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TECHNICAL
MEMORANDUM

MINIMIZATION OF SM RCS FUEL FOR
SKYLAB ATTITUDE MANEUVERS

Bellcomm

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ABSTRACT

The Service Module Reaction Control System is an
existing Apollo system that can be conveniently used to apply
control torques to the orbiting Skylab. Thruster firing times
can be optimized such that any specified angular impulse will
be provided with a minimum expenditure of RCS fuel. In this
paper, analytic solutions for optimum firing times are derived,
and are presented such that numerical results can be obtained
by direct substitution into a set of formulas.

The results form the basis for an attitude maneuvering
scheme. The optimum firing times would be evaluated and seg-
mented into an appropriate firing sequence by either the
Command Module Computer or Mission Control. The maneuver
would then be executed by the on-board computer using the
stored firing sequence and its internal timer. A possible
application is the spin-up of Skylab B for an artificial
gravity experiment.

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ABSTRACT ONLY

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SUBJECT: Minimization of SM RCS Fuel for
Skylab Attitude Maneuvers - Case 620

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TECHNICAL MEMORANDUM

INTRODUCTION

The Service Module Reaction Control System can be used
to apply control torques to the orbiting Skylab. Maneuvers
between solar inertial and earth pointing attitudes in the first
series of Skylab missions and spin-up of the second Skylab for
an artificial gravity experiment are examples of possible appli-
cations of the SM RCS. The purpose of this memorandum is to
detail a thruster firing logic which is based on minimization
of the fuel required to apply a specified control torque.

A minimum solution for the fuel required to apply a
specified control torque for a specified time can always be
obtained, but the solution is not always unique. In fact, if
the 10° offsets of the thrusters from the local SM-skin tan-
gential planes are neglected, unique optimum solutions occur
for only a few specific sets of conditions. When the 10° off-
sets are included, unique optimum solutions occur over a wide
range of conditions. However, a great deal of insight into
the problem may be gained by considering the case where the
10° is neglected, and this simplified case will be presented
first.

PRELIMINARIES

There are 16 reaction jets mounted on the outside of
the SM. The jets are clustered in four quads, four jets to a
quad, as depicted in Figure 1. The quads are located in the
same longitudinal plane and spaced at 90° increments around the
circumference. One jet in each quad is directed nominally for-
ward, and one nominally aft. The other two jets in each quad
are directed nominally tangential to the local SM circumference.
The directions of the axes of all jets differ from the nominal
directions because of a 10° outward cant of each nozzle.

Expulsion of combustion products from a jet will
produce a reaction force of approximately 100 pounds. Reaction
forces, if not directed through the center of mass, will torque

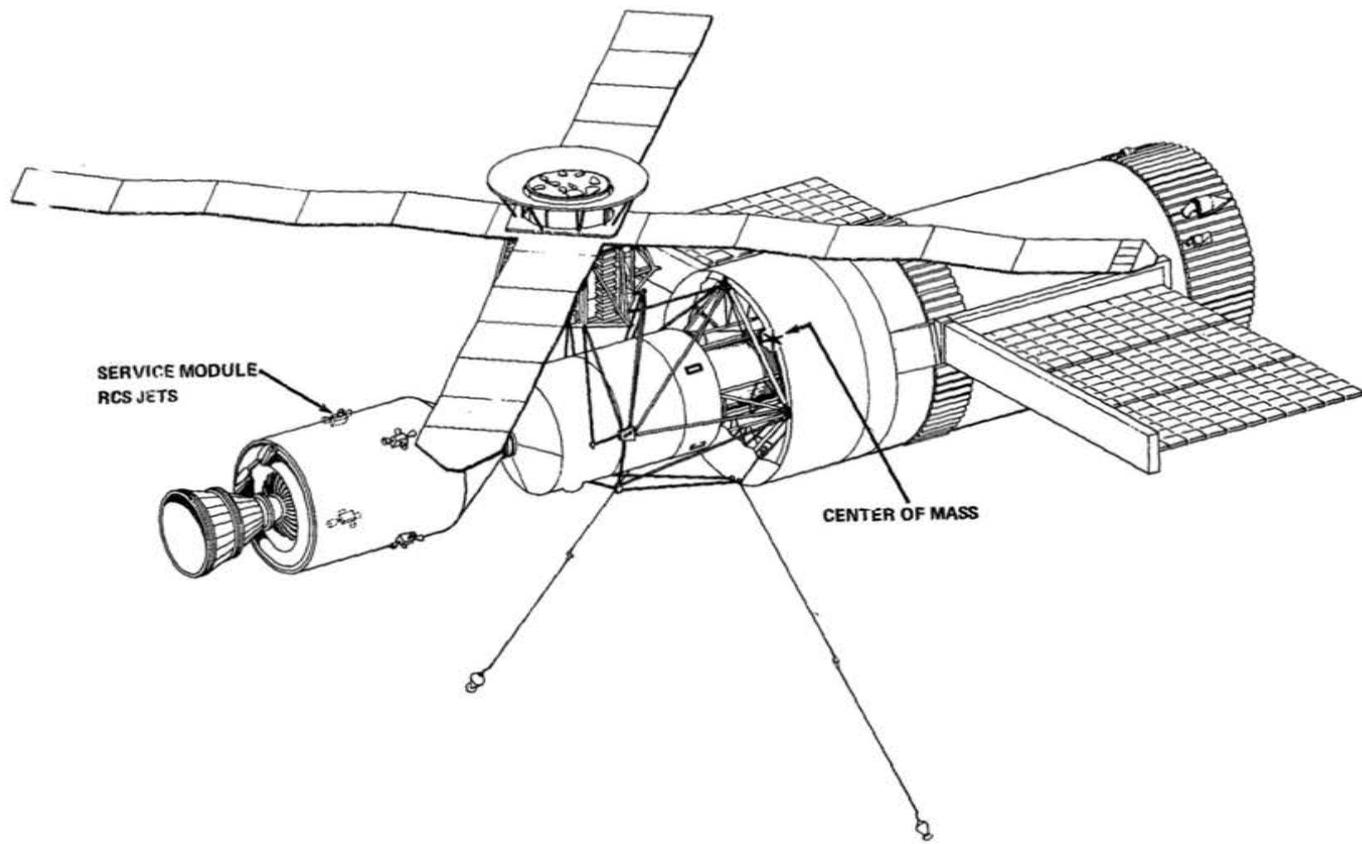SERVICE MODULE
RCS JETS

CENTER OF MASS

FIGURE 1 - SKYLAB CLUSTER

the vehicle about that center and will also translate the vehicle. Translational acceleration can be avoided if necessary by firing opposite jets in opposite quads to obtain a pure couple* (i.e., a torque produced by forces whose vector sum vanishes). In the case of Skylab, small orbital perturbations which might occur from individual jet firings will not affect the success of the mission. Therefore the large moment arm between the 8 SM thrusters in the plane of the quads and the Skylab center of mass can be used to advantage. This moment arm is approximately 40 feet, whereas the comparable moment arm of a fore-and-aft coupled pair that produce equivalent directional torque is 6.65 feet. The CSM lateral-firing jets therefore require only one-sixth as much fuel as the axial-firing coupled pairs to provide the same pitch or yaw control torque. Noting that these same 8 lateral-firing thrusters are the only ones available to (efficiently) provide a roll torque, we can conclude that only these 8 need be considered in the minimum fuel problem.

Let us, for the moment, forget these 8 thrusters and make believe we have instead a single thruster and want to place it in an optimum location in the plane of the quads to provide a specified control torque. Imagine the plane through the center of mass perpendicular to the axis about which the torque is to be applied. Any force vector in this plane (except those through the center of mass) will apply torque about the correct axis only. Therefore, we would place the axis of our single thruster along the intersection of this plane and the plane of the quads. The particular thruster location along this line is arbitrary, but naturally the reaction force vector must be directed so that the torque is in the specified sense. Our objective, then, is to make the eight real thrusters operate in a fashion that optimally simulates the operation of the single ideal one. To do this, we must introduce time into the problem.

When a force is applied over a time, the product (force x time) is a linear impulse. Similarly, the product of a torque times the time it acts is the angular impulse that acts. Since the control torque will act for some time, Δt, the required angular impulse over Δt can be as readily specified as the torque. Dividing by the moment arm, the equivalent linear impulse, $\bar{I}$, that must act along the intersection of the two previously defined planes is determined.

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*In Apollo 11, the docked CSM-LM vehicle was rotated by lateral translation jets prior to the landing. The resulting perturbations of the lunar orbit contributed to the inaccuracy of the LM landing. Apollo 12 used only couples to rotate the CSM-LM and with improved navigation techniques achieved a pin-point landing next to Surveyor as planned.

We must therefore determine the impulse required from each thruster so that they combine to have the same magnitude, direction and line of action as $\bar{I}$. Recalling that the magnitude of reaction force of each thruster is a constant, the problem becomes one of determining how long each thruster should fire within the time Δt. If the result for any thruster is greater than Δt, the specified torque cannot be provided. But the specified impulse can be provided by letting Δt be at least as great as the maximum resulting thruster on-time.

THE SIMPLIFIED PROBLEM

Figure 2 depicts the geometry of the simplified problem in which the 10° outward cant of the thrusters is neglected. The vector $\bar{I}$ is the required total impulse determined, in part, by the intersection of the plane of the RCS quads (the plane of the paper in Figure 2) and the plane through the mass center that is perpendicular to the control axis. The coordinate axes p and q originate at the center of the SM and pass through adjacent quads. These axes are chosen such that the angle ϕ between the positive p axis and the desired impulse vector $\bar{I}$ is in the range $0° \leq \phi \leq 45°$ which is possible because the plane of the RCS quads can be viewed from either side. This condition assures that $\bar{I}$ intersects the q axis at q_0. $\overline{Tt}_1$ through $\overline{Tt}_8$ are the unknown impulses provided by each reaction jet, with $\bar{T}$ being the constant thrust of each and t_i being the time it is applied. R is the radius from the center of the SM to any quad. The lines of action of the jets form a square box with sides 2R.

We can quickly see that, to minimize fuel, at most four of the eight thrusters would be fired. Firing both of a pair that are directly opposite each other (e.g., 1 and 8, or 2 and 3) would be wasteful as impulse from one would directly cancel impulse from the other. We can therefore pick one of each opposite pair to enter into the equations, and if the result is negative we know that the opposite thruster is the one that should be used. We might expect that the proper ones to pick are the four that have reaction force components in the direction of $\bar{I}$, and therefore choose 3, 5, 6, and 8.

The three kinematical conditions that must be met for the reaction impulses of the four jets to be equivalent to the desired impulse $\bar{I}$ are:

The sum of the components parallel to $\bar{I}$ must equal I (magnitude of $\bar{I}$).

$$T((t_3 + t_6) \cos\phi + (t_5 + t_8) \sin\phi) = I \qquad (1)$$

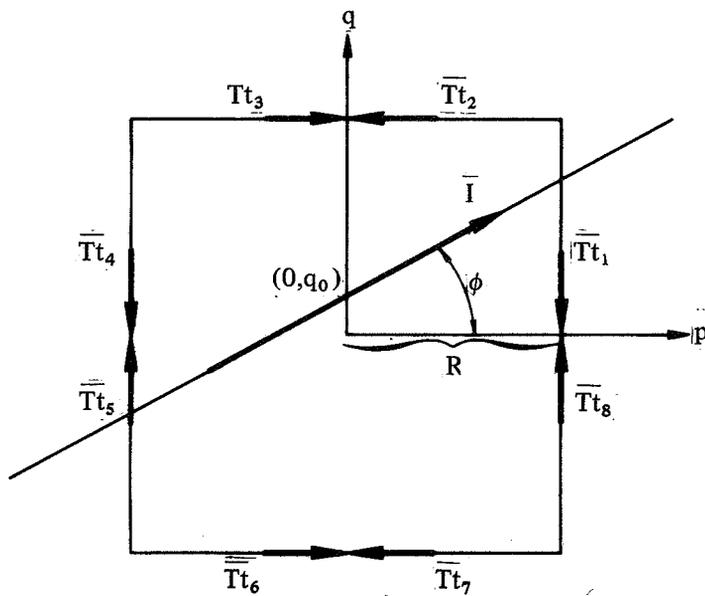where T is the magnitude of $\bar{T}$, and a constant.

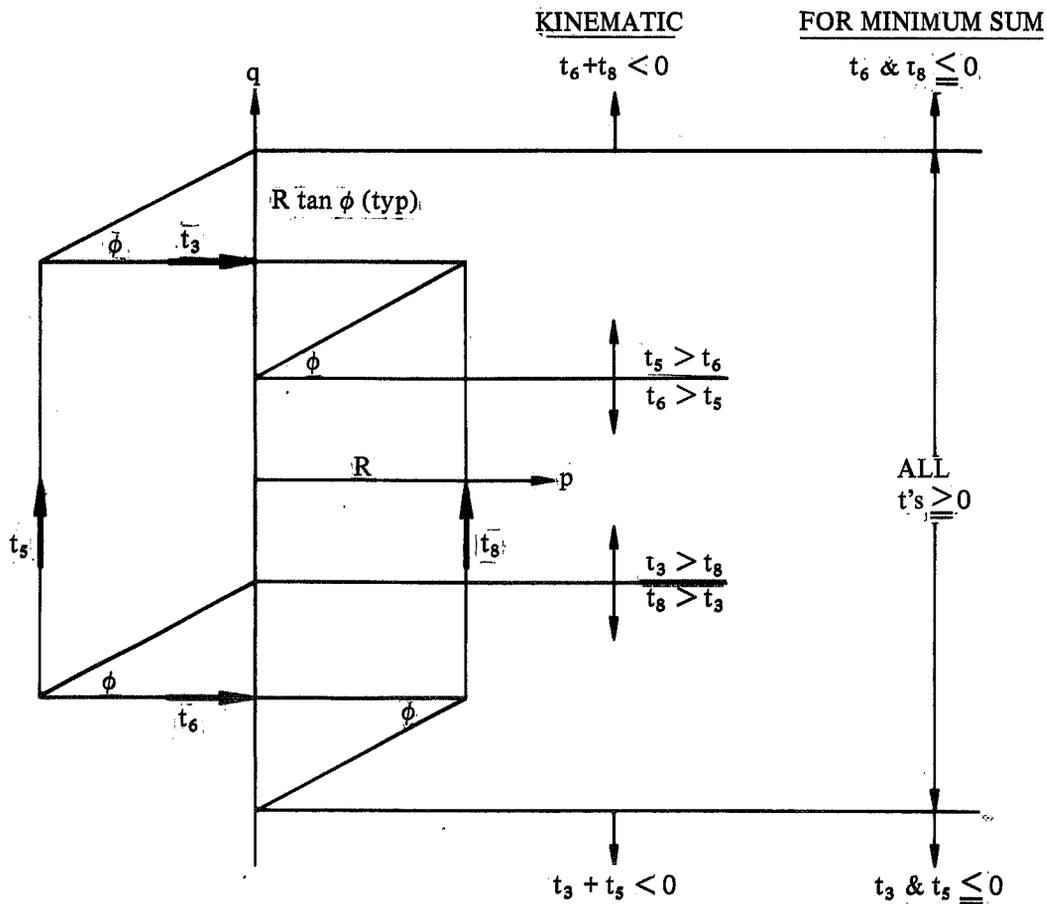FIGURE 2 - GEOMETRY FOR SIMPLIFIED PROBLEM

FIGURE 3 - DERIVED CONDITIONS ON THRUSTER FIRING TIMES

The sum of the components normal to $\overline{I}$ must vanish.

$$T((t_3 + t_6)\sin\phi - (t_5 + t_8)\cos\phi) = 0 \qquad (2)$$

Finally, the sum of the moments about any point, say $(0, q_o)$, on the line of action of $\overline{I}$ must vanish.

$$T((t_8 - t_5)R + t_6(R + q_o) - t_3(R - q_o)) = 0 \qquad (3)$$

Multiplying (1) by $\cos\phi$ and (2) by $\sin\phi$ and adding we obtain

$$t_3 + t_6 = I\cos\phi/T \qquad \text{(a positive constant)} \qquad (4)$$

Doing the opposite, we obtain

$$t_5 + t_8 = I\sin\phi/T \qquad \text{(a non-negative constant)} \qquad (5)$$

Adding (4) and (5) gives

$$t_3 + t_5 + t_6 + t_8 = \frac{I}{T}(\sin\phi + \cos\phi) \qquad (6)$$

The other condition to be met is that the sum of the lengths of time the jets are on is a minimum. We recall that negative solutions for time are admissible--meaning that the opposite thruster should be fired, and therefore write

$$|t_3| + |t_5| + |t_6| + |t_8| = \text{MINIMUM} \qquad (7)$$

We know by (6) that the sum of the t's is a positive constant. The sum of the absolute values can never be less than this constant, and will be greater if any t is negative. Any set of non-negative t's that satisfy the three kinematical relations will therefore give the minimum. If there is not such a set, the absolute value of the sum of the negative t's (two at the most by (4) and (5)) must be minimum to satisfy (7).

The three kinematic equations may be combined to obtain other constant relationships between sets of two t's, such as (4) and (5). Rewriting (3)

$$t_8 - t_5 + t_6 - t_3 = \frac{-q_o}{R} (t_3 + t_6)$$

and substituting (4) gives

$$t_6 - t_3 + t_8 - t_5 = \frac{I}{T} \cos \phi \left(\frac{-q_o}{R}\right) \tag{8}$$

Adding (6) and (8) gives

$$t_6 + t_8 = \frac{I}{2T} \left\{ \sin\phi + \cos\phi \left(1 - \frac{q_o}{R}\right) \right\} \tag{9}$$

which is non-negative if $q_o \leq R(1 + \tan\phi)$. Subtracting (8) from (6) gives

$$t_3 + t_5 = \frac{I}{2T} \left\{ \sin\phi + \cos\phi \left(1 + \frac{q_o}{R}\right) \right\} \tag{10}$$

which is non-negative if $q_o \geq -R(1 + \tan\phi)$. Thus, a non-negative set of t's will satisfy the kinematic conditions only if

$$-R(1 + \tan\phi) \leq q_o \leq R(1 + \tan\phi)$$

or, in words, if the line of action of $\bar{I}$ touches or passes through the square box formed by the lines of action of the thrusters (i.e., lies on or between the points $(p,q) = (-R,R)$ and $(R,-R)$). This conclusion is illustrated by Figure 3. If q_o lies outside this range, (7) is satisfied only when both of the two t's that make up the negative sum, by either (9) or (10), is non-positive.

Two more useful relations can be obained by subtracting (9) from (4),

$$t_3 - t_8 = \frac{I}{2T} \{-\sin\phi + \cos\phi (1 + \frac{q_o}{R})\} \qquad (11)$$

which is positive if $q_o > R(\tan\phi - 1)$, and by subtracting (5) from (9)

$$t_6 - t_5 = \frac{I}{2T} \{-\sin + \cos (1 - \frac{q_o}{R})\} \qquad (12)$$

which is positive if $q_o < R(1 - \tan\phi)$. These conditions are also illustrated in Figure 3, and it is seen that they too can be related to the box formed by the lines of action of the thrusters; they are dependent on whether or not $\overline{I}$ passes through opposite sides of the box.

Recall that we started with three kinematic equations and four unknowns. The condition that the sum of the absolute values of the unknowns be a minimum has led to some limits on the unknown times, but has not in general provided a fourth independent equation that forces a unique solution. The fact that there are non-unique sets of t's which yield minimum solutions can be demonstrated throughout the range of q_o if $\phi \neq 0°$, except at two points. These points are $q_o = \pm R(1 + \tan\phi)$, equivalent to $\overline{I}$ passing through one of the outside corners of the box of Figures 2 and 3. In each of these cases, the right side of either (9) or (10) is zero, and for a minimum sum, both t's which make up the sum on the left side will be zero. If $\phi = 0°$ and $-R \leq q_o \leq R$, then there exists a unique minimum solution involving only t_3 and t_6. However if $|q_o| > R$, there is no longer a unique solution.

When a pure roll maneuver is called for, the plane normal to the roll axis and the plane of the CSM thrusters do not intersect. But the case can be treated when it is noted that at an infinite radius, the linear impulse I to give a specified angular impulse is zero. Therefore, by (4) and (5), $t_3 = -t_6$ and $t_5 = -t_8$, which are two pure couples. Note that $RT(t_6 + t_8 - t_3 - t_5)$ is the total angular impulse, which is not denied by (8).

THE COMPLETE PROBLEM

In the complete problem, the 10° outward cant of the thrusters is included, and unique minimum solutions for firing times occur over a wide range of parameters. Figure 4 depicts the geometry; all coordinates and quantities, except α, are defined the same way as in the simplified problem. The engine cant-angle, α, is 10° for each thruster. The axes p and q are again chosen so $0° \leq \phi \leq 45°$ and $\bar{I}$ intersects q at q_o. The geometry is such that the impulse from any one thruster cannot directly cancel the impulse from any other thruster and therefore, unlike the simplified problem, all eight must be considered. Furthermore, a negative solution for any t is not admissible. (An individual thruster cannot be fired in the opposite direction.) It is noted, however, that the direction of $\overline{Tt}_{i+4}$ is directly opposite that of $\overline{Tt_i}$, i=1,2,3,4, and the lines of action of each are separated by a distance $2R \cos \alpha$.

The same three kinematic conditions must be met: The sum of the components parallel to $\bar{I}$ must equal I.

$$(t_1 - t_5)(-\sin(\phi+\alpha)) + (t_2 - t_6)(-\cos(\phi-\alpha)) +$$

$$+ (t_3 - t_7)(\cos(\phi+\alpha)) + (t_4 - t_8)(-\sin(\phi-\alpha)) = I/T \qquad (13)$$

The sum of the components perpendicular to $\bar{I}$ must vanish.

$$(t_1 - t_5)(-\cos(\phi+\alpha)) + (t_2 - t_6)(\sin(\phi-\alpha)) +$$

$$+ (t_3 - t_7)(-\sin(\phi+\alpha)) + (t_4 - t_8)(-\cos(\phi-\alpha)) = 0 \qquad (14)$$

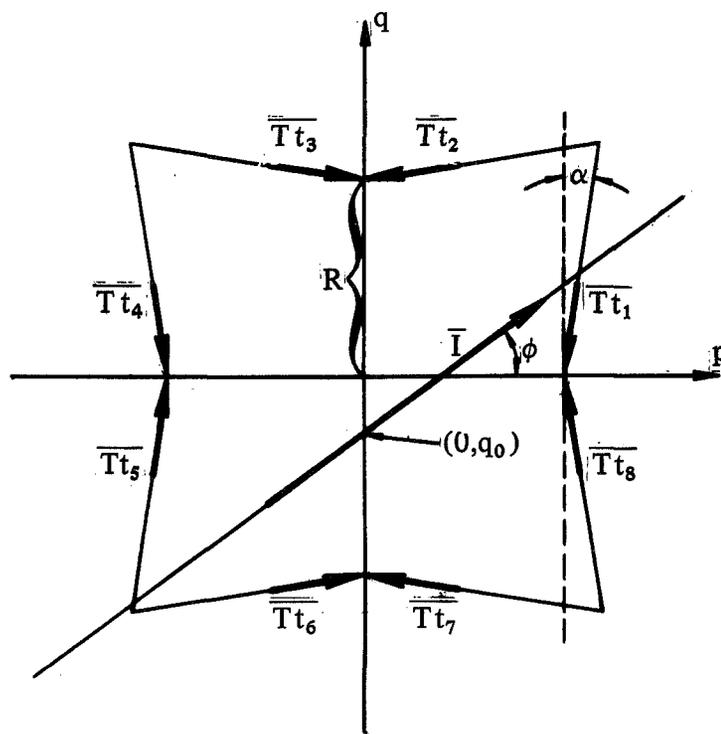The sum of the moments about any point, say $(0, q_o)$, on $\bar{I}$ must vanish.

FIGURE 4 - GEOMETRY FOR COMPLETE PROBLEM

$$(t_1 - t_5)(-q_o \sin\alpha) + (t_1 + t_5)(-R\cos\alpha)$$

$$+ (t_2 - t_6)(-q_o \cos\alpha) + (t_2 + t_6)(R\cos\alpha)$$

$$+ (t_3 - t_7)(q_o \cos\alpha) + (t_3 + t_7)(-R\cos\alpha)$$

$$+ (t_4 - t_8)(q_o \sin\alpha) + (t_4 + t_8)(R\cos\alpha) = 0 \qquad (15)$$

Obtaining A Minimum Solution

Using formulas from trigonometry and substitution the problem can be rewritten as

$$\text{Minimize} \left[\sum_{i=1}^{8} t_i \right] \qquad (16)$$

subject to

$$(t_5 - t_1 + t_8 - t_4)\cos\alpha + (t_6 - t_2 + t_7 - t_3)\sin\alpha = \frac{I}{T}\sin\phi, \qquad (17)$$

$$(t_5 - t_1 - t_8 + t_4)\sin\alpha + (t_6 - t_2 - t_7 + t_3)\cos\alpha = \frac{I}{T}\cos\phi, \qquad (18)$$

$$(t_1 + t_3 + t_5 + t_7) - (t_2 + t_4 + t_6 + t_8) = \frac{I\, q_o\, \cos\phi}{T\, R\, \cos\alpha} \qquad (19)$$

and

$$t_i \geq 0, \quad i = 1, 2, \ldots, 8. \qquad (20)$$

This is a linear programming problem in the variables t_1, t_2, ..., t_8 and in the parameters α, ϕ, I/T, R and q_o. A solution to (17), (18), (19) and (20) is called a feasible

solution and if that solution also minimizes $\sum\limits_{i=1}^{8} t_i$ (i.e.,

satisfies (16)) then it is an optimal feasible solution (or just an optimal solution for short). To make exposition more convenient the following representations will be used:

$$c = \cos \phi, \quad s = \sin \phi, \quad a = \cos \alpha, \quad b = \sin \alpha,$$

$$I^* = I/T \quad \text{and} \quad k = \frac{q_o \cos \phi}{R \cos \alpha} \quad .$$

Since ϕ and α are both between 0 and 45°, $c \geq s \geq 0$ and $a \geq b \geq 0$. I^* is a magnitude ratio and therefore positive and k will have the same sign as q_o.

We know that the kinematic conditions can be satisfied by using three of the eight thrusters, and might expect that numbers 3, 5, and 6 will produce an optimal solution for the conditions pictured in Figure 4 as all have components in the direction of $\bar{I}$. Solving the kinematic equations, (17), (18) and (19), for t_3, t_5 and t_6 in terms of the remaining variables yields

$$t_3 = \frac{I^*}{2a}[c-s+k(a-b)] - \frac{a-b}{a} t_1 + \frac{a-b}{a} t_2 - \frac{b}{a} t_4 + \frac{b}{a} t_7 + t_8 \qquad (21)$$

$$t_5 = I^*[\frac{s+kb}{a+b}] + \frac{a-b}{a+b} t_1 + \frac{2b}{a+b} t_2 + t_4 - \frac{2b}{a+b} t_7 - \frac{a-b}{a+b} t_8 \qquad (22)$$

$$t_6 = \frac{I^*}{2a}[c + \frac{s(a-b)-k}{a+b}] + \frac{t_1}{a(a+b)} + \frac{b(a-b)}{a(a+b)} t_2 - \frac{b}{a} t_4 + \frac{t_7}{a(a+b)} - \frac{a-b}{a+b} t_8 \qquad (23)$$

After substitution, (16) requires minimization of

$$\sum_{i=1}^{8} t_i = \frac{I^*}{a}[c + \frac{a-b}{a+b} (s+kb)] + 2[\frac{t_1}{a(a+b)} + (1 + \frac{b(a-b)}{a(a+b)})t_2 + \frac{a-b}{a} t_4$$

$$+ \frac{t_7}{a(a+b)} + \frac{2b}{a+b} t_8] \quad . \qquad (24)$$

If all the constant terms in (21)-(23) are non-negative, then $t_1=t_2=t_4=t_7=t_8=0$,

$$t_3 = \frac{I^*}{2a} [c-s+k(a-b)], \quad t_5 = I^*[\frac{s+kb}{a+b}], \quad t_6 = \frac{I^*}{2a} [c + \frac{s(a-b)-k}{a+b}]$$

is a feasible solution. Furthermore, since the coefficients of the t's in (24) are non-negative ($a \geq b \geq 0$) the above solution would be an optimal solution since no feasible solution can have

$$\sum_{i=1}^{8} t_i < \frac{I^*}{a} [c + \frac{a-b}{a+b} (s + kb)] .$$

The constant terms in (21)-(23) are non-negative when

$$\max \{- \frac{s}{b} , - \frac{c-s}{a-b}\} \leq k \leq c(a+b) + s(a-b)$$

This implies that as long as q_o is in range

$$\frac{Ra}{c} \max \{- \frac{s}{b} , - \frac{c-s}{a-b}\} \leq q_o \leq \frac{Ra}{c} [c(a+b) + s(a-b)] ,$$

(call this region B) the solution above is an optimal solution to the problem with

$$\sum_{i=1}^{8} t_i = \frac{I^*}{a} [c + \frac{a-b}{a+b} (s + \frac{q_o cb}{Ra})] .$$

For the case at hand $\alpha=10°$ and since all variables other than the three dependent variables t_3, t_5 and t_6 appear in the $\sum t_i$ with positive coefficients, this optimal solution is unique.

Regions of Solutions

   For the complete problem q_0 has the range between $-\infty$ and $+\infty$, and ϕ can be greater or less than α. The relation between ϕ and α and divisions of the range of q_0 will serve to define regions of solutions. The solutions will be given to terms of the parameters and proved to be optimal feasible solutions within their respective regions by putting the problem in the format of (21)-(24) having solved for the potentially positive variables in the solution. To find other regions, we first determine which independent variables must become positive as k crosses the boundary of a known region.

   Consider the case where k reaches the lower limit of region B with $\alpha < \phi$. When k has reached $-\frac{c-s}{a-b}$ from above

$$\frac{I^*}{2a} [c - s + k(a-b)] = 0$$

As soon as k becomes less than $-\frac{c-s}{a-b}$ by an ε-amount, the constant in (21) becomes negative while the constants in (22) and (23) remain positive (for small enough ε). Hence, the variable t_3 can become 0 and must be aided or replaced by a variable with a positive coefficient in (21). The candidates are t_2, t_7 and t_8; the choice is made by choosing the variable which will, after substitution, leave all coefficients in (24) non-negative. To make this obvious let $t_2^* = \frac{a-b}{a} t_2$, $t_7^* = \frac{b}{a} t_7$ and $t_8^* = t_8$. With k less than $-\frac{c-s}{a-b}$ by ε, and t_1, t_3, $t_4 \geq 0$, (21) gives,

$$t_2^* + t_7^* + t_8^* \geq -\frac{I^*}{2a} [c-s+k(a-b)] > 0 \tag{25}$$

From (24), it is desired to minimize

$$\sum_{i=1}^{8} t_i = \frac{I^*}{a} [c + \frac{a-b}{a+b} (s+kb)] + 2[\frac{t_1}{a(a+b)} + (1 + \frac{2ab}{a^2-b^2})t_2^* +$$

$$\frac{a-b}{a} t_4 + (1 + \frac{a(a-b)}{b(a+b)}) t_7^* + (1 - \frac{a-b}{a+b}) t_8^*]$$

and since <u>all</u> coefficients are non-negative, we must minimize

$$[(1 + \frac{2ab}{a^2+b^2})t_2^* + (1 + \frac{a(a-b)}{a(a+b)})t_7^* + (1 - \frac{a-b}{a+b})t_8^*] \ . \tag{26}$$

The one chosen then is the one with the smallest coefficient in (26), or t_8^*, since it adds the least to Σt_i when satisfying (25). This is precisely the variable which leaves the coefficients in (24) non-negative after solving for that variable in (21) and substituting into (24). This is true for the following reasons. The variables with negative coefficients in (21) will have positive coefficients in the equation solved for t_8 and substitution only adds something to their coefficients in (24). The variables with positive coefficients in (21) have negative coefficients when (21) is solved for t_8 and substitution subtracts something from their coefficients in (24); this something is less than or equal to the coefficient in (24) since t_8^* had the minimum coefficient in (26). It is noted that ε must be small enough to keep the constant terms of (22) and (23) non-negative after substitution.

The above process yields

$$t_5 = \frac{I^*}{2a} [\frac{c(a-b)+s(a+b)+k}{a+b}] + \frac{b(a-b)}{a(a+b)} t_1 + \frac{t_2}{a(a+b)} - \frac{a-b}{a+b} t_3 + \frac{t_4}{a(a+b)} - \frac{b}{a} t_7$$

$$t_6 = I^* [\frac{c-bk}{a+b}] + \frac{2b}{a+b} t_1 + \frac{a-b}{a+b} t_2 - \frac{a-b}{a+b} t_3 - \frac{2b}{a+b} t_4 + t_7$$

$$t_8 = \frac{I^*}{2a} [-c+s-k(a-b)] + \frac{a-b}{a} t_1 - \frac{a-b}{a} t_2 + t_3 + \frac{b}{a} t_4 - \frac{b}{a} t_7$$

and the sum to be minimized is

$$\sum_{i=1}^{8} t_i = \frac{I^*}{a} [s+\frac{a-b}{a+b} (c-bk)] + 2[(1+\frac{b(a-b)}{a(a+b)}) t_1 + \frac{t_2}{a(a+b)} + \frac{2b}{a+b} t_3$$

$$+ \frac{t_4}{a(a+b)} + \frac{a-b}{a} t_7] \ .$$

The above equations give an optimal solution and define the region over which that solution is optimal. The solution is $t_1 = t_2 = t_3 = t_4 = t_7 = 0$,

$$t_5 = \frac{I^*}{2a} \left[\frac{c(a-b)+s(a+b)+k}{a+b}\right] \ , \quad t_6 = I^* \left[\frac{c-bk}{a+b}\right] \ ,$$

$$t_8 = \frac{I^*}{2a} [-c+s-k(a-b)]$$

with

$$\sum_{i=1}^{8} t_i = \frac{I^*}{a} \left[s + \frac{a-b}{a+b}(c-bk)\right]$$

and the region over which this is the optimal feasible solution is

$$-[c(a-b) + s(a+b)] \leqq k \leqq - \frac{c-s}{a-b}$$

This region exists only if $\alpha \lneqq \phi$ and will be called $C_{\alpha \lneqq \phi}$. This optimal solution corresponds to a vertex on the convex feasible solution space. The solution is unique when $0 < \alpha < \pi/2$.

The remaining regions and corresponding solutions are arrived at in a similar manner.

Solutions for Remaining Regions

Solutions for the remaining regions are shown below in the format proving their optimality.

Region $D_{\alpha \lneqq \phi}$: $\alpha \lneqq \phi$, $-\infty < k \lneqq -c(a-b) - s(a+b)$. Coming from Region $C_{\alpha \lneqq \phi}$ the variable t_2 replaces t_5 giving

$$t_2 = \frac{I^*}{2} [-c(a-b)-s(a-b)-k] - b(a-b)t_1 + a(a-b)t_3 - t_4 + a(a+b)t_5$$

$$+ \ b(a+b)t_7$$

$$t_6 = \frac{I*}{2} \; [c(a+b)-s(a-b)-k] \; + \; b(a+b)t_1 \; - \; b(a-b)t_3 \; - \; t_4 \; + \; a(a-b)t_5$$

$$+ \; a(a+b)t_7$$

$$t_8 = I*[-cb+sa] \; + \; (a^2-b^2)t_1 \; + \; 2abt_3 \; + \; t_4 \; - \; (a^2-b^2)t_5 \; - \; 2abt_7$$

and the sum to be minimized is

$$\sum_{i=1}^{8} t_i = -I*k \; + \; 2(t_1+t_3+t_5+t_7) \; .$$

Since t_4 does not appear in Σt_i expressed above the minimum, $\Sigma t_i = -I*k$, is attained by any feasible solution satisfying

$$t_2 \; + \; t_4 = \frac{I*}{2} \; [-c(a-b) \; - \; s(a+b) \; - \; k] \; ,$$

$$t_6 \; + \; t_4 = \frac{I*}{2} \; [c(a+b) \; - \; s(a-b) \; - \; k] \; ,$$

$$t_8 \; - \; t_4 = I*[-cb \; + \; sa] \; ,$$

$$t_1 = t_3 = t_5 = t_7 = 0 \; .$$

An appropriate minimum solution can be found by choosing any value for t_4 such that $0 \le t_4 \le \frac{I*}{2} \; [-c(a-b) \; - \; s(a+b) \; - \; k]$; the variables t_2, t_6 and t_8 will thus be determined and be non-negative. These non-unique solutions correspond to an edge of the convex feasible solution space.

Region A: $\phi \lessgtr \alpha$, $+\infty > k \geq c(a+b) + s(a-b)$. Coming from Region B, t_1 replaces t_5 giving

$$t_1 = \frac{I*}{2}[-c(a+b)-s(a-b)+k] - b(a-b)t_2 + b(a+b)t_4 + a(a+b)t_6 -$$

$$t_7 + a(a-b)t_8$$

$$t_3 = I*(ca-sb) + (a+b)(a-b)t_2 - 2abt_4 - (a-b)(a+b)t_6 + t_7 + 2abt_8$$

$$t_5 = \frac{I*}{2}[-c(a-b)+s(a+b)+k] + b(a+b)t_2 + a(a+b)t_4 + a(a-b)t_6 -$$

$$t_7 - b(a-b)t_8$$

and the sum to be minimized is

$$\sum_{i=1}^{8} t_i = I*k + 2(t_2 + t_4 + t_6 + t_8) \ .$$

Therefore the minimum is

$$\sum_{i=1}^{8} t_i = I*k$$

realizable by any solution satisfying

$$t_1 + t_7 = \frac{I*}{2}[-c(a+b) - s(a-b) + k] \ ,$$

$$t_3 - t_7 = I*(ca - sb) \ ,$$

$$t_5 + t_7 = \frac{I*}{2}[-c(a-b) + s(a+b) + k] \ ,$$

and $t_2 = t_4 = t_6 = t_8 = 0$; t_1, t_3, t_5, $t_7 \geq 0$. These conditions

will be satisfied by choosing any value for t_7 such that $0 \leqq t_7 \leqq \frac{I^*}{2}[-c(a+b) - s(a-b) + k]$; the variables t_1, t_3 and t_5 will thus be determined and non-negative giving an optimal solution. These non-unique solutions correspond to an edge of the convex feasible solution space.

Region $C_{\phi < \alpha}$: $\phi < \alpha$, $-c(a+b) + s(a-b) \leqq k < -\frac{s}{b}$.

Coming from Region B, t_4 replaces t_5 giving

$$t_3 = \frac{I^*}{2a} [\frac{c(a+b)-s(a-b)+k}{a+b}] - \frac{a-b}{a+b} t_1 + \frac{t_2}{a(a+b)} - \frac{b}{a} t_5 + \frac{b(a-b)}{a(a+b)} t_7 + \frac{t_8}{a(a+b)}$$

$$t_4 = -I^* (\frac{s+kb}{a+b}) - \frac{a-b}{a+b} t_1 - \frac{2b}{a+b} t_2 + t_5 + \frac{2b}{a+b} t_7 + \frac{a-b}{a+b} t_8 ,$$

$$t_6 = \frac{I^*}{2a} [c+s-k(a-b)] + t_1 + \frac{b}{a} t_2 - \frac{b}{a} t_5 + \frac{a-b}{a} t_7 - \frac{a-b}{a} t_8 ,$$

and the sum to be minimized is

$$\sum_{i=1}^{8} t_i = \frac{I^*}{a}[c - \frac{a-b}{a+b} (s+kb)] + 2[\frac{2b}{a+b} t_1 + \frac{t_2}{a(a+b)} + \frac{a-b}{a} t_5 +$$

$$(1 + \frac{b(a-b)}{a(a+b)}) t_7 + \frac{t_8}{a(a+b)}]$$

The minimum is therefore

$$\sum_{i=1}^{8} t_i = \frac{I^*}{a}[c - \frac{a-b}{a+b} (s+kb)]$$

attained by using the solution: $t_1 = t_2 = t_5 = t_7 = t_8 = 0$,

$$t_3 = \frac{I^*}{2a} [\frac{c(a+b)-s(a-b)+k}{a+b}] , \quad t_4 = -I*(\frac{s+kb}{a+b}) ,$$

$$t_6 = \frac{I^*}{2a} [c + s - k(a-b)] .$$

This optimum solution is unique in the region $C_{\phi < \alpha}$.

Region $D_{\phi < \alpha}$: $\phi < \alpha$, $-\infty < k < -c(a+b) + s(a-b)$.
Coming from Region $C_{\phi < \alpha}$, t_2 replaces t_3 giving

$$t_2 + \frac{I^*}{2} [-c(a+b) + s(a-b)-k] + a(a-b)t_1 + a(a+b)t_3 + b(a+b)t_5$$

$$-b(a-b)t_7 - t_8$$

$$t_4 = I^*(bc-as)-(a^2-b^2)t_1 - 2abt_3 + (a^2-b^2)t_5 + 2abt_7 + t_8$$

$$t_6 = \frac{I^*}{2} [c(a-b) + s(a+b)-k] + a(a+b)t_1 + b(a+b)t_3 - b(a-b)t_5$$

$$+ a(a-b)t_7 - t_8$$

and the sum to be minimized is

$$\sum_{i=1}^{8} t_i = -I^*k + 2[t_1 + t_3 + t_5 + t_7] \ .$$

Therefore, the minimum is

$$\sum_{i=1}^{8} t_i = -I^*k$$

attained by any solution satisfying

$$t_2 + t_8 = \frac{I^*}{2} [-c(a+b) + s(a-b) - k]$$

$$t_4 - t_8 = I^*(bc - as)$$

$$t_6 + t_8 = \frac{I^*}{2} [c(a-b) + s(a+b) - k]$$

any $t_1 = t_3 = t_5 = t_7 = 0$. A non-unique solution is determined by choosing any value for t_8 such that $0 \leqq t_8 \leqq \frac{I*}{2}[-c(a+b) + s(a-b) -k]$, this will suffice to fix t_2, t_4 and t_6 as non-negative numbers to complete the solution.

An Optimal Solution for each Region in Terms of the Parameters

For each region, one optimal solution is given in terms of the parameters R, α, T, ϕ, q_o, I on the following three pages. Recall that the thrusters need to be labeled such that the vector $\overline{I}$ has angle ϕ between 0° and 45° from the positive p axis. The angle α is 10° for the Apollo CSM. The relation of regions are shown in Figures 5, 6 and 7.

Region A: $0 \leqq \phi \lessgtr \alpha$, $R \frac{\cos\alpha}{\cos\phi} [\cos(\phi+\alpha) + \sin(\phi+\alpha)] < q_o < +\infty$

Solution (a non-unique solution for minimum):

$$t_2 = t_4 = t_6 = t_8 = t_7 = 0,$$

$$t_1 = \frac{I}{2T} [-\cos(\phi+\alpha) - \sin(\phi+\alpha) + \frac{q_o}{R} \frac{\cos\phi}{\cos\alpha}]$$

$$t_3 = \frac{I}{T} \cos(\phi+\alpha)$$

$$t_5 = \frac{I}{2T} [-\cos(\phi+\alpha) + \sin(\phi+\alpha) + \frac{q_o}{R} \frac{\cos\phi}{\cos\alpha}]$$

with minimum

$$\sum_{i=1}^{8} t_i = \frac{I}{T} \frac{q_o}{R} \frac{\cos\phi}{\cos\alpha} .$$

Region B: $0 \leqq \phi \leqq \alpha$, $-R \frac{\tan\phi}{\tan\alpha} \leq q_o \leqq \frac{R \cos\alpha}{\cos\phi} [\cos(\phi+\alpha) + \sin(\phi+\alpha)]$

or

$0 \leqq \alpha \leqq \phi$, $-R \frac{(1-\tan\phi)}{(1-\tan\alpha)} \leqq q_o \leqq \frac{R \cos\alpha}{\cos\phi} [\cos(\phi+\alpha) + \sin(\phi+\alpha)]$

Solution (a unique solution for minimum when $\alpha > 0$):

$$t_1 = t_2 = t_4 = t_7 = t_8 = 0,$$

$$t_3 = \frac{I \cos\phi}{2T \cos\alpha} [(1 - \tan\phi) + \frac{q_o}{R} (1 - \tan\alpha)]$$

$$t_5 = \frac{I \cos\phi}{T(\cos\alpha + \sin\alpha)} [\tan\phi + \frac{q_o}{R} \tan\alpha]$$

$$t_6 = \frac{I}{2T \cos\alpha(\cos\alpha + \sin\alpha)} [\cos(\phi+\alpha) + \sin(\phi+\alpha) - \frac{q_o}{R} \frac{\cos\phi}{\cos\alpha}]$$

with minimum

$$\sum_{i=1}^{8} t_i = \frac{I}{T \cos\alpha(\cos\alpha + \sin\alpha)} [\cos(\phi+\alpha) + \sin(\phi+\alpha)$$
$$+ \frac{q_o}{R} \cos\phi \tan\alpha(\cos\alpha - \sin\alpha)].$$

Region $C_{\phi<\alpha}$: $0 \leqq \phi < \alpha$, $\dfrac{-R\cos\alpha}{\cos\phi}[\cos(\alpha-\phi) + \sin(\alpha-\phi)] \leqq q_o < \dfrac{-R\tan\phi}{\tan\alpha}$

Solution (a unique solution for minimum):

$t_1 = t_2 = t_5 = t_7 = t_8 = 0$,

$t_3 = \dfrac{I}{2T\cos\alpha(\cos\alpha + \sin\alpha)}[\cos(\alpha-\phi) + \sin(\alpha-\phi) + \dfrac{q_o\cos\phi}{R\cos\alpha}]$,

$t_4 = \dfrac{-I\cos\phi}{T(\cos\alpha + \sin\alpha)}[\tan\phi + \dfrac{q_o}{R}\tan\alpha]$,

$t_6 = \dfrac{I\cos\phi}{2T\cos\alpha}[(1 + \tan\phi) - \dfrac{q_o}{R}(1 - \tan\alpha)]$

with minimum

$$\sum_{i=1}^{8} t_i = \dfrac{I}{T\cos\alpha(\cos\alpha + \sin\alpha)}[\cos(\alpha-\phi) + \sin(\alpha-\phi) -$$

$$- \dfrac{q_o}{R}\cos\phi\tan\alpha(\cos\alpha - \sin\alpha)] \ .$$

Region $D_{\phi<\alpha}$: $0 \leqq \phi < \alpha$, $-\infty < q_o < \dfrac{-R\cos\alpha}{\cos\phi}[\cos(\alpha-\phi) + \sin(\alpha-\phi)]$

Solution (a non-unique solution for minimum):

$t_1 = t_3 = t_5 = t_7 = t_8 = 0$,

$t_2 = \dfrac{I}{2T}[-\cos(\alpha-\phi) - \sin(\alpha-\phi) - \dfrac{q_o}{R}\dfrac{\cos\phi}{\cos\alpha}]$,

$t_4 = \dfrac{I}{T}\sin(\alpha-\phi)$,

$t_6 = \dfrac{I}{2T}[\cos(\alpha-\phi) - \sin(\alpha-\phi) - \dfrac{q_o}{R}\dfrac{\cos\phi}{\cos\alpha}]$

with minimum

$$\sum_{i=1}^{8} t_i = -\dfrac{I\,q_o\,\cos\phi}{T\,R\,\cos\alpha}$$

<u>Region $C_{\alpha < \phi}$</u>: $0 < \alpha < \phi$, $\dfrac{-R\cos\alpha}{\cos\phi}$ $[\cos(\phi-\alpha) + \sin(\phi-\alpha)] \leq q_O < -\dfrac{R(1-\tan\phi)}{(1-\tan\alpha)}$

Solution (a unique solution for minimum):

$t_1 = t_2 = t_3 = t_4 = t_7 = 0$,

$$t_5 = \frac{I}{2T\cos\alpha(\cos\alpha+\sin\alpha)} \; [\cos(\phi-\alpha) + \sin(\phi-\alpha) + \frac{q_O\cos\phi}{R\cos\alpha}\;]$$

$$t_6 = \frac{I\cos\phi}{T(\cos\alpha+\sin\alpha)} \; [1 - \frac{q_O}{R}\tan\alpha]$$

$$t_8 = \frac{-I\cos\phi}{2T\cos\alpha} \; [(1 - \tan\phi) + \frac{q_O}{R}(1 - \tan\alpha)]$$

with minimum

$$\sum_{i=1}^{8} t_i = \frac{I}{T\cos\alpha(\cos\alpha+\sin\alpha)} \; [\cos(\phi-\alpha) + \sin(\phi-\alpha) - \frac{q_O}{R}\cos\phi\tan\alpha(\cos\alpha-\sin\alpha)]$$

<u>Region $D_{\alpha \leq \phi}$</u>: $0 \leq \alpha \leq \phi$, $-\infty < q_O < \dfrac{-R\cos\alpha}{\cos\phi}$ $[\cos(\phi-\alpha) + \sin(\phi-\alpha)]$

Solution (a non-unique solution for minimum):
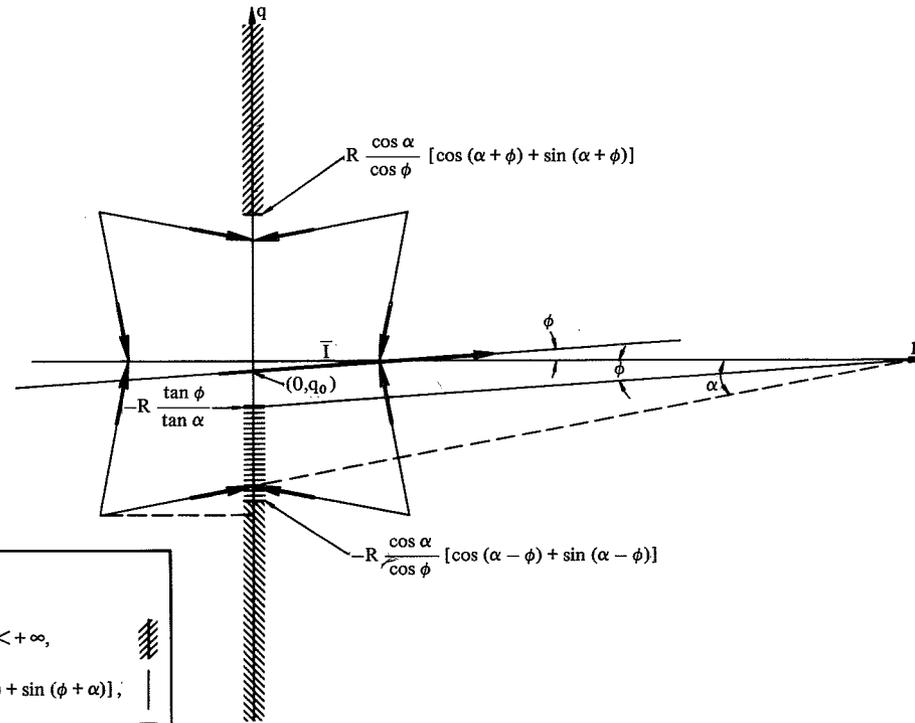
$t_1 = t_3 = t_4 = t_5 = t_7 = 0$,

$$t_2 = \frac{I}{2T} \; [-\cos(\phi-\alpha) - \sin(\phi-\alpha) - \frac{q_O}{R}\frac{\cos\phi}{\cos\alpha}\;]$$

$$t_6 = \frac{I}{2T} \; [\cos(\phi-\alpha) - \sin(\phi-\alpha) - \frac{q_O}{R}\frac{\cos\phi}{\cos\alpha}\;]$$

$$t_8 = \frac{I}{T}\sin(\phi-\alpha)$$

with minimum

$$\sum_{i=1}^{8} t_i = -\frac{I}{T}\frac{q_O}{R}\frac{\cos\phi}{\cos\alpha} \;.$$

$$R \frac{\cos \alpha}{\cos \phi} \ [\cos (\alpha + \phi) + \sin (\alpha + \phi)]$$

$$-R \frac{\cos \alpha}{\cos \phi} \ [\cos (\alpha - \phi) + \sin (\alpha - \phi)]$$

REGIONS FOR $0 \leqq \phi < \alpha \leqq \dfrac{\pi}{4}$

A: $R \dfrac{\cos \alpha}{\cos \phi} \ [\cos (\phi + \alpha) + \sin (\phi + \alpha)] < q_0 < +\infty,$

B: $-R \dfrac{\tan \phi}{\tan \alpha} \leqq q_0 \leqq R \dfrac{\cos \alpha}{\cos \phi} \ [\cos (\phi + \alpha) + \sin (\phi + \alpha)],$

C$_{\phi < \alpha}$: $-R \dfrac{\cos \alpha}{\cos \phi} \ [\cos (\alpha - \phi) + \sin (\alpha - \phi)] \leqq q_0 < -R \dfrac{\tan \phi}{\tan \alpha},$

D$_{\phi < \alpha}$: $-\infty < q_0 < \dfrac{-R \cos \alpha}{\cos \phi} \ [\cos (\alpha - \phi) + \sin (\alpha - \phi)],$

FIGURE 5 - REGIONS FOR $\phi < \alpha$

$$-R \frac{\cos \alpha}{\cos \phi} [\cos (\alpha + \phi) + \sin (\alpha + \phi)]$$

$-R \dfrac{1-\tan \phi}{1-\tan \alpha}$

$(0,q_0)$

$$-R \frac{\cos \alpha}{\cos \phi} [\cos (\phi - \alpha) + \sin (\phi - \alpha)]$$

REGIONS FOR $0 \leqq \alpha < \phi \leqq \dfrac{\pi}{4}$

A: $\quad R\dfrac{\cos \alpha}{\cos \phi} [\cos (\alpha + \phi) + \sin (\alpha + \phi)] < q_0 < +\infty$,

B: $\quad -R \dfrac{1-\tan \phi}{1-\tan \alpha} \leqq q_0 \leqq R \dfrac{\cos \alpha}{\cos \phi} [\cos (\alpha + \phi) + \sin (\alpha + \phi)]$,

$C_{\alpha < \phi}: \left| -R \dfrac{\cos \alpha}{\cos \phi} [\cos (\phi - \alpha) + \sin (\phi - \alpha)] \leqq q_0 < -R \dfrac{1 - \tan \phi}{1 - \tan \alpha} \right.$,

$D_{\alpha < \phi}: -\infty < q_0 < -R \dfrac{\cos \alpha}{\cos \phi} [\cos (\phi - \alpha) + \sin (\phi - \alpha)]$,

FIGURE 6 - REGIONS FOR $\alpha < \phi$

FIGURE 7 - REGIONS FOR $\phi = \alpha$
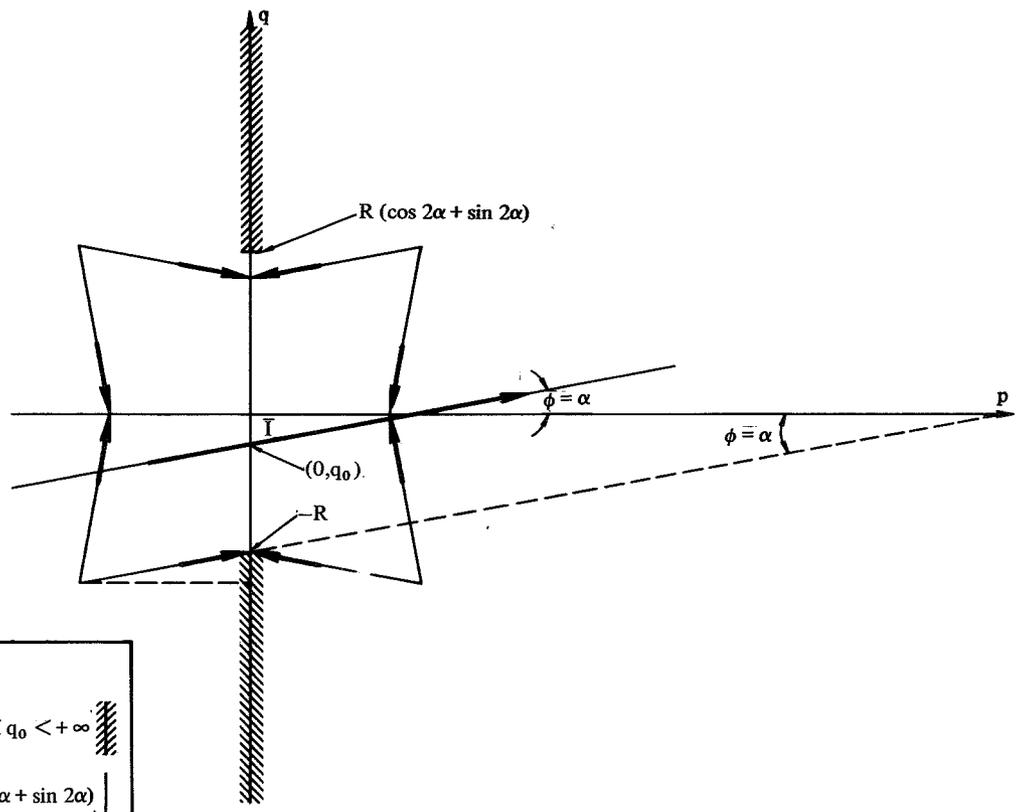
REGIONS FOR $\phi = \alpha$

A: $R (\cos 2\alpha + \sin 2\alpha) < q_0 < +\infty$

B: $-R \leqq q_0 \leqq R (\cos 2\alpha + \sin 2\alpha)$

$C_{\alpha=\phi}$ DISAPPEARS

$D_{\alpha=\phi} : -\infty < q_0 < -R$

The three regions, A, $D_{\phi < \alpha}$, and $D_{\alpha \leq \phi}$, where non-unique minimum solutions occur correspond to cases where at least one jet with a thrust component opposite to $\bar{I}$ must be fired to cancel the torque caused by the two that do have components in the direction of $\bar{I}$. This amounts to use of a pure couple, and pure couples in the appropriate direction can be equally well obtained by either or both of two opposing (i and i+4) pairs of jets. For example, the Region A solution given on page 22 indicates that jets 1 and 5 can be on simultaneously, but while on they form a pure couple that could as well have been formed by 3 and 7. Similarly, for Regions D, the couple formed by simultaneous firing of 2 and 6 could as well have been formed by 4 and 8. These regions are comparable to $\bar{I}$ lying outside the box of the simplified problem but, as can be seen from Figures 6, 7 and 8, do not correspond directly to projections of the· lines of action of the eight thrusters in the complete problem.

The case of non-intersecting planes where the line of action of $\bar{I}$ is at infinity and its magnitude is zero with known direction is handled in an identical fashion to the simplified problem. The magnitude of the required angular impulse is Iq_O (choose $\phi = 0$) and if the direction of $\bar{I}$ is such that q_O is $+\infty$ then Region A applies and $t_1 = t_5 = Iq_O/T$ $(2R \cos \alpha)$, which is an obviously appropriate answer.

DISCUSSION

The results given on pages 22, 23 and 24 form the basis for several possible automatic attitude maneuvering schemes to torque the Skylab about a specified axis with a minimum amount of CSM RCS fuel. First, the magnitude, direction, and location of the vector $\bar{I}$ are determined. It lies in the plane of the RCS quads, and if applied as a linear impulse would result in the required angular impulse about only the desired axis through the mass center. The coordinate system p-q is then chosen so that the angle ϕ between $\bar{I}$ and +p is between 0 and 45°, and the correct one of eight possible transformations between the Skylab coordinate system and p-q is recorded. This latter step is required so that the jet numbers and their firing times, which are found in the p-q system, can be identified in the unique Skylab system. With ϕ and q_O evaluated, the appropriate region is chosen and the optimum feasible solution for thruster firing times is known.

The question of the sequence of thruster firings that should be adopted to give the correct total times has not been addressed in this paper. One can think of many possible sequences,

such as firing each thruster in turn, firing all for variable
time increments in a repetitive sequence, or firing each for a
different number of equal-time increments. The particular
sequence chosen will affect the dynamics of the Skylab because
the applied moment at any time is (except in special cases)
never about the desired axis. (It is the time-integral of the
moment--the total angular impulse--that is applied about the
desired axis.) Further, for the instantaneous angular accelera-
tion vector to coincide with the applied torque vector, the
torque axis must be an axis of principal moment of inertia and
there must be no existing angular velocity components about
other axes. These facts can be deduced from Euler's equation
for rotation of a rigid body. However, for known Skylab applica-
tions, the effects of the $\underline{\omega} \times I\underline{\omega}$ terms in Euler's equation can
be shown to be small as long as an appropriate thruster firing
sequence is chosen.

The thruster firing sequence must be based on the
desired dynamic behavior. For instance, if the thrusters are
used to desaturate the ATM control moment gyros, these will
counter the applied torque during firing so the vehicle will
not rotate, and any sequence will suffice. On the other hand,
if the thrusters are fired to spin-up Skylab B about its axis
of maximum moment of inertia for an artificial gravity experi-
ment, the time-average of applied torques should be about the
correct axis over relatively short time increments so excessive
angular velocity does not develop about incorrect axes. Taken
to a limit, there would be infinite repetition of infinitesimal
firing times giving the effect of throttleable thrusters applying
the correct torque all of the time. However, there is a physically
constrained minimum thruster on-time of .014 seconds, and with
this so-called minimum impulse bit, maximum specific impulse of
the thrusters is not achieved. To approach maximum specific
impulse, minimum firing times on the order of a second are desir-
able. Simulations using a sequence that repeats every 10 seconds,
with one jet firing a minimum of 0.7 seconds and another a maxi-
mum of 8.4 seconds, have indicated that the dynamic behavior of
Skylab B during spin-up to 4 RPM is satisfactory (Reference 1).
The simulations include the cross-product terms in Euler's equa-
tions and parametrically investigate the effect of errors in cal-
culating the location and direction of the linear impulse vector,
$\overline{I}$, with respect to the Skylab center of mass and axis of maximum
moment of inertia.

<div style="text-align: center;">

W. W. Hough

L. D. Nelson
</div>

WWH-1022
LDN-1033 -jf

Attachment
  Reference

BELLCOMM, INC.

REFERENCE

1. L. E. Voelker, "Spinup of Skylab B for Artificial Gravity", TM-70-1022-14, Bellcomm, Inc., to be released.