X	317	44.07	4.4	3.9	47.9	40.6	4d,11	2d,6	42
					44.66	11.0	5.3	49.1	40.6	4d,11	0	
		251	X	775	51.16	7.7	6.8	58.7	44.6	5d,12	2d,6	64
					53.07	5.8	3.6	57.1	49.4	5d,13	2d,7	

(1) FIRE BEFORE IGNITION

TABLE C11 - CBGS, LO_2/LH_2, L/D = 1.8, SUMMARY OF TEST DATA

W	V	TEST NO.	IGNITION TYPE	IGNITION TIME	Y	$S_{P(I)}\%$	$S_Y\%$	Y_U	Y_L	AVAILABLE DATA d's, ST'NS	DATA OMITTED d's, ST'NS	Y_{URS}
1000	44	211	SP(1)	0	4.94	13.3	6.7	5.6	4.4	4d,12	(1)	12
					12.36	12.7	5.8	13.7	11.1	4d,12	0	
		266	SP(2)	0	6.76	2.4	1.9	7.0	6.5	5d,14	2d,5,(1)	14
					17.40	7.3	4.0	18.8	16.1	5d,14	2d,5,(1)	
		264	SP(3)	21	11.36	17.9	8.0	13.1	9.9	5d,14	0	22
					19.26	15.8	7.5	22.0	16.8	5d,14	1d,3	
		217	X	1490	28.22	14.8	13.3	37.5	21.2	5d,11	2d,6	33
					33.67	5.0	2.9	35.6	31.8	5d,10	1d,2,(1)	
		262	X	900	36.86	6.3	4.2	39.9	34.1	5d,15	2d,6,(1)	'2
					38.15	8.2	4.5	41.6	35.0	5d,15	2d,6,(1)	
25000	44	289	SP(4)	166	2.67	4.1	2.3	2.8	2.6	5d,13	(2)	4
					3.87	3.5	1.7	4.0	3.7	5d,13	(3)	
		290	SP(2)	105	2.15	11.0	8.3	2.5	1.8	5d,14	2d,5	4
					4.46	10.1	5.1	4.9	4.1	5d,14	2d,5	
		288C	(4)	365	10.17	7.5	6.9	11.7	8.9	5d,14	3d,8	13
					11.42	6.6	2.9	12.0	10.9	5d,14	(1)	

(1) IGNITION AT IMPACT
(2) FIRE
(3) SUSPECT DIAPHRAGM BREAK
(4) IGNITION SOURCE UNKNOWN

TABLE C12 - CBGS, LO_2/LH_2, L/D = 1.8, SUMMARY OF TEST DATA

W	V	TEST NO.	IGNITION TYPE	IGNITION TIME	Y	$S_{P(I)}\%$	$S_Y\%$	Y_U	Y_L	AVAILABLE DATA d's, ST'NS	DATA OMITTED d's, ST'NS	Y_{URS}
200	23	105	XSP	0	6.04	5.6	3.8	6.5	5.6	3d,7	0	7
					6.76	8.5	5.0	7.5	6.1	3d.7	0	
		152	X	480	11.88	11.5	8.9	14.2	9.9	3d,9	1d,3	14
					13.67	6.3	3.3	14.5	12.8	3d,9	0	
		153	X	121	12.78	13.4	7.6	14.7	11.1	3d,9	0	14
					—	—	—	—	—	—	—	
		184	X	810	16.30	9.0	7.5	19.1	13.9	3d,6	1d,1	17
					15.79	6.7	4.0	17.1	14.6	3d,7	0	
		225	X	933	23.84	11.5	8.3	27.8	20.4	5d,15	2d,6	34
					26.31	9.0	4.9	28.9	24.0	5d,14	2d,6	
		201	X	1524	26.0	14.4	7.2	29.6	22.8	4d,11	0	26
					28.22	11.7	5.6	31.2	25.5	4d,11	0	
	78	226	SP(1)	283	37.04	12.5	9.3	44.1	31.1	5d,14	2d,5,(1)	51
					34.41	10.7	4.4	37.2	31.8	5d,14	0	
		150	X	40	34.18	3.7	4.0	38.5	30.4	3d,9	2d,6	35
					34.77	4.0	2.1	36.2	33.4	3d,9	0	
		151	X(2)	167	35.18	6.0	4.8	39.4	31.5	3d,7	1d,3	46
					44.28	3.5	2.1	46.1	42.5	3d,7	0	
		114	X	74	52.01	7.1	4.6	57.1	47.4	3d.9	1d,3	54
					51.84	4.5	2.9	55.0	48.9	3d,9	1d,3	
		195(3)	X	292	53.60	24.9	13.1	68.7	41.8	4d,11	1d,3	104
					69.47	17.0	9.6	83.4	57.9	4d,11	1d,3	

(1) FIRE....TEST USED IN URS ANALYSIS

(2) TANK FELL THROUGH STOPPER

(3) d_3 WAS OMITTED BECAUSE OF ERRONEOUS VALUES IN THE PRELIMINARY REPORT.

TABLE C13 - CBGS, LO_2/LH_2, CORRELATIONS BETWEEN YIELD AND IGNITION TIME

		LOGS				DIRECT					
W	V	r	r_u	r_L	b	r	r_u	r_L	b	NO PTS	TESTS EXCL.
200	23	.908	.952	.773	.176	.952	.971	.907	.014	5	NONE
		.896	.948	.733	.170	.932	.962	.852	.015	5	
		.933	.969	.785	.710	.895	.958	.561	.013	4	SPURIOUS (1)
		.881	.954	.467	.672	.859	.949	.324	.014	4	
200	44	.51	.749	.0605	.212	.343	.669	-.168	.012	9	NONE
		.49	.739	.0280	.177	.365	.680	-.141	.012	9	
		.068	.548	.461	.060	.148	.594	-.401	.006	8	SPURIOUS (1)
		.132	.585	.414	.105	.191	.616	-.366	.007	8	
200	78	.233	.764	.577	.059	.209	.757	-.590	.017	5	NONE
		.353	.798	.494	.120	.364	.801	-.485	.046	5	
		.450	.878	.638	.125	.473	.882	-.623	.044	4	SPURIOUS (1)
		.816	.940	.078	.270	.846	.946	.248	.110	4	
1000	44	.979	.985	.968	.242	.852	.932	.565	.018	5	NONE
		.950	.970	.903	.127	.876	.940	.656	.014	5	
		—	—	—	-.530	—	—	—	—	2	SPURIOUS (1)
		—	—	—	-.248	—	—	—	—	2	

N.B. FIRST LINE REFERS TO PRESSURE, SECOND LINE TO IMPULSE, ETC.

TABLE C14 - CBGS, LO_2/LH_2, L/D = 1.8, SUMMARY OF TEST DATA

W	V	TEST NO.	IGNITION TYPE	IGNITION TIME	Y	$S_{P(I)}\%$	$S_Y\%$	Y_U	Y_L	AVAILABLE DATA d's, ST'NS	DATA OMITTED d's, ST'NS	Y_{URS}
1000	78	215	SP(1)	20	12.13 24.05	9.2 4.5	4.0 2.4	13.0 25.1	11.3 [illegible]	5d,15 5d,15	0 2d,6	20
		216	SP	0	5.48 11.35	9.6 8.5	4.7 4.7	6.0 12.4	5.0 10.4	5d,15 5d.14	(2) 2d,6	9

(1) TANK BROKE AT IMPACT

TABLE C15 - CBGS, LO_2/LH_2, V = 44 FPS. REGRESSION SUMMARY, $Y = AW^B$

TESTS	NUMBER OF TESTS	$\sigma_Y\%$	A	B	$A_{.90}$[a]	$B_{.90}$	$A_{.95}$[a]	$B_{.95}$	W = 25,000 LBS $\hat{Y}$	W = 25,000 LBS $Y_{.90}$	W = 25,000 LBS $Y_{.95}$	S-II $\hat{Y}$	S-II $Y_{.90}$	S-II $Y_{.95}$
NON-SPURIOUS	11,P	18.68	73.0	-.172	34.7	-.047	27.1	-.006	12.93	28.94	37.61	6.94	21.88	31.79
	11,I	17.51	72.1	-.157	35.6	-.039	28.3	-.001	14.75	31.38	40.12	8.37	24.55	34.82
NON-ZERO IGNITION	15,P	25.88	212.0	-.382	102.3	-.271	81.5	-.237	4.44	11.13	14.82	1.12	3.52	5.03
	15,I	21.31	159.1	-.312	82.3	-.221	72.4	-.192	6.77	14.44	18.28	2.20	5.65	7.59
HIGH YIELDS	4, P	7.65	274.9	-.319	153.7	-.235	111.7	-.189	10.85	17.19	22.11	3.42	6.79	9.88
	4, I	7.39	253.7	-.300	144.6	-.219	106.2	-.174	12.12	18.91	24.12	4.09	7.93	11.41

TABLE C16 - CBGS, LO_2/LH_2, W = 200 LBS. REGRESSION SUMMARY, $Y = AV^B$

TESTS	NUMBER OF TESTS	$\sigma_Y\%$	A	B	$A^{a}_{.90}$	$B_{.90}$	$A^{a}_{.95}$	$B_{.95}$	V = 78 FPS $\hat{Y}$	V = 78 FPS $Y_{.90}$	V = 78 FPS $Y_{.95}$	V = 160 FPS $\hat{Y}$	V = 160 FPS $Y_{.90}$	V = 160 FPS $Y_{.95}$
DROP TESTS	16,P	15.82	2.17	.680	.747	.963	.537	1.050	41.97	71.47	84.26	68.40	128.0	155.4
	16,I	15.30	2.0	.721	.715	.995	.519	1.080	46.44	77.70	91.12	77.97	143.0	172.5
DROP TESTS & FLAT WALL TEST	17,P	15.70	3.93	.519	2.12	.677	1.75	.725	37.73	62.51	73.00	54.78	93.28	109.88
	17,I	15.49	4.16	.524	2.26	.679	1.88	.727	40.75	66.96	78.01	59.38	100.24	117.77

TABLE C17 - CBGS, LO_2/LH_2, SUMMARY OF TEST DATA

W	V	TEST NO.	IGNITION TYPE	IGNITION TIME	Y	$S_{P(I)}\%$	$S_Y\%$	Y_U	Y_L	AVAILABLE DATA d's, ST'NS	DATA OMITTED d's, ST'NS	Y_{URS}
200	597	079		0	90.36 94.59	18.9 16.8	12.3 8.7	113.6 111.2	71.9 80.5	4d,11 4d,11	1d,2 1d,2	121
	569	080		0	93.25 119.78	45.2 34.3	29.4 17.8	161.0 166.7	54.0 86.0	4d,11 4d,11	1d,2 1d,2	163

TABLE C18 - CBGS, LO_2/LH_2, MULTIPLE REGRESSION SUMMARY, $Y = AW^B V^C$

TESTS	NUMBER OF TESTS	σ_Y%	A	B	C	$B_{.90}$	$C_{.90}$	$B_{.95}$	$C_{.95}$		25,000 LBS 44 FPS	S-II 44 FPS	S-II 160 FPS
DROP TESTS	19,P	16.24	5.41	-.169	.683	-.069	.972	-.038	1.060	$\hat{Y}$	12.99	7.05	17.03
										$Y_{.90}$	25.52	18.25	47.18
										$Y_{.95}$	31.38	24.42	64.43
	19,I	15.78	4.78	-.159	.724	-.062	1.005	-.033	1.091	$\hat{Y}$	14.70	8.26	21.05
										$Y_{.90}$	28.33	20.81	56.64
										$Y_{.95}$	34.64	27.60	76.68
DROP TESTS & FLAT WALL TEST	20,P	16.20	9.96	-.165	.512	-.065	.673	-.035	.722	$\hat{Y}$	13.06	7.20	13.94
										$Y_{.90}$	25.56	18.53	36.94
										$Y_{.95}$	31.37	24.72	49.70
	20,I	16.00	9.99	-.154	.517	-.056	.676	-.027	.724	$\hat{Y}$	14.79	8.47	16.50
										$Y_{.90}$	28.70	21.52	43.14
										$Y_{.95}$	35.13	28.59	57.81

TABLE C 18a - CBGS, LO_2/LH_2, COMPARISON BETWEEN URS OBSERVED AND PREDICTED YIELDS

TEST NO.	W	V	t	URS PREDICTED	OBSERVED Y_{URS}	Y_P	Y_I	% DEVIATION
105^{S}	200	23	0	18.4	7	6.0	6.8	163
153			121	19.2	14	12.8	-	37
152			480	21.2	14	11.9	13.7	51
184			810	22.8	17	16.3	15.8	34
225			933	23.3	34	23.8	26.3	-31
201			1524	25.4	26	26.0	28.2	- 2
			(2619)*	(26.9)*				
230^{S}	200	44	24	19.5	21	12.9	16.1	- 7
204			317	31.1	42	44.1	44.7	-26
252			325	31.3	38	32.6	30.0	-18
197			500	36.9	19	17.0	17.1	94
231			525	37.6	24	16.6	19.7	57
254			533	37.8	32	20.3	25.8	18
251			775	43.5	64	51.2	53.1	-32
203			800	44.0	31	22.9	25.5	42
			(1369)*	(49.4)*				
229			1374	49.4	53	35.2	36.4	- 7
150	200	78	40	28.2	35	34.2	34.8	-19
114			74	36.1	54	52.0	51.8	-33
151			167	55.9	46	35.2	44.3	22
226^{S}			283	76.6	51	37.0	34.4	50
195			292	78.0	104	53.6	69.5	25
			(772)*	(115.8)*				

TABLE C 18a - CBGS, LO_2/LH_2, COMPARISON BETWEEN URS OBSERVED AND PREDICTED YIELDS (Continued)

TEST NO.	W	V	t	URS PREDICTED	OBSERVED Y_{URS}	OBSERVED Y_P	OBSERVED Y_I	% DEVIATION
211[S]	1,000	44	0	18.4	12	4.9	12.4	53
266[S]			0	18.4	14	6.8	17.4	31
264[S]			21	19.0	22	11.4	19.3	- 14
262			900	37.6	42	36.9	38.2	- 10
217			1490	45.3	33	28.2	33.7	37
			(2341)*	(49.4)*				
216[S]	1,000	78	0	18.4	9	5.5	11.4	104
215[S]			20	21.3	20	12.1	24.0	6
			(1321)*	(115.8)*				
290[S]	25,000	44	105	19.3	4	2.2	4.5	382
289[S]			166	19.9	4	2.7	3.9	398
288C			365	21.6	13	10.2	11.4	66
			(6845)*	(49.4)*				

S DESIGNATES SPURIOUS TEST

* DESIGNATES URS PREDICTED OPTIMUM TIME AND MAXIMUM YIELD

But this equation does not have a maximum; that is, according to this regression there is no optimal ignition time (or pool diameter) and consequently yield increases without limit as D* increases. Whereas, using equation (1°), URS obtains an optimal ignition time (pool diameter) of D* = 10.3 and, hence, the following "prediction" equation for t* unknown:

$$(3°) \qquad \hat{Y}_{max} = 18.4 + 0.016 V^2.$$

To summarize, the geometric scaling postulated by URS, but not visible in the data, forces the use of variables which lead statistically to the improbable prediction equation (2°). Therefore, the statistically nonsignificant quadratic term must be retained in order to obtain a prediction equation that is consistent with the simplest of physical requirements.

Returning to equation (3°), we see that it is independent of weight and appears to be applied up to about 80 fps where its value is approximately 120%. The corresponding upper 90% prediction limit reaches 120% at about 63 fps. For comparison, the corresponding values obtained in the present analysis (for W=200 lbs.) may be determined from Figures 9P and 9I: e.g., the expected value at 80 fps is 43% with an upper 90% limit of 61% at V=63 fps for overpressure. The upper 90% at 80 fps is 73%.

In order to treat the HVI tests, URS returns to equation (1°) sets t=0 to obtain Y=18.4 and connects this equation at 70 fps with a straight line to the yield of the flat wall HVI test.

Finally, equation (3°) gives a constant value of 49.5% for the expected yield when V=44 fps. This may be compared with the regression equations given in Section III. The value of $\hat{Y}$ for several weights together with the corresponding 90% upper prediction limits is also given there.

IV. CBGS, LO_2/RP-1

1. Summary of Data

1.1 Weight*, V=44 fps (Table C19, Figure C6)

1.2 Velocity (Tables C20, C21, C23, C24)

2. Correlation (Table C22)

3. Regression (Table C24)

4. Geometric Scaling and the URS Prediction Model (Table C25)

It is difficult to discuss the effect of ignition delay time on yield from the figures in the URS report since they combine all three velocities and weights (Figure 5-39); the relationship between yield and ignition time is further obscured by plotting $Y/V^{.9}$ against $D^*/V^{.2}$.

We have already discussed the relationship between yield and ignition time in Section III.4. and only point out that the increase of 76% for the maximum 1000 lb., 44 fps yield over the corresponding maximum yield at 200 lbs. is sufficient to preclude any assumption of geometric scaling of yield.

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*See Section III, 4.

DIFFERENCES IN PRESSURE AND IMPULSE VALUES
BETWEEN URS PRELIMINARY AND FINAL REPORTS

Test No.	Station	P or I	Prel'y Report	Final Report	Remarks*
095A	115	P	18.8	18.1	
	116	I	21.2	31.2	Final value appears erroneous
238	129	P	6.7	0.7	Rejected as extreme
	139	P	6.6	0.7	Rejected as extreme
096	Ten typographical errors in final report.				
206	116	P	20.0	--	
	126	P	18.0	--	
201	135	P	23.1	--	
197	115	P	11.5	--	Omitted in estimating yield.
	125	P	20.9	--	Omitted in estimating yield.
	116	P	4.7	--	Omitted in estimating yield.
	117	P	3.1	--	Omitted in estimating yield.
203	115	P	18.5	--	
	125	P	21.2	--	
	126	P	9.4	--	
	136	P	8.9	--	
	115	I	45.7	--	
	125	I	45.0	--	
254	137	P	1.5	--	Rejected as extreme

*Indicates whether the preliminary report value was used in estimating least squares yield--blank implies yes.

DIFFERENCES IN PRESSURE AND IMPULSE VALUES
BETWEEN URS PRELIMINARY AND FINAL REPORTS (Cont.)

Test No.	Station	P or I	Prel'y Report	Final Report	Remarks
262	137	P	3.8	--	Rejected as extreme
	137	I	22.0	--	Rejected as extreme
266	137	P	1.8	--	Rejected as extreme
	137	I	13.5	--	Rejected as extreme
114	116	P	15.2	57.2	Final value appears erroneous
195	117	P	12.9	6.3	Rejected as extreme
	127	P	12.7	5.0	Rejected as extreme
	137	P	11.3	5.0	Rejected as extreme
226	128	P	.8	--	
216	119	P	.3	--	Rejected as extreme

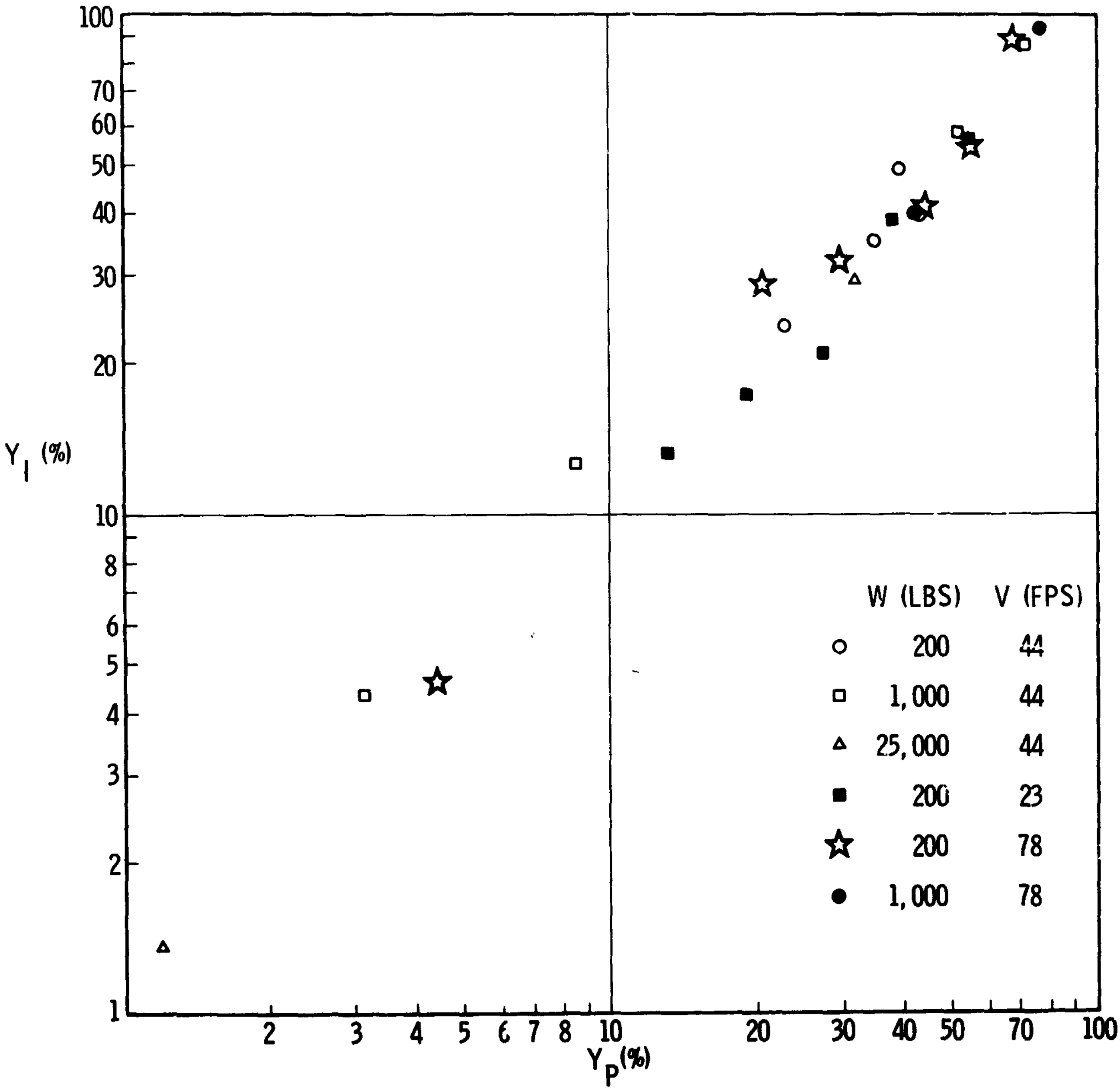

FIGURE C 9 - CBGS, LO_2/RP-1, L/D = 1.8, Y_I VS Y_P, SCATTER DIAGRAM

TABLE C19 - CBGS, LO_2/RP-1, L/D = 1.8, SUMMARY OF TEST DATA

W	V	TEST NO.	IGNITION TYPE	IGNITION TIME	Y	$S_{P(I)}$%	S_Y%	Y_U	Y_L	AVAILABLE DATA d's, ST'NS	DATA OMITTED d's, ST'NS	Y_{URS}
200	44	232	X	1220	22.91	5.1	5.6	26.1	20.1	5d,10	2d,6	30
					23.65	7.8	3.9	25.4	22.0	5d,10	0	
		208	X	460	39.26	22.2	19.5	59.5	25.9	4d,9	2d,4	62
					49.49	10.0	6.9	57.4	42.7	4d,10	2d,5	
		249	X	710	35.00	7.4	5.5	38.8	31.6	5d,14	2d,6	50
					35.31	7.3	3.8	37.9	32.9	5d,15	2d,6	
		250	X	200	43.30	14.3	10.0	52.1	36.0	5d,15	2d,6	52
					40.04	6.7	3.6	42.9	37.4	5d,15	2d,6,(1)	
1000	44	218	SP(1)	0	3.10	14.9	7.5	3.5	2.7	5d,14	0d,0	4
					4.37	12.5	5.6	4.8	4.0	5d,12	0d,0	
		219	X	1835	8.49	13.0	9.1	10.0	7.2	5d,14	2d,5	14
					13.53	9.4	4.8	14.8	12.4	5d,15	2d,6	
		267	X	1170[d]	51.87	12.3	7.9	60.2	44.7	5d,15	2d,6(1)	64
					58.73	9.7	4.3	65.4	54.4	5d,14	(1)	
		268	X	340	54.47	16.8	10.7	66.7	44.5	5d,14	2d,6	70
					55.9	7.2	3.3	59.3	52.7	5d,14	(2)	
		220	X	525	70.97	14.8	7.7	81.8	61.6	4d,10	0	96
					87.65	7.8	3.9	94.2	81.6	4d,11	(1)	
25000	44	284	SP(2)	0[d]	1.19	7.5	4.7	1.3	1.1	5d,15	1d,3	2
					1.39	3.5	1.5	1.4	1.4	5d,14	(1)	
		285	(3)	465	31.93	6.8	4.6	34.8	29.3	5d,13	2d,5	37
					29.43	3.7	1.8	30.4	28.5	5d,13	(2)	

(1) IGNITION ON IMPACT
(2) DIAPHRAGM RUPTURE IGNITION
(3) IGNITION SOURCE UNKNOWN

TABLE C20 - CBGS, LO_2/RP-1, L/D = 1.8, SUMMARY OF TEST DATA

W	V	TEST NO.	IGNITION TYPE	IGNITION TIME	Y	$S_{P(I)}$%	S_Y%	Y_U	Y_L	AVAILABLE DATA d.s, ST'NS	DATA OMITTED d's, ST'NS	Y_{URS}
200	23	096	X	50	13.03	5.4	3.0	13.8	12.3	3d,9	0	14
					13.22	9.4	6.0	14.9	11.7	3d,9	1d,3	
		248	X	210	18.99	12.4	6.1	21.2	17.1	5d,15	(1)	25
					17.15	9.7	3.9	18.4	16.0	5d,15	0	
		144	X	190	27.28	5.6	6.3	32.7	22.7	3d,7	2d,4	24
					21.04	3.2	2.0	21.9	20.2	4d,9	1d,3	
		202	X	870	37.99	9.4	8.4	45.4	31.8	4d,11	2d,6	42
					39.14	7.3	4.1	42.3	36.2	4d,11	1d,3	
	78	141	SP (1)	0	4.38	6.5	4.5	4.8	4.0	3d,7	0	5
					4.66	9.2	7.1	5.5	3.9	3d,7	1d,3	
		110	X	35	20.58	3.4	2.5	21.6	19.6	3d,9	1d,3	26
					28.94	6.7	3.7	31.1	27.0	3d,9	(1)	
		207	SP (2)	28	29.72	4.9	2.4	31.0	28.4	4d,11	0	38
					32.26	2.6	1.4	33.1	31.4	4d,11	(2)	
		205	SP (3)	40	44.25	6.8	5.9	50.2	39.0	4d,11	2d,6	41
					42.03	6.9	3.3	44.6	39.6	4d,11	0	
		236	X	720	54.76	19.0	12.9	69.6	43.1	5d.15	2d.6	74
					54.51	10.8	5.0	59.7	49.7	5d,14	1d,3	
		206	X	350	67.28	8.7	5.1	74.1	61.1	3d,8	0	84
					90.06	4.2	2.4	94.2	86.1	3d,8	0	

(1) PROBABLE DIAPHRAGM RUPTURE IGNITION
(2) NO OBVIOUS IGNITION SOURCE....USED IN URS ANALYSIS
(3) FIRE...USED IN URS ANALYSIS

TABLE C21 - CBGS, LO_2/RP-1, L/D = 1.8, SUMMARY OF TEST DATA

W	V	TEST NO.	IGNITION TYPE	IGNITION TIME	Y	$S_{P(I)}$%	S_Y%	Y_U	Y_L	AVAILABLE DATA d's, ST'NS	DATA OMITTED d's, ST'NS	Y_{URS}
1000	78	190	SP(1)	570	75.94	10.2	6.4	86.4	66.7	4d,11	2d,5	96
					94.61	16.7	10.8	117.7	76.1	4d,10	2d,4	
		269A	SP(2)	77	41.92	8.3	5.4	46.5	37.8	5d,14	2d,6	44
					40.06	3.7	2.0	41.6	38.5	5d,14	2d,6	

(1) CAP FIRED EARLY, PROPELLANT FOUND OTHER IGNITION SOURCE–USED IN URS ANALYSIS

(2) TANK FAILED AT IMPACT–USED IN URS ANALYSIS

TABLE C22 - CBGS, LO_2/RP-1, CORRELATIONS BETWEEN YIELD AND IGNITION TIME

		LOGS				DIRECT					
W	V	r	r_u	r_L	b	r	r_u	r_L	b	NO PTS	
200	78	.931	.937	.872	.402	.694	.866	.192	.056	6	NONE
		.933	.958	.876	.410	.604	.836	-.008	.060	6	
		.923	.984	.557	.370	.673	.970	-.908	.047	3	SPURIOUS (3)
		.773	.974	.881	.278	.374	.961	-.937	.033	3	
	44	-.898	-.576	.959	-.329	-.994	-.992	-.996	-.020	4	NONE
		-.709	.312	-.919	-.288	-.817	-.083	-.940	-.020	4	
1000	44	.673	.880	-.363	.298	-.095	.648	-.721	-.003	5	NONE
		.701	.888	.038	.283	-.070	.659	-.713	-.003	5	
		-.648	.435	-.909	-.718	-.745	.211	-.926	-.024	4	SPURIOUS (1)
		-.623	.473	-.905	-.601	-.694	.345	-.917	-.025	4	
200	23	.934	.969	.789	.369	.904	.960	.613	.027	4	NONE
		.955	.977	.884	.379	.986	.990	.976	.031	4	

N.B. FIRST LINE REFERS TO PRESSURE, SECOND LINE TO IMPULSE, ETC.

TABLE C23 - CBGS, LO_2/RP-1, L/D = 1.8, SUMMARY OF TEST DATA, HVI

W	V	TEST NO.	IGNITION TYPE	IGNITION TIME	Y	$S_{P(I)}$%	S_Y%	Y_U	Y_L	AVAILABLE DATA d.s, ST'NS	DATA OMITTED d's, ST'NS	Y_{URS}
200	526	075		0	17.77	[illegible]6.9	12.6	22.5	14.1	4d,11	1d,2	21
					15.69	25.1	12.8	19.9	12.4	4d,11	1d,2	
	523	077		0	8.74	23.5	5.9	12.5	6.1	4d,11	1d,2,(1)	20
					16.37	17.1	8.7	19.2	13.9	4d,11	1d,2	
	523	076		0	31.37	46.4	33.4	58.4	16.9	4d,11	1d,2	57
					44.55	31.0	15.9	59.9	33.2	4d,11	1d,2	
	518	078		0	37.25	35.3	31.8	67.3	20.6	4d,11	1d,2	77
					49.30	34.0	17.5	63.2	35.6	4d,11	1d,2	

TABLE C24 - CBGS, LO_2/RP-1, W = 200 LBS. REGRESSION SUMMARY, $Y = AV^B$

TESTS	NUMBER OF TESTS	$\sigma_Y\%$	A	B	$A^a_{.90}$	$B_{.90}$	$A^a_{.95}$	$B_{.95}$	$\hat{Y}$ (V = 78 FPS)	$Y_{.90}$ (V = 78 FPS)	$Y_{.95}$ (V = 78 FPS)
NON-SPURIOUS DROP TESTS	11,P	18.84	4.41	.527	1.11	.900	.710	1.021	43.89	85.94	106.9
	11,I	18.23	1.95	.761	.512	1.121	.332	1.238	53.51	102.6	126.7

TABLE C 25 - CBGS, LO_2/RP-1, COMPARISION BETWEEN URS OBSERVED AND PREDICTED YIELDS

TEST NO.	W	V	t	URS PREDICTED	OBSERVED Y_{URS}	Y_P	Y_I	% DEVIATION
096	200	23	50	8.5	14	13.0	13.2	- 39
144			190	21.8	24	27.3	21.0	- 9
248			210	23.7	25	19.0	17.2	- 5
			(524)*	(53.6)*				
202			870	35.0	42	38.0	39.1	- 17
250	200	44	200	64.1	52	43.3	40.0	23
			(312)*	(96.4)*				
208			460	69.3	62	39.3	49.5	12
249			710	48.4	50	35.0	35.3	- 3
232			1220	32.3	30	22.9	23.7	8
141^S	200	78	0	11.1	5	4.4	4.7	122
207			28	32.4	38	29.7	32.3	- 15
110			35	37.7	26	20.6	28.9	45
205^S			40	41.5	41	44.2	42.0	1
			(197)*	(161.5)*				
206			350	99.3	84	67.3	90.1	18
236			720	56.8	74	54.8	54.5	-23

TABLE C 25 - CBGS, LO_2/RP-1, COMPARISON BETWEEN URS OBSERVED AND PREDICTED YIELDS (Continued)

TEST NO.	W	V	t	URS PREDICTED	OBSERVED Y_{URS}	OBSERVED Y_P	OBSERVED Y_I	% DEVIATION
218[S]	1,000	44	0	6.6	4	3.1	4.4	65
268			340	63.7	70	54.5	55.9	- 9
220			525	94.8	96	71.0	87.6	- 1
			(533)*	(96.4)*				
267			1170[d]	49.9	64	51.9	58.7	- 22
219			1835	35.4	14	8.5	12.7	153
269A[S]	1,000	78	77	37.9	44	41.9	40.1	- 14
			(337)*	(161.5)*				
190[S]			570	92.3	96	75.9	94.6	- 4
284[S]	25,000	44	0	6.6	2	1.2	1.4	230
285			465	33.3	37	31.9	29.4	- 10
			(1558)*	(96.4)*				

S DESIGNATES SPURIOUS TEST

* DESIGNATES URS PREDICTED OPTIMUM TIME AND MAXIMUM YIELD

APPENDIX D

SPECIALIZED REGRESSION FORMULAS

1. Approximate prediction limits

Let the least squares regression equation, estimated from the observations $(x_1,y_1),\ldots,(x_n,y_n)$, be

$$y = \bar{y} + b(x-\bar{x}) \tag{1}$$

where $\bar{x} = \sum x_i/n$, $\bar{y} = \sum y_i/n$. Let also $\sigma_x{}^2 = \sum(x_i-\bar{x})^2/n$. When the x's and y's are (common) logs,

$$\begin{aligned} x &= \log X \\ y &= \log Y \end{aligned} \tag{2}$$

then (1) becomes

$$\begin{aligned} Y &= \bar{Y}(X/\bar{X})^b \\ &= AX^b \\ A &= \bar{Y}/\bar{X}^b \end{aligned} \tag{3}$$

where $\bar{X}$, $\bar{Y}$ are antilogs of $\bar{x}$, $\bar{y}$.

Let s^2 be the estimate of the variance about the regression line. As is well known, for fixed but arbitrary x, the variance of the predicted value of y is

$$s_y^2 = s^2\left(1 + \frac{1}{n} + \frac{(x-\bar{x})^2}{n\sigma_x^2}\right) \tag{4}$$

$$= s^2\left(1 + \frac{1}{n}\right) + [(x-\bar{x})s_b]^2$$

$$s_b^2 = \frac{s^2}{n\sigma_x^2} \tag{5}$$

Equation (4) leads to the following upper prediction limit for y. (For confidence limits on the expected value of y, equation (1), one need only replace in (4) ' + 1/n by 1/n.)

$$y_U = \bar{y} + b(x-\bar{x}) + t_{\nu,P}s_y \tag{6}$$

$t_{\nu,P}$ is the (one-sided) value of the t-distribution with ν=n-2 degrees of freedom and confidence probability P. (For this the y_i are required to be normally distributed.)

The upper confidence value for the slope is

$$b_U = b + t_{\nu,P}s_b \tag{7}$$

$$= b + \Delta b$$

$$\Delta b = t_{\nu,P}s_b \tag{8}$$

The line through $(\bar{x},\bar{y})$ with slope b_U yields the quantity

$$y_U{}^* = \bar{y} + b_U(x-\bar{x}) \tag{9}$$

$$= y + (x-\bar{x})\Delta b$$

$$= y + \Delta y$$

$$\Delta y = (x-\bar{x})\Delta b \tag{10}$$

Δy is the difference between the ordinates of the two lines, (1) and (9).

Substituting (5)-(10) into (6) gives, after simple algebra,

$$y_U = y_U{}^* + \delta y \tag{11}$$

where

$$\delta y = \sqrt{(\Delta y)^2 + s_{\bar{y}}{}^2} - \Delta y \tag{12}$$

$$s_{\bar{y}} = t_{\nu,P}\, s \sqrt{1 + \frac{1}{n}} \tag{13}$$

Note that $s_{\bar{y}}$ represents the prediction error in $\bar{y}$ when $x = \bar{x}$.

When Δy is large (i.e., $x >> \bar{x}$) equation (12) can be approximated by the first two terms of the binomial expansion to give

$$\delta y \simeq \frac{1}{2} \frac{s_{\bar{y}}{}^2}{\Delta y} \tag{14}$$

As $\Delta y \to \infty$ (i.e., $x - \bar{x} \to \infty$), $\delta y \to 0$ so that y_U^* is an approximate prediction limit. In fact, y_U vs. x is a hyperbola with asymptote y_U^*.

In terms of the X and Y, one gets (capital symbols are defined as the antilogs of corresponding lower case quantities, e.g., $\bar{X} = 10^{\bar{x}}$)

$$\Delta Y = \left(\frac{X}{\bar{X}}\right)^{\Delta b} \tag{15}$$

The asymptotic prediction equation is

$$Y_U^* = \Delta Y \cdot Y \tag{16}$$

$$= A_U^* X^{b_U} \tag{17}$$

$$A_U^* = \bar{Y}/\bar{X}^{b_U} \tag{18}$$

$$= A/\bar{X}^{\Delta b}$$

The exact prediction limits are given by

$$Y_U = \delta Y \cdot Y_U^* \tag{19}$$

$$= A_U^* \delta Y \cdot X^{b_U}$$

where

$$\delta Y = 10^{\sqrt{(\Delta y)^2 + s_{\bar{y}}^2} - \Delta y}$$

$$= \Delta Y^{\left(\sqrt{1 + (s_{\bar{y}}/\Delta y)^2} - 1\right)} \tag{20}$$

Using the approximation (14) gives

$$\delta Y \simeq 10^{s_{\bar{y}}^{2}/2\Delta y} \tag{21}$$

For ξ small, $10^{\xi} \simeq 1 + 2.3026\xi$, so that

$$\begin{aligned}\delta Y &\simeq 1 + 1.1513\ s_{\bar{y}}^{2}/\Delta y \\ &= 1 + \frac{1.1513\ s_{\bar{y}}^{2}}{\Delta b\ \log(X/\bar{X})} \\ &= 1 + \frac{1.1513\ t_{\nu,P} s\sigma_{x}(n^{1/2}+n^{-1/2})}{\log\ (X/\bar{X})}\end{aligned} \tag{22}$$

As a numerical example, consider the CBM LO_2/RP-1 regression (high yield tests) of pressure yield (Y) on weight (X). The relevant constants are

$$\bar{X} = 1308$$

$$\sigma_x = 1.04$$

$$s = .081$$

$$n = 6$$

Let also P = .95 so that $t_{4,.95} = 2.1318$. Equation (22) then becomes

$$\delta Y \simeq 1 + \frac{.534}{\log\ (X/1308)}$$

For X = 94,000 this equation gives $\delta Y \simeq 1.288$, the exact value being $\delta Y = 1.241$. For $X = 4.606\times10^{6}$, $\delta Y \simeq 1.150$ compares with $\delta Y = 1.146$.

2. Confidence limits for correlation coefficient

The formulas derived here for confidence limits for the correlation coefficient differ from the usual approximation in that the dependent variable (ignition time) is assumed to be non-randomly rather than randomly selected.

Let the regression line, estimated from (x_i, y_i), $i=1,\ldots,n$, be

$$y = \bar{y} + b\,(x-\bar{x})$$

where

$$b = \frac{\sum (x_i-\bar{x})(y_i-\bar{y})}{\sum (x_i-\bar{x})^2} \qquad (1)$$

The variance of b is

$$s_b^{\,2} = \frac{s^2}{\sum (x_i-\bar{x})^2} \qquad (2)$$

where s^2 is the estimated variance about the regression line, with $\nu=n-2$ degrees of freedom.

By definition

$$r = \frac{\sum (x_i-\bar{x})(y_i-\bar{y})}{\left[\sum (x_i-\bar{x})^2 \sum (y_i-\bar{y})^2\right]^{1/2}} \qquad (3)$$

Using (1) and (2), together with the equality

$$\sum (y_i-\bar{y})^2 = \nu s^2 + b^2 \sum (x_i-\bar{x})^2 \qquad (4)$$

one can readily show that (3) can be written

$$r = \frac{c}{\sqrt{\nu + c^2}} \qquad (5)$$

where

$$c = b/s_b \qquad (6)$$

The same argument applied to the parameters (represented by the Greek analogs for Roman letters) rather than the estimates gives

$$\rho = \frac{\gamma}{\sqrt{n+\gamma^2}} \tag{7}$$

where

$$\gamma = \beta/\sigma_b \tag{8}$$

Confidence limits on ρ can be obtained by substituting into (7) the corresponding confidence limits, γ_U and γ_L, for γ. The exact solution is quite complicated, yielding a non-tabulated distribution similar to the non-central t. A heuristic approximation can be obtained in the following manner. Note that $(b-\beta)/s_b$ has the t-distribution. If $\pm t_{\nu,P}$ represent the 2-sided probability P limits, then, since $Es_b^2 = \sigma_b^2$,

$$\gamma_U = \frac{s_b}{\sigma_b}(c + t_{\nu,P}) \simeq c + t_{\nu,P} \tag{9}$$

$$\gamma_L = \frac{s_b}{\sigma_b}(c - t_{\nu,P}) \simeq c - t_{\nu,P} \tag{10}$$

The above limits are too narrow, since they do not take into account the randomness of s_b/σ_b. To compensate for this, we have used ν rather than n, i.e., equation (5) rather than (7).

3. <u>Relation between individual and joint regression</u>

Suppose the regression of z on x and y is linear

$$Ez = \alpha + \beta x + \gamma y \tag{1}$$

Further, suppose the experimental design varies one variable at a time, i.e., for $y = y_0$, x takes on the values x_i $(i=1,\ldots,n)$, and for $x = x_0$, y takes on values y_j $(j=1,\ldots,m)$. The x_i and y_j need not all be distinct; moreover, some of the values may equal x_0 or y_0. It is desired to determine the relation between the estimated coefficients under the joint regression (1) compared with the simple regressions of z on x with $y = y_0$, and z on y with $x=x_0$.

Let z_k $(k=1,\ldots,m+n)$ be the observed values of z when $x = x_k$ and $y = y_k$, where $y_k = y_0$ for $k=1,\ldots, n$ and $x_k = x_0$ for $k = n+1,\ldots,n+m$. Then (1) can be written either as

$$Ez_k = \alpha + \beta x_k + \gamma y_k \quad , k = 1,\ldots, n+m \tag{2}$$

or as

$$Ez_i = \alpha + \beta x_i + \gamma y_0 \qquad (i=1,\ldots,n) \tag{3}$$

and

$$Ez_j = \alpha + \beta x_0 + \gamma y_j \qquad (j=1,\ldots,m)$$

Let

$$\bar{x}_n = \frac{1}{n}\sum x_i \qquad \bar{y}_m = \frac{1}{m}\sum y_j \qquad \bar{z}_n = \frac{1}{n}\sum z_i$$

$$\bar{x} = \frac{1}{n+m}\sum x_k \qquad \bar{y} = \frac{1}{m+n}\sum y_k \qquad \bar{z}_m = \frac{1}{m}\sum z_j \tag{4}$$

$$= \frac{1}{n+m}(n\bar{x}_n+mx_0) \qquad = \frac{1}{m+n}(ny_0+m\bar{y}_m) \qquad \bar{z} = \frac{1}{m+n}\sum z_k$$

$$= \frac{1}{m+n}(n\bar{z}_n + m\bar{z}_m)$$

The general least squares solution for the parameters in (2) is well known.

$$\hat{\alpha} = \bar{z} - \hat{\beta}\bar{x} - \hat{\gamma}\bar{y} \tag{5}$$

and $\hat{\beta}$ and $\hat{\gamma}$ are solutions of the simultaneous normal equations

$$\begin{aligned} \hat{\beta}S_{xx} + \hat{\gamma}S_{xy} &= S_{xz} \\ \hat{\beta}S_{xy} + \hat{\gamma}S_{yy} &= S_{yz} \end{aligned} \qquad (6)$$

where

$$\begin{aligned} S_{xz} &= \sum (x_k-\bar{x})(z_k-\bar{z}) \\ &= \sum x_k z_k - (m+n)\overline{xz} \end{aligned} \qquad (7)$$

and similarly for S_{yz}, S_{xy}, S_{xx}, and S_{yy}. Solving (6) for $\hat{\beta}$ gives

$$\hat{\beta} = \frac{S_{xz}S_{yy} - S_{yz}S_{xy}}{S_{xx}S_{yy} - S_{xy}^2} \qquad (8)$$

and similarly for $\hat{\gamma}$.

Now, using equations (5), S_{xz} in (7) can be written as follows:

$$\begin{aligned} S_{xz} &= \sum x_i z_i + m x_0 \bar{z}_m - \frac{1}{n+m} (n\bar{x}_n + m x_0)(n\bar{z}_n + m\bar{z}_m) \\ &= \sum x_i z_i - n\bar{x}_n\bar{z}_n + \frac{nm}{n+m} (x_0 - \bar{x}_n)(\bar{z}_m - \bar{z}_n) \qquad (9) \\ &= \sigma_{xz} + \tau_{xz} \end{aligned}$$

where σ_{xz} is the covariance for the regression on x alone

$$\sigma_{xz} = \sum x_i z_i - n\bar{x}_n \bar{z}_n \tag{10}$$

and τ_{xz} is the covariance between the means $(\bar{x}_n, \bar{z}_n)$ and $(x_0, \bar{z}_m)$:

$$\tau_{xz} = \frac{nm}{n+m} (x_0 - \bar{x}_n)(\bar{z}_m - \bar{z}_n) \tag{11}$$

Similarly,

$$\begin{aligned} S_{yz} &= \sigma_{yz} + \tau_{yz} \\ S_{xx} &= \sigma_{xx} + \tau_{xx} \\ S_{yy} &= \sigma_{yy} + \tau_{yy} \end{aligned} \tag{12}$$

However,

$$S_{xy} = \tau_{xy} \tag{13}$$

The σ's and τ's in (12) and (13) are defined as in (10) and (11) with an obvious permutation of symbols and appropriate subscripts. The estimates of the coefficients for the simple regression of z on x alone (equation (3a)) and z on y alone (equation (3b)), are given by

$$\tilde{\beta} = \frac{\sigma_{xz}}{\sigma_{xx}} \tag{14}$$

$$\tilde{\gamma} = \frac{\sigma_{yz}}{\sigma_{yy}} \tag{15}$$

Note also that when the x_i and y_j are symmetric about x_0, y_0, then all the τ's vanish.

Substituting (9), (12), and (13) into (8) and simplifying gives

$$\hat{\beta} = \tilde{\beta}\left[\frac{1 + \tau_{yy}/\sigma_{yy} + (\tau_{xz}-\tilde{\gamma}\tau_{xy})/\sigma_{xz}}{1 + \tau_{yy}/\sigma_{yy} + \tau_{xx}/\sigma_{xx}}\right] \quad (16)$$

$$= \tilde{\beta} + \delta\tilde{\beta} \quad (17)$$

where

$$\delta\tilde{\beta} = \frac{\tau_{xz} - \tilde{\beta}\tau_{xx} - \tilde{\gamma}\tau_{xy}}{\sigma_{xx} + \tau_{xx} + \sigma_{xx}\tau_{yy}/\sigma_{yy}} \quad (18)$$

By symmetry, the formula for $\hat{\gamma}$ interchanges the subscripts x and y, and also $\tilde{\beta}$ and $\tilde{\gamma}$, in (16) - (18).

It is important to emphasize that if the point (x_0,y_0) is included in the simple regression of z on x, say, it must be excluded from the regression of z on y. More generally, if (x_0,y_0) is replicated, any arbitrary non-overlapping allocation is permissible.

When m=1, i.e., there is only one y value, y_1, other than y_0, it can be shown that the joint solution is equivalent to first performing the simple regression of z on x to determine $\tilde{\beta} = \hat{\beta}$, and then obtaining the regression value of z at $x = x_0$ (and $y = y_0$). This value together with the z value at $y=y_1$ then determines $\hat{\gamma}$.

It is of interest to determine the effect of the y variable upon the constant term of the simple regression of z on x for $y=y_0$. Let

$$Ez_i = \xi + \beta x_i \quad (19)$$

where

$$\xi = \alpha + \gamma y_0$$

The uncorrected estimate $\tilde{\xi}$ is of course

$$\tilde{\xi} = \overline{z}_n - \tilde{\beta}\overline{x}_n \tag{20}$$

The corrected estimate $\hat{\xi}$ is then

$$\begin{aligned}\hat{\xi} &= \hat{\alpha} + \hat{\gamma} y_0 \\ &= \overline{z} - \hat{\beta}\overline{x} - \hat{\gamma}(\overline{y} - y_0) \\ &= \frac{1}{n+m}\left[n(\overline{z}_n - \hat{\beta}\overline{x}_n) + m[\overline{z}_m - \hat{\beta}x_0 - \hat{\gamma}(\overline{y}_m - y_0)]\right] \qquad (21) \\ &= \tilde{\xi} - \delta\tilde{\beta}\overline{x}_n + \frac{m}{n+m}\left[(\overline{z}_m - \overline{z}_n) - \hat{\beta}(x_0 - \overline{x}_n) - \hat{\gamma}(\overline{y}_m - y_0)\right]\end{aligned}$$