TM-70-1031-3

TECHNICAL MEMORANDUM

TIME SERIES ANALYSIS PACKAGE (TSAP) FOR TERMINAL USE

Bellcomm

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COVER SHEET FOR TECHNICAL MEMORANDUM

TITLE- Time Series Analysis Package (TSAP) for Terminal Use

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DATE- December 31, 1970

FILING CASE NO(S)- 103-7, 630

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FILING SUBJECT(S) (ASSIGNED BY AUTHOR(S))- Time Series Analysis
Remote Terminal

ABSTRACT

The time series analysis package (TSAP) presented here in the form of a user's manual, is a set of eleven interactive, functionally dependent time series programs which are accessible in any order from a remote terminal. TSAP has been designed so that an analyst can conduct a basic portion of the statistical calculations involved in multiple time series analysis during one session at a terminal. This allows the analyst to avoid the waiting times between outputs that occur with standard batch processing. In addition, it takes advantage of the sequential nature of the statistical computations involved in the analysis of time series.

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INTRODUCTION

The result of any particular statistical computation produces two types of information, one type leading to a conclusion and the other suggesting a particular direction for further analysis. With respect to the analysis of time series, the analyst usually has a general idea of the avenues of investigation likely to provide a broad preliminary understanding of the particular problem, e.g., how two series are statistically related or what is the statistical mechanism generating a single series. Since the direction of the investigation after the completion of any particular segment of the computations depends on the results of that segment, he must view the results of a large number of intermediate steps of the analysis before he can reach a point where a detailed evaluation and summary of the data can begin. Unfortunately,each segment usually requires at least one run on a computer and, more often than not, several hours elapse before the results can be viewed. On the other hand, the decision concerning the next step of the analysis can usually be made immediately, say in the length of time it would take the results to print out at a terminal.

TSAP, the terminal time series analysis package presented here, is designed to eliminate this miss-match between the computer and analyst by furnishing him with a set of interactive, functionally dependent time series programs; these programs are accessible in any order from a remote terminal.

The particular set of programs which are presently available in TSAP are necessary for the first steps in the analysis of any time series data. Further reduction of such data may require programs that are best kept in a batch mode (e.g., the method using the cross-spectral information of several series to improve the least squares estimates of the coefficients in a multiple time series regression). More programs will be added as they prove useful or are required in the solution of a particular problem. For this reason TSAP is provided with an index (cf., Appendix III) that may be consulted to determine what programs are available. The Bellcomm Univac 1108 implementation of TSAP is in file RRS*TSAP.

Descriptions of the TSAP programs are given in section 1 together with a brief nontechnical explanation of their use or meaning. This section also contains a diagram indicating the principal ways the programs in TSAP may be combined. The way in which data files are generated and used is described in Section 2.

Section 3 contains a series of examples (pages, 10, 22 and 30) that illustrate most of the present capabilities of TSAP and is intended as a users guide.[1]

Section 4 contains a discussion of program techniques and the flow charts of each of the programs; the listings are given in Appendix III. Some elementary remarks concerning time series together with the computational formulae used in the various programs are given in Appendix I. Details concerning the various filters available in TSAP are given in Appendix II.

1.0 PROGRAMS

This section contains a list of the programs in TSAP together with a brief description of their use.

(1) Data Transformations and Filters (FILTER)

The purpose of this program is to allow the user to prepare the data for time series analysis. For example, this might include transforming the series into the logarithms of its absolute values and then centering the resulting series at its mean in an effort to obtain a series that approximates a zero mean Gaussian time series. A series could be pre-whitened for spectral estimation by applying an appropriate high-pass filter; alternately, a trend in the series could be estimated or a cycle removed by using one or more of the available filters. The program is designed so that any subset of its transformations or filters may be applied sequentially to a particular time series with a single request for the element labeled FILTER. The transformations that are available in this program are listed when the user calls @filter; details concerning these transformations are given in Appendix II.

(2) Auto-Covariance and Auto-Correlation (ACF)

The sample auto-correlation function calculated with this program is a measure of the "memory" or inertia of the process generating the time series; this information can be used to some extent in model discrimination. The primary purpose of the auto-covariance function is in calculating the spectrum of the series.

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[1] Although some remarks concerning statistics have been made to prevent misunderstanding and to motivate the use of a particular sequence of programs, this section is not meant to be tutorial in statistics; it assumes an elementary knowledge of this area.

(3) Partial-Auto-Correlation Function (PACF)

This function describes the correlation between two distinct members of a time series with the linear affect of intervening members removed. The program allows the user to calculate (up to) the first sixty values of the partial-auto-correlation function. It also computes the residual variance of auto-regression (a.r.) models,

$$y(t) = \alpha_0 + \alpha_1 y(t-1) + \ldots + \alpha_p y(t-p) + \varepsilon(t) \quad ,$$

up to order p = 60. On the basis of these results the user may determine the order of the best fitting a.r. model. Once the order is furnished, the program will print out the coefficients of the corresponding a.r. process. The appropriateness of the model may be judged by submitting the residuals of the fitted a.r. model to ACF and ASF.

(4) Auto-Spectrum (ASF)

An estimate of the auto-spectrum of a stationary time series is calculated and plotted at the terminal; if requested, its upper and lower 90% confidence limits are superimposed on the spectrum plot. One of the main purposes of this program is to determine which frequency bands contribute significantly to the total variance of the series. Another is in suggesting and testing the appropriateness of parametric models for the series being studied.

(5) Cross-Covariance (XCF)

This program is used primarily as data for the cross-spectral routine.

(6) Cross-Spectrum (XSF)

This program calculates estimates of the co-spectrum and the quadrature spectrum of pairs of (jointly stationary) time series. These quantities measure the covariance between the "in-phase" and the "$\pi/2$ out-of-phase" frequency components of the two series, respectively. The co- and quadrature spectrums, together with results from ASF, are used to calculate the coherence and phase of the pair of series. The coherence is a measure of linear relationships between frequency components of the series, while the phase measures lead and lag relationships. The arc-tanh of the coherence is plotted to obtain confidence bands which are independent of frequency.

(7) Least Squares Regression (LSRA and LSRP)

Least squares regression is included in the system for several purposes. It provides a method of estimating the trend component of a series, furnishes the standard errors of auto-regression coefficients (the order of the a.r. is determined with PACF) and gives preliminary estimates of the coefficients in a multiple regression on time series.

Two separate least squares regression programs[1] are included in TSAP: LSRA and LSRP. LSRA is a forward stepwise regression program that includes an analysis of variance table at each step together with a table of observed values, expected values and percent deviation, printed out at the request of the user. LSRP is a backward regression program whose main feature is that it yields predicted values with upper and lower prediction limits.

(8) Save

After an application of the program FILTER the user is asked whether or not he wants the transformed series stored. If the answer is yes and the data format is specified, then the series is automatically stored, but the user must supply the statement @SAVE FILTER,[series name] to be able to reference the series for later use.[2] Similar statements are required after executing the LSRA, LSRP or COMB programs (e.g., @SAVE LSRA,[series name]).

(9) Combination of Series (COMB)

Given two time series of the same length, this program enables the user to perform term by term addition (ADD), subtraction (SUB), multiplication (MPY) or division (DIV) of the two series. COMB also allows one to concatenate (CON) series of arbitrary length.

(10) MATH Conversion (MATHCON)

In some applications, the user requires a series of values of some known or constructed function and a way to easily combine this series algebraically with an observed series

---

[1] Both of these programs are modifications of regression programs available in the UNIVAC Stat-pack program library.

[2] This control statement stores the series in the TPF; if the user has a catalogued file he may store the series in this file by typing @SAVE FILTER,[FILE.SERIES NAME].

being analyzed with TSAP. This can be accomplished without leaving the terminal by generating the required series with MATH and combining it in TSAP (using COMB) with the observed series. MATHCON eliminates the incompatibilities in I/O[1] between the two series.

Additional programs that are available for batch runs, but have not been included in TSAP at the present time include

(A) Time Series Regression (TSR)[2]

This program combines the preliminary least squares estimates in a multiple regression on time series with cross-spectral information between series to produce asymptotically efficient estimates of the regression coefficients.

(B) Histogram and Scatter Plot Programs (HIST, SCAT)[2]

Work is in progress on the following programs: a means of estimating mixed regression, auto-regression and moving average models and a program for forecasting with such models.

The efficiency and effectiveness of TSAP would be enhanced with the addition of CRT (Cathode Ray Tube) display capabilities to allow more outputs. This would provide the user with a more comprehensive system containing programs dealing with graphics.

Another possible improvement which will be looked into concerns the formatting structure. At present each input or output series requires a format; this reduces data format conversions at the expense of answering questions. An alternative method would be to have a fixed format, say G20.10, in which all data is read and written. This would reduce the number of user responses but also reduce the generality of the input data structures. A way around this is to add an option, say F, to allow the present dialogue to be used.

The following diagram indicates the ways in which the TSAP programs may be linked together in applications. These will be further described by means of the examples given in Section 3.

---

(1) MATH "SAVED" elements are in fixed point and floating point notation, using *10**n instead of the FORTRAN E+n. A format of (G20.10) will suffice to read the converted series.

(2) TSR will be documented in the near future; HIST is a corrected UNIVAC Stat-pack routine and SCAT is obtainable from R. R. Singers.

TSAP

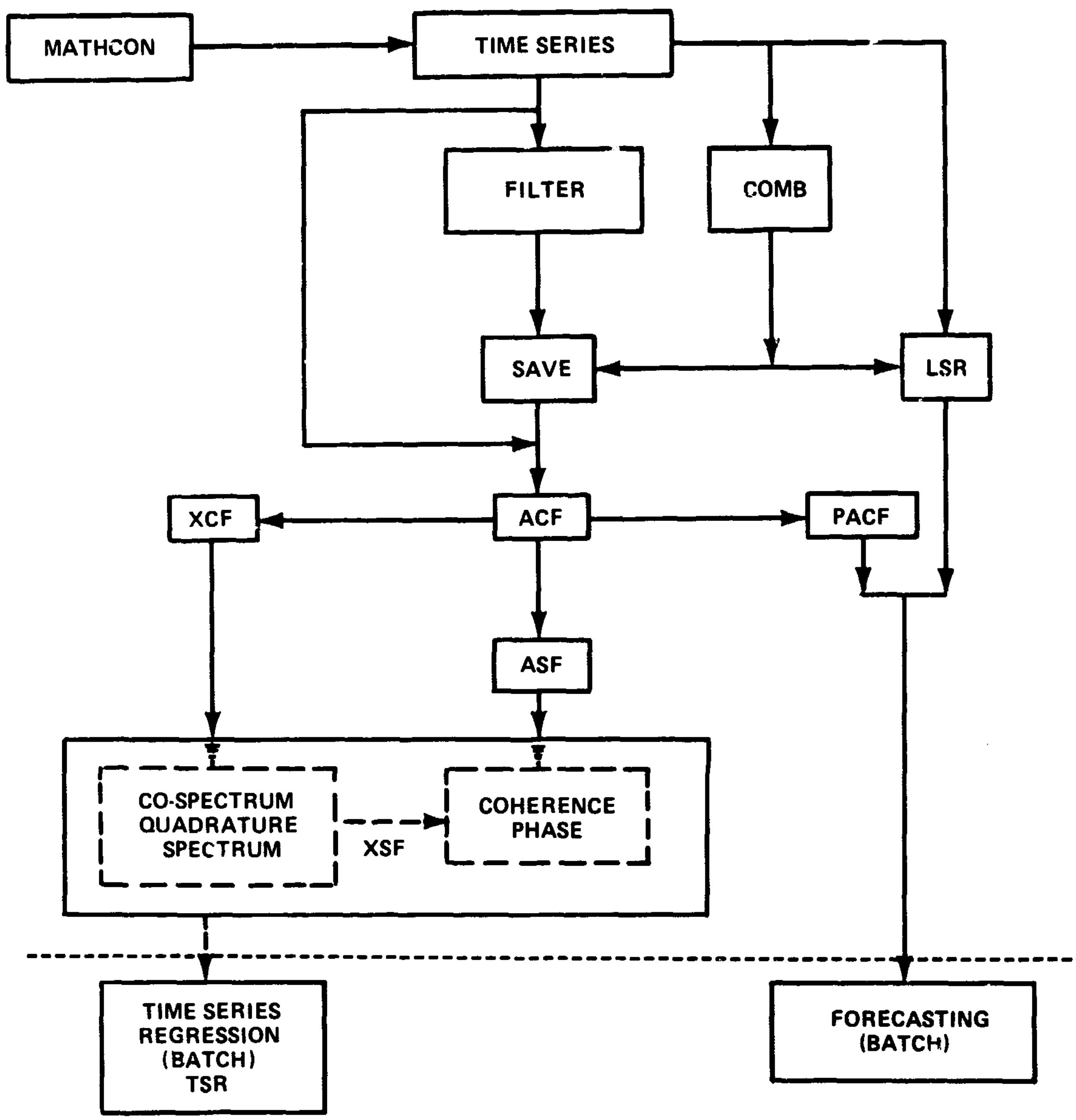

FIGURE 1

2.0 GENERATION AND USE OF STORED DATA ELEMENTS

The first step in applying TSAP to a particular set of time series data is to make this data accessible from the terminal. This can be accomplished by using the EXEC 8 control statements @ELT and @ADD. The control statement ELT allows the user to place the data into the machine, via the card reader or the terminal, with a name attached to it. This name together with the @ADD statement makes the data available for use in TSAP.

The ELT processor allows one to generate a data element in a file using the following control card (starting in column 1):

@ELT,I‸QUAL*FILE.ELEMENT

where ‸ indicates one or mare spaces.

QUAL*FILE. is the name of the catalogued file[(1)] into which the user wants the data stored. This field may be left blank, in which case the user temporary file, TPF$, is used.

ELEMENT is the name of the data series. The data follows immediately after this card and the element is terminated when the next control card is read.

When the data is asked for within TSAP, the element can be obtained by using the following control card (starting in column 1):

@ADD‸QUAL*FILE.ELEMENT

The following are four examples of ELT processor use:

1) For external card deck

(a) without previously catalogued file

1. @RUN,/NR‸INIT,ID,PROJECT,15,150

2. @ASG,CP‸QUAL*FILE.,F2

3. @ELT,IL‸QUAL*FILE.ELEMENT

4. (Data cards)

5. @FIN

---

(1) See [14] for definition of catalogued file.

(b) with previously catalogued file

1. @RUN,/NR^INIT,ID,PROJECT,15,150
2. @ASG,A^QUAL*FILE.
3. @ELT,IL QUAL*FILE.ELEMENT
4. (Data cards)
5. @FIN

2) For terminal input

(a) without previously catalogued file

1. @RUN^INIT,ID,PROJECT,70
2. @ASG,CP^QUAL*FILE.,F2
3. @ELT,I^QUAL*FILE.ELEMENT
4. (Type in data)

(b) with previously catalogued file

1. @RUN^INIT,ID,PROJECT,70
2. @ASG,A^QUAL*FILE.
3. @ELT,I^QUAL*FILE.ELEMENT
4. (Type in data)

It is necessary for all data to be in FORTRAN formatted I/O.[1]

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[1] For information on formats see [13].

3.0 USING TSAP

The mechanics of using TSAP will now be explained by means of several examples. These examples fall into essentially three groups and begin on pages 10, 22 and 30.

The first group involves a single time series and uses that portion of TSAP indicated in Figure 2. The first example within this group illustrates spectral estimation using a Tukey window at two different bandwidths. These spectra together with their 90% confidence bands are plotted against frequency in cycles per unit time. The spectrum of the time series is then estimated by means of both the Daniell and Tukey windows and plotted against period with no confidence bands. The second and third examples within the first group illustrate two types of Goodman filters applied to the series and the spectra of the resulting series.

To aid the reader, an arrow has been added to the left margin of the terminal printout each time a new request is made for a TSAP program.

Finally, it should be remembered that each group of examples was executed in one session at a remote terminal; depending on the user, the amount of his time required for computations similar to those in one of these groups would be between 10 and 30 minutes.

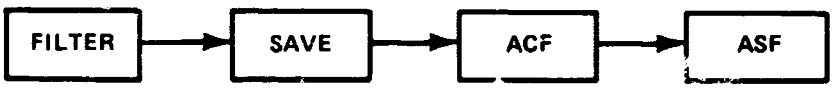


FIGURE 2

We will assume that the series has been stored in the machine, as explained in the previous section, under the name y10e; it contains 362 terms.

In order to activate the terminal, assign the file and copy it over into a temporary file we type the terminal number, say

BTL1,

followed by

```
@RUN‸INITIALS,ACCOUNT,PROJECT,MAXTIME,MAXPAGES

@ASG,AX‸RRS*TSAP.

@COPY,P‸RRS*TSAP.,TPF$.
```

The first program called is @filter(1):

```
➔ @filter

 'FILTER' CONTAINS THE FOLLOWING FILTERS:
 CENTER,EXP,LOG,DIFF,10**,LOG10,SQRT,LAG,ABS,TUKEY(HIGH AND LOW
 PASS),GOODMAN,SCALE.
 TO TERMINATE THE FILTER SEQUENCE,REPLY WITH 'END'.
 THE FINAL SERIES IS STORED ON UNIT 8(IF DESIRED).
 LENGTH OF SERIES TO BE READ IN?
362                              ←  The entire series that is stored
 INPUT DATA FORMAT?                 in the data file must be read in
(4e15.8)                            at this point.
 INPUT DATA:
@add yl0e
 LENGTH OF SERIES TO BE USED?       Here we could have deleted any
362                              ←  number, m, of consecutive terms from
 MEAN=    .37433816+01              the end of the series by typing 362-m.
 LENGTH=   362

 FILTER?                            After each request for a filter the
center                              mean of the filtered series together
 MEAN=    .35338186-05           ←  with its length is automatically
 LENGTH=   362                      printed out.

 FILTER?
end
 DO YOU WANT TO SEE PART OF THE SERIES(YES/NO)?
no
 DO YOU WANT THE FINAL SERIES STORED(YES/NO)?
yes
 OUTPUT DATA FORMAT?
(4e15.8)
 SERIES STORED.
 FILTER TERMINATES.
➔ @save filter, yl000
```

---

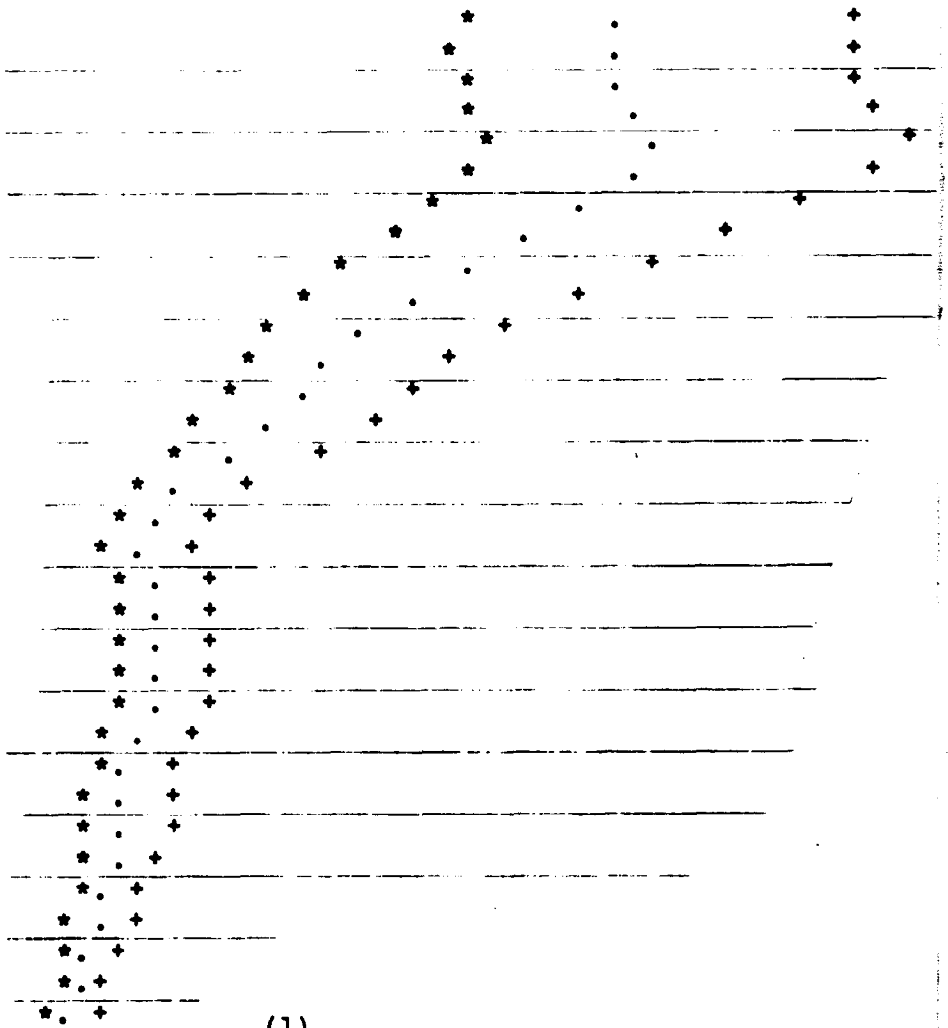
(1) The underlined quantities are the users responses to the preceeding program questions. The italicized type indicates explanatory comments that have been placed onto the terminal printout to aid the reader of this memorandum.

The name of the yl0e series after the above transformations is yl000. The auto-correlation function of yl000 is calculated next:

```
→ @acf

 ACF COMPUTES THE AUTO-COVARIANCE/CORRELATION FUNCTION OF A SERIES.
 THE COVARIANCES ARE STORED ON UNIT 1 IN BINARY(IF DESIRED)
 IN CONSECUTIVE ORDER.

 DO YOU WANT COVARIANCE(COV),CORRELATION(COR) OR END?
cor
 LENGTH OF SERIES?
362
 NUMBER OF LAGS(ZERO INCLUDED)?
41
 INPUT DATA FORMAT?
(4e15.8)
 INPUT DATA:
@add yl000
   0  .10000000+01    13  .18489365-01    26  .11289490+00
   1  .46415033+00    14  .11077084+00    27  .15901410+00
   2  .17615860+00    15  .17462706+00    28  .11500745+00
   3  .49235427-01    16  .35404109-01    29  .93957893-01
   4 -.50011224-01    17 -.16164341-01    30  .97580075-01
   5 -.26090787-01    18  .35023679-03    31  .59549038-01
   6 -.19535766-01    19  .60941407-01    32 -.80215645-02
   7 -.31009051-01    20  .61706792-01    33 -.28728484-01
   8 -.47952917-01    21  .17506226-01    34  .53134954-01
   9  .23769772-02    22 -.79266704-01    35  .18498416-01
  10  .21561115-01    23 -.79293265-01    36  .18898459-01
  11  .10452646-02    24  .13103552-01    37 -.31084192-02
  12  .25509858-02    25  .82080608-01    38  .88336019-01
                                          39  .14070644+00
                                          40  .96740634-01
 VARIANCE=    .34657791+00
 MEAN=    .35338186-05
 DO YOU WANT THE COVARIANCES STORED(YES/NO)?
yes
 COVARIANCE STORED.

 DO YOU WANT COVARIANCE(COV),CORRELATION(COR) OR END?
end
 ACF TERMINATES.
→ @asf

 ASF COMPUTES THE AUTO SPECTRUM FUNCTION.
 IT IS ASSUMED THAT ACF WAS CALLED AND THE DESIRED COVARIANCES STORED.
 THE AUTO-SPECTRA ARE STORED ON UNIT 2(IF DESIRED) CONSECUTIVELY.
 DATA SPACING?
1.
 WINDOW TO BE USED(TUKEY/DANIELL)?
```

```
tukey
 BANDWIDTH PARAMETER?
18
 NUMBER OF FREQUENCY POINTS TO BE VIEWED?
33
 DO YOU WANT CONFIDENCE LIMITS(YES/NO)?
yes
 DO YOU WANT FREQUENCY(FREQ.) OR PERIOD(PERIOD) DISPLAYED?
freq.
 AUTO-SPECTRUM:
```

FREQ.	SPECTRUM
.0000	.74730+00
.0156	.74350+00
.0313	.74541+00
.0469	.76602+00
.0625	.78415+00
.0781	.76448+00
.0938	.70337+00
.1094	.62974+00
.1250	.56432+00
.1406	.50291+00
.1563	.44168+00
.1719	.39207+00
.1875	.35903+00
.2031	.32513+00
.2188	.27371+00
.2344	.21620+00
.2500	.17947+00
.2656	.17243+00
.2813	.18018+00
.2969	.18700+00
.3125	.18969+00
.3281	.18850+00
.3438	.18017+00
.3594	.16534+00
.3750	.15253+00
.3906	.14782+00
.4063	.14651+00
.4219	.14004+00
.4375	.12689+00
.4531	.11207+00
.4688	.99940-01
.4844	.91983-01
.5000	.89126-01



THE 90% CONFIDENCE LIMITS FOR NU=57[1] ARE: CU= 1.39722 , CL= .75097

[1] NU, the number of degrees of freedom of the spectral estimator, is defined in Appendix I.

```
 THE SQRT OF VARIANCE RATIO IS:  19.311%
 DO YOU WANT THE SPECTRUM STORED(YES/NO)?
no
 DO YOU HAVE ANOTHER SERIES(YES/NO)?
yes
 IS THE SERIES THE SAME(YES/NO)?
yes(1)
 DATA SPACING?
1.
 WINDOW TO BE USED(TUKEY/DANIELL)?
tukey
 BANDWIDTH PARAMETER?
33
 NUMBER OF FREQUENCY POINTS TO BE VIEWED?
33
 DO YOU WANT CONFIDENCE LIMITS(YES/NO)?
yes
 DO YOU WANT FREQUENCY(FREQ.) OR PERIOD(PERIOD) DISPLAYED?
freq.
```

---

(1) This illustrates the usual method of calculating the spectrum at successively smaller bandwidths ("window closing") in order to determine the structure of the true but unknown spectrum.

```
AUTO-SPECTRUM:
 FREQ.     SPECTRUM
  .0000   .88953+00                      *        .                   +
  .0156   .73221+00                   *      .               +
  .0313   .62610+00                *     .            +
  .0469   .73374+00                   *      .               +
  .0625   .90672+00                       *         .                 +
  .0781   .84932+00                      *        .                +
  .0938   .62948+00                 *     .             +
  .1094   .59679+00                *     .            +
  .1250   .56681+00               *     .           +
  .1406   .56516+00               *     .           +
  .1563   .45911+00             *    .        +
  .1719   .26265+00        *  .     +
  .1875   .35751+00           *  .       +
  .2031   .45512+00             *    .        +
  .2188   .27104+00         * .     +
  .2344   .12386+00     * .+
  .2500   .15475+00      * .  +
  .2656   .20220+00       * .   +
  .2813   .19860+00       * .   +
  .2969   .16452+00      * .  +
  .3125   .16827+00      * .   +
  .3281   .22187+00        * .   +
  .3438   .20527+00       * .    +
  .3594   .14452+00      *.  +
  .3750   .12858+00     * . +
  .3906   .14862+00      *.   +
  .4063   .16991+00      * .   +
  .4219   .14789+00      *.  +
  .4375   .11107+00     *.  +
  .4531   .10553+00     *. +
  .4688   .11495+00     *.  +
  .4844   .91251-01     *.+
  .5000   .64649-01    *.+
 THE 90% CONFIDENCE LIMITS FOR NU=30 ARE:  CU=    1.63145 ,CL=    .68925
 THE SQRT OF VARIANCE RATIO IS:  26.148 %
 DO YOU WANT THE SPECTRUM STORED(YES/NO)?
no
```

In this last example we see that the square root of the variance ratio is about 26%; that is, fractional errors of about 26% in either direction can be expected. It was decided that with this length of series it would be difficult to obtain both high resolution (small bandwidth) and reliability. However, suppose for illustration that a bandwidth parameter of 33 gives us the proper resolution and then compare the spectral estimate obtained using the Daniell window with that obtained using the Tukey window.

```
@asf,n(1)
 DATA SPACING?
1.
 WINDOW TO BE USED(TUKEY/DANIELL)?(2)
daniell
 BANDWIDTH PARAMETER?
33
 NUMBER OF COVARIANCES TO BE USED?
362
 NUMBER OF FREQUENCY POINTS TO BE VIEWED?
33
 DO YOU WANT CONFIDENCE LIMITS(YES/NO)?
no
 DO YOU WANT FREQUENCY(FREQ.) OR PERIOD(PERIOD) DISPLAYED?
period
```

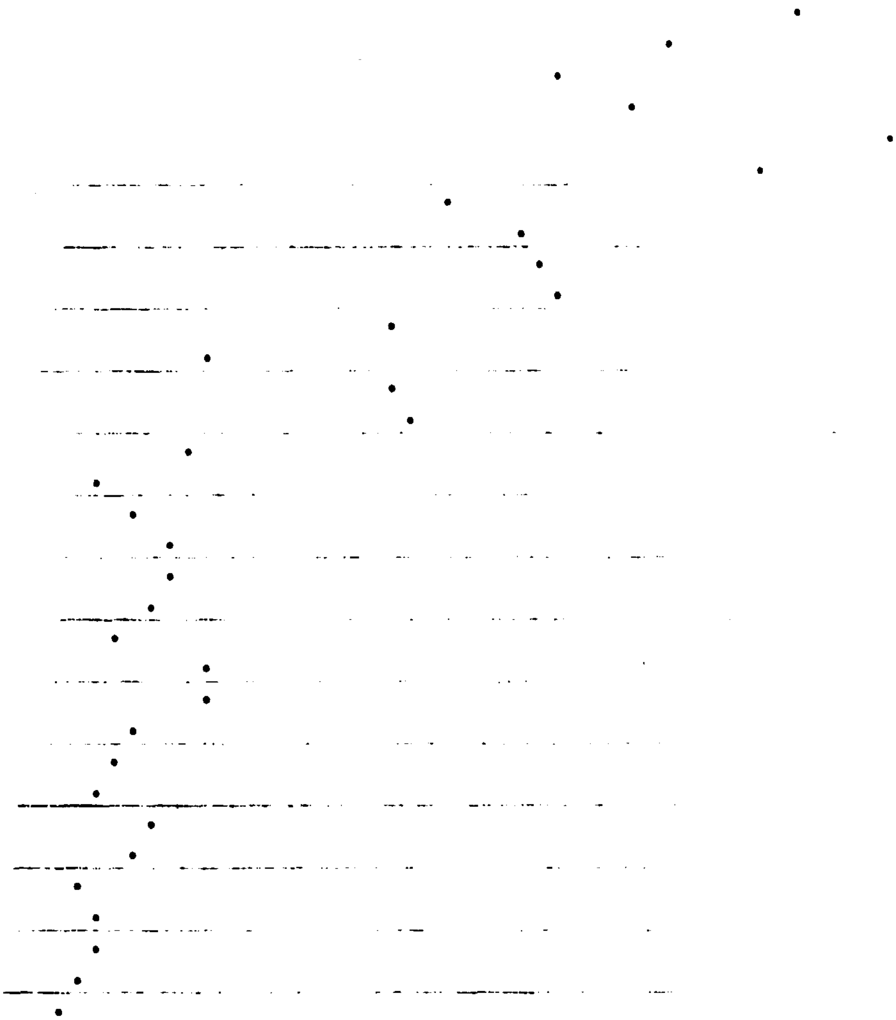
---

(1) The "n" option is supplied to suppress the initial commentary for the experienced user.

(2) If there are N terms in the series, then to employ the Daniell window, N covariances (including the variance) must be calculated in ACF and used in responding to the 4th question in ASF. If any number of covariances less than this number are either calculated or used, a request for the Daniell window will yield a truncated Daniell window.

AUTO-SPECTRUM:

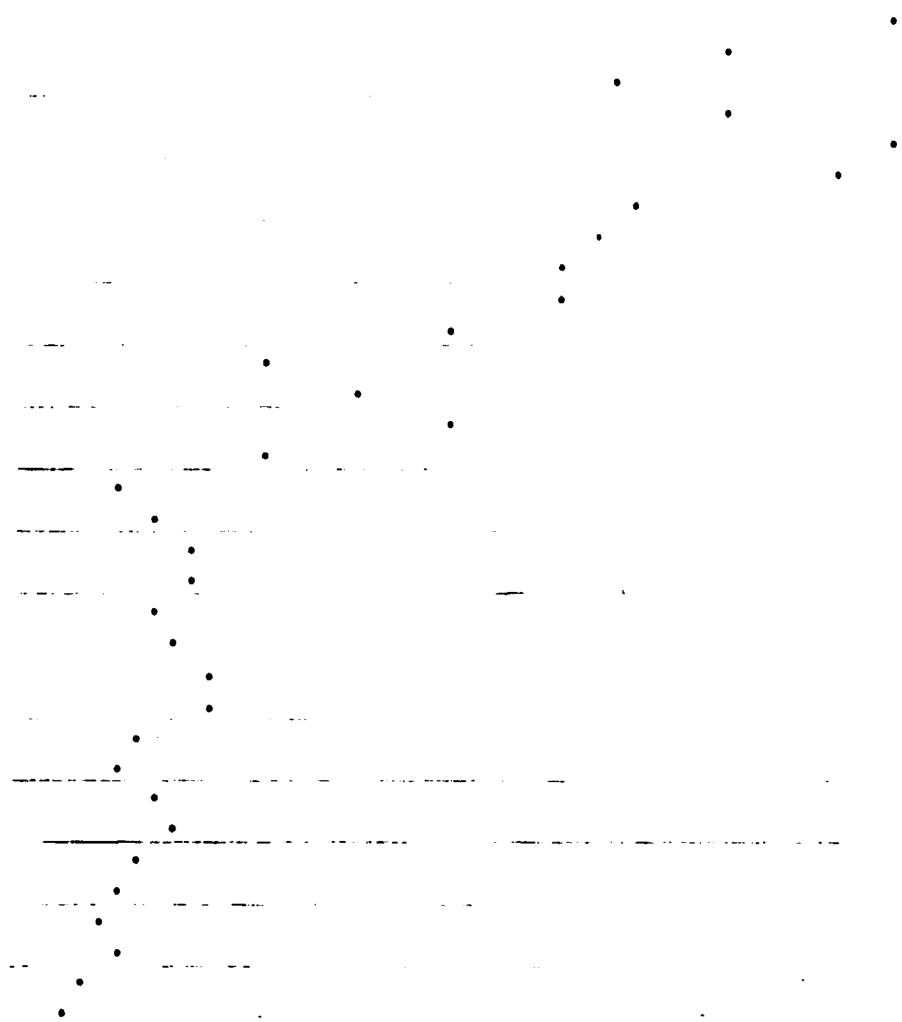
PERIOD	SPECTRUM
99.9999	.88676+00
64.0000	.75138+00
32.0000	.63410+00
21.3333	.69779+00
16.0000	.10041+01
12.8000	.85815+00
10.6667	.50042+00
9.1429	.57420+00
8.0000	.59914+00
7.1111	.62584+00
6.4000	.43762+00
5.8182	.23736+00
5.3333	.43552+00
4.9231	.45420+00
4.5714	.21520+00
4.2667	.10984+00
4.0000	.16376+00
3.7647	.19306+00
3.5556	.19815+00
3.3684	.16836+00
3.2000	.13186+00
3.0476	.24439+00
2.9091	.23136+00
2.7826	.14539+00
2.6667	.13965+00
2.5600	.12276+00
2.4615	.16503+00
2.3704	.15110+00
2.2857	.10209+00
2.2069	.11633+00
2.1333	.11029+00
2.0645	.95078-01
2.0000	.67187-01



DO YOU WANT THE SPECTRUM STORED(YES/NO)?
no
DO YOU HAVE ANOTHER SERIES(YES/NO)?
yes
IS THE SERIES THE SAME(YES/NO)?
yes
DATA SPACING?
1.
WINDOW TO BE USED(TUKEY/DANIELL)?
tukey
BANDWIDTH PARAMETER?
33
NUMBER OF FREQUENCY POINTS TO BE VIEWED?
33
DO YOU WANT CONFIDENCE LIMITS(YES/NO)?
no

```
 DO YOU WANT FREQUENCY(FREQ.) OR PERIOD(PERIOD) DISPLAYED?
period
 AUTO-SPECTRUM
```

PERIOD	SPECTRUM
99.9999	.88953+00
64.0000	.73221+00
32.0000	.62610+00
21.3333	.73374+00
16.0000	.90672+00
12.8000	.84932+00
10.6667	.62948+00
9.1429	.59679+00
8.0000	.56681+00
7.1111	.56516+00
6.4000	.45911+00
5.8182	.26265+00
5.3333	.35751+00
4.9231	.45512+00
4.5714	.27104+00
4.2667	.12386+00
4.0000	.15475+00
3.7647	.20220+00
3.5556	.19860+00
3.3684	.16452+00
3.2000	.16827+00
3.0476	.22287+00
2.9091	.20527+00
2.7826	.14452+00
2.6667	.12858+00
2.5600	.14862+00
2.4615	.16991+00
2.3704	.14789+00
2.2857	.11107+00
2.2069	.10553+00
2.1333	.11495+00
2.0645	.91251-01
2.0000	.64649-01



```
 DO YOU WANT THE SPECTRUM STORED(YES/NO)?
no
 DO YOU HAVE ANOTHER SERIES(YES/NO)?
no
 ASF TERMINATES.
```

We now return to the filter program and illustrate the use of the Goodman filters (cf., Appendix III):

```
@filter,n
 LENGTH OF SERIES TO BE READ IN?
362
 INPUT DATA FORMAT?
```

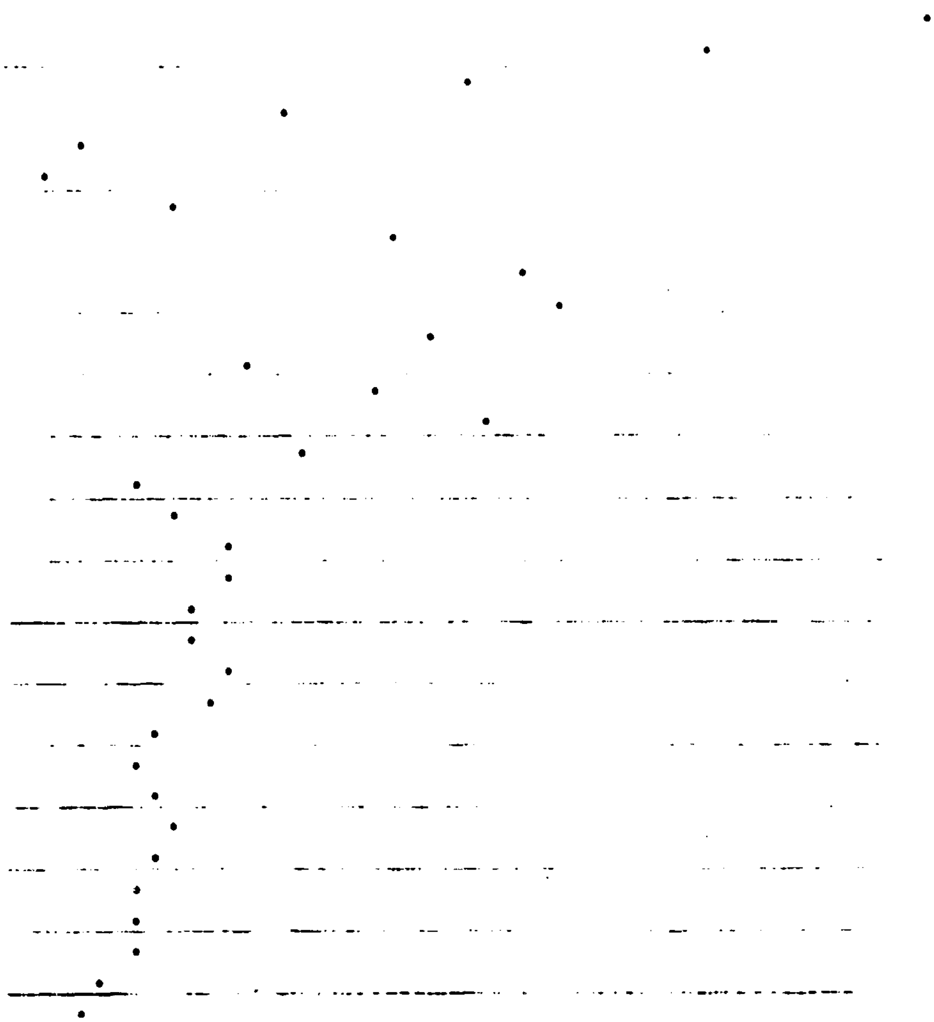
```
(4e15.8)
 INPUT DATA:
@add y1000
 LENGTH OF SERIES TO BE USED?
362
 MEAN=    .35338186-05
 LENGTH=   362

 FILTER?
goodman
 LENGTH OF MOVING AVERAGE(ODD NUMBER)?
41
 NUMBER OF TRIANGLES?
1
 DATA SPACING?
1.
 HIGH PASS(HIGH), LOW PASS(LOW), PASS BAND(PASS) OR REJECT BAND(REJECT)?
reject
 CENTER FREQUENCY?
.074
 MEAN=    -.25713835-01
 LENGTH=    322

 FILTER?
end
 DO YOU WANT TO SEE PART OF THE SERIES(YES/NO)?
no
 DO YOU WANT THE FINAL SERIES STORED(YES/NO)?
yes
 OUTPUT DATA FORMAT?
(4e15.8)
 SERIES STORED.
 FILTER TERMINATES.
→ @save filter, y1000rl3
```

Thus, after filtering the series consists of 322 terms; 20 terms off of each end of the series y10b were required to remove the spectral content at the frequency .074 (cycles/day) or at 13.5 (days). The filter is triangular and frequencies in a neighborhood, of length .1, centered at .074 are attenuated. The new series was saved, and its auto covariance function was calculated and stored. The known affect (Appendix I) of this filter on the spectrum may be observed by calculating its spectrum:

```
@asf,n
 DATA SPACING?
1.
 WINDOW TO BE USED(TUKEY/DANIELL)?
tukey
 BANDWIDTH PARAMETER?
33
 NUMBER OF FREQUENCY POINTS TO BE VIEWED?
33
 DO YOU WANT CONFIDENCE LIMITS(YES/NO)?
no
 DO YOU WANT FREQUENCY(FREQ.) OR PERIOD(PERIOD) DISPLAYED?
period
```

AUTO-SPECTRUM

PERIOD	SPECTRUM
99.9999	.95803+00
64.0000	.73078+00
32.0000	.46987+00
21.3333	.28421+00
16.0000	.77602-01
12.8000	.29739-01
10.6667	.17326+00
9.1429	.40914+00
8.0000	.53168+00
7.1111	.57579+00
6.4000	.44403+00
5.8182	.25050+00
5.3333	.37526+00
4.9231	.49792+00
4.5714	.30044+00
4.2667	.13079+00
4.0000	.16472+00
3.7647	.22356+00
3.5556	.21855+00
3.3684	.18145+00
3.2000	.18210+00
3.0476	.23152+00
2.9091	.20816+00
2.7826	.14103+00
2.6667	.13133+00
2.5600	.14793+00
2.4615	.16166+00
2.3704	.14852+00
2.2857	.11957+00
2.2069	.11832+00
2.1333	.12439+00
2.0645	.91969-01
2.0000	.63272-01



```
 DO YOU WANT THE SPECTRUM STORED(YES/NO)?
no
 DO YOU HAVE ANOTHER SERIES(YES/NO)?
no
 ASF TERMINATES.
```

In order to illustrate the use of a low pass filter and view its affect on the spectrum of the filtered series we return to FILTER and call an adaptation of Goodman's trapezoidal filter (cf., Appendix II):

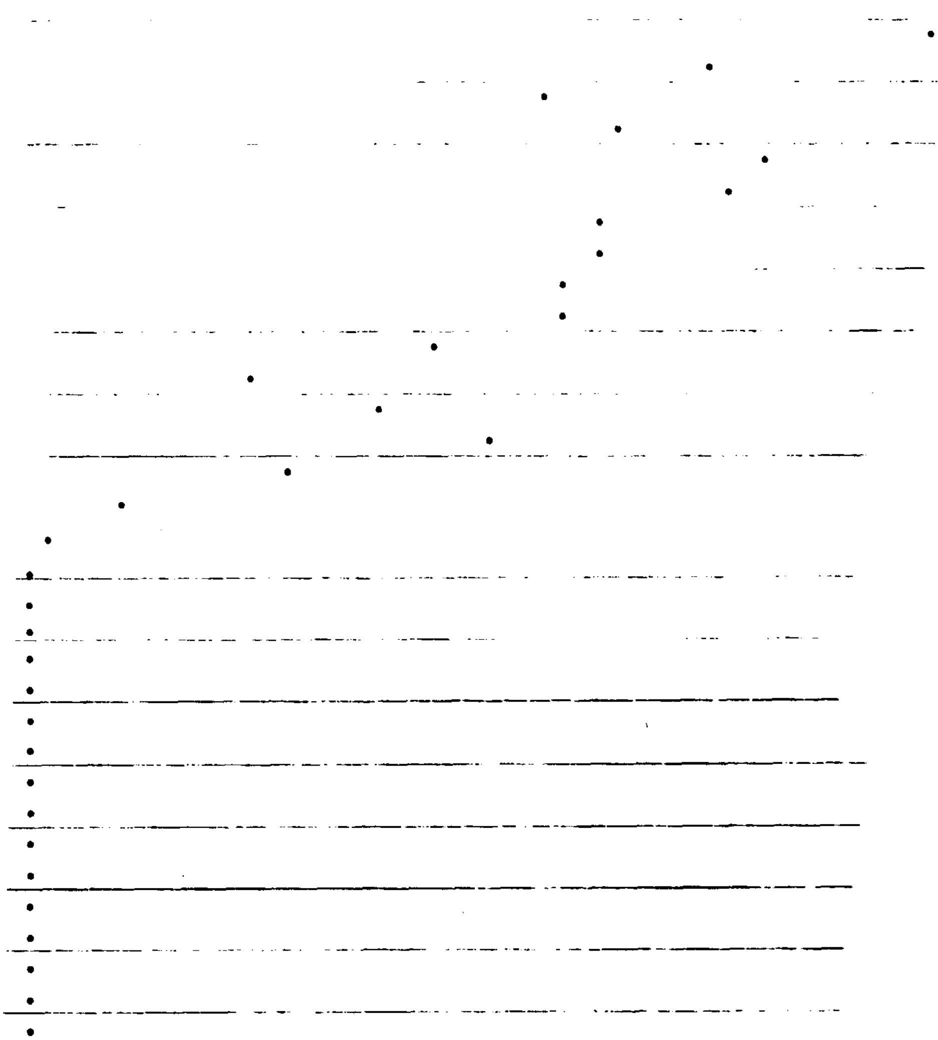
```
 FILTER?
goodman
 LENGTH OF MOVING AVERAGE(ODD NUMBER)?
41
 NUMBER OF TRIANGLES?
5
 HIGH PASS(HIGH), LOW PASS(LOW), PASS BAND(PASS) OR REJECT BAND(REJECT)?
low
 DATA SPACING?
1.
 CUT-OFF FREQUENCY IS    .2250000
 DO YOU WANT TO CHANGE INPUT VALUES(YES/NO)?
no
 MEAN=    -.27138643-01
 LENGTH=    322

 FILTER?
end
 DO YOU WANT TO SEE PART OF THE SERIES(YES/NO)?
no
 DO YOU WANT THE FINAL SERIES STORED(YES/NO)?
yes
 OUTPUT DATA FORMAT?
(4e15.8)
 SERIES STORED.
 FILTER TERMINATES.
@save filter, y10low
@acf,n

 DO YOU WANT COVARIANCE(COV),CORRELATION(COR) OR END?
cor
 LENGTH OF SERIES?
322
 NUMBER OF LAGS(ZERO INCLUDED)?
33
 INPUT DATA FORMAT?
(4e15.8)
 INPUT DATA:
@add y10low
```

0	.10000000+01	12	-.25210146-01	24	-.73984930-02
1	.74639370+00	13	.52257420-01	25	.10968976+00
2	.24772014+00	14	.16964401+00	26	.16955650+00
3	-.62077395-01	15	.18732763+00	27	.17834130+00
4	-.58770868-01	16	.83261370-01	28	.18380984+00
5	.31103434-01	17	-.17561690-01	29	.19426458+00
6	.13050401-01	18	-.22876046-02	30	.17436985+00
7	-.49688995-01	19	.92481258-01	31	.10661660+00
8	-.21093979-01	20	.12817274+00	32	.32856451-01
9	.69109890-01	21	.42624820-01		
10	.87358192-01	22	-.76860528-01		
11	.14558297-01	23	-.10038714+00		

```
 VARIANCE=    .26590697+00
 MEAN=  -.27138641-01
 DO YOU WANT THE COVARIANCES STORED(YES/NO)?
yes
 COVARIANCE STORED.

 DO YOU WANT COVARIANCE(COV),CORRELATION(COR) OR END?
end
 ACF TERMINATES.
@asf,n
 DATA SPACING?
1.
 WINDOW TO BE USED(TUKEY/DANIELL)?
tukey
 BANDWIDTH PARAMETER?
33
 NUMBER OF FREQUENCY POINTS TO BE VIEWED?
33
 DO YOU WANT CONFIDENCE LIMITS (YES/NO)?
no
 DO YOU WANT FREQUENCY(FREQ.) OR PERIOD(PERIOD) DISPLAYED?
freq.
```

AUTO-SPECTRUM:

FREQ.	SPECTRUM
.0000	.97734+00
.0156	.75601+00
.0313	.56642+00
.0469	.64053+00
.0625	.79873+00
.0781	.76774+00
.0938	.62412+00
.1094	.62409+00
.1250	.58374+00
.1406	.58306+00
.1563	.45604+00
.1719	.25828+00
.1875	.39156+00
.2031	.50674+00
.2188	.29786+00
.2344	.10037+00
.2500	.37529-01
.2656	.94984-02
.2813	.17673-02
.2969	.11131-02
.3125	.14980-02
.3281	.15829-02
.3438	.15418-02
.3594	.12205-02
.3750	.11983-02
.3906	.10520-02
.4063	.11808-02
.4219	.11860-02
.4375	.10052-02
.4531	.98564-03
.4688	.12136-02
.4844	.10427-02
.5000	.88202-03



DO YOU WANT THE SPECTRUM STORED(YES/NO)?
<u>no</u>
DO YOU HAVE ANOTHER SERIES(YES/NO)?
<u>no</u>
ASF TERMINATES.

The second group involves two time series and uses that portion of TSAP shown in Figure 3. This example shows the sequence required to obtain the coherence and phase.

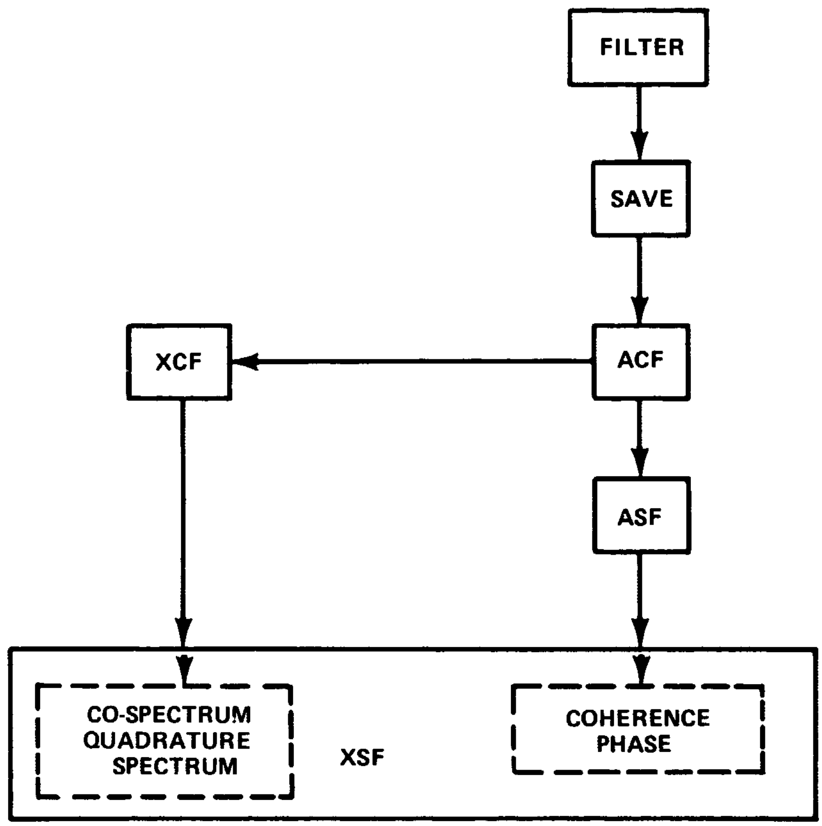


FIGURE 3

Since the filter program has already been demonstrated, we will assume that both series are prepared for spectral analysis and have been saved under the names ybh and sskp[(1)]. The first program called is ACF:

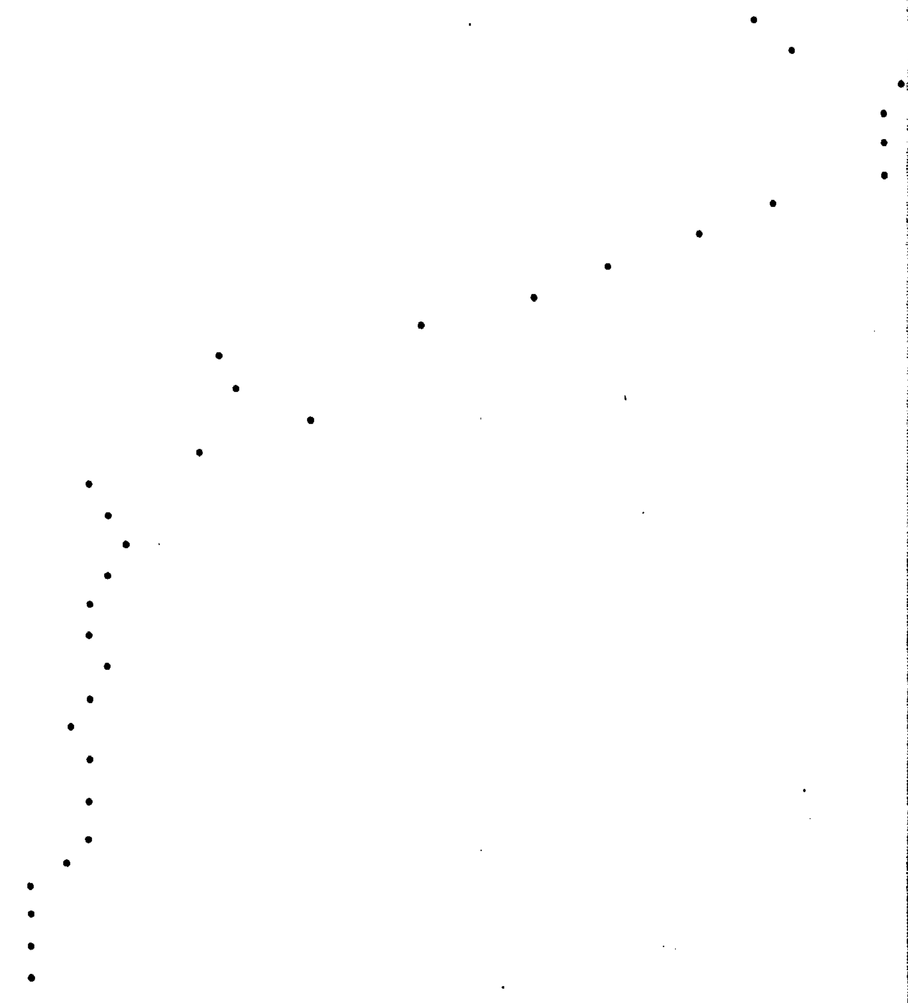
```
► @acf,n

 DO YOU WANT COVARIANCE(COV),CORRELATION(COR) OR END?
cor
 LENGTH OF SERIES?
362
 NUMBER OF LAGS(ZERO INCLUDED)?
33
 INPUT DATA FORMAT?
(4e15.8)
 INPUT DATA:
@add ybh
```

---

(1) SSKP is saved in the file gra while ybh is in TPF$.

```
   0    .10000000+01   11   -.31216709-01   22   -.63481236-01
   1    .60598906+00   12   -.34091541-01   23   -.51319714-01
   2    .26923198+00   13   -.21461296-01   24    .28734243-01
   3    .75024208-01   14    .30060796-01   25    .97583456-01
   4   -.63190014-01   15    .52854193-01   26    .11406376+00
   5   -.58960785-01   16   -.37186533-01   27    .13826372+00
   6   -.56973809-01   17   -.79431379-01   28    .12823801+00
   7   -.35041797-01   18   -.37290229-01   29    .14123514+00
   8   -.18516959-01   19    .24989689-01   30    .14814627+00
   9   -.64888234-02   20    .19790754-01   31    .98589633-01
  10    .41145700-02   21   -.21608890-01   32    .20618195-01
 VARIANCE=    .14191380+00
 MEAN=    .40113709-05
 DO YOU WANT THE COVARIANCES STORED(YES/NO)?
yes
 COVARIANCE STORED.

 DO YOU WANT COVARIANCE(COV),CORRELATION(COR) OR END?
cor
 LENGTH OF SERIES?
362
 NUMBER OF LAGS(ZERO INCLUDED)?
33
 INPUT DATA FORMAT?
(4e15.8)
 INPUT DATA:
@add gra.sskp
   0    .10000000+01   11    .34387418-01   22    .15657139-01
   1    .48397417+00   12    .85019480-01   23   -.32998001-02
   2    .16358888+00   13    .18485049-01   24    .19564041-02
   3    .34291061-01   14    .71320571-01   25    .20206913-01
   4   -.82355165-01   15    .29714237-01   26    .47620085-01
   5   -.48359534-01   16   -.17095344-01   27    .65758524-01
   6   -.44122326-01   17    .44739600-02   28    .61266199-01
   7   -.32026744-01   18    .17245420-02   29    .77879018-02
   8   -.34230735-01   19    .19832755-01   30   -.39413126-01
   9   -.71012059-02   20    .87463330-02   31   -.93015461-02
  10    .96968840-02   21    .34905837-02   32   -.10927683-01
 VARIANCE=    .11447779+01
 MEAN=    .40104705-05
 DO YOU WANT THE COVARIANCES STORED(YES/NO)?
yes
 COVARIANCE STORED.

 DO YOU WANT COVARIANCE(COV),CORRELATION(COR) OR END?
end
 ACF TERMINATES.
@asf,n
 DATA SPACING?
1.
```

```
 WINDOW TO BE USED(TUKEY/DANIELL)?
tukey
 BANDWIDTH PARAMETER?
33
 NUMBER OF FREQUENCY POINTS TO BE VIEWED?
33
 DO YOU WANT CONFIDENCE LIMITS(YES/NO)?
no
 DO YOU WANT FREQUENCY(FREQ.) OR PERIOD(PERIOD) DISPLAYED?
period
 AUTO-SPECTRUM:
  PERIOD    SPECTRUM
 99.9999    .33286+00
 64.0000    .34424+00
 32.0000    .39179+00
 21.3333    .38674+00
 16.0000    .38696+00
 12.8000    .39049+00
 10.6667    .33785+00
  9.1429    .31070+00
  8.0000    .26813+00
  7.1111    .23930+00
  6.4000    .18930+00
  5.8182    .10181+00
  5.3333    .10485+00
  4.9231    .13927+00
  4.5714    .91497-01
  4.2667    .44466-01
  4.0000    .50774-01
  3.7647    .62834-01
  3.5556    .55293-01
  3.3684    .41898-01
  3.2000    .44758-01
  3.0476    .53232-01
  2.9091    .46325-01
  2.7826    .38710-01
  2.6667    .40210-01
  2.5600    .44652-01
  2.4615    .46159-01
  2.3704    .35595-01
  2.2857    .23144-01
  2.2069    .18414-01
  2.1333    .19327-01
  2.0645    .18276-01
  2.0000    .15677-01
```



```
 DO YOU WANT THE SPECTRUM STORED(YES/NO)?
yes
 SPECTRUM STORED.
 DO YOU HAVE ANOTHER SERIES(YES/NO)?
yes
 IS THE SERIES THE SAME(YES/NO)?
no
 DATA SPACING?
1.
```

WINDOW TO BE USED(TUKEY/DANIELL)?

tukey

BANDWIDTH PARAMETER?

33

NUMBER OF FREQUENCY POINTS TO BE VIEWED?

33

DO YOU WANT CONFIDENCE LIMITS(YES/NO)?

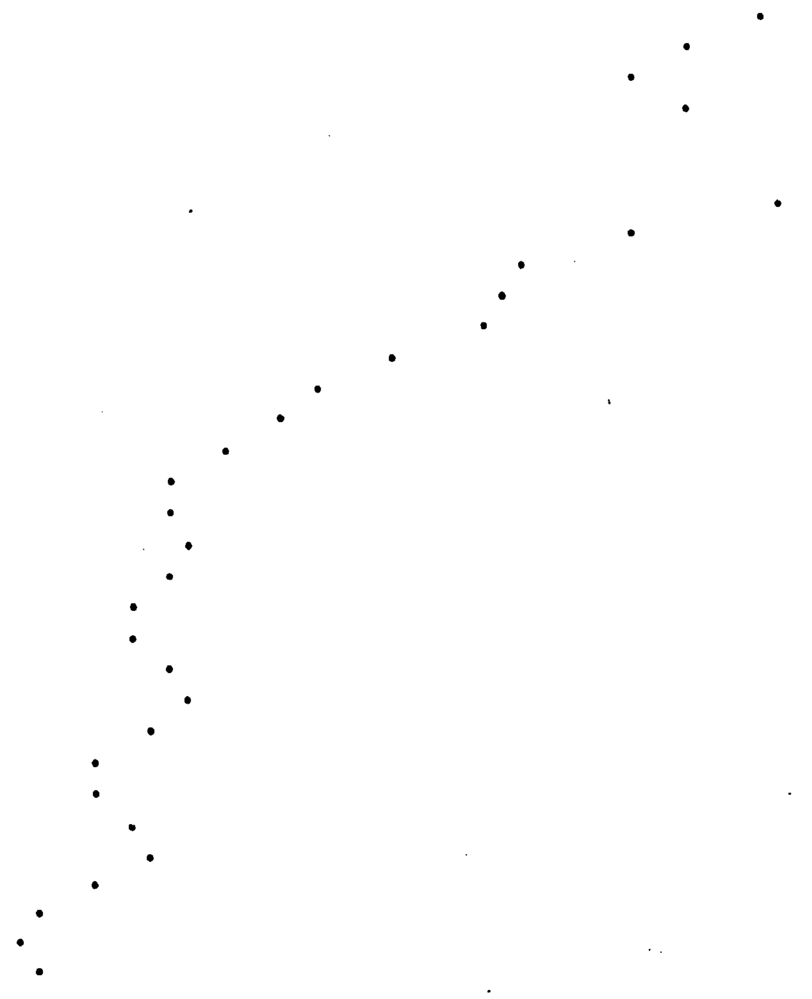
no

DO YOU WANT FREQUENCY(FREQ.) OR PERIOD(PERIOD) DISPLAYED?

period

AUTO-SPECTRUM:

PERIOD	SPECTRUM
99.9999	.25931+01
64.0000	.23221+01
32.0000	.21251+01
21.3333	.23342+01
16.0000	.27527+01
12.8000	.29730+01
10.6667	.26473+01
9.1429	.21566+01
8.0000	.17926+01
7.1111	.17529+01
6.4000	.16743+01
5.8182	.13744+01
5.3333	.11421+01
4.9231	.97125+00
4.5714	.83534+00
4.2667	.66678+00
4.0000	.61252+00
3.7647	.67283+00
3.5556	.62783+00
3.3684	.50060+00
3.2000	.49504+00
3.0476	.61500+00
2.9091	.69453+00
2.7826	.60262+00
2.6667	.39595+00
2.5600	.36967+00
2.4615	.53734+00
2.3704	.56806+00
2.2857	.37592+00
2.2069	.20381+00
2.1333	.17313+00
2.0645	.23317+00
2.0000	.27656+00



DO YOU WANT THE SPECTRUM STORED(YES/NO)?

yes

SPECTRUM STORED.

DO YOU HAVE ANOTHER SERIES(YES/NO)?

no

```
ASF TERMINATES.
@xcf

XCF COMPUTES THE CROSS-COVARIANCE/-CORRELATION FUNCTION OF TWO
SERIES.  IT IS ASSUMED THAT THE AUTO-COVARIANCES OF THE TWO SERIES
ARE STORED ON UNIT 1.  THE CROSS-COVARIANCES ARE STORED ON UNIT 1
(IF DESIRED) AFTER THE AUTO-COVARIANCES.
DO YOU WANT COVARIANCE(COV) OR CORRELATION(COR)?
cor
INPUT DATA FORMAT OF SERIES 1?
(4e15.8)
INPUT DATA OF SERIES 1:
@add ybh
INPUT DATA FORMAT OF SERIES 2?
(4e15.8)
INPUT DATA OF SERIES 2:
@add gra.sskp
```

CROSS-COR. FOR SERIES 1 VS. 2.

0	-.31068495+00	11	.68149552-01	22	-.19024821-01
1	-.32688250+00	12	.19405418-01	23	.28423370-01
2	-.23031710+00	13	.67258818-01	24	.65661986-01
3	-.15878268+00	14	.33894985-01	25	.71599943-01
4	-.97850593-02	15	.15917252-01	26	.77692746-01
5	.70688121-02	16	.44837943-01	27	.54928598-01
6	-.78098371-02	17	.68729260-01	28	.34432033-01
7	.32410393-04	18	.30546963-01	29	-.31944079-02
8	-.14137039-01	19	-.21630687-01	30	-.27489146-01
9	.92356610-02	20	-.22341036-01	31	-.65063784-01
10	.56534438-01	21	-.36277876-01	32	-.22737283-01

CROSS-COR. FOR SERIES 2 VS. 1.

0	-.31068495+00	11	.68149863-02	22	.22976052-01
1	.16895355+00	12	-.29895622-01	23	-.29432392-01
2	.40202706+00	13	.19673600-01	24	-.10881614+00
3	.40670389+00	14	-.28525013-01	25	-.13978173+00
4	.38908781+00	15	-.15708253-01	26	-.98344548-01
5	.27146963+00	16	.39682694-01	27	-.48471882-01
6	.16915689+00	17	.24792291-01	28	.39428172-01
7	.90782260-01	18	-.34359416-01	29	.11597794+00
8	.41205884-01	19	-.67074667-01	30	.10655324+00
9	.65060452-02	20	-.42766543-01	31	.97206097-01
10	.64103368-02	21	.89084057-02	32	.10046362+00

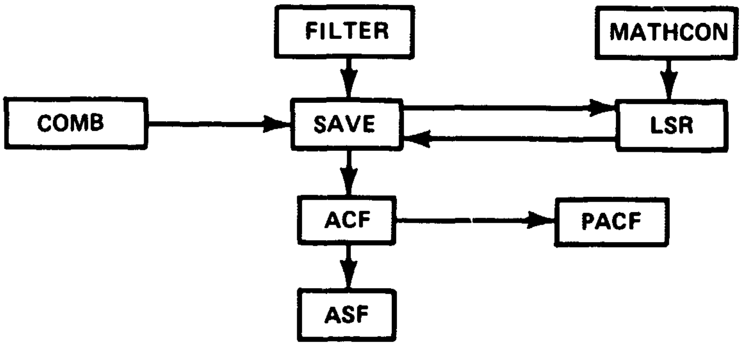
```
 DO YOU WANT THE COVARIANCES STORED(YES/NO)?
yes
 COVARIANCES STORED.
  XCF TERMINATES.
@xsf
 XSF COMPUTES THE CROSS-SPECTRA FUNCTIONS FOR TUKEY WINDOW.
 IT IS ASSUMED THAT THE AUTO- AND CROSS-COVARIANCES ARE STORED
 ON UNIT 1 AND THAT THE AUTO-SPECTRA ARE STORED ON UNIT 2.
 THE CROSS-SPECTRA ARE STORED ON UNIT 2 AFTER THE AUTO-SPECTRA(IF DESIRED).
 DO YOU WANT FREQUENCY(FREQ.) OR PERIOD (PERIOD)
 DISPLAYED?
freq


FREQ.    ARCTANH(COHERENCE) FOR SERIES 1 VS. 2 .
 .0000   .45577+00                     +
 .0156   .37510+00                 +
 .0313   .14873+00       +
 .0469   .17172+00        +
 .0625   .24303+00            +
 .0781   .33788+00                +
 .0938   .52812+00                        +
 .1094   .63082+00                            +
 .1250   .67882+00                              +
 .1406   .71782+00                                +
 .1563   .73696+00                                 +
 .1719   .55091+00                         +
 .1875   .35149+00                +
 .2031   .45226+00                    +
 .2188   .51547+00                       +
 .2344   .60792+00                           +
 .2500   .75909+00                                  +
 .2656   .69457+00                               +
 .2813   .49093+00                      +
 .2969   .44707+00                    +
 .3125   .84579+00                                     +
 .3281   .12234+01                                                      +
 .3438   .10487+01                                              +
 .3594   .78859+00                                   +
 .3750   .64568+00                             +
 .3906   .59831+00                           +
 .4063   .56528+00                         +
 .4219   .54029+00                        +
 .4375   .41398+00                   +
 .4531   .11776+00      +
 .4688   .13592+00       +
 .1844   .36685+00                 +
 .5000   .45305+00                     +
NU=  30.             *           *
```

FREQ.	PHASE FOR SERIES 1 VS. 2 .
. 000	.00000
.0156	-.52353+01
.0313	-.39472+02
.0469	-.13202+03
.0625	-.14719+03
.0781	-.16059+03
.0938	-.16853+03
.1094	-.17007+03
.1250	-.17200+03
.1406	-.17227+03
.1563	-.17250+03
.1719	-.17193+03
.1875	-.16744+03
.2031	-.17176+03
.2188	-.17493+03
.2344	-.17545+03
.2500	-.17624+03
.2656	-.17626+03
.2813	-.17471+03
.2969	-.17732+03
.3125	-.17998+03
.3281	-.17938+03
.3438	-.17893+03
.3594	-.17812+03
.3750	-.17657+03
.3906	-.17693+03
.4063	-.17884+03
.4219	-.17872+03
.4375	-.17625+03
.4531	-.17222+03
.4688	.17008+03
.4844	.17622+03
.5000	.18000+03

-180 0 180

```
 DO YOU WANT THE CO-SPECTRA AND QUADRATURES
 STORED FOR TIME SERIES REGRESSION(YES/NO)?
no
 XSF TERMINATES.
```

THE NEXT GROUP USES THE FOLLOWING PARTS OF TSAP:



The main object of this group of examples is to illustrate the regression and partial auto-correlation programs. This is accomplished by regressing one series onto two others with LSRA and then applying ACF and PACF to the residuals. On the basis of the output from PACF, the residual series is modeled as a second order autoregression. The residuals corresponding to this model are obtained by successive applications of FILTER and COMB and then examined using PACF and ASF.

The final example of this group illustrates LSRP.

The forward stepwise regression program, LSRA, is illustrated first by regressing a series xl, consisting of 123 terms, onto two series, yl and time123, of the same length. The series time 123 consists of the positive integers from 1 to 123 and was generated using MATH and MATHCON:

```
    @asg,a gra.
--> @math
    MATH VERSION 3.  WHAT CAN I DO FOR YOU?

    1.1 set a(i)=i.
    do part 1 for i=1(1)123.
    use file gra.
    ROGER.
    save a as a.
    DONE.
--> @mathcon,i ,gra*gra.time123
    READY.
    @add gra*gra.a
    @nop
    MATHCON  TERMINATES.
```

```
@lsra

 LSRA IS A STEPWISE MULTIPLE REGRESSION PROGRAM
 WITH ANALYSIS OF VARIANCE PRINTOUT.
 THE RESIDUALS ARE STORED ON UNIT 8(IF DESIRED).

 NUMBER OF OBSERVATIONS.
123
 NUMBER OF VARIABLES(DEPENDENT AND INDEPENDENT)?
3
 F LEVEL
3.25
 DEPENDENT VARIABLE FORMAT?

(4e15.8)
 DEPENDENT VARIABLE DATA:
@add xl
 INDEPENDENT VARIABLE 1 FORMAT?
(4e15.8)
 INDEPENDENT VARIABLE 1 DATA:
@add yl
 INDEPENDENT VARIABLE 2 FORMAT?
(e15.8)
 INDEPENDENT VARIABLE 2 DATA:
@add gra.time123
 MEANS:
    1    .39642049+01    2    .62000000+02

 CORRELATION COEFFICIENTS:
     1 VS 2 =    -.116816
     1 VS Y =     .787702     2 VS Y(1)= .515992
 STANDARD ERROR OF Y =    .478109+00

 SSVAR=*************  SSRES=      27.887794   DFRES=   122.000000
 MSRES=        .228588      T=        .000000  MULTCOR=      .000000

 STEP NO.  1
      VARIABLE ENTERING:   1
      FLEVEL   197.819456
      STANDARD ERROR OF Y     .295757
      CONSTANT   .289935+00

            VARIABLE     COEFFICIENT   STD. ERROR OF COEFF.
                 X 1    .92686272+00         .65899315-01

 SSVAR=   17.303675  SSRES=   10.584119  DFRES=   121.000000
 MSRES=     .087472      T=   14.064830  MULTCOR=     .620475
```

(1) In the computer responses, Y is a generic symbol for the dependent variable while the symbol X, with subscripts, represents an independent variable. The first X-variable entered is that independent variable having the highest correlation with the dependent variable.

```
 STEP NO.  2
      VARIABLE ENTERING:  2
      FLEVEL   9494.080078
      STANDARD ERROR OF Y    .033180
      CONSTANT  -.558482+00

             VARIABLE     COEFFICIENT  STD. ERROR OF COEFF.
                  X 1    .10115818+01       .74439476-02
                  X 2    .82666745-02       .84840725-04

 SSVAR=     10.452011  SSRES=      .132108    DFRES=   120.000000
 MSRES=       .001101       T=   97.437571  MULTCOR=      .995263

 DO YOU WANT TO SEE SOME OF THE DEVIATIONS(YES/NO)?
yes
 WHAT PART(INITIAL,FINAL,STEP)?
1,120,12

 PREDICTED VS. ACTUAL RESULTS:
 OB. NO.     ACTUAL     PREDICTED     %DEVIATION
    1    .385775+01   .377441+01       2.16
   13    .321840+01   .317747+01       1.27
   25    .337288+01   .336292+01        .30
   37    .342785+01   .347028+01      -1.24
   49    .432563+01   .437717+01      -1.19
   61    .344014+01   .348335+01      -1.26
   73    .400280+01   .399166+01        .28
   85    .357452+01   .357611+01       -.04
   97    .464195+01   .463681+01        .11
  109    .479612+01   .476218+01        .71
 DO YOU WANT THE RESIDUALS STORED(YES/NO)?
yes
 OUTPUT DATA FORMAT?
(4e15.8)
 SERIES STORED.
 LSR TERMINATES.
→ @save lsra, rl
```

We have not printed out enough of the deviations of predicted from actual values to decide if the residuals are close to being white. However, this can be determined by submitting the residuals, saved under the name rl, to ACF and PACF:

```
→ @acf,n

 DO YOU WANT COVARIANCE(COV),CORRELATION(COR) OR END?
cor
 LENGTH OF SERIES?
123
 NUMBER OF LAGS(ZERO INCLUDED)?
21
```

```
 INPUT DATA FORMAT?
(4e15.8)
 INPUT DATA:
@add rl
   0   .10000000+01    7   .63026889+00   14   .30409733+00
   1   .91360913+00    8   .58674648+00   15   .27090824+00
   2   .86974473+00    9   .53077113+00   16   .24346690+00
   3   .82899847+00   10   .48667117+00   17   .21895623+00
   4   .77865961+00   11   .45664223+00   18   .18133523+00
   5   .72503306+00   12   .41310194+00   19   .16843886+00
   6   .66401155+00   13   .35039130+00   20   .14462271+00
 VARIANCE=   .10740904-02
 MEAN=  -.14518410-06
 DO YOU WANT THE COVARIANCES STORED(YES/NO)?
yes
 COVARIANCE STORED.

 DO YOU WANT COVARIANCE(COV),CORRELATION(COR) OR END?
end
 ACF TERMINATES.

@pacf
 PACF COMPUTES THE PARTIAL AUTO-CORRELATION AND A.R. COEFFICIENTS
 UP TO ORDER K TOGETHER WITH THE RESIDUAL VARIANCES OF THE
 FITTED A.R.'S UP TO ORDER K AND THE FIRST ORDER M.A. COEFFICIENTS.

 MAXIMUM ORDER OF K?
15

 PACF:

   1   .91360912+00    6  -.99956975-01   11   .72968422-01
   2   .21209436+00    7   .10971080+00   12  -.60200907-01
   3   .57967539-01    8  -.56858845-02   13  -.15911209+00
   4  -.57376178-01    9  -.99147042-01   14  -.18779177-01
   5  -.71001749-01   10  -.14166919-01
 DO YOU WANT RESIDUAL VARIANCES(YES/NO)?
yes
```

```
RESIDUAL VARIANCES:

   1   .18052634-03     6   .17648832-03    11   .18078887-03
   2   .17389055-03     7   .17607510-03    12   .18216872-03
   3   .17483730-03     8   .17784500-03    13   .17962297-03
   4   .17584129-03     9   .17792252-03    14   .18171295-03
   5   .17658256-03    10   .17978376-03
ORDER OF A.R. PROCESS TO BE VIEWED(0=NONE OR NO MORE)?
2

  .71983778+00   .21209436+00
ORDER OF A.R. PROCESS TO BE VIEWED(0=NONE OR NO MORE)?
0
DO YOU WANT THE FIRST ORDER M.A. COEFFICIENTS(YES/NO)?
no
PACF TERMINATES.
```

Because the acf attenuates, the pacf appears to truncate[(1)] at the 2nd lag and the 2nd residual variance is minimum for a 2nd order a.r. model, it is reasonable to examine the hypothesis that the residuals rl satisfy the following 2nd order a.r. model:

$$\varepsilon(t) = rl(t) - .7198rl(t-1) - .2121rl(t-2)$$

for t=3,4,...,123.

The construction of $\varepsilon(t)$ allows us to illustrate the use of the "length of series to be used", "lag", and "scale" transformations in FILTER together with the series subtraction operator in COMB. (Of course, once it is noticed that the residuals, rl, are auto-correlated the regression of xl onto yl and time123 must be re-examined along the lines suggested in Durbin [5] or Hannan [10].)

```
@filter,n
 LENGTH OF SERIES TO BE READ IN?
123
 INPUT DATA FORMAT?
(4e15.8)
 INPUT DATA:
@add rl
 LENGTH OF SERIES TO BE USED?
123
```

---

(1) Under the hypothesis that the process is a p^{th} order autoregression the k^{th} value of the pacf has a normal distribution with mean zero and standard deviation equal to $1/\sqrt{N}$, where the N is the length of the series. In this particular example, the two standard deviation band for significance is $\pm.18$.

```
 MEAN=    -.14518410-06
 LENGTH=    123

 FILTER?
lag
 NUMBER OF LAGS?
2
 MEAN=    -.12660986-02
 LENGTH=    121

 FILTER?
end
 DO YOU WANT TO SEE PART OF THE SERIES(YES/NO)?
no
 DO YOU WANT THE FINAL SERIES STORED(YES/NO)?
yes
 OUTPUT DATA FORMAT?
(4e15.8)
 SERIES STORED.
 FILTER TERMINATES.
@save filter, rll


@filter,n
 LENGTH OF SERIES TO BE READ IN?
123
 INPUT DATA FORMAT?
(4e15.8)
 INPUT DATA:
@add rl
 LENGTH OF SERIES TO BE USED?
122
 MEAN=    -.42588965-03
 LENGTH=    122

 FILTER?
lag
 NUMBER OF LAGS?
1
 MEAN=    -.11181355-02
 LENGTH=    121

 FILTER?
scale
 SCALE OPERATOR(ADD/MPY)?
mpy
 SCALE FACTOR?
.7198
 MEAN=    -.80483410-03
 LENGTH=    121
```

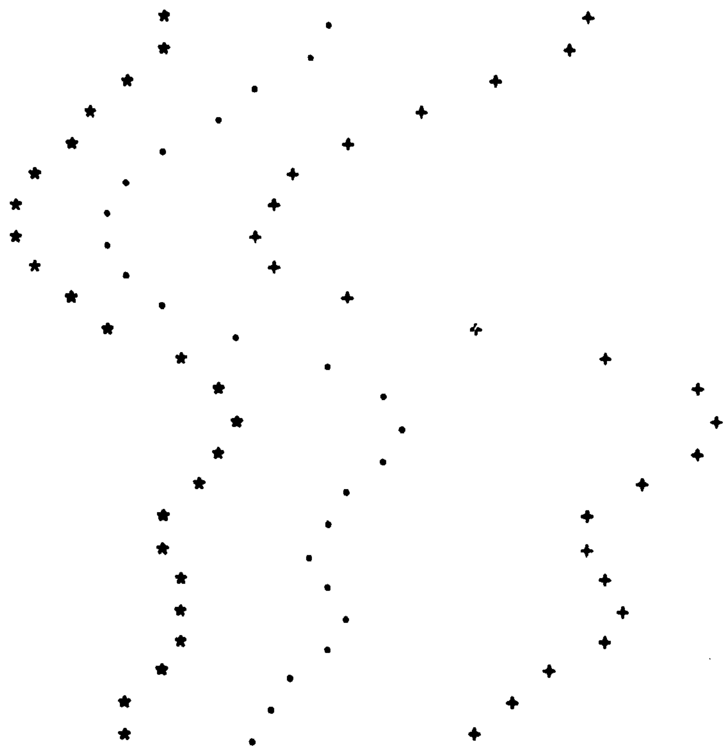
```
 FILTER?
end
 DO YOU WANT TO SEE PART OF THE SERIES(YES/NO)?
no
 DO YOU WANT THE FINAL SERIES STORED(YES/NO)?
yes
 OUTPUT DATA FORMAT?
(4e15.8)
 SERIES STORED.
 FILTER TERMINATES.
→ @save filter, r12

→ @filter,n
 LENGTH OF SERIES TO BE READ IN?
123
 INPUT DATA FORMAT?
(4e15.8)
 INPUT DATA:
@add r1
 LENGTH OF SERIES TO BE USED?
121
 MEAN=    -.16543380-03
 LENGTH=    121

 FILTER?
scale
 SCALE OPERATOR(ADD/MPY)?
mpy
 SCALE FACTOR?
.2121
 MEAN=    -.16543380-03
 LENGTH=    121

 FILTER?
end
 DO YOU WANT TO SEE PART OF THE SERIES(YES/NO)?
no
 DO YOU WANT THE FINAL SERIES STORED(YES/NO)?
yes
 OUTPUT DATA FORMAT?
(4e15.8)
 SERIES STORED.
 FILTER TERMINATES.
→ @save filter, r13
```

The COMB program is now employed to subtract the series r12 and r13 from r11 to form ε1.

```
@comb,n
 LENGTH OF SERIES 1 TO BE READ IN?
121
 INPUT DATA 1 FORMAT?
(4e15.8)
 INPUT DATA 1 :
@add rl1
 LENGTH OF SERIES 2 TO BE READ IN?
121
 INPUT DATA 2 FORMAT?
(4e15.8)
 INPUT DATA 2 :
@add rl2
 LENGTH OF OUTPUT SERIES?
121
 OPERATOR TO BE USED?
sub
 DO YOU WANT TO SEE PART OF THE SERIES(YES/NO)?
no
 DO YOU WANT THE RESULTANT SERIES STORED(YES/NO)?
yes
 OUTPUT DATA FORMAT?
(4e15.8)
 SERIES STORED.
 COMB TERMINATES.
@save comb, kl
@comb,n
 LENGTH OF SERIES 1 TO BE READ IN?
121
 INPUT DATA 1 FORMAT?
(4e15.8)
 INPUT DATA 1 :
@add kl
 LENGTH OF SERIES 2 TO BE READ IN?
121
 INPUT DATA 2 FORMAT?
(4e15.8)
 INPUT DATA 2 :
@add rl3
 LENGTH OF OUTPUT SERIES?
121
 OPERATOR TO BE USED?
sub
 DO YOU WANT TO SEE PART OF THE SERIES(YES/NO)?
no
 DO YOU WANT THE RESULTANT SERIES STORED(YES/NO)?
yes
 OUTPUT DATA FORMAT?
(4e15.8)
 SERIES STORED.
 COMB TERMINATES.
@save comb, el
```

```
→ @pacf,n

 MAXIMUM ORDER OF K?
10

 PACF:

   1  -.85908396-01     4    .12719820+00     7    .14001438+00
   2  -.24426123-01     5    .31113816-01     8    .59418262-01
   3    .12837213+00    6  -.12368082+00     9    .17003972-01
 DO YOU WANT RESIDUAL VARIANCES(YES/NO)?
no
 ORDER OF A.R. PROCESS TO BE VIEWED(0=NONE OR NO MORE)?
0
 DO YOU WANT THE FIRST ORDER M.A. COEFFICIENTS(YES/NO)?
yes

 FIRST ORDER M.A. COEFFICIENTS:

   1  -.92659724-01     4    .12719820+00     7  -.14001438+00
   2  -.92405158-01     5    .31113816-01     8    .59418262-01
   3  -.77068748-01     6  -.12368082+00     9    .17003972-01

→ @asf,n
 DATA SPACING?
1.
 WINDOW TO BE USED(TUKEY/DANIELL)?
tukey
 BANDWIDTH PARAMETER?
10
 NUMBER OF FREQUENCY POINTS TO BE VIEWED?
24
 DO YOU WANT CONFIDENCE LIMITS(YES/NO)?
yes
 DO YOU WANT FREQUENCY(FREQ.) OR PERIOD(PERIOD) DISPLAYED?
freq.
```

```
AUTO-SPECTRUM:
 FREQ.     SPECTRUM
  .0000    .10650-03
  .0217    .10318-03
  .0435    .94606-04
  .0652    .83922-04
  .0870    .74053-04
  .1087    .66422-04
  .1304    .61409-04
  .1522    .58782-04
  .1739    .63310-04
  .1957    .73549-04
  .2174    .89675-04
  .2391    .10746-03
  .2609    .12082-03
  .2826    .12535-03
  .3043    .12115-03
  .3261    .11286-03
  .3478    .10652-03
  .3696    .10551-03
  .3913    .10848-03
  .4130    .11091-03
  .4348    .10888-03
  .4565    .10228-03
  .4783    .94936-04
  .5000    .91759-04
```



```
THE 90% CONFIDENCE LIMITS FOR NU=36 are:  CU=  1.54095 , CL=  .70309
THE SQRT OF VARIANCE RATIO IS:  24.896 %
DO YOU WANT THE SPECTRUM STORED(YES/NO)?
no
DO YOU HAVE ANOTHER SERIES(YES/NO)?
no
ASF TERMINATES.
```

The final example involves an application of the second LSR program, LSRP:

```
@lsrp

LSRP IS A STEPWISE MULTIPLE REGRESSION PROGRAM WITH PREDICTIONS.
THE PREDICTION RESULTS ARE STORED(IF DESIRED) ON UNIT 8.
NUMBER OF OBSERVATIONS?
7
NUMBER OF VARIABLES(DEPENDENT AND INDEPENDENT)?
2
F LEVEL?
3.25
DEPENDENT VARIABLE FORMAT?
(e20.10)
DEPENDENT VARIABLE DATA:
@add g35log
```

```
 INDEPENDENT VARIABLE 1 FORMAT?
(e20.10)
 INDEPENDENT VARIABLE 1 DATA:
@add gra.time
 T-VALUE FOR NU=  5 DEGREES OF FREEDOM?
2.015
 STANDARD ERROR OF Y       .0108512
 VARIABLE COEFFICIENT         S.D.     UPPER T    LOWER T
 (0=CONST.)
         0  .76394+01  .91710-02  .76579+01  .76209+01
         1  .34170-01  .20507-02  .38302-01  .30038-01
 PREDICTION DATA:
 NUMBER OF PREDICTION VALUES?
17
 INDEPENDENT VARIABLE 1 FORMAT?
(e20.10)
 INDEPENDENT VARIABLE 1 DATA:
@add gra.time17
 PREDICTED VALUES:
 NUMBER  PREDICTION       UPPER T        LOWER T
      1  .7673548+01   .7700006+01   .7647089+01
      2  .7707718+01   .7732510+01   .7682925+01
      3  .7741888+01   .7765625+01   .7718150+01
      4  .7776058+01   .7799432+01   .7752683+01
      5  .7810228+01   .7833965+01   .7786490+01
      6  .7844398+01   .7869190+01   .7819605+01
      7  .7878568+01   .7905026+01   .7852109+01
      8  .7912738+01   .7941366+01   .7884109+01
      9  .7946908+01   .7978105+01   .7915711+01
     10  .7981078+01   .8015152+01   .7947003+01
     11  .8015248+01   .8052437+01   .7978059+01
     12  .8049418+01   .8089904+01   .8008931+01
     13  .8083588+01   .8127513+01   .8039663+01
     14  .8117758+01   .8165232+01   .8070283+01
     15  .8151928+01   .8203039+01   .8100816+01
     16  .8186098+01   .8240917+01   .8131279+01
     17  .8221268+01   .8278851+01   .8161685+01
 DO YOU WANT THE PREDICTION VALUES AND LIMITS SAVED(YES/NO)?
yes
 OUTPUT FORMAT?
(e20.10)

 SERIES STORED IN FOLLOWING ORDER:
 PREDICTION SERIES,LOWER LIMIT SERIES,UPPER LIMIT SERIES.
 LSRP TERMINATES.
→ @save lsrp, pul
```

4.0 PROGRAM TECHNIQUES AND FLOW CHARTS

The programs are designed to operate in an interactive mode for user ease and convenience. The system, TSAP, is a set of individual main programs to allow interaction with system processors, e.g., FURPUR or MATH, without the need to exit TSAP. This also allows the user to devise similar programs to be used in conjunction with TSAP, e.g., tape I/O or unpacking data. Data output at the terminal is kept minimal since terminal I/O is so much slower than printer I/O.

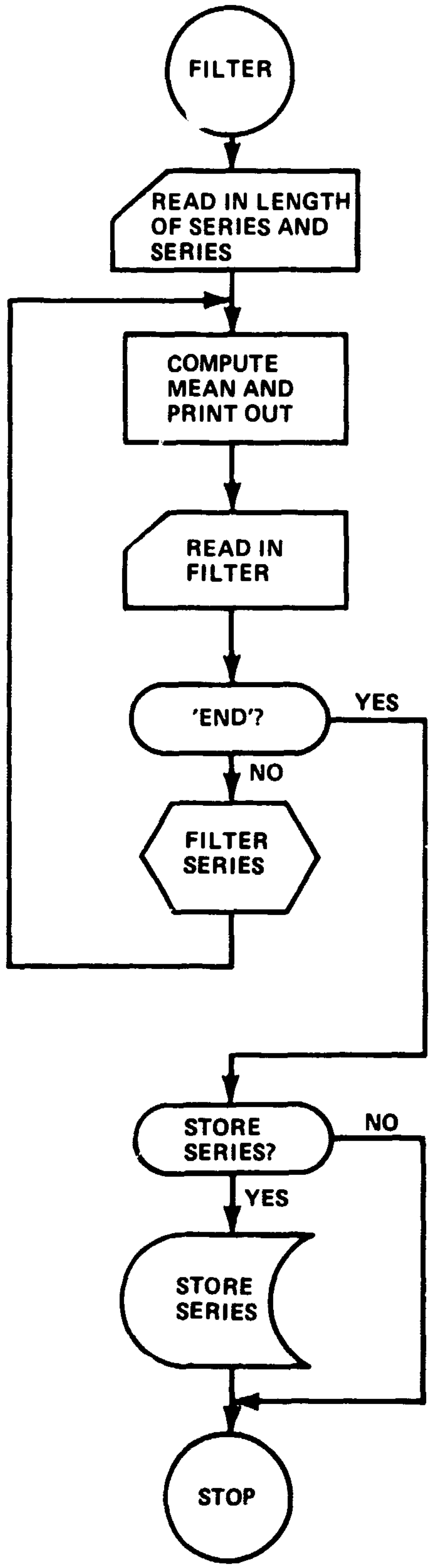
Intermediate results of interdependent programs are stored in binary on dynamically assigned FASTRAND files 1, 2 and 8 to save redundant calculations and/or I/O. Two such "linked" programs are ACF and PACF (PACF needs the covariances generated by ACF). This also prevents errors in answering the same questions in both programs.

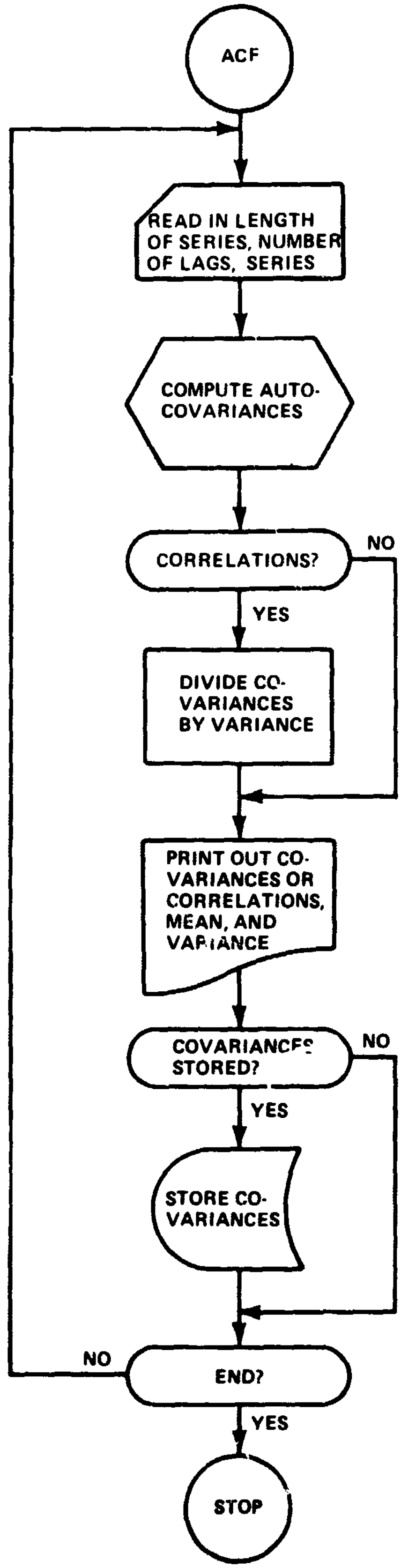
At Bellcomm, terminal core is restricted so a maximum series length of 1000 is built into all programs. This length is more of a "practical computation" size than an upper core limit and may be changed fairly easily. LSR is presently restricted to 5 series of length 500, again changeable.

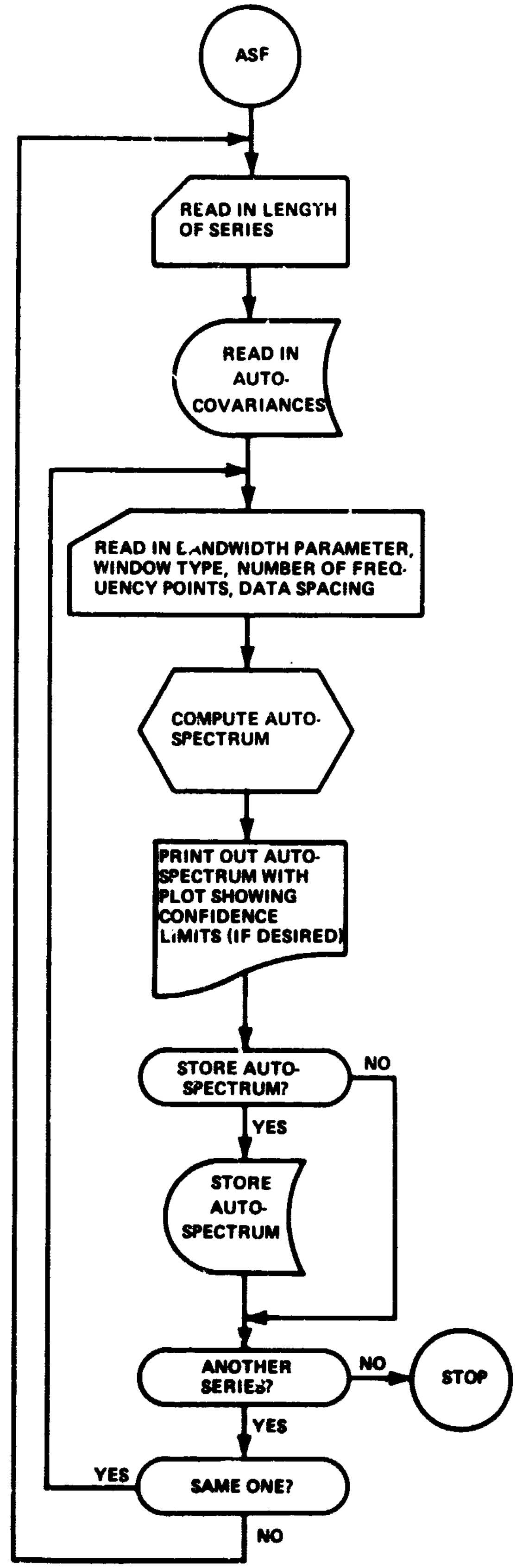
An N option is available on most of the programs to skip the introductory commentary for the experienced user.

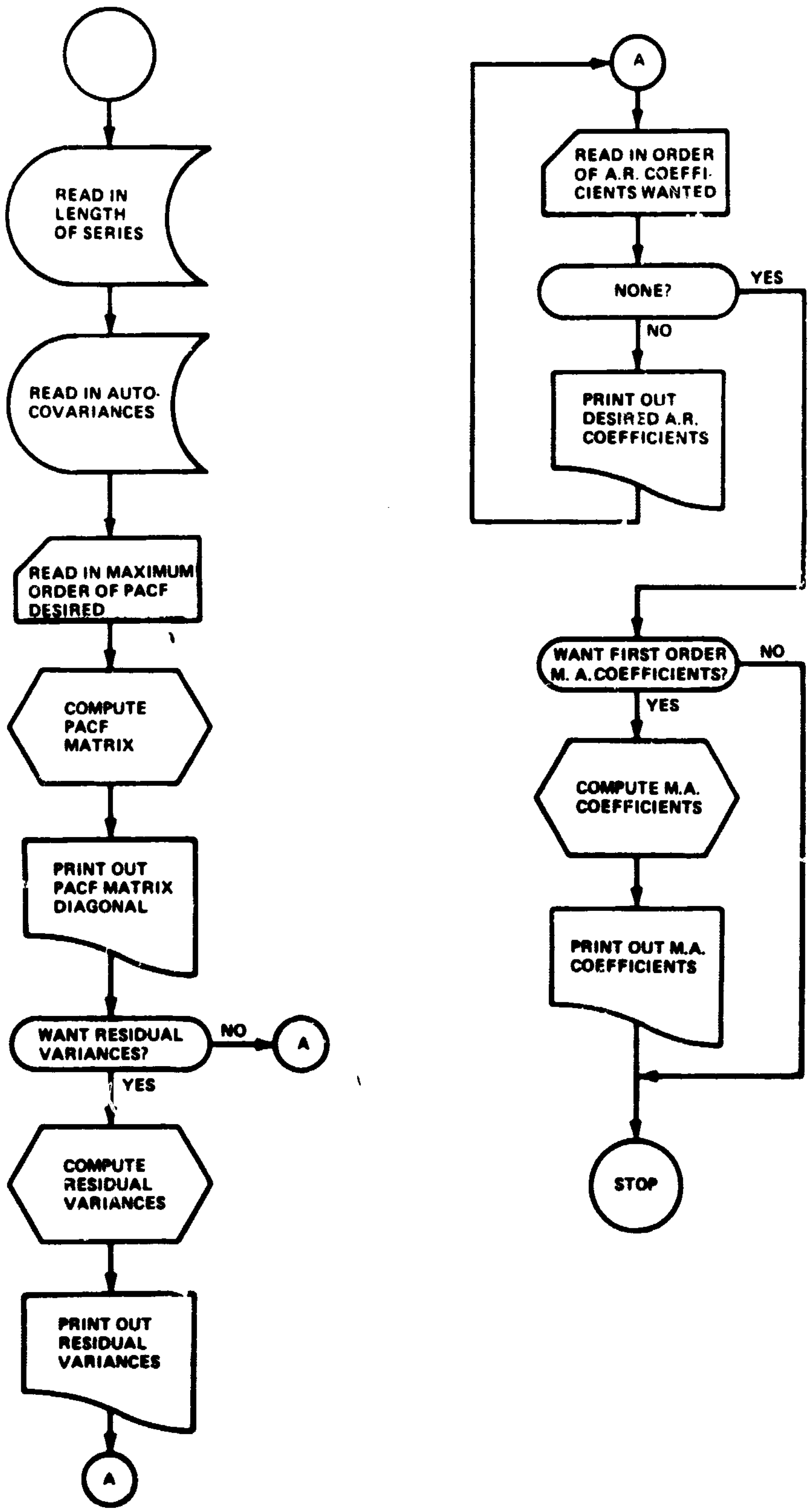
In ASF, the confidence limits use 90% chi square tables for $\gamma \leq 60$ degrees of freedom. Exceeding this limit may cause problems which can best be avoided by not asking for confidence limits for such series. The on-line plots of ASF (and XSF) consist of a 50 x NQ (NQ = number of frequency points to be viewed) grid which may cause distortion problems. It also makes comparisons between different series more difficult. XSF plots only the coherence ($\tanh^{-1}$) and phase to retain terminal efficiency while giving the most pertinent data output. These problems will be removed if a CRT display is incorporated into the system.

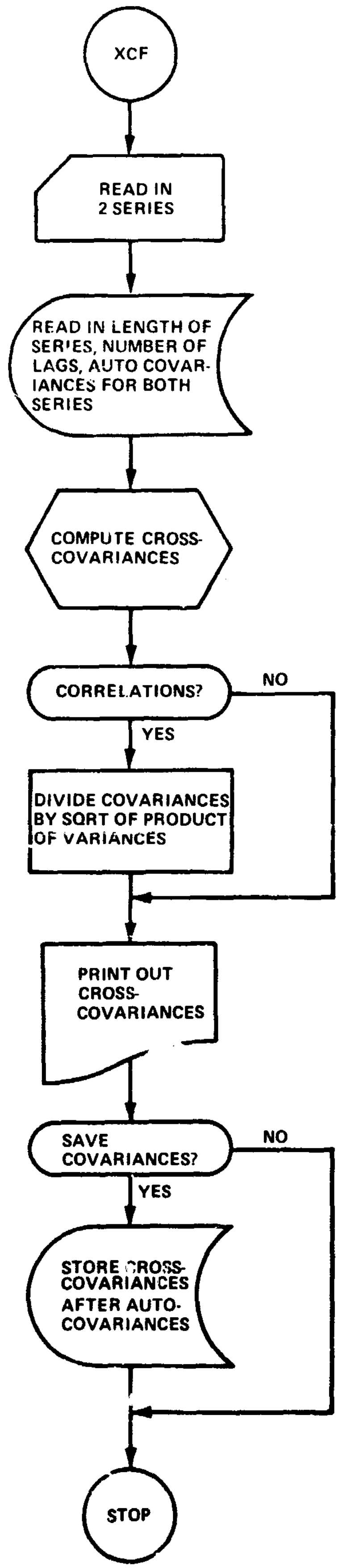
Flow charts of the main programs follow.

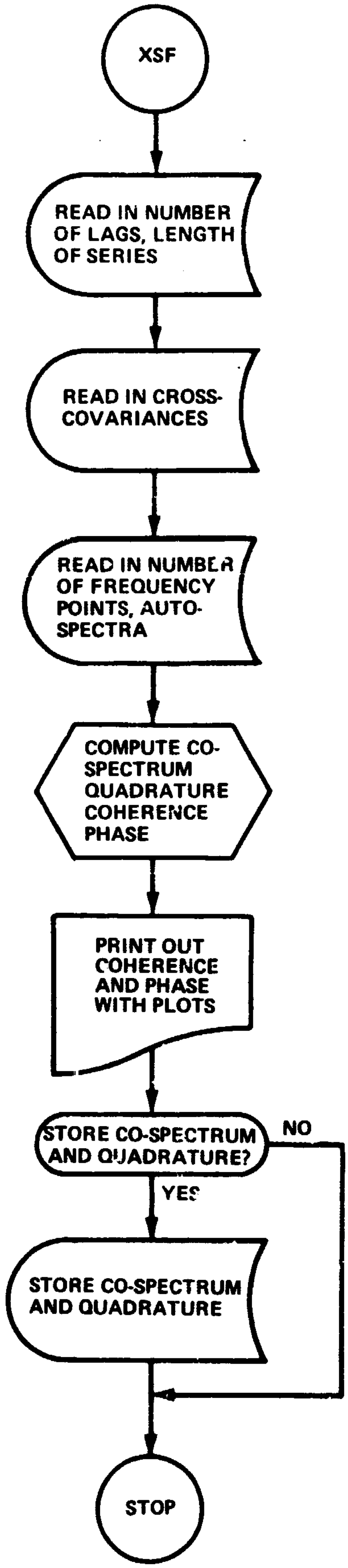

FILTER
READ IN LENGTH OF SERIES AND SERIES
COMPUTE MEAN AND PRINT OUT
READ IN FILTER
'END'?
YES
NO
FILTER SERIES
STORE SERIES?
NO
YES
STORE SERIES
STOP

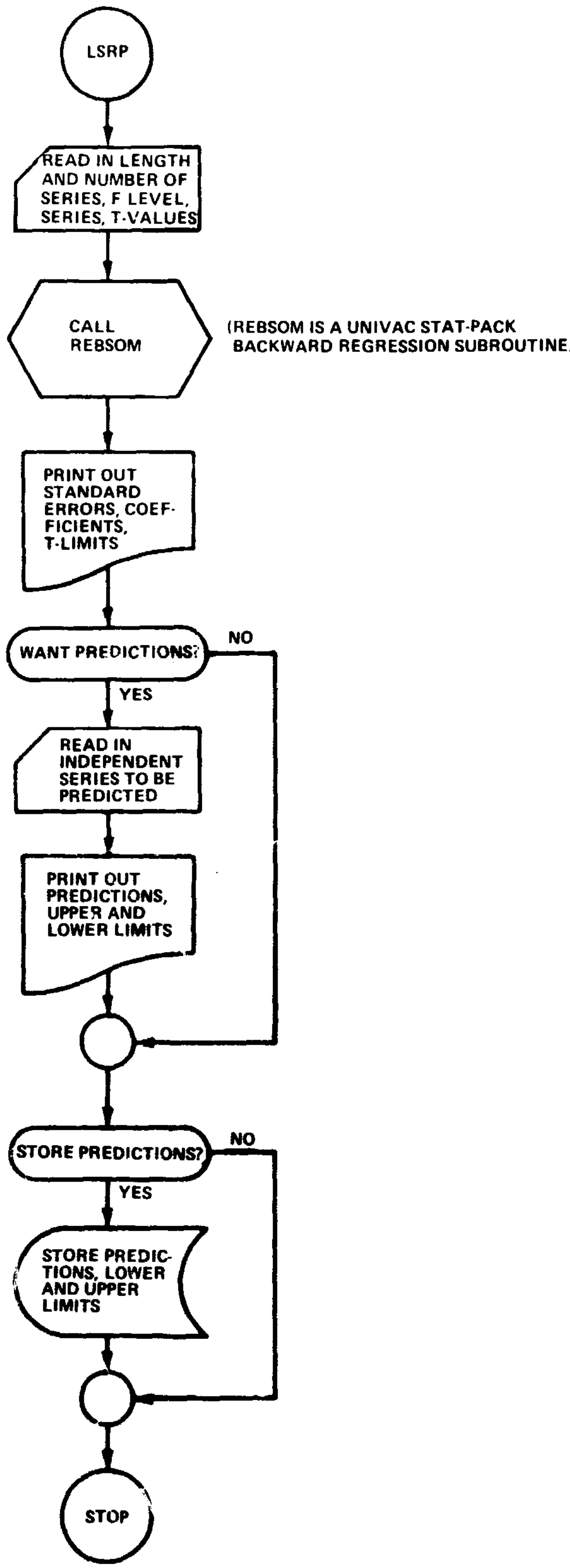

ACF
READ IN LENGTH OF SERIES, NUMBER OF LAGS, SERIES
COMPUTE AUTO-COVARIANCES
CORRELATIONS?
NO
YES
DIVIDE CO-VARIANCES BY VARIANCE
PRINT OUT CO-VARIANCES OR CORRELATIONS, MEAN, AND VARIANCE
COVARIANCES STORED?
NO
YES
STORE CO-VARIANCES
NO
END?
YES
STOP

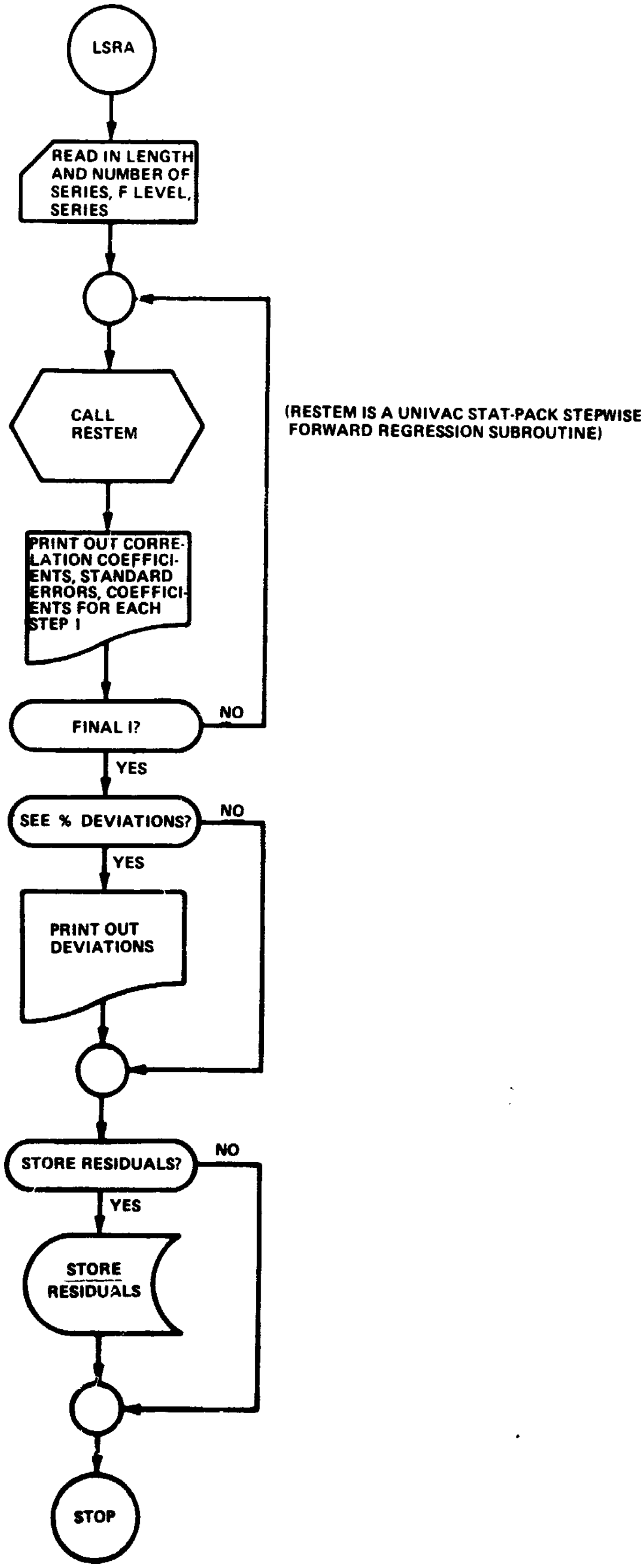

ASF
READ IN LENGTH OF SERIES
READ IN AUTO-COVARIANCES
READ IN BANDWIDTH PARAMETER, WINDOW TYPE, NUMBER OF FREQUENCY POINTS, DATA SPACING
COMPUTE AUTO-SPECTRUM
PRINT OUT AUTO-SPECTRUM WITH PLOT SHOWING CONFIDENCE LIMITS (IF DESIRED)
STORE AUTO-SPECTRUM?
NO
YES
STORE AUTO-SPECTRUM
ANOTHER SERIES?
NO
STOP
YES
SAME ONE?
YES
NO

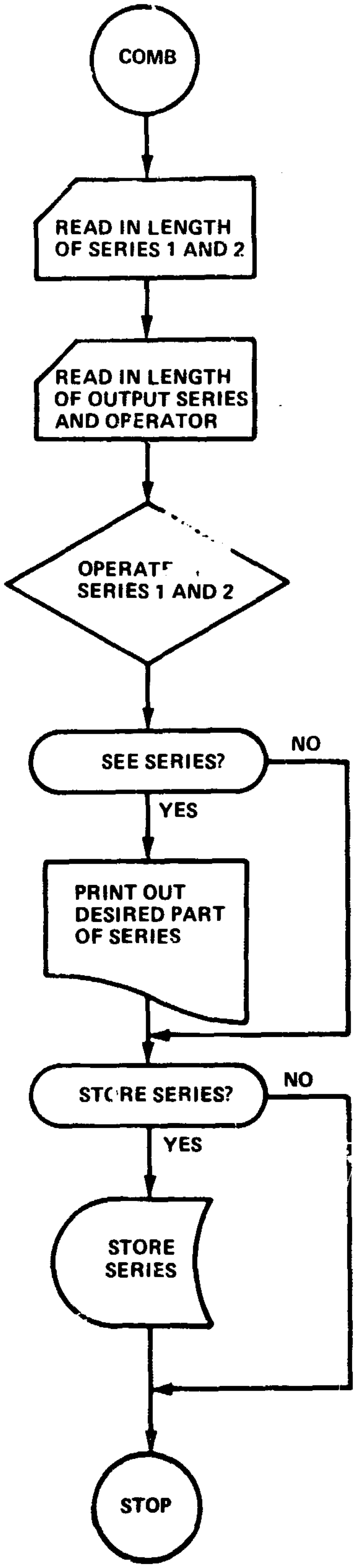

READ IN LENGTH OF SERIES
READ IN AUTO-COVARIANCES
READ IN MAXIMUM ORDER OF PACF DESIRED
COMPUTE PACF MATRIX
PRINT OUT PACF MATRIX DIAGONAL
WANT RESIDUAL VARIANCES?
NO
A
YES
COMPUTE RESIDUAL VARIANCES
PRINT OUT RESIDUAL VARIANCES
A
A
READ IN ORDER OF A.R. COEFFI-CIENTS WANTED
NONE?
YES
NO
PRINT OUT DESIRED A.R. COEFFICIENTS
WANT FIRST ORDER M. A. COEFFICIENTS?
NO
YES
COMPUTE M.A. COEFFICIENTS
PRINT OUT M.A. COEFFICIENTS
STOP


XCF
READ IN 2 SERIES
READ IN LENGTH OF SERIES, NUMBER OF LAGS, AUTO COVARIANCES FOR BOTH SERIES
COMPUTE CROSS-COVARIANCES
CORRELATIONS?
NO
YES
DIVIDE COVARIANCES BY SQRT OF PRODUCT OF VARIANCES
PRINT OUT CROSS-COVARIANCES
SAVE COVARIANCES?
NO
YES
STORE CROSS-COVARIANCES AFTER AUTO-COVARIANCES
STOP


XSF
READ IN NUMBER OF LAGS, LENGTH OF SERIES
READ IN CROSS-COVARIANCES
READ IN NUMBER OF FREQUENCY POINTS, AUTO-SPECTRA
COMPUTE CO-SPECTRUM QUADRATURE COHERENCE PHASE
PRINT OUT COHERENCE AND PHASE WITH PLOTS
STORE CO-SPECTRUM AND QUADRATURE?
NO
YES
STORE CO-SPECTRUM AND QUADRATURE
STOP


LSRP
READ IN LENGTH AND NUMBER OF SERIES, F LEVEL, SERIES, T-VALUES
CALL REBSOM
(REBSOM IS A UNIVAC STAT-PACK BACKWARD REGRESSION SUBROUTINE)
PRINT OUT STANDARD ERRORS, COEF-FICIENTS, T-LIMITS
WANT PREDICTIONS?
NO
YES
READ IN INDEPENDENT SERIES TO BE PREDICTED
PRINT OUT PREDICTIONS, UPPER AND LOWER LIMITS
STORE PREDICTIONS?
NO
YES
STORE PREDIC-TIONS, LOWER AND UPPER LIMITS
STOP


LSRA
READ IN LENGTH AND NUMBER OF SERIES, F LEVEL, SERIES
CALL RESTEM
(RESTEM IS A UNIVAC STAT-PACK STEPWISE FORWARD REGRESSION SUBROUTINE)
PRINT OUT CORRELATION COEFFICIENTS, STANDARD ERRORS, COEFFICIENTS FOR EACH STEP I
FINAL I?
NO
YES
SEE % DEVIATIONS?
NO
YES
PRINT OUT DEVIATIONS
STORE RESIDUALS?
NO
YES
STORE RESIDUALS
STOP


COMB
READ IN LENGTH OF SERIES 1 AND 2
READ IN LENGTH OF OUTPUT SERIES AND OPERATOR
SERIES 1 AND 2
SEE SERIES?
NO
YES
PRINT OUT DESIRED PART OF SERIES
STORE SERIES?
NO
YES
STORE SERIES
STOP

SUMMARY

The original TSAP system was developed to aid in the study of electron flux data from the earth's outer trapped radiation belts. Since then it has been applied to a variety of time series problems and each of these problems has influenced its present form. As noted in the introduction, the present content of TSAP will be increased in accordance with the needs of future applications.

The principal feature of TSAP is the fact that it is a terminal system. Time series analysis is ideally suited to such a mode of operation because it requires relatively long sequences of statistical computations which ordinarily depend directly either on the previous computations or on statistical judgements derived from these computations.

Simply stated, TSAP allows real time statistical judgements to be made in time series analysis on direction of the analysis. The versatility of TSAP is enhanced by its capability of interacting with machine processors and user designed terminal programs.

G. R. Andersen

G. R. Andersen

R. R. Singers

1031-GRA/RRS-jf

R. R. Singers

Attachments
Appendices I - III
References

APPENDIX I.

The stochastic properties of stationary stochastic processes are well known and are concisely presented in U. Grenander and M. Rosenblatt [8]. A recent practical approach to the subject may be found in the book by G. Jenkins and D. Watts [12]. The definitive text in the area of multiple time series analysis is the work by E. J. Hannan[1].

A.1 Spectral Analysis (ACF, ASF, XCF, XSF):

Let $\{X_j(t)\}$, j=1,2, and t=1,2,...,N, be finite realizations of two jointly stationary stochastic processes. These time series may be realizations of discrete processes or discrete series formed by sampling a realization, $\xi_j(t)$, of a continuous time stochastic process at equidistant time points, $t\cdot\Delta$, $(\Delta>0)$, and defining $X_j(t) = \xi_j(t\cdot\Delta)$, t=1,...,N.

In either case, let the centered series be denoted by

$$\tilde{X}_j(t) = X_j(t) - \overline{X}_j \qquad (1)$$

where

$$\overline{X}_j = \frac{1}{N}\sum_{t=1}^{N} X_j(t), \quad (j=1,2) \qquad (2)$$

and form the lagged (sample) cross covariance, C_{jk}, by setting

$$C_{jk}(\nu) = \frac{1}{N}\sum_{d=1}^{N-\nu} \tilde{X}_j(d)\tilde{X}_k(d+\nu) \quad , \qquad (3)$$

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(1) Hannan's book will be published by J. Wiley and Sons late in 1970.

for $\nu=0,1,\ldots,N-1$ and extend this definition to $-1,\ldots,-N+1$ by setting

$$C_{jk}(-\nu) = C_{kj}(\nu) \quad . \tag{4}$$

In an attempt to be consistent with most mathematical treatments of stationary stochastic processes, we will introduce the basic computational definitions concerning spectra for a unit sampling interval and consider frequency to have the dimensions of radians per unit of time. It is then a simple matter to transform the spectral estimators to the more practical form used in the programs where the data spacing is an arbitrary positive constant, Δ, and frequency is in terms of cycles per unit time.

The sample auto-spectrum, $\hat{g}_{jj}$, of the $j^{\underline{th}}$ time series is defined in terms of the sample auto-covariance function, C_{jj}, and a lag window, ω, by setting

$$\hat{g}_{jj}(\lambda) = \hat{g}_{jj}(\lambda;\omega) =$$

$$= \frac{1}{2\pi}\left(C_{jj}(0) + 2\sum_{\nu=1}^{N-1} \omega(\tfrac{\nu}{M})C_{jj}(\nu)\cos(\lambda\nu)\right) \quad , \tag{5}$$

where $|\lambda| \leq \pi$. The positive integer M, the bandwidth parameter, determines the bandwidth of the spectral window with Fourier coefficients $\omega(\frac{\nu}{M}) = \omega(\frac{-\nu}{M})$, $\nu=0,1,\ldots,N-1$.

Two types of spectral windows are available in TSAP: the Tukey window and the Daniell window. The lag windows corresponding to these spectral windows are as follows:

(i) Tukey:

$$\omega(y) = \omega_T(y) = \begin{cases} \frac{1}{2}\,(1+\cos(\pi y)), & |y| \le 1 \\ 0, & |y| > 1 \end{cases}$$

(ii) Daniell:

$$\omega(y) = \omega_D(y) = \begin{cases} \frac{\sin \pi y}{\pi y}, & y \ne 0 \\ 1, & y = 0\ . \end{cases}$$

The use of the Tukey window in (5) requires that only M lagged covariances (including the variance) be calculated; whereas, using the Daniell window requires all lagged covariances that can be calculated from a series of length N, namely N (including zero). For this reason the Daniell window should only be used on relatively short series (say, $N \leqq 500$).

If we denote the spectral window corresponding to $\omega(\frac{\cdot}{M})$ by W_M, then

$$W_M(s) = \frac{1}{2\pi} \sum_{-N+1}^{N-1} \omega(\tfrac{\nu}{M}) e^{i\nu s}\ , \tag{6}$$

$|s| \le \pi$.

Although the reader should consult any one of the above mentioned references for details on spectral analysis (cf. the bibliography for additional works), we will indicate the relationship between the spectral window and $\hat{g}_{jj}$.

If we set ω identically equal to 1 in (5) we obtain the (continuous) periodogram $\hat{g}_{jj}(\lambda;1)$. Integrating the product $W_M(\lambda-s)\hat{g}_{jj}(s;1)$ with respect to s over the interval $(-\pi,\pi)$ we obtain

$$\hat{g}_{jj}(\lambda;\omega) = \int_{-\pi}^{\pi} W_M(\lambda-s)\hat{g}_{jj}(s;1)ds \quad , \qquad (7)$$

$|\lambda| \leq \pi$. This exhibits the sample auto-spectrum as a weighted average of the periodogram. With the Daniell window this averaging is approximately uniform over the interval $(\lambda-h,\lambda+h)$ where $h = \pi/M$. The relationship between the sample auto-spectrum formed with the Tukey window and the periodogram is more easily seen directly from (5) where a simple cosine identity leads to

$$\hat{g}_{jj}(\lambda_k;\omega_T) = \begin{cases} \frac{1}{4}\hat{g}_{jj}(\lambda_{k-1};1)+\frac{1}{2}\hat{g}_{jj}(\lambda_k;1)+\frac{1}{4}\hat{g}_{jj}(\lambda_{k+1};1) & \text{for } k=1,2,\ldots,Q-1 \\ \frac{1}{2}\hat{g}_{jj}(0;1)+\frac{1}{4}\hat{g}_{jj}(\lambda_1;1) \quad , & \text{for } k = 0 \\ \frac{1}{2}\hat{g}_{jj}(\pi\ ;1)+\frac{1}{4}\hat{g}_{jj}(\lambda_{Q-1};1) \quad , & \text{for } k = Q \end{cases} \qquad (8)$$

where $\lambda_k = \pi k/Q$, $k=0,1,\ldots,Q$.

Since $\hat{g}_{jj}(\cdot;1)$ is proportional to the modulus squared of the discrete Fourier transform of the time series, (8) is often used for computation when a fast Fourier transform (FFT) is available. The first extension of TSAP will include a FFT routine for use in model estimation. At that time either (8) will be used or, following Gentleman and Sande [6], (5) will be used with the lagged covariances calculated using the FFT and the convolution theorem.

If the time series $\{X_j(t)\}$, $t = 1,2,\ldots,N$, is related to a finite realization of $\xi_j(\tau)$, $0 < \tau \leq T$, of a stationary process by $X_j(t) = \xi_j(\Delta t)$, $\Delta>0$, $N\Delta \leq T$, then the right-hand side of (5), multiplied by Δ and taking λ in $(-\pi/\Delta, \pi/\Delta)$, defines an estimate (call it $g_j^*(\lambda)$) of

$$g_j(\lambda) = \sum_{r=-\infty}^{+\infty} \phi_j(\lambda + \frac{2\pi r}{\Delta}), \ |\lambda| < \pi/\Delta \ ,$$

where ϕ_j is the spectrum of ξ_j.

Hence, g_j will be approximately equal to ϕ_j only if Δ is chosen in such a way that $\phi_j(s) \cong 0$ for $|s| > \frac{\pi}{\Delta}$, the so-called Nyquist frequency (cf., R. B. Blackman and J. W. Tukey [3]). Then

$$\phi_j(\lambda) \cong \begin{cases} g_j(\lambda) & , |\lambda| \leq \pi/\Delta \\ 0 & , |\lambda| > \pi/\Delta \end{cases}$$

and $g_j^*(\lambda)$, $|\lambda| \leq \pi/\Delta$ is an estimate of the spectrum of ξ_j.

We now state the computational forms used in TSAP, allowing for data spacing, Δ, and requiring that frequency be in dimensions of cycles per unit time[(1)]:

$$\hat{f}_{jj}(\lambda_k) = \Delta(C_{jj}(0)+2\sum_{\nu=1}^{M-1} \omega_T(\tfrac{\nu}{M})C_{jj}(\nu)\cos(\tfrac{\pi k\nu}{M_1})) \quad , \qquad (10)$$

where $\lambda_k = \frac{k}{2M_1\Delta}$[(2)], $k=0,1,\ldots,M_1$. M_1 is selected by the user and is generally between M and 2M. The proper bandwidth parameter, M, depends on the unknown spectrum and is usually found (if possible) by the technique of "window closing" (cf., [12]). Taking M to be 5, 10 and 20% of the record length N is a reasonable rule of thumb.

A compromise must usually be made between resolution and accuracy. As a guide to the accuracy of the estimate, the square root of the variance ratio, $\mathrm{var}(\hat{f}_{jj}(\lambda)/f_{jj}(\lambda))$, is printed out at the end of ASF. This quantity is (3/4)(M/N) for the Tukey window and M/N for the Daniell window. The user also has the option of having 90% confidence limits plotted with the sample auto-spectrum. These are the usual limits based on the approximation which states that $\nu \cdot \hat{f}_{jj}(\lambda)/f_{jj}(\lambda)$, where f_{jj} denotes the true (but unknown) auto-spectrum, is a chi-square random variable with ν degrees of freedom, with $\nu = (8/3)(N/M)$ in the case of the Tukey window and $\nu = 2\,N/M$ for the Daniell window. (Cf., Grenander and Rosenblatt [8].)

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(1) For simplicity we will state them for the Tukey window only since this is all that is available in the cross-spectral segment of TSAP.

(2) The user may choose to plot f_{jj} against period, λ_k^{-1}, $k=1,2,\ldots,M_1$.

The sample co-spectrum and quadrature spectrum of $\{X_j(t)\}$ and $\{X_k(t)\}$ are computed by

$$\hat{p}_{jk}(\lambda_\ell) = \Delta\{C_{jk}(0) + \sum_{\nu=1}^{M-1} (C_{jk}(\nu) + C_{kj}(\nu))\omega_T(\frac{\nu}{M})\cos(\frac{\pi\ell\nu}{M_1})\} \qquad (11)$$

and

$$\hat{q}_{jk}(\lambda_\ell) = \Delta \sum_{\nu=1}^{M-1} (C_{jk}(\nu) - C_{kj}(\nu))\omega_T(\frac{\nu}{M})\sin(\frac{\pi\ell\nu}{M_1}) \qquad (12)$$

respectively, where $\lambda_\ell = \frac{\ell}{2M_1\Delta}$, $\ell=0,1,\ldots,M_1$.

These functions are used to compute the sample coherence [2]

$$\hat{\gamma}_{jk}(\lambda_\ell) = \begin{cases} \dfrac{\hat{p}^2_{jk}(\lambda_\ell) + \hat{q}^2_{jk}(\lambda_\ell)}{\hat{f}_{jj}(\lambda_\ell)\hat{f}_{kk}(\lambda_\ell)}, & \hat{f}_{jj}(\lambda_\ell) \cdot \hat{f}_{kk}(\lambda_\ell) > 0 \\ 0, & \text{otherwise} \end{cases} \qquad (13)$$

and the sample phase

$$\hat{\phi}_{jk}(\lambda_\ell) = \begin{cases} \tan^{-1}\left(\dfrac{\hat{q}_{jk}(\lambda_\ell)}{\hat{p}_{jk}(\lambda_\ell)}\right), & \hat{p}_{jk}(\lambda_\ell) > 0 \\ \\ \tan^{-1}\left(\dfrac{\hat{q}_{jk}(\lambda_\ell)}{\hat{p}_{jk}(\lambda_\ell)}\right) - \Pi, & \hat{p}_{jk}(\lambda_\ell) < 0 \end{cases}$$

$\lambda_\ell = \frac{\ell}{2M_1\Delta}$, $\ell=0,1,\ldots,M_1$. For definiteness the phase is set equal to $\frac{\pi}{2}$ if $\hat{p}_{jk}(\lambda_\ell) = 0$.

A.2 Auto-Regression Coefficients, Residual Variance, and Partial Auto-Correlation Function (PACF)

All estimates in this section[(1)] are based on the sample auto-correlation function $r(\nu) = C(\nu)/C(0)$, where

$$C(\nu) = N^{-1} \sum_{t=1}^{N-\nu} X_t \, X_{t+\nu}.$$

We use the results of Durbin [5]:

Letting $a_1^{(1)} = r(1)$, $V_1 = 1$, choose a positive integer $k \leq N-1$, and define $a_1^{(k)}, a_2^{(k)}, \ldots, a_k^{(k)}$ by setting

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(1) We assume that the time series under discussion is $\{X(t):\ t=1,\ldots,N\}$, with $\bar{X}=0$.

$$a_s^{(s)} = (r(s) - a_1^{(s-1)}r(s-1) - a_2^{(s-1)}r(s-2) - \ldots - a_{s-1}^{(s-1)}r(1))/V_{s-1} \quad (1)$$

where

$$V_{s-1} = 1 - a_1^{(s-1)}r(1) - \ldots - a_{s-1}^{(s-1)}r(s-1) \quad (2)$$

for $s=2,\ldots,k$ and

$$a_r^{(s)} = a_r^{(s-1)} - a_s^{(s)}a_{s-r}^{(s-1)} \quad , r=1,\ldots,s-1 \ . \quad (3)$$

Then $a_s^{(s)}$, $s=2,\ldots,k$, are the partial correlation coefficients measuring the correlation between members of the time series $2,\ldots,k$ time units apart with the linear effect of intervening observations removed. The terms $a_1^{(s)},\ldots,a_s^{(s)}$ are the least squares estimators of the autoregression coefficients in the $s^{\underline{th}}$ order a. r. model.

$$y(t) = \alpha_1 y(t-1) + \ldots + \alpha_s y(t-s) + \varepsilon(t) \ . \quad (4)$$

The quantity

$$\sigma_\varepsilon^2(s) = \frac{N-s}{N-2s-1} C(0) \ V_s \quad (5)$$

is an estimator of the residual variance of the model in (4), i.e., the variance of $\varepsilon(t)$. V_s satisfies

$$V_s = V_{s-1}(1-(a_s^{(s)})^2)$$

APPENDIX II

This appendix contains an explanation of the transformations available in FILTER. Let $\{X(t): t=1,...,N\}$ be a given time series, then a request for "center" transforms $X(t)$ into $X(t) - \bar{X}$, where $\bar{X}$ is the sample mean (cf. Appendix I); symbolically

Center: $X(t) \to X(t) - \bar{X}$.

Similarly,

Exp: $X(t) \to e^{X(t)}$;

Log: $X(t) \to \log_e X(t)$, if $X(t) > 0$, for all t;

Log10 $X(t) \to \log_{10} X(t)$, if $X(t) > 0$, for all t;

Diff: $X(t) \to X(t) - X(t-1)$;

10**: $X(t) \to 10^{X(t)}$

Sqrt: $X(t) \to \sqrt{X(t)}$, if $X(t) \geq 0$, for all t;

Abs: $X(t) \to |X(t)|$;

Scale: $X(t) \to \alpha X(t)$, where the real number α is supplied by the user.

The transformation "LAG", accompanied by a user response k, $1 \leq k \leq N-1$, deletes the first k term from the series, i.e, it maps

$$(X(1), X(2), ..., X(k), X(k+1), ..., X(N))$$

into

$$(X(k+1), ..., X(N))$$

When the high pass Tukey filter (A. S. Alvia and G. M. Jenkins, [1]) is requested, the following linear transformation

$$(F_T^{(m)}X)(t) = \sum_{j=-m}^{m} \lambda_j X(t-j) \quad , \qquad (1)$$

is applied to the series $\{X(\tau): \tau=1,\dots,N\}$, where $t=m+1,\dots,N-m$[(1)] and $\lambda_0 = 1 - \frac{1}{m+1}$,
$\lambda_j = \lambda_{-j} = \frac{-1}{2(m+1)} (1 + \cos(\frac{j\pi}{m+1}))$, $(j=1,2,\dots,m)$.

The corresponding frequency response function $A_m(\lambda) = \sum_{-m}^{m} \lambda_j \cos(\lambda_j)$ constitutes the Tukey high pass filter; its effect on the auto-spectrum, f, of the process generating the time series $\{X(t)\}$ is well-known and given by $f_X(\lambda) = |A_m(\lambda)|^2 f(\lambda)$. The gain $|A_m|^2$ is shown in Figure A.II.1 for m=7.

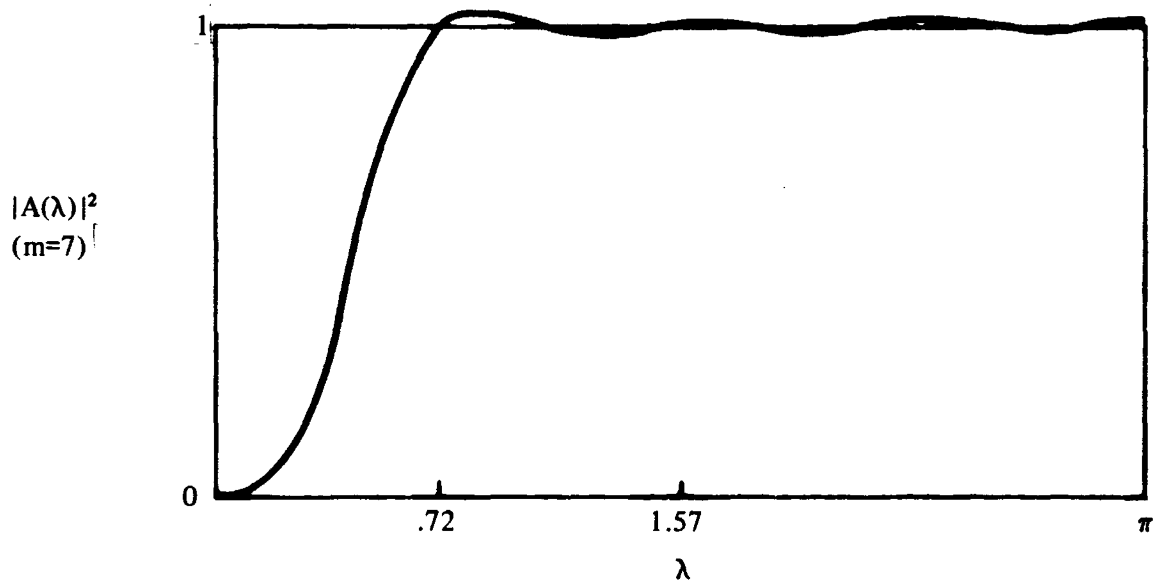

FIGURE A.II.1

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(1) The length, 2m+1, of the moving average (1) is specified by the user.

The low frequency portion of the series $X(t)$ corresponding to the low pass filter with frequency response $1 - A_m(\lambda)$ is just $X(t) - (F_T^{(m)} X)(t)$, $t=m+1,\ldots,N-m$.

The Goodman band pass filter, as presented by N. R. Goodman [7] and extended in the Biomedical Computer Program [4], is a trapezoidal band pass filter constructed from a series of overlapping triangles of unit height with base length $2/m\Delta$ (1), using 2m+1 pieces of data. The high pass filter that we refer to as a Goodman high pass filter is a simple modification of Goodman's results constructed for use in TSAP.

The linear transformation of $\{X(t)\}$ which produces a bandpass filter with center frequency α_0 using n triangles (2) and 2m+1 pieces of data is given by

$$(F_{N,\alpha_0}^{(m)} X)(t) = \sum_{j=-m}^{m} \beta_j X(t-j) ,$$

$t=m+1,\ldots,N-m$, where $\beta_0 = 2n/m$,

$$\beta_j = \beta_{-j} = \frac{2}{m}[.54 + .46 \cos(\frac{j\pi}{m})]\cos(j\alpha_0) \frac{\sin(\frac{nj\pi}{m})}{\sin(\frac{j\pi}{m})}$$

$(j=1,2,\ldots,m-1)$ and

$$\beta_m = (-1)^{n+1} \frac{n}{m} (.08)\cos(m\alpha_0) ,$$

---

(1) When frequency is in dimensions of cycles per unit time and data spacing is $\Delta>0$.

(2) See Figures A.II.2 and 3 for examples with n=2 and m=3.

provided that $(n+1)/2m\Delta \leq \alpha_0 \leq \frac{1}{2\Delta}(1 - \frac{n+1}{m})$

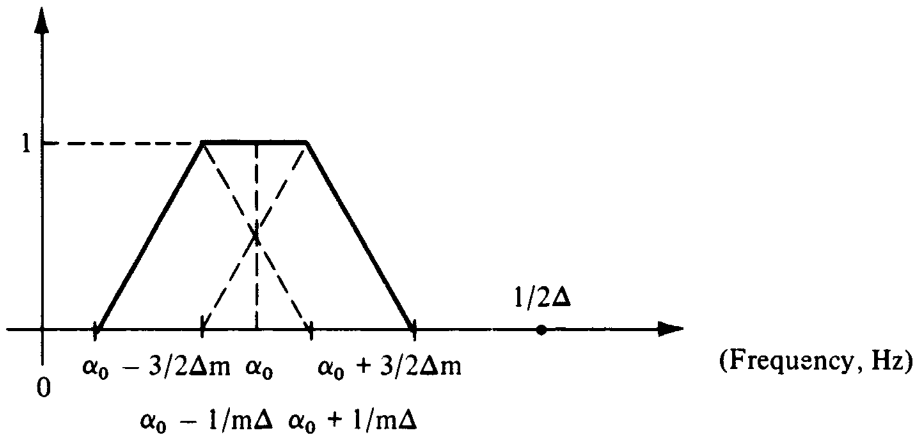

FIGURE A.II.2

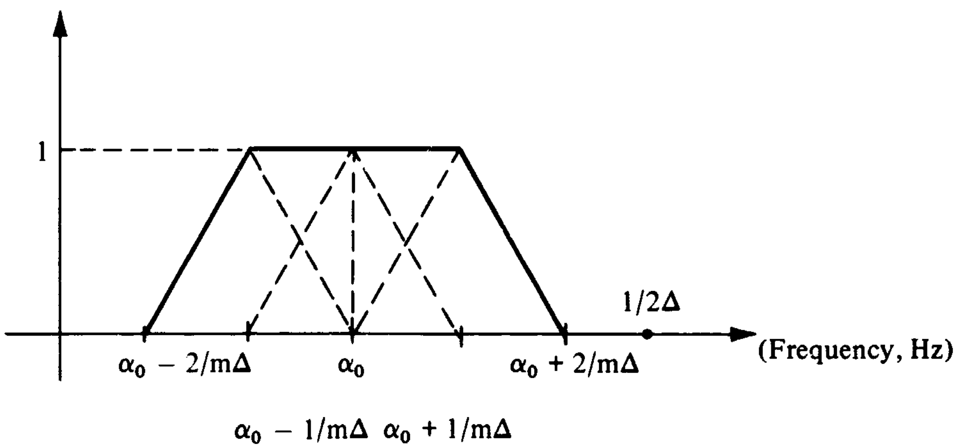

FIGURE A.II.3

The length of the base of the trapezoid is then $(n+1)/m\Delta$.

The modification of the band pass to high pass filter mentioned earlier is accomplished by observing that the Goodman filters are symmetric with respect to the origin and add if the base of the trapezoid defining the band pass filter on $(0, 1/2\Delta)$ overlaps $1/2\Delta$ (or the origin).

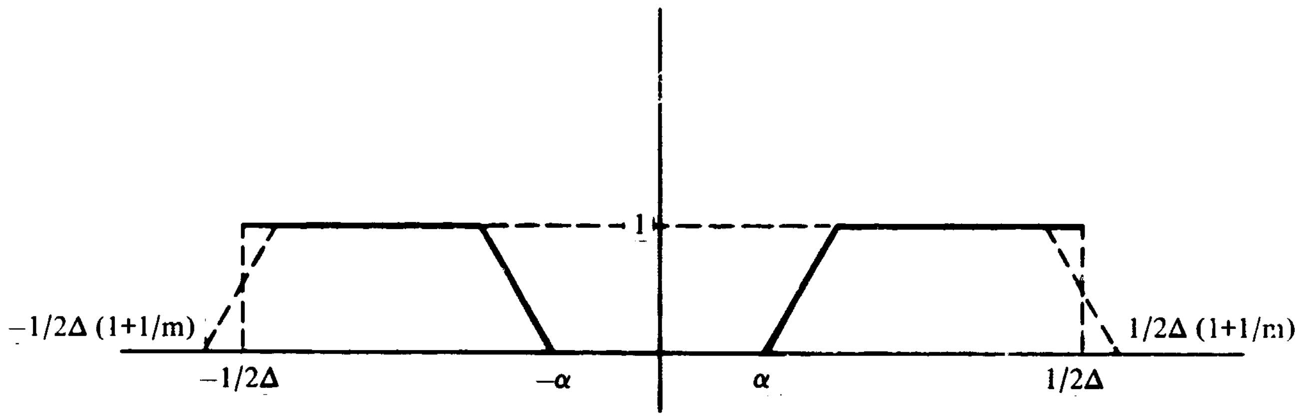

FIGURE A.II.4

The program requests the number of triangles, n, and 2m+1 and tells the user the cut-off point:

$$\alpha = \frac{1}{2\Delta m}\,[m - 2n - 1]\ .$$

APPENDIX III

This appendix contains listings of all main programs presently implemented in TSAP except for SAVE[1] and MATHCON[1] both to be in BAPL (BELLCOMM APPLICATIONS PROGRAM LIBRARY) on SYS*BLIB along with the TSAP INDEX.

```
@PRT     R.INDEX
           000     DATE OF PRESENT ENTRY - 12/1/70
           000     DATE OF LAST ENTRY -    11/30/70
           000
           000     PRESENT CONTENTS:
           000
           000     0) SAVE    - PROGRAM TO CONVERT A FORTRAN GENERATED DATA
           000                  FILE TO SDF(SYSTEM DATA FORMAT) ELEMENT FILE.
           000     1) FILTER* - PROGRAM TO FILTER OR TRANSFORM A DATA SERIES.
           000     2) ACF     - AUTO-COVARIANCE/CORRELATION PROGRAM.
           000     3) PACF    - PARTIAL AUTO-CORRELATION PROGRAM.
           000     4) ASF*    - AUTO-SPECTRUM PROGRAM.
           000     5) LSRA    - LEAST SQUARES MULTIPLE REGRESSION PROGRAM
           000                  WITH AN ANALYSIS OF VARIANCE PRINT OUT.
           000     6) XCF     - CROSS-COVARIANCE/CORRELATION PROGRAM.
           000     7) XSF*    - CROSS-SPECTRUM PROGRAM.
           000     8) COMB    - PROGRAM TO COMBINE TWO TIME SERIES.
           000     9) MATHCON* - PROGRAM TO CONVERT MATH DATA SERIES TO
           000                  FORTRAN DATA SERIES.
           000    10) LSRP¤ - LEAST SQUARES MULTIPLE REGRESSION PROGRAM
           000                WITH PREDICTION LIMITS.
           000     * - REVISED ENTRY
           000     ¤ - NEW ENTRY
```

[1] Both programs were written by C. Mee.

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SINGERS/TSAP - TIME SERIES ANALYSIS PACKAGE//

    R.FILTER,J,J
UUU COMPILED BY 1201 BCS7E ON 01 DEC 70 AT 12:28:17.

N PROGRAM

RAGE USED: CODE(1) 001565; DATA(0) 002410; BLANK COMMON(2) 000000

ERNAL REFERENCES (BLOCK, NAME)

U3     OPTION
U4     CSF
U5     EXIT
U6     NINTR$
U7     NRDU$
10     NIO1$
11     NIO2$
12     NWDU$
13     ALOG
14     NEXP2$
15     ALOG10
16     SQRT
17     EXP
20     COS
21     NEXP1$
22     SIN
23     NSTOP$

RAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

U1     000030 1L       0001   000245 10L     0000   002012 100F    0000   002065 1
U0     002076 103F     0000   002103 104F    0000   002114 105F    0000   002125 1
U0     002141 108F     0000   002234 109F    0000   002274 110F    0000   002300 1
U0     002312 112F     0000   002321 113F    0000   002177 114F    0000   002150 1
U0     002203 117F     0000   002211 118F    0000   002222 119F    0001   000264 1
U0     002255 121F     0000   002324 122F    0000   002246 123F    0000   002155 1
U0     002106 140F     0001   000071 142G    0001   000111 153G    0001   000325 1
U1     000345 18L      0001   000132 2L      0001   000365 20L     0000   002074 2
U1     000204 211G     0001   000424 22L     0001   000221 221G    0001   000235 2
U1     000257 242G     0001   000614 25L     0001   000274 252G    0001   000315 2
U1     000335 272G     0001   000746 28L     0001   001022 29L     0001   001270 3
U1     000420 321G     0001   001440 33L     0001   000500 346G    0001   000545 3
U1     001066 455G     0001   001117 463G    0001   001355 50L     0001   001531 5
U1     001336 523G     0001   001554 53L     0001   001430 554G    0001   001500 5
U1     001517 606G     0001   000225 8L      0000 R 001777 A       0000 R 002001 A
U0 R   002005 CP       0000 R 002003 DEL     0000 R 001750 FMI     0000 R 001756 F
U0 I   001773 IFL      0000 I 002000 II      0000 I 002006 ISO     0000 I 001766 I
U0 I   001775 K        0000 I 001767 L       0000 I 001776 M       0000 I 001770 N
U3 L   000000 OPTION   0000 R 002007 SF      0000 R 001772 SM      0000 R 002004 W


    1*    C
    2*    C TITLE          FILTER - A PROGRAM TO FILTER OR TRANSFORM A
```

```
      C                              DATA SERIES.
      C
      C AUTHOR          R.R.SINGERS
      C
      C SPONSOR         G.R.ANDERSEN
      C
      C DATE            AUGUST,1970
      C
      C KEY WORDS       FILTER
      C                 TRANSFORMATION
      C
      C PURPOSE         TO APPLY ONE OR MORE OF A LIST OF FILTERS OR
      C                 TRANSFORMS TO A DATA SERIES.
      C
      C METHOD          THE PROGRAM IS DESIGNED IN AN INTERACTIVE MODE
      C                 ASKING FOR CERTAIN PARAMETERS AND APPLYING THEM TO
      C                 THE INPUT DATA SERIES. THE PROCESS IS SEQUENTIAL
      C                 ALLOWING MORE THAN ONE FILTER OR TRANSFORMATION
      C                 TO BE APPLIED TO THE SAME SERIES.
      C
      C NOTE            1) A DYNAMICALLY ASSIGNED FILE (UNIT 8) IS
      C                 USED TO STORE THE FINAL SERIES.
      C
                        PARAMETER MX=1000
                        DIMENSION X(MX),FMI(6),FMO(6)
                        DIMENSION ASG8(2)/'@ASG,T 8,F'/
                        LOGICAL OPTION
                        READ(5,201,END=53) FMI
                        IF(OPTION('N'))GO TO 1
                        L=MX
                        WRITE(6,100)
        100             FORMAT(/' ''FILTER'' CONTAINS THE FOLLOWING FILTERS:'/' C
            .ENTER,EXP,LOG,DIFF,10**,LOG10,SQRT,LAG,ABS,TUKEY(HIGH AND LOW PASS
            .),GOODMAN,'/' SCALE.'/' TO TERMINATE THE FILTER SEQUENCE,REPLY WIT
            .H ''END''.'/' THE FINAL SERIES IS STORED ON UNIT 8(IF DESIRED).')
        1               WRITE(6,101)
        101             FORMAT(' LENGTH OF SERIES TO BE READ IN?')
                        READ(5,200,END=53)N
        200             FORMAT( )
                        IF(N.GT.MX)GO TO 51
        201             FORMAT(6A6)
                        WRITE(6,103)
        103             FORMAT(' INPUT DATA FORMAT?')
                        READ(5,201,END=53)FMI
                        WRITE(6,104)
        104             FORMAT(' INPUT DATA:')
                        READ(5,FMI,END=53)(X(I),I=1,N)
                        WRITE(6,140)
        140             FORMAT(' LENGTH OF SERIES TO BE USED?')
                        READ(5,200,END=53)N
        2               SM=0.
                        DO 3 I=1,N
        3               SM=SM+X(I)
                        SM=SM/N
                        WRITE(6,105)SM,N
        105             FORMAT(' MEAN=',E15.8/' LENGTH=',I5///' FILTER?')
        4               READ(5,201,END=53) IFL
```

```
                    IF(IFL.EQ.'END    ')GO TO 50
                    IF(IFL.NE.'CENTER')GO TO 6
                    DO 5 I=1,N
          5         X(I)=X(I)-SM
                    GO TO 2
          6         IF(IFL.NE.'ABS    ')GO TO 8
                    DO 7 I=1,N
          7         X(I)=ABS(X(I))
                    GO TO 2
          8         IF(IFL.NE.'LOG    ')GO TO 10
                    DO 9 I=1,N
          9         X(I)=ALOG(X(I))
                    GO TO 2
          10        IF(IFL.NE.'DIFF   ')GO TO 12
                    N=N-1
                    DO 11 I=1,N
          11        X(I)=X(I+1)-X(I)
                    GO TO 2
          12        IF(IFL.NE.'10**   ')GO TO 14
                    DO 13 I=1,N
          13        X(I)=10**X(I)
                    GO TO 2
          14        IF(IFL.NE.'LOG10  ')GO TO 16
                    DO 15 I=1,N
          15        X(I)=ALOG10(X(I))
                    GO TO 2
          16        IF(IFL.NE.'SQRT   ')GO TO 18
                    DO 17 I=1,N
          17        X(I)=SQRT(X(I))
                    GO TO 2
          18        IF(IFL.NE.'EXP    ')GO TO 20
                    DO 19 I=1,N
          19        X(I)=EXP(X(I))
                    GO TO 2
          20        IF(IFL.NE.'LAG    ')GO TO 22
                    WRITE(6,106)
          106       FORMAT(' NUMBER OF LAGS?')
                    READ(5,200,END=53)J
                    N=N-J
                    DO 21 I=1,N
          21        X(I)=X(I+J)
                    GO TO 2
          22        IF(IFL.NE.'TUKEY  ')GO TO 25
                    WRITE(6,107)
          107       FORMAT(' LENGTH OF MOVING AVERAGE(ODD NUMBER)?')
                    READ(5,200,END=53)J
                    K=(J-1)/2
                    WRITE(6,108)
          108       FORMAT(' HIGH PASS(HIGH) OR LOW PASS(LOW)?')
                    READ(5,201,END=53)M
                    DO 24 I=1,N
                    IF(I.LE.K.OR.I.GT.N-K)GO TO 24
                    A=0.
                    DO 23 II=-K,K
                    AL=-1./(K+1.)*(.5+.5*COS(II*3.14159/(K+1.)))
                    IF(II.EQ.0)AL=1.-1./(K+1.)
          23        A=A+AL*X(I+II)
```

```
              IF(M.EQ.'LOW    ')A=X(I)-A
              X(I-K)=A
   24         CONTINUE
              N=N-J+1
              GO TO 2
   25         IF(IFL.NE.'GOODMA')GO TO 30
              WRITE(6,107)
              READ(5,200,END=53) J
              K=(J-1)/2
              WRITE(6,115)
  115         FORMAT(' NUMBER OF TRIANGLES?')
              READ(5,200,END=53) NO
              WRITE(6,124)
  124         FORMAT(' DATA SPACING?')
              READ(5,200,END=53)DEL
              WRITE(6,116)
  116         FORMAT(' HIGH PASS(HIGH), LOW PASS(LOW), PASS BAND',
     .        '(PASS) OR REJECT BAND(REJECT)?')
              READ(5,201,END=53)M
              IF(M.EQ.'HIGH'.OR.M.EQ.'LOW') GO TO 28
              WRITE(6,114)
  114         FORMAT(' CENTER FREQUENCY?')
              READ(5,200,END=53) W
              W=W*DEL*6.28318
              GO TO 29
   28         CP=.5*(K-1.-2.*NO)/K/DEL
              WRITE(6,117) CP
  117         FORMAT(' CUT-OFF FREQUENCY IS ',F10.7)
              WRITE(6,118)
  118         FORMAT(' DO YOU WANT TO CHANGE INPUT VALUES(YES/NO)?')
              READ(5,201,END=53)I
              IF(I.EQ.'YES') GO TO 25
              W=3.14159*(1.-FLOAT(NO)/FLOAT(K))
   29         DO 27 I=1,N
              IF(I.LE.K.OR.I.GT.N-K) GO TO 27
              A=0.
              DO 26 II=-K,K
              IF(II.EQ.ABS(K))AL=(-1)**(NO+1)*.08*NO/K*COS(K*W)
              IF(II.EQ.0) AL=2.*NO/K
              IF(II.NE.0.AND.II.NE.ABS(K))AL=2./K*(.54+.46*COS(
     .        II*3.14159/K))*COS(II*W)*SIN(NO*II*3.14159/K)/SIN
     . (3.14159*II/K)
   26         A=A+AL*X(I+II)
              IF(M.EQ.'LOW'.OR.M.EQ.'REJECT')A=X(I)-A
              X(I-K)=A
   27         CONTINUE
              N=N-J+1
              GO TO 2
   30         IF(IFL.NE.'SCALE')GO TO 52
              WRITE(6,119)
  119         FORMAT(' SCALE OPERATOR(ADD/MPY)?')
              READ(5,201)ISO
              WRITE(6,120)
  120         FORMAT(' SCALE FACTOR?')
              READ(5,200)SF
              DO 31 I=1,N
              IF(ISO.EQ.'ADD')X(I)=X(I)+SF
```

```
     31               IF(ISO.EQ.'MPY')X(I)=X(I)*SF
                      GO TO 2
     50               WRITE(6,109)
     109              FORMAT(' DO YOU WANT TO SEE PART OF THE SERIES',
         .            '(YES/NO)?')
                      READ(5,201,END=53)IFL
                      IF(IFL.NE.'YES')GO TO 33
                      WRITE(6,123)
     123              FORMAT(' WHAT PART(INITIAL,FINAL,STEP)?')
                      READ(5,200,END=53)I,J,K
                      DO 32 L=I,J,K
     32               WRITE(6,200)X(L)
     33               WRITE(6,121)
     121              FORMAT(' DO YOU WANT THE FINAL SERIES STORED',
         .            '(YES/NO)?')
                      READ(5,201,END=53)IFL
                      IF(IFL.NE.'YES   ')GO TO 53
                      WRITE(6,102)
     102              FORMAT(' OUTPUT DATA FORMAT?')
                      READ(5,201,END=53)FMO
                      CALL CSF(2,ASG8,I)
                      WRITE(8,FMO)(X(I),I=1,N)
                      WRITE(6,110)
     110              FORMAT(' SERIES STORED.')
                      GO TO 53
     51               WRITE(6,111)L
     111              FORMAT(' LENGTH OF SERIES MUST BE LESS THAN',I5,',TRY AGA
         .IN.')
                      GO TO 1
     52               WRITE(6,112)
     112              FORMAT(' FILTER NOT RECOGNIZED,TRY AGAIN.')
                      WRITE(6,113)
     113              FORMAT(' FILTER?')
                      GO TO 4
     53               WRITE(6,122)
     122              FORMAT(' FILTER TERMINATES.')
                      CALL EXIT
              END

END OF COMPILATION:          NO  DIAGNOSTICS.
```

```
SINGERS/TSAP - TIME SERIES ANALYSIS PACKAGE//

    R.ACF,J,J
000 COMPILED BY 1201 BCS7E ON 01 DEC 70 AT 12:28:49.

N PROGRAM

RAGE USED: CODE(1) 000501; DATA(0) 004145; BLANK COMMON(2) 000000

ERNAL REFERENCES (BLOCK, NAME)

03    OPTION
04    CSF
05    EXIT
06    NINTR$
07    NRDU$
10    NIO1$
11    NIO2$
12    NREW$
13    NWDU$
14    NABU$
15    NWEF$
16    NSTOP$

RAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

01      000040 1L       0001    000422 10L     0000    004004 100F    0000    004017 1
00      004033 103F     0000    004040 104F    0000    004043 105F    0000    004046 1
00      004055 108F     0000    004066 109F    0000    004073 110F    0000    004105 1
00      004121 122F     0001    000160 163G    0001    000200 174G    0001    000061 2
00      004023 201F     0001    000211 202G    0001    000226 207G    0001    000274 2
01      000340 251G     0001    000103 3L      0001    000416 301G    0001    000431 3
01      000452 51L      0001    000462 52L     0001    000470 53L     0000    003750 9
00 R    003726 AC       0000 R  003744 B       0000 R  000000 C       0000 R  003720 F
00 I    003745 IC       0000 I  003733 ICO     0000 I  003743 II      0000 I  003730 I
00 I    003731 L        0000 I  003735 M       0000 I  003742 MR      0000 I  003736 M
00 I    003737 NI       0003 L  000000 OPTION  0000 R  001750 Y

      C
      C TITLE        ACF - AUTO-COVARIANCE/CORRELATION PROGRAM.
      C
      C AUTHOR       R.R.SINGERS
      C
      C SPONSOR      G.R.ANDERSEN
      C
      C DATE         AUGUST,1970
      C
      C KEY WORDS    COVARIANCE
      C              CORRELATION
      C
      C PURPOSE      TO COMPUTE THE AUTO-COVARIANCES OF A DATA SERIES
      C              AND IF DESIRED, THE AUTO-CORRELATIONS.
      C
```

```
      C METHOD        THE PROGRAM IS DESIGNED IN AN INTERACTIVE MODE
      C               ASKING FOR CERTAIN PARAMETERS AND A DATA SERIES.
      C               THE STANDARD METHOD OF COMPUTING THE AUTO-COVARIANCES
      C               IS THEN APPLIED.
      C
      C NOTE          1) A DYNAMICALLY ASSIGNED FILE (UNIT 1) IS
      C               USED TO SAVE THE AUTO-COVARIANCES (WHEN
      C               SPECIFIED) FOR LATER USE.
      C
                      PARAMETER MX=1000
                      DIMENSION C(MX),Y(MX),FMT(6),AC(2)/'@ASG,T 1,F'/
                      LOGICAL OPTION
                      L=MX
                      READ(5,200,END=53)FMT
                      CALL CSF(2,AC,I)
                      REWIND 1
                      IF(OPTION('N'))GO TO 1
                      WRITE(6,99)
        99            FORMAT(/' ACF COMPUTES THE AUTO-COVARIANCE/CORRELATION FU
             .NCTION OF A SERIES.'/' THE COVARIANCES ARE STORED ON UNIT 1 IN BINA
             .RY(IF DESIRED)'/' IN CONSECUTIVE ORDER.')
        1             WRITE(6,100)
        100           FORMAT(/' DO YOU WANT COVARIANCE(COV),CORRELATION(COR) OR
             . END?')
                      READ(5,200,END=53)ICO
        200           FORMAT(6A6)
                      IF(ICO.EQ.'END')GO TO 50
        2             WRITE(6,101)
        101           FORMAT(' LENGTH OF SERIES?')
                      READ(5,201,END=53)N
        201           FORMAT( )
                      IF(N.GT.MX)GO TO 51
        3             WRITE(6,102)
        102           FORMAT(' NUMBER OF LAGS(ZERO INCLUDED)?')
                      READ(5,201,END=53)M
                      IF(M.GT.N)GO TO 52
        4             WRITE(6,103)
        103           FORMAT(' INPUT DATA FORMAT?')
                      READ(5,200,END=53)FMT
                      WRITE(6,104)
        104           FORMAT(' INPUT DATA:')
                      READ(5,FMT,END=53)(Y(I),I=1,N)
                      M1=M-1
                      DO 6 I=0,M1
                      NI=N-I
                      A=0.
                      DO 5 J=1,NI
        5             A=A+Y(J)*Y(J+I)
                      C(I+1)=A/N
        6             IF(ICO.EQ.'COR'.AND.I.NE.0)C(I+1)=C(I+1)/C(1)
                      A=C(1)
                      IF(ICO.EQ.'COR') C(1)=1.
                      MR=M/4
                      II=MR+1
                      IF(MR.GT.10)MR=10
                      DO 7 I=0,MR
        7             WRITE(6,105)(J,C(J+1),J=I,M1,II)
```

```
 105            FORMAT(4(I4,E15.8))
                IF(ICO.EQ.'COR')WRITE(6,106)A
 106            FORMAT(' VARIANCE=',E15.8)
                B=0.
                DO 8 I=1,N
 8              B=B+Y(I)
                B=B/N
                WRITE(6,107)B
 107            FORMAT(' MEAN=',E15.8)
                WRITE(6,108)
 108            FORMAT(' DO YOU WANT THE COVARIANCES STORED(YES/NO)?')
                READ(5,200,END=53)IC
                IF(IC.NE.'YES')GO TO 1
                WRITE(1)N,M
                IF(ICO.NE.'COR')GO TO 10
                DO 9 I=1,M
 9              C(I)=A*C(I)
 10             WRITE(1)(C(I),I=1,M)
                WRITE(6,109)
 109            FORMAT(' COVARIANCE STORED.')
                GO TO 1
 50             END FILE 1
                REWIND 1
                GO TO 53
 51             WRITE(6,110)L
 110            FORMAT(' SERIES LENGTH MUST BE LESS THAN',I10,' -TRY AGAI
     .N.')
                GO TO 2
 52             WRITE(6,111)
 111            FORMAT(' NUMBER OF LAGS CANNOT BE GREATER THAN LENGTH OF
     .SERIES-TRY AGAIN.')
                GO TO 3
 53             WRITE(6,122)
 122            FORMAT(' ACF TERMINATES.')
                CALL EXIT
          END
```

END OF COMPILATION: NO DIAGNOSTICS.

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SINGERS/TSAP - TIME SERIES ANALYSIS PACKAGE//

    R.PACF,J,J
)00 COMPILED BY 1201 BCS7E ON 01 DEC 70 AT 12:29:01.

N PROGRAM

RAGE USED: CODE(1) 000555; DATA(0) 007457; BLANK COMMON(2) 000000

ERNAL REFERENCES (BLOCK, NAME)

03    OPTION
04    EXIT
05    NINTR$
06    NRDU$
07    NIO1$
10    NIO2$
11    NWDU$
12    NRBU$
13    NSTOP$

RAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

)1     000026 1L        0001   000413 10L     0000   007253 100F    0000   007317 1
)0     007330 104F      0000   007343 105F    0000   007376 106F    0000   007405 1
)0     007350 109F      0000   007363 110F    0001   000011 110G    0000   007420 1
)1     000110 147G      0001   000155 157G    0001   000213 172G    0000   007250 2
)1     000243 206G      0001   000273 224G    0001   000310 230G    0001   000350 2[illegible]
)1     000455 277G      0001   000470 304G    0001   000525 323G    0001   000354 5[illegible]
)1     000544 53L       0000 R 000000 A       0000 D 007224 A0      0000 D 007226 A
)0 D 007232 DEN         0000 R 007210 FMT     0000 I 007240 I       0000 I 007234 I[illegible]
)0 I 007241 K           0000 I 007246 K0      0000 I 007242 L       0000 I 007243 L
)0 I 007247 MA          0000 I 007237 ML      0000 I 007245 M1      0000 I 007236 N
)3 L 000030 OPTION      0000 D 007216 R0      0000 D 007220 R1      0000 D 007222 R[illegible]
```

```
C
C TITLE         PACF - PARTIAL AUTO-CORRELATION PROGRAM.
C
C AUTHOR        R.R.SINGERS
C
C SPONSOR       G.R.ANDERSEN
C
C DATE          AUGUST,1970
C
C KEY WORDS     PARTIAL AUTO-CORRELATION
C
C PURPOSE       TO COMPUTE THE PARTIAL AUTO-CORRELATION
C               COEFFICIENTS AND THE CORRESPONDING VARIANCES
C               OF UP TO 30 LAGS GIVEN THE AUTO-COVARIANCES.
C               THE FIRST ORDER MOVING AVERAGE COEFFICIENT
C               IS ALSO COMPUTED.
C
C METHOD        THE PROGRAM IS DESIGNED IN AN INTERACTIVE MODE
```

```
C                  ASKING FOR CERTAIN PARAMETERS. IT IS ASSUMED
C                  THAT ACF WAS EXECUTED AND THE AUTO-COVARIANCES
C                  SAVED. THE STANDARD METHOD OF COMPUTATION IS USED.
C
C NOTE             1) THE AUTO-COVARIANCES ARE ASSUMED TO
C                  BE STORED (ON UNIT 1) FROM ACF.
C
                   PARAMETER MX=60
                   DIMENSION A(MX,MX),CO(MX),V(MX),FMT(6)
                   DOUBLE PRECISION R0,R1,R2,A0,A1,NUM,DEN
                   LOGICAL OPTION
                   READ(5,200,END=53)FMT
  200              FORMAT(6A6)
                   IF(OPTION('N'))GO TO 1
                   WRITE(6,100)
  100              FORMAT(' PACF COMPUTES THE PARTIAL AUTO-CORRELATION AND A
         ..R. COEFFICIENTS'/' UP TO ORDER K TOGETHER WITH THE RESIDUAL',
         .' VARIANCES OF THE'/' FITTED A.R.''S UP TO ORDER K AND THE FIRST',
         .' ORDER M.A. COEFFICIENTS.')
  1                WRITE(6,101)
  101              FORMAT(/' MAXIMUM ORDER OF K?')
                   READ(5,201,END=53)M
  201              FORMAT( )
                   IF(M.GT.MX)GO TO 51
                   READ(1)N,ML
                   READ(1)(CO(I),I=1,ML)
                   A(1,1)=CO(2)/CO(1)
                   DO 4 K=3,M
                   L=K-1
                   L1=L-1
                   R0=CO(1)
                   NUM=CO(K)/R0
                   DEN=1.D0
                   DO 2 I=1,L1
                   A1=A(L1,I)
                   R1=CO(L-I+1)
                   R2=CO(I+1)
                   NUM=NUM-A1*R1/R0
  2                DEN=DEN-A1*R2/R0
                   A0=NUM/DEN
                   A(L,L)=A0
                   DO 3 J=1,L1
  3                A(L,J)=A(L1,J)-A(L,L)*A(L1,L-J)
  4                CONTINUE
                   M1=M-1
                   WRITE(6,103)
  103              FORMAT(/' PACF:')
                   WRITE(6,104)(A(J,J),J=1,M1)
  104              FORMAT(/(5E15.8))
                   WRITE(6,108)
  108              FORMAT(' DO YOU WANT RESIDUAL VARIANCES',
         .         '(YES/NO)?')
                   READ(5,200,END=53)I
                   IF(I.NE.'YES')GO TO 5
                   DO 7 L=1,M1
                   V(L)=CO(1)
                   DO 6 J=1,L
```

```
      6          V(L)=V(L)-A(L,J)*CO(J+1)
      7          V(L)=(N-L)/(N-2.*L-1.)*V(L)
                 WRITE(6,105)
      105        FORMAT(/' RESIDUAL VARIANCES:')
                 WRITE(6,104)(V(I),I=1,M1)
      5          WRITE(6,109)
      109        FORMAT(' ORDER OF A.R. PROCESS TO BE VIEWED',
              .  '(0=NONE OR NO MORE)?')
                 READ(5,201,END=53)K0
                 IF(K0.EQ.0)GO TO 10
                 WRITE(6,104)(A(K0,I),I=1,K0)
                 GO TO 5
      10         WRITE(6,110)
      110        FORMAT(' DO YOU WANT THE FIRST ORDER M.A.',
              .  ' COEFFICIENTS(YES/NO)?')
                 READ(5,200,END=53) MA
                 IF(MA.NE.'YES') GO TO 53
                 DO 9 K=1,M1
                 DEN=1.D0
                 NUM=A(M1,1)
                 DO 8 I=1,K
                 R1=A(M1,1)
                 R2=A(M1,I+1)
                 DEN=DEN+R1**2
      8          NUM=NUM+R1*R2
                 DEN=DEN+(DBLE(A(M1,M1)))**2
      9          V(K)=NUM/DEN
                 WRITE(6,106)
      106        FORMAT(/' FIRST ORDER M.A. COEFFICIENTS:')
                 WRITE(6,104)(V(I),I=1,M1)
                 GO TO 53
      51         L=MX
                 WRITE(6,107)L
      107        FORMAT(' NUMBER OF LAGS CANNOT BE GREATER THAN',I3,' -TRY
              .AGAIN.')
                 GO TO 1
      53         WRITE(6,111)
      111        FORMAT(' PACF TERMINATES.')
                 CALL EXIT
            END

END OF COMPILATION:        NO DIAGNOSTICS.
```

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SINGERS/TSAP - TIME SERIES ANALYSIS PACKAGE//

     R.ASF,J,J
)00 COMPILED BY 1201 BCS7E ON 01 DEC 70 AT 12:29:30.


N PROGRAM

RAGE USED: CODE(1) 001037; DATA(0) 004532; BLANK COMMON(2) 000000


ERNAL REFERENCES (BLOCK, NAME)

)3    OPTION
)4    CSF
)5    EXIT
)6    NINTR$
)7    NRDU$
.0    NIO1$
.1    NIO2$
.2    NWDU$
.3    NRBU$
14    COS
15    SIN
16    SQRT
17    NWBU$
20    NWEF$
21    NSTOP$


RAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

)1    000151 1L        0000   004250 100F      0000   004317 102F      0000   004327 1
)0    004377 105F      0000   004407 106F      0000   004413 107F      0000   004440 1
)0    004454 110F      0000   004334 111F      0000   004463 112F      0000   004472 1
)0    004353 115F      0000   004313 116F      0001   000011 121G      0001   000170 2
)0    004326 201F      0001   000221 214G      0001   000322 240G      0001   000347 2
)1    000500 302G      0001   000637 327G      0001   000754 371G      0001   001026 5
)1    001023 7L        0001   000026 99L       0000 R 004214 ANU       0000 R 003722 A
)0 R  000002 C         0000 R 003733 CHI       0000 R 004224 CL        0000 R 004231 C
)0 R  004210 DEL       0000 R 004242 EF        0000 R 004235 EL1       0000 R 004213 E
)0 R  001752 F         0000 R 004226 FM        0000 R 003725 FMT       0000 R 004241 F
)0 I  004220 I         0000 I 004222 ICL       0000 I 004244 IF        0000 I 004205 I
)0 I  004243 IU        0000 I 004211 IW        0000 I 003724 I$        0000 I 004227 K
)0 I  004212 M         0000 I 004206 N         0000 I 004216 NM        0000 I 004207 N
)0 I  004225 NU        0003 L 000000 OPTION    0000 R 000001 PI        0000 R 000000 P
)0 R  004245 VR        0000 R 004232 V0        0000 R 004233 V1        0000 R 004237 V



   1*    C
   2*    C TITLE         ASF - AUTO-SPECTRUM PROGRAM.
   3*    C
   4*    C AUTHOR        R.R.SINGERS
   5*    C
   6*    C SPONSOR       G.R.ANDERSEN
   7*    C
   8*    C DATE          AUGUST,1970
```

```
      C
      C KEY WORDS      AUTO-SPECTRUM
      C                SPECTRAL DENSITY
      C
      C PURPOSE        TO COMPUTE THE AUTO-SPECTRUM OF A DATA SERIES
      C                AND TO PLOT THE SPECTRUM WITH ITS CONFIDENCE
      C                LIMITS.
      C
      C METHOD         THE PROGRAM IS DESIGNED IN AN INTERACTIVE MODE
      C                ASKING FOR CERTAIN PARAMETERS AND A DATA SERIES.
      C                IT IS ASSUMED THAT ACF WAS EXECUTED AND THE
      C                AUTO-COVARIANCES SAVED. THE STANDARD METHOD OF
      C                COMPUTATION IS USED.
      C
      C NOTES          1) THE COVARIANCES ARE ASSUMED TO BE
      C                STORE (ON UNIT 1) FROM ACF.
      C                2) A DYNAMICALLY ASSIGNED FILE ( UNIT 2)
      C                IS USED TO STORE THE AUTO-SPECTRUM
      C                (WHEN SPECIFIED).
      C
                       PARAMETER MX=1000
                       DATA PIV,PI/.1591549,3.1415926/
                       DIMENSION C(MX),F(MX),AS(2)/'@ASG,T 2,F'/,FMT(6)
                       DIMENSION CHI(60,2)/
            . 0.004,0.103,0.352,0.711,1.145,1.635,2.167,2.733,3.325,3.940,4.575
            . ,5.226,5.892,6.571,7.261,7.962,8.672,9.390,10.117,10.851,11.591,1
            . 2.338,13.091,13.848,14.611,15.379,16.151,16.928,17.708,18.493,19.
            . 281,20.072,20.867,21.664,22.465,23.269,24.075,24.884,25.695,26.50
            . 9,27.326,28.144,28.965,29.787,30.612,
            . 31.439,32.268,33.098,33.930,34.764,35.6,36.437,37.276,38.116,38.9
            . 58,39.801,40.646,41.492,42.339,43.188,
            .                                         3.841,5.991,7.815,9.488,11.0
            . 71,12.592,14.067,15.507,16.919,18.307,19.675,21.026,22.362,23.685
            . ,24.996,26.296,27.587,28.869,30.144,31.410,32.671,33.924,35.172,3
            . 6.415,37.652,38.885,40.113,41.337,42.557,43.773,44.985,46.194,47.
            . 400,48.602,49.802,50.998,52.192,53.384,54.572,55.758,56.942,58.12
            . 4,59.304,60.481,61.656,
            . 62.83,64.001,65.171,66.339,67.505,68.669,69.832,70.993,72.153,73.
            . 311,74.468,75.624,76.778,77.931,79.082/
                       DIMENSION IPT(50)/50*' '/
                       DATA IL/'NO'/
                       LOGICAL OPTION
                       READ(5,201,END=53)FMT
                       IF(OPTION('N'))GO TO 99
                       WRITE(6,100)
        100            FORMAT(/' ASF COMPUTES THE AUTO SPECTRUM FUNCTION.'/
               .       ' IT IS ASSUMED THAT ACF WAS CALLED AND THE DESIRED'
               .       ' COVARIANCES STORED.'/' THE AUTO-SPECTRA ARE'
               .       ' STORED ON UNIT 2(IF DESIRED) CONSECUTIVELY.')
        99             IF(IL.NE.'YES')READ(1)N,NN
        200            FORMAT( )
                       WRITE(6,116)
        116            FORMAT(' DATA SPACING?')
                       READ(5,200,END=53) DEL
                       WRITE(6,102)
        102            FORMAT(' WINDOW TO BE USED(TUKEY/DANIELL)?')
                       READ(5,201,END=53)IW
```

```
201   FORMAT(6A6)
      WRITE(6,103)
103   FORMAT(' BANDWIDTH PARAMETER?')
      READ(5,200,END=53)M
      IF(IW.EQ.'TUKEY')GO TO 1
      EM1=M-1
      ANU=2*N/EM1
      AU=1.
      WRITE(6,111)
111   FORMAT(' NUMBER OF COVARIANCES TO BE USED?')
      READ(5,200,END=53) NM
      GO TO 2
1     EM1=M-1.
      ANU=2.667*N/EM1
      AU=.75
      NM=M
2     WRITE(6,104)
104   FORMAT(' NUMBER OF FREQUENCY POINTS TO BE VIEWED?')
      READ(5,200,END=53)NQ
      IF(IL.NE.'YES')READ(1)(C(I),I=1,NN)
      ENQ1=NQ-1
      WRITE(6,115)
115   FORMAT(' DO YOU WANT CONFIDENCE LIMITS(YES/NO)?')
      READ(5,201,END=53)ICL
      CU=1.
      CL=1.
      IF(ICL.NE.'YES')GO TO 25
      NU=IFIX(ANU+.5)
      CU=ANU/CHI(NU,1)
      CL=ANU/CHI(NU,2)
25    FM=0.
      DO 4 K=1,NQ
      TAU=K-1
      CO=COS(PI*TAU/ENQ1)
      V0=0.
      V1=0.
      DO 3 L=NM,2,-1
      EL1=L-1
      W=1.
      IF(IW.EQ.'DANIEL')W=SIN(EL1*PI/EM1)/(EL1*PI/EM1)
      IF(IW.EQ.'TUKEY')W=.5*(1.+COS(PI*EL1/EM1))
      V2=2.*CO*V1-V0+W*C(L)
      V0=V1
3     V1=V2
      F(K)=DEL*(C(1)+2.*(V1*CO-V0))
4     FM=AMAX0(FM,F(K))
      FX=FM*CU/49.
      WRITE(6,114)
114   FORMAT(' DO YOU WANT FREQUENCY(FREQ.) OR'
      ' PERIOD(PERIOD) DISPLAYED?')
      READ(5,201,END=53)FP
      WRITE(6,105)FP
105   FORMAT(' AUTO-SPECTRUM:'/2X,A6,'   SPECTRUM')
      DO 5 I=1,NQ
      L=I-1
      IF(FP.NE.'PERIOD')EF=.5*L/ENQ1/DEL
      IF(FP.EQ.'PERIOD'.AND.L.EQ.0)EF=99.9999
```

```
                  IF(FP.EQ.'PERIOD'.AND.L.NE.0)EF=2.*ENQI/L*DEL
                  IU=IFIX(F(I)*CU/FX)+1
                  IL=IFIX(F(I)*CL/FX)+1
                  IF=IFIX(F(I)/FX)+1
                 IF(IU.GT.50)IU=50
                  IPT(IU)='+'
                  IPT(IL)='*'
                  IPT(IF)='.'
                  WRITE(6,106)EF,F(I),IPT
      106         FORMAT(F8.4,E12.5,2X,50A1)
                  IPT(IU)=' '
                  IPT(IF)=' '
      5           IPT(IL)=' '
                  VR=SQRT(AU*M/N)*100.
                  IF(ICL.EQ.'YES')WRITE(6,107)NU,CU,CL,VR
      107         FORMAT(' THE 90% CONFIDENCE LIMITS FOR NU=',I2,' ARE: CU=
           ,'F10.5,' ,CL=',F10.5/' THE SQRT OF VARIANCE RATIO IS:',F8.3,' %')
                  WRITE(6,108)
      108         FORMAT(' DO YOU WANT THE SPECTRUM STORED(YES/NO)?')
                  READ(5,201,END=53)IW
                  IF(IW.NE.'YES')GO TO 6
                  CALL CSF(2,AS,I)
                  WRITE(2)M,DEL,NQ
                  WRITE(2)(F(I),I=1,NQ)
                  WRITE(6,109)
      109         FORMAT(' SPECTRUM STORED.')
      6           WRITE(6,110)
      110         FORMAT(' DO YOU HAVE ANOTHER SERIES(YES/NO)?')
                  READ(5,201,END=53)IW
                  IF(IW.NE.'YES')GO TO 7
                  WRITE(6,112)
      112         FORMAT(' IS THE SERIES THE SAME(YES/NO)?')
                  READ(5,201) IL
                  GO TO 99
      7           END FILE 2
      53          WRITE(6,113)
      113         FORMAT(' ASF TERMINATES.')
                  CALL EXIT
            END
```

END OF COMPILATION: NO DIAGNOSTICS.

```
SINGERS/TSAP - !IME SERIES ANALYSIS PACKAGE//

     R.LSRA,J,J
000 COMPILED BY 1201 BCS7E ON 01 DEC 70 AT 12:29:18.


N PROGRAM

RAGE USED: CODE(1) 001076; DATA(0) 007410; BLANK COMMON(2) 000000


ERNAL REFERENCES (BLOCK, NAME)

03    OPTION
04    RESTEM
05    EXIT
06    CSF
07    NINTR$
10    NRDU$
11    NIO1$
12    NIO2$
13    NWDU$
14    SQRT
15    NSTOP$


RAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

01     000026 1L      0001   000617 10L     0000   007003 100F    0000   007042 1
00     007217 103F    0000   007053 105F    0000   007056 107F    0000   007064 1
00     007102 110F    0001   000011 110G    0000   007116 111F    0000   007124 1
00     007137 114F    0000   007145 115F    0000   007151 116F    0000   007156 1
01     000732 12L     0000   007265 120F    0000   007306 121F    0000   007335 1
00     007243 124F    0000   007256 125F    0000   007311 126F    0000   007324 1
00     007363 129F    0001   000140 160G    0001   000160 171G    0001   000173 1
00     007000 200F    0000   007041 201F    0001   000212 207G    0001   000232 2
01     000345 253G    0001   000366 260G    0001   000412 272G    0001   000455 3
01     000603 366G    0001   000705 423G    0001   000770 447G    0001   001015 4
01     000246 5L      0001   001041 51L     0001   001053 52L     0001   001065 5
01     000511 8L      0001   000520 9L      0000   007034 99F     0000 R 005675 A
00 R   006765 C       0000 R 006744 D       0000 R 006777 DEV     0000 R 006756 E
00 R   006767 FL      0000 R 006736 FMT     0000 I 006760 I       0000 I 006763 I
00 I   005733 IVAR    0000 I 006755 IW      0000 I 006752 I$      0000 I 006774 J
00 I   006762 L       0000 I 006772 M       0000 I 006761 N       0000 I 006753 N
00 I   006754 NV      0000 I 006766 NVAR    0003 L 000000 OPTION  0000 R 005752 R
00 R   005726 SIG     0000 R 006776 SSK     0000 R 006775 SSO     0000 R 006770 S
00 R   000000 X       0000 R 005670 XBAR


     C
     C TITLE       LSRA - LEAST SQUARES MULTIPLE REGRESSION PROGRAM
     C                       WITH ANALYSIS OF VARIANCE PRINTOUT.
     C
     C AUTHOR      R.R.SINGERS
     C
     C SPONSOR     G.R.ANDERSEN
     C
```

```
      C DATE           AUGUST,1970
      C
      C KEY WORDS      REGRESSION
      C                LEAST SQUARES
      C
      C PURPOSE        TO COMPUTE THE LEAST SQUARES REGRESSION COEFFICIENTS
      C                AND THE ANALYSIS OF VARIANCE DATA OF TWO OR MORE
      C                DATA SERIES.
      C
      C METHOD         THE PROGRAM IS DESIGNED IN AN INTERACTIVE MODE
      C                ASKING FOR CERTAIN PARAMETERS AND THE DATA SERIES.
      C                THE STANDARD FORWARD LEAST SQUARES REGRESSION
      C                TECHNIQUE IS USED.
      C
      C SUBPROGRAMS    RESTEM - UNIVAC 1108 STAT-PACK FORWARD
      C USED           REGRESSION SUBROUTINE.
      C
      C NOTE           1) A DYNAMICALLY ASSIGNED FILE (UNIT 8) IS
      C                USED TO SAVE THE RESIDUALS (WHEN SPECIFIED).
      C
                       PARAMETER MXL=500,MXV=5
                       DIMENSION X(MXL,MXV),W(MXL),XBAR(MXV),A(MXV,MXV),SIG(MXV)
             .         ,IVAR(MXV),B(MXV),SB(MXV),R(MXL),FMT(6)
                       DIMENSION D(6)
                       LOGICAL OPTION
                       READ(5,200,END=53)FMT
        200            FORMAT(6A6)
                       IF(OPTION('N'))GO TO 1
                       WRITE(6,100)
        100            FORMAT(/' LSRA IS A STEPWISE MULTIPLE REGRESSION PROGRAM'
             .         /' WITH ANALYSIS OF VARIANCE PRINTOUT.'
             .         /' THE RESIDUALS ARE STORED ON UNIT 8(IF DESIRED).'/)
        1              WRITE(6,99)
        99             FORMAT(' NUMBER OF OBSERVATIONS?')
                       READ(5,201,END=53)NL
        201            FORMAT( )
                       IF(NL.GT.MXL)GO TO 51
                       WRITE(6,101)
        101            FORMAT(' NUMBER OF VARIABLES(DEPENDENT AND INDEPENDENT)?'
             .         )
        2              READ(5,201,END=53)NV
                       IF(NV.GT.MXV)GO TO 52
                       IW=0
        3              WRITE(6,105)
        105            FORMAT(' F LEVEL?')
                       READ(5,201,END=53)EFIN
                       EFOUT=EFIN
                       WRITE(6,107)
        107            FORMAT(' DEPENDENT VARIABLE FORMAT?')
                       READ(5,200,END=53)FMT
                       WRITE(6,108)
        108            FORMAT(' DEPENDENT VARIABLE DATA:')
                       READ(5,FMT)(X(I,NV),I=1,NL)
                       N=NV-1
                       DO 4 L=1,N
                       WRITE(6,109)L
        109            FORMAT(' INDEPENDENT VARIABLE',I2,' FORMAT?')
```

```
             READ(5,200,END=53)FMT
             WRITE(6,110)L
 110         FORMAT(' INDEPENDENT VARIABLE',I2,' DATA:')
 4           READ(5,FMT)(X(I,L),I=1,NL)
             IND=0
             ISTEP=-1
 5           CALL RESTEM(X,NL,NV,MXL,MXV,W,IW,EFIN,EFOUT,XBAR,A,SIG,C,
          .  NVAR,FL,SY,NOIN,IVAR,B,SB,R,IND)
             IF(IND.EQ.0)GO TO 10
             ISTEP=ISTEP+1
             IF(ISTEP.NE.0)GO TO 7
             WRITE(6,102)(I,XBAR(I),I=1,N)
 102         FORMAT(/' MEANS:'/(3(I5,E15.8)))
             WRITE(6,111)
 111         FORMAT(//' CORRELATION COEFFICIENTS:')
             M=N-1
             DO 6 I=1,M
             K=I+1
 6           WRITE(6,112)(I,J,A(I,J),J=K,N)
 112         FORMAT(3(4X,I2,' VS',I2,' = ',F10.6))
             WRITE(6,113)(I,A(I,NV),I=1,N)
 113         FORMAT(3(4X,I2,' VS Y = ',F10.6))
             WRITE(6,114)SY
 114         FORMAT(' STANDARD ERROR OF Y =',E12.6)
             D(4)=SY**2
             D(3)=NL-1.
             D(2)=D(4)*D(3)
             D(1)=1.E30
             SS0=D(2)
             WRITE(6,103)D
             GO TO 5
 7           WRITE(6,115)ISTEP
 115         FORMAT(/' STEP NO.',I3)
             IF(NVAR.GE.0)GO TO 8
             NVAR=-NVAR
             FL=-FL
             WRITE(6,116)NVAR
 116         FORMAT(5X,' VARIABLE REMOVED:',I3)
             GO TO 9
 8           WRITE(6,117)NVAR
 117         FORMAT(5X,' VARIABLE ENTERING:',I3)
 9           WRITE(6,118)FL,SY,C,(IVAR(I),B(I),SB(I),I=1,NOIN)
 118         FORMAT(5X,' FLEVEL',F13.6/6X,'STANDARD ERROR OF Y',F13.6/
          .  6X,'CONSTANT',E13.6//13X,'VARIABLE     COEFICIENT   STD.
          .ERROR OF COEFF'/(17X,' X',I2,E16.8,E18.8))
             D(4)=SY**2
             D(3)=D(3)-1.
             SSK=D(4)*D(3)
             D(1)=D(2)-SSK
             D(2)=SSK
             D(5)=SQRT(D(1)/D(4))
             D(6)=1.-D(2)/SS0
             WRITE(6,103)D
 103         FORMAT(/' SSVAR=',F13.6,2X,'SSRES=',F13.6,2X
          .  ,'DFRES=',F13.6/' MSRES=',F13.6,6X,'T=',
          .  F13.6,' MULTCOR=',F12.6/)
             IF(ISTEP.LT.NV)GO TO 5
```

```
                  IND=-1
                  GO TO 5
      10          WRITE(6,124)
      124         FORMAT(' DO YOU WANT TO SEE SOME OF THE ',
         .        'DEVIATIONS(YES/NO)?')
                  READ(5,200,END=53)I
                  IF(I.NE.'YES') GO TO 12
                  WRITE(6,125)
      125         FORMAT(' WHAT PART(INITIAL,FINAL,STEP)?')
                  READ(5,201,END=53)J,K,L
                  WRITE(6,120)
      120         FORMAT(/' PREDICTED VS. ACTUAL RESULTS:'/' OB. NO.',4X,
         .        'ACTUAL',4X,'PREDICTED',4X,'%DEVIATION')
                  DO 11 I=J,K,L
                  DEV=(X(I,NV)-R(I))/X(I,NV)*100.
      11          WRITE(6,121)I,X(I,NV),R(I),DEV
      121         FORMAT(I5,2E13.6,F10.2)
      12          WRITE(6,126)
      126         FORMAT(' DO YOU WANT THE RESIDUALS STORED(YES/NO)?')
                  READ(5,200,END=53) I
                  IF(I.NE.'YES') CALL EXIT
                  DO 13 I=1,NL
      13          R(I)=X(I,NV)-R(I)
                  CALL CSF(2,'@ASG,T 8,F',I)
                  WRITE(6,127)
      127         FORMAT(' OUTPUT DATA FORMAT?')
                  READ(5,200,END=53)FMT
                  WRITE(8,FMT)(R(I),I=1,NL)
                  WRITE(6,128)
      128         FORMAT(' SERIES STORED.')
                  GO TO 53
      51          I=MXL
                  WRITE(6,122)I
      122         FORMAT(' NUMBER OF OBSERVATIONS MUST BE LESS THAN',I5,' T
         .RY AGAIN')
                  GO TO 1
      52          I=MXV
                  WRITE(6,123)I
      123         FORMAT(' NUMBER OF VARIABLES MUST BE LESS THAN',I5,' TRY
         .AGAIN')
                  GO TO 2
      53          WRITE(6,129)
      129         FORMAT(' LSR TERMINATES.')
                  CALL EXIT
             END

END OF COMPILATION:        NO  DIAGNOSTICS.
```

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SINGERS/TSAP - TIME SERIES ANALYSIS PACKAGE//

    R.XCF,J,J
000 COMPILED BY 1201 BCS7E ON 01 DEC 70 AT 12:29:45.

N PROGRAM

RAGE USED: CODE(1) 000502; DATA(0) 010063; BLANK COMMON(2) 000000

ERNAL REFERENCES (BLOCK, NAME)

03    OPTION
04    EXIT
05    NINTR$
06    NRDU$
07    NIO1$
10    NIO2$
11    NWDU$
12    NREW$
13    NRBU$
14    SQRT
15    NWBU$
16    NWEF$
17    NSTOP$

RAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

00      007667 100F      0000    007746 101F      0000    007761 103F      0000    007771 1
00      010011 106F      0000    010014 107F      0001    000011 107G      0000    010025 1
01      000061 133G      0001    000104 143G      0001    000144 161G      0001    000156 1
01      000213 200G      0000    007760 201F      0001    000232 204G      0001    000241 2
01      000332 240G      0001    000356 244G      0001    000421 267G      0001    000422 2
01      000446 303G      0001    000471 53L       0001    000432 8L        0001    000026 9
00 R 007650 B            0000 R 000000 C          0000 R 007640 FMT        0000 I 007654 I
00 I 007647 ICO          0000 I 007662 II         0000 I 007646 I$         0000 I 007651 J
00 I 007653 M            0000 I 007661 MR         0000 I 007655 M1         0000 I 007652 N
03 L 000000 OPTION       0000 R 003720 Y


C
C TITLE         XCF - CROSS-COVARIANCE/CORRELATION PROGRAM.
C
C AUTHOR        R.R.SINGERS
C
C SPONSOR       G.R.ANDERSEN
C
C DATE          AUGUST,1970
C
C KEY WORDS     CROSS-COVARIANCE
C               CROSS-CORRELATION
C
C PURPOSE       TO COMPUTE THE CROSS-COVARIANCES OF TWO DATA SERIES
C               AND IF DESIRED, THE CROSS-CORRELATIONS OF TWO DATA
C               SERIES.
```

```
C
C METHOD         THE PROGRAM IS DESIGNED IN AN INTERACTIVE MODE
C                ASKING FOR CERTAIN PARAMETERS AND TWO DATA SERIES.
C                IT IS ASSUMED THAT ACF WAS EXECUTED AND THE AUTO-
C                COVARIANCES SAVED. THE STANDARD METHOD OF COMPUTATION
C                IS USED.
C
C NOTES          1) IT IS ASSUMED THAT THE AUTO-COVARIANCES
C                ARE STORED (ON UNIT 1) FROM ACF.
C                2) THE CROSS-COVARIANCES ARE STORED AFTER
C                THE AUTO-COVARIANCES (ON UNIT 1).
C
                 PARAMETER MX=1000
                 DIMENSION C(MX,2),Y(MX,2),FMT(6)
                 LOGICAL OPTION
                 READ(5,200,END=53)FMT
  200            FORMAT(6A6)
                 IF(OPTION('N'))GO TO 99
                 WRITE(6,100)
  100           FORMAT(/' XCF COMPUTES THE CROSS-COVARIANCE/-CORRELATION',
     .           ' FUNCTION OF TWO SERIES.'/' IT IS ASSUMED THAT THE ',
     .           'AUTO-COVARIANCES OF THE TWO SERIES ARE'/' STORED ON ',
     .           'UNIT 1. THE CROSS-COVARIANCES ARE STORED ON '/' UNIT',
     .           ' 1(IF DESIRED) AFTER THE AUTO-COVARIANCES.')
  99             REWIND 1
                 WRITE(6,101)
  101          FORMAT(' DO YOU WANT COVARIANCE(COV) OR CORRELATION(COR)?')
                 READ(5,200,END=53)ICO
  201            FORMAT( )
                 B=1.
                 DO 3 J=1,2
                 WRITE(6,103)J
  103            FORMAT(' INPUT DATA FORMAT OF SERIES',I2,'?')
                 READ(5,200,END=53)FMT
                 WRITE(6,104)J
  104            FORMAT(' INPUT DATA OF SERIES',I2,':')
                 READ(1)N,M
                 READ(5,FMT,END=53)(Y(I,J),I=1,N)
                 READ(1)(C(I,J),I=1,M)
  3              B=B*C(1,J)
                 B=SQRT(B)
                 M1=M-1
                 DO 5 I=0,M1
                 NI=N-I
                 DO 5 J=1,2
                 A=0.
                 DO 4 K=1,NI
  4              A=A+Y(K,J)*Y(K+I,3-J)
                 C(I+1,J)=A/N
  5              IF(ICO.EQ.'COR')C(I+1,J)=C(I+1,J)/B
                 MR=M/4
                 II=MR+1
                 IF(MR.GT.10)MR=10
                 DO 6 K=1,2
                 I=3-K
                 WRITE(6,105)ICO,K,I
  105            FORMAT(/' CROSS-',A3,'. FOR SERIES',I3,' VS.',I3,'.')
```

```
                    DO 6 I=0,MR
      6             WRITE(6,106)(J,C(J+1,K),J=I,M1,II)
      106           FORMAT(4(I4,E15.8))
                    WRITE(6,107)
      107           FORMAT(' DO YOU WANT THE COVARIANCES STORED(YES/NO)?')
                    READ(5,200,END=53)IC
                    IF(IC.NE.'YES')GO TO 53
                    IF(ICO.NE.'COR')GO TO 8
                    DO 7 J=1,2
                    DO 7 I=1,M
      7             C(I,J)=B*C(I,J)
      8             WRITE(1)((C(I,J),I=1,M),J=1,2)
                    WRITE(6,108)
      108           FORMAT(' COVARIANCES STORED.')
                    END FILE 1
                    REWIND 1
      53            WRITE(6,111)
      111           FORMAT(' XCF TERMINATES.')
                    CALL EXIT
              END
```

END OF COMPILATION: NO DIAGNOSTICS.

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    SINGERS/TSAP - TIME SERIES ANALYSIS PACKAGE//

       R.XSF,J,J
J00 COMPILED BY 1201 BCS7E ON 01 DEC 70 AT 12:30:29.

N PROGRAM

RAGE USED: CODE(1) 001174; DATA(0) 014152; BLANK COMMON(2) 000000

ERNAL REFERENCES (BLOCK, NAME)

03    OPTION
04    ATANH
05    EXIT
06    NINTR$
07    NRDU$
10    NIO1$
11    NIO2$
12    NWDU$
13    NRBU$
14    SQRT
15    COS
16    SIN
17    ATAN2
20    NWBU$
21    NSTOP$

RAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

J0     013713 100F     0000     013774 105F     0000     014010 106F     0000     014024 1
J0     014060 109F     0000     014101 110F     0000     014030 111F     0000     014045 1
J1     000011 113G     0001     000050 134G     0001     000077 146G     0001     000110 1
J1     000210 175G     0000     013710 200F     0000     013773 201F     0001     000223 2
J1     000413 244G     0001     000436 257G     0001     000445 264G     0001     000452 2
J1     000657 334G     0001     000723 351G     0001     000750 367G     0001     001100 4
J1     001146 442G     0001     001163 53L      0001     000026 99L      0000 R 013660 A
J4 R 000000 ATANH    0000 R 000000 C        0000 R 013664 CO       0000 R 013702 D
J0 R 013703 EF       0000 R 013673 EL1      0000 R 013656 EM1      0000 R 013657 E
J0 R 003720 F        0000 R 013560 FMT      0000 R 013701 FP       0000 I 013651 I
J0 I 013706 IP       0000 I 013562 IPT      0000 I 013644 IS       0000 I 013652 J
00 I 013672 L        0000 I 013653 M        0000 I 013647 N        0000 I 013650 N
00 R 013676 OD       0003 L 000000 OPTION   0000 R 013705 PH       0000 R 013646 P
00 R 007640 Q        0000 R 013665 SI       0000 R 013663 TAU      0000 R 013707 T
00 R 013667 V1       0000 R 013677 V2       0000 R 013674 W        0000 R 013670 Z
00 R 013700 Z2

      C
      C TITLE         XSF - CROSS-SPECTRUM FUNCTION.
      C
      C AUTHOR        R.R.SINGERS
      C
      C SPONSOR       G.R.ANDERSEN
      C
```

```
C DATE         SEPTEMBER,1970
C
C KEY WORDS    CO-SPECTRUM
C              QUADRATURE
C
C PURPOSE      TO COMPUTE THE CO-SPECTRUM,QUADRATURE,PHASE,AND
C              COHERENCE BETWEEN TWO DATA SERIES.
C
C METHOD       THE PROGRAM IS DESIGNED IN AN INTERACTIVE MODE
C              ASKING FOR CERTAIN PARAMETERS AND TWO DATA SERIES.
C              IT IS ASSUMED THAT ACF,XCF,AND ASF HAVE BEEN
C              EXECUTED AND THE AUTO-COVARIANCES,CROSS-COVARIANCES,
C              AND THE AUTO-SPECTRA(USING THE TUKEY WINDOW)
C              HAVE BEEN SAVED. THE STANDARD METHOD OF COMPUTATION
C              IS USED.
C
C NOTES        1) THE AUTO AND CROSS COVARIANCES ARE ASSUMED TO BE
C              STORED(ON UNIT 1) FROM ACF AND XCF.
C              2) THE AUTO SPECTRA ARE ASSUMED TO BE STORED(ON
C              UNIT 2) FROM ASF.
C
               PARAMETER MX=1000
               DIMENSION C(MX,2),F(MX,2),Q(MX,2),FMT(2),IPT(50)/50*
     .         ' '/
               DATA PIV,PI/.1591549,3.1415926/
               LOGICAL OPTION
               READ(5,200,END=53)FMT
  200          FORMAT(2A6)
               IF(OPTION('N'))GO TO 99
               WRITE(6,100)
  100          FORMAT(/' XSF COMPUTES THE CROSS-SPECTRA FUNCTIONS FOR',
     .        /' THE TUKEY WINDOW.'/' IT IS ASSUMED THAT THE AUTO- AND C
     .ROSS-COVARIANCES ARE STORED ON'/' UNIT 1 AND THAT THE AUTO-SPECTRA
     .ARE STORED ON UNIT 2.'/' THE CROSS SPECTRA ARE STORED ON UNIT 2 AF
     .TER THE AUTO-SPECTRA(IF DESIRED).')
  201          FORMAT( )
  99           READ(1)N,NN
               READ(1)(C(I,1),I=1,NN)
               READ(1)N,NN
               READ(1)(C(I,1),I=1,NN)
               READ(1)((C(I,J),I=1,NN),J=1,2)
               READ(2)M,DEL,NQ
               EM1=M-1
               ENQ1=NQ-1.
               AMU=2.667*N/EM1
               ANU=1.65/SQRT(AMU)
               DO 2 I=1,2
               J=MOD(I,2)+1
               DO 2 K=1,NQ
               TAU=K-1
               CO=COS(PI*TAU/ENQ1)
               SI=SIN(PI*TAU/ENQ1)
               V0=0.
               V1=0.
               Z0=0.
               Z1=0.
               DO 1 L=M,2,-1
```

```
                    EL1=L-1
                    W=.5*(1.+COS(PI*EL1/EM1))
                    EV=.5*(C(L,I)+C(L,J))
                    OD=.5*(C(L,I)-C(L,J))
                    V2=2.*CO*V1-V0+W*EV
                    Z2=2.*CO*Z1-Z0+W*OD
                    V0=V1
                    V1=V2
                    Z0=Z1
      1             Z1=Z2
                    F(K,I)=DEL*(C(1,I)+2.*(V1*CO-V0))
      2             Q(K,I)=2.*PIV*Z1*SI
                    WRITE(6,105)
      105           FORMAT(' DO YOU WANT FREQUENCY(FREQ.) OR PERIOD(PERIOD)',
                    /' DISPLAYED?')
                    READ(5,200,END=53)FP
                    READ(2)(C(I,1),I=1,NQ)
                    READ(2)I,D,J
                    READ(2)(C(I,2),I=1,NQ)
                    DO 3 I=1,NQ
                    D=C(I,1)*C(I,2)
                    DO 3 J=1,2
                    C(I,J)=ATANH(SQRT((F(I,J)**2+Q(I,J)**2)/D))
      3             FMT(J)=MAX(ABS(C(I,J)),FMT(J))
                    FMT(1)=FMT(1)/49.
                    FMT(2)=FMT(2)/49.
                    I=1
                    J=2
                    WRITE(6,106)FP,I,J
      106          FORMAT(/2X,A6,'   ARCTANH(COHERENCE) FOR SERIES',I2,' VS.'
                    ,I2,' .')
                    DO 4 K=1,NQ
                    L=K-1
                    IF(FP.NE.'PERIOD')EF=.5*L/ENQ1/DEL
                    IF(FP.EQ.'PERIOD'.AND.L.NE.0)EF=2.*ENQ1/L*DEL
                    IF(FP.EQ.'PERIOD'.AND.L.EQ.0)EF=99.9999
                    IF=IFIX(C(K,I)/FMT(I))+1
                    IF(IF.GT.50)IF=50
                    IF(IF.LT.1)IF=1
                    IPT(IF)='+'
                    WRITE(6,107)EF,C(K,I),IPT
      107           FORMAT(F8.4,E12.5,2X,50A1)
      4             IPT(IF)=' '
                    IF=IFIX(ANU/FMT(I))+1
                    IPT(1)='*'
                    IPT(IF)='*'
                    WRITE(6,111)AMU,IPT
      111           FORMAT(' NU='F5.0,13X,50A1)
                    IPT(1)=' '
                    IPT(IF)=' '
                    WRITE(6,108)FP,I,J
      108           FORMAT(/3X,A6,'   PHASE FOR SERIES',I2,' VS.',I2,' .')
                    DO 5 K=1,NQ
                    PH=57.2958*ATAN2(Q(K,I),F(K,I))
                    L=K-1
                    IF(FP.NE.'PERIOD')EF=.5*L/ENQ1/DEL
                    IF(FP.EQ.'PERIOD'.AND.L.EQ.0)EF=99.9999
```

```
                     IF(FP.EQ.'PERIOD'.AND.L.NE.0)EF=2.*ENQ1/L*DEL
                     IP=IFIX(PH/7.16)+25
                     IF(IP.GT.50)IP=50
                     IF(IP.LT.1)IP=1
                      IPT(25)='*'
                     IPT(IP)='+'
                     WRITE(6,107)EF,PH,IPT
                     IPT(25)=' '
    5                IPT(IP)=' '
                     WRITE(6,112)
  112                FORMAT(T21,'*',25X,'*',25X,'*'/T19,'-180',
         .           24X,'0',24X,'180')
                     WRITE(6,109)
  109                FORMAT(/' DO YOU WANT THE CO-SPECTRA AND QUADRATURES',/
         .           ' STORED FOR TIME SERIES REGRESSION(YES/NO)?')
                     READ(5,200,END=53)TS
                     IF(TS.NE.'YES')GO TO 53
                     WRITE(2)((F(I,J),I=1,NQ),J=1,2)
                     WRITE(6,110)
  110                FORMAT(' SERIES STORED.')
  53                 WRITE(6,113)
  113                FORMAT(' XSF TERMINATES.')
                     CALL EXIT
            END
```

END OF COMPILATION: NO DIAGNOSTICS.

```
SINGERS/TSAP - TIME SERIES ANALYSIS PACKAGE77

      R.COMB,J,J
J00 COMPILED BY 1201 BCS7E ON 01 DEC 70 AT 12:31:15.

N PROGRAM

RAGE USED: CODE(1) 000522; DATA(0) 004161; BLANK COMMON(2) 000000

ERNAL REFERENCES (BLOCK, NAME)

J3   OPTION
J4   CSF
J5   EXIT
J6   NINTR$
07   NRDU$
10   NIO1$
11   NIO2$
12   NWDU$
.3   NSTOP$

RAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

)1     000026 1L       0001     000240 10L       0000     003743 100F      0000       003776 1
J0     004015 103F     0000     004022 104F      0000     004030 105F      0000       004035 1
)0     004055 108F     0000     004066 109F      0000     004073 110F      [illegible] 000011 1
)0     004110 112F     0000     004122 113F      0000     004136 114F      [illegible] 000255 1
J1     000301 14L      0001     000102 145G      0001     000364 15L       [illegible] 000124 1
J0     004006 201F     0001     000201 205G      0001     000216 215G      0001       000233 2
)1     000272 247G     0001     000360 276G      0001     000132 3L        0001       000424 3
)1     000154 4L       0001     000455 50L       0001     000465 51L       0001       000477 5
)1     000206 6L       0001     000223 8L        0000 R   003726 ASG8      0000 R     003720 F
)0 I   003735 IOP      0000 I   003730 I$        0000 I   003736 J         0000 I     003733 K
)0 I   003732 M        0000 I   003734 N         0003 L   000000 OPTION    0000 R     000000 X

      C
      C TITLE        COMB - A PROGRAM TO COMBINE TWO DATA SERIES.
      C
      C AUTHOR       R.R.SINGERS
      C
      C SPONSOR      G.R.ANDERSEN
      C
      C DATE         NOVEMBER,1970
      C
      C KEY WORDS    COMBINE,
      C              CONCATINATE
      C
      C PURPOSE      TO COMBINE TWO DATA SERIES USING ONE OF FIVE
      C              OPERATORS;ADD,SUB,MPY,DIV,CON.
      C
      C METHOD       THE PROGRAM IS DESIGNED IN AN INTERACTIVE MODE
      C              ASKING FOR CERTAIN PARAMETERS AND APPLYING THEM TO
      C              TWO INPUT DATA SERIES. THE RESULTANT SERIES MAY THEN
```

```
C                   BE STORED.
C
C NOTE              1) A DYNAMICALLY ASIGNED FILE (UNIT 8) IS USED TO
C                   STORE THE FINAL SERIES.
C
                    PARAMETER MX=1000
                    DIMENSION X(MX,2),FMT(6),
         .          ASG8(2)/'@ASG,T 8,F'/
                    LOGICAL OPTION
                    READ(5,200,END=53)FMT
      200           FORMAT(6A6)
                    IF(OPTION('N'))GO TO 1
                    WRITE(6,100)
      100           FORMAT(' COMB COMBINES TWO SERIES BY ONE OF THE FOLLOWING
         . SERIES:'/' ADD,SUB,MPY,DIV,CON(CONCATINATE)'/' THE RESULTANT SERI
         .ES IS STORED ON UNIT 8(IF DESIRED).')
      1             DO 2 I=1,2
                    WRITE(6,101)I
      101           FORMAT(' LENGTH OF SERIES',I2,' TO BE READ IN?')
                    READ(5,201,END=53)M
      201           FORMAT( )
                    IF(M.GT.MX)GO TO 51
                    WRITE(6,102)I
      102           FORMAT(' INPUT DATA',I2,' FORMAT?')
                    READ(5,200,END=53)FMT
                    WRITE(6,103)I
      103           FORMAT(' INPUT DATA',I2,' :')
      2             READ(5,FMT)(X(K,I),K=1,M)
      3             WRITE(6,104)
      104           FORMAT(' LENGTH OF OUTPUT SERIES?')
                    READ(5,201,END=53)N
                    IF(N.GT.2*MX)GO TO 52
      4             WRITE(6,105)
      105           FORMAT(' OPERATOR TO BE USED?')
                    READ(5,200,END=53)IOP
                    IF(IOP.NE.'ADD')GO TO 6
                    DO 5 J=1,N
      5             X(J,1)=X(J,1)+X(J,2)
                    GO TO 14
      6             IF(IOP.NE.'SUB')GO TO 8
                    DO 7 J=1,N
      7             X(J,1)=X(J,1)-X(J,2)
                    GO TO 14
      8             IF(IOP.NE.'MPY')GO TO 10
                    DO 9 J=1,N
      9             X(J,1)=X(J,1)*X(J,2)
                    GO TO 14
      10            IF(IOP.NE.'DIV')GO TO 12
                    DO 11 J=1,N
      11            X(J,1)=X(J,1)/X(J,2)
                    GO TO 14
      12            IF(IOP.NE.'CON')GO TO 50
                    K=N-M+1
                    L=0
                    DO 13 J=K,N
                    L=L+1
      13            X(J,1)=X(L,2)
```

```
      14          WRITE(6,106)
      106         FORMAT(' DO YOU WANT TO SEE PART OF THE SERIES(YES/NO)?')
                  READ(5,200,END=53)IOP
                  IF(IOP.NE.'YES')GO TO 15
                  WRITE(6,107)
      107         FORMAT(' WHAT PART (INITIAL,FINAL,STEP)?')
                  READ(5,201,END=51)I,J,K
                  WRITE(6,201)(X(L,1),L=I,J,K)
      15          WRITE(6,108)
      108         FORMAT(' DO YOU WANT THE RESULTANT SERIES STORED(YES/NO)'
           .      )
                  READ(5,200,END=53)IOP
                  IF(IOP.NE.'YES')GO TO 53
                  WRITE(6,109)
      109         FORMAT(' OUTPUT DATA FORMAT?')
                  READ(5,200,END=53)FMT
                  CALL CSF(2,ASG8,I)
                  WRITE(8,FMT)(X(I,1),I=1,N)
                  WRITE(6,110)
      110         FORMAT(' SERIES STORED.')
                  GO TO 53
      50          WRITE(6,111)IOP
      111         FORMAT(1X,A6,' IS NOT A RECOGNIZED OPERATOR,TRY AGAIN.')
                  GO TO 4
      51          L=MX
                  WRITE(6,112)L
      112         FORMAT(' LENGTH OF SERIES MUST BE LESS THAN',I2,
           .      ' TRY AGAIN.')
                  GO TO 1
      52          L=2*MX
                  WRITE(6,113)L
      113         FORMAT(' LENGTH OF OUTPUT SERIES MUST BE LESS THAN',I5,
           .      ' TRY AGAIN.')
                  GO TO 3
      53          WRITE(6,114)
      114         FORMAT(' COMB TERMINATES.')
                  CALL EXIT
                  END

END OF COMPILATION:         NO  DIAGNOSTICS.
```

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SINGERS/TSAP - TIME SERIES ANALYSIS PACKAGE//

 R.LSRP,J,J
000 COMPILED BY 1201 BCS7E ON 03 DEC 70 AT 08:37:28.

N PROGRAM

RAGE USED: CODE(1) 001012; DATA(0) 007326; BLANK COMMON(2) 000000

ERNAL REFERENCES (BLOCK, NAME)

03     OPTION
04     REBSOM
05     CSF
06     EXIT
07     NINTR$
10     NRDU$
11     NIO1$
12     NIO2$
13     NWDU$
14     SQRT
15     NSTOP$

RAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

01     000026 1L        0000   006764 100F      0000   007014 101F      0000   007022 1
00     007036 104F      0000   007044 105F      0000   007052 106F      0000   007062 1
00     007101 109F      0000   007116 110F      0001   000011 110G      0000   007120 1
00     007141 113F      0000   007156 114F      0000   007160 115F      0000   007174 1
00     007225 118F      0000   007240 119F      0000   007253 120F      0000   007257 1
01     000150 157G      0001   000171 170G      0001   000205 176G      0001   000055 2
00     007021 201F      0001   000224 206G      0001   000245 217G      0001   000260 2
01     000455 301G      0001   000474 310G      0001   000515 322G      0001   000530 3
01     000551 343G      0001   000553 347G      0001   000574 356G      0001   000703 4
01     000723 427G      0001   000741 51L       0001   000753 52L       0001   000765 5
01     001002 55L       0000 R 006654 ATA       0000 R 006731 EFOUT     0000 R 006717 F
00 I 006736 INDIC       0000 I 006712 IVAR      0000 I 006725 I$        0000 I 006745 I
00 I 006752 JK          0000 I 006750 JL        0000 I 006760 K         0000 I 006751 K
00 I 006754 LM          0000 I 006733 N         0000 I 006726 NL        0000 I 006746 N
00 I 006730 NV          0000 I 006727 NV1       0003 L 000000 OPTION    0000 R 005670 R
00 R 006737 SY          0000 R 006744 TL        0000 R 006741 TNU       0000 R 006743 T
00 R 000000 X           0000 R 006753 Y         0000 R 006757 YL        0000 R 006755 Y

      C
      C TITLE       LSRP - LEAST SQUARES MULTIPLE REGRESSION PROGRAM
      C                       WITH PREDICTIONS.
      C
      C AUTHOR      R.R.SINGERS
      C
      C SPONSOR     G.R.ANDERSEN
      C
      C DATE        DECEMBER,1970
      C
```

```
C KEY WORDS      REGRESSION
C                LEAST SQUARES
C                PREDICTIONS
C
C PURPOSE        TO COMPUTE THE LEAST SQUARES REGRESSION COEFFICIENTS
C                AND PREDICTION VALUES WITH UPPER AND LOWER
C                PREDICTION LIMITS.
C
C METHOD         THE PROGRAM IS DESIGNED IN AN INTERACTIVE MODE
C                ASKING FOR CERTAIN PARAMETERS AND DATA SERIES.
C                THE STANDARD BACKWARD REGRESSION TECHNIQUE IS
C                USED.
C
C SUBROUTINES    REBSOM - UNIVAC 1108 STAT-PACK BACKWARD
C USED           REGRESSION SUBROUTINE.
C
C NOTE           1) A DYNAMICALLY ASSIGNED FILE (UNIT 8) IS
C                USED TO SAVE THE PREDICTED VALUES AND THE
C                UPPER AND LOWER PREDICTION LIMITS.
C
                 PARAMETER MXN=500,MXNP=5
                 PARAMETER MXNP1=MXNP+1
                 DIMENSION X(MXN,MXNP1),R(MXN),ATA(MXNP,MXNP1),
        .        IVAR(MXNP),FMT(6)
                 LOGICAL OPTION
                 READ(5,200,END=53)FMT
    200          FORMAT(6A6)
                 IF(OPTION('N'))GO TO 1
                 WRITE(6,100)
    100          FORMAT(/' LSRP IS A STEPWISE MULTIPLE REGRESSION PROGRAM'
        .        ,' WITH PREDICTIONS.'/' THE PREDICTION RESULTS ARE STORED
        .'       ,'(IF DESIRED) ON UNIT 8.')
    1            WRITE(6,101)
    101          FORMAT(' NUMBER OF OBSERVATIONS?')
                 READ(5,201,END=53)NL
    201          FORMAT( )
                 IF(NL.GT.MXN)GO TO 51
                 WRITE(6,102)
    102          FORMAT(' NUMBER OF VARIABLES(DEPENDENT AND INDEPENDENT)?'
        .        )
    2            READ(5,201,END=53)NV1
                 NV=NV1+1
                 IF(NV1.GT.MXNP)GO TO 52
                 WRITE(6,103)
    103          FORMAT(' F LEVEL?')
                 READ(5,201,END=53)EFOUT
                 WRITE(6,104)
    104          FORMAT(' DEPENDENT VARIABLE FORMAT?')
                 READ(5,200,END=53)FMT
                 WRITE(6,105)
    105          FORMAT(' DEPENDENT VARIABLE DATA:')
                 READ(5,FMT,END=53)(X(I,NV),I=1,NL)
                 N=NV1-1
                 DO 3 L=1,N
                 WRITE(6,106)L
    106          FORMAT(' INDEPENDENT VARIABLE',I2,' FORMAT?')
                 READ(5,200,END=53)FMT
```

```
                    WRITE(6,112)L
      3             READ(5,FMT,END=53)(X(I,L+1),I=1,NL)
                    DO 9 I=1,NL
      9             X(I,1)=1.
                    CALL REBSOM(X,NL,NV1,MXN,MXNP,EFOUT,VARY,R,ATA,INDIC,IVAR
                    ,$54,$55)
                    SY=SQRT(VARY)
                    NU=NL-INDIC
                    WRITE(6,107)NU
      107           FORMAT(' T-VALUE FOR NU=',I3,' DEGREES OF FREEDOM?')
                    READ(5,201,END=53)TNU
                    WRITE(6,108)SY
      108           FORMAT(' STANDARD ERROR OF Y',F13.7)
                    WRITE(6,109)
      109           FORMAT(' VARIABLE COEFFICIENT        S.D.    UPPER T    LOWE
           .R T'/' (0=CONST.)')
                    DO 4 I=1,INDIC
                    SDC=SY*SQRT(ATA(I,I))
                    TU=ATA(I,NV)+TNU*SDC
                    TL=ATA(I,NV)-TNU*SDC
                    I1=I-1
      4             WRITE(6,110)I1,ATA(I,NV),SDC,TU,TL
      110           FORMAT(I10,4E11.5)
                    WRITE(6,111)
      111           FORMAT(' PREDICTION DATA:'/' NUMBER OF PREDICTION VALUES?
           .')
                    READ(5,201,END=53)NPR
                    DO 5 I=1,N
                    WRITE(6,106)I
                    READ(5,200,END=53)FMT
                    WRITE(6,112)I
      112           FORMAT(' INDEPENDENT VARIABLE',I2,' DATA:')
      5             READ(5,FMT,END=53)(X(J,I+1),J=1,NPR)
                    DO 10 J=1,NPR
      10            X(J,1)=1.
                    WRITE(6,113)
      113           FORMAT (' PREDICTED VALUES:'/' NUMBER  PREDICTION      UPP
           .ER T       LOWER T')
                    DO 8 JL=1,NPR
                    DO 6 KL=1,NV
                    R(KL)=0.
                    DO 6 JK=1,INDIC
      6             R(KL)=R(KL)+X(JL,JK)*ATA(JK,KL)
                    Y=0.
                    DO 7 LM=1,INDIC
      7             Y=Y+R(LM)*X(JL,LM)
                    Y=SY*SQRT(1.+Y)
                    YPR=R(NV)
                    YU=YPR+TNU*Y
                    YL=YPR-TNU*Y
                    X(JL,1)=YPR
                    X(JL,2)=YL
                    X(JL,3)=YU
      8             WRITE(6,114)JL,YPR,YU,YL
      114           FORMAT(I7,3E13.7)
                    WRITE(6,115)
      115           FORMAT(/' DO YOU WANT THE PREDICTION VALUES AND LIMITS SA
```

```
          .VED(YES/NO)?')
               READ(5,200,END=53)I
               IF(I.NE.'YES')GO TO 53
               WRITE(6,116)
  116          FORMAT(' OUTPUT FORMAT?')
               READ(5,200,END=53)FMT
               CALL CSF(2,'@ASG,T 8,F',J)
               WRITE(8,FMT)((X(J,K),J=1,NPR),K=1,3)
               WRITE(6,117)
  117          FORMAT(/' SERIES STORED IN FOLLOWING ORDER:'/
          .    ' PREDICTION SERIES,LOWER LIMIT SERIES,UPPER LIMIT SERIES
          ..')
               GO TO 53
  51           I=MXN
               WRITE(6,118)I
  118          FORMAT(' NUMBER OF OBSERVATIONS MUST BE LESS THAN',I5,
          .    ' TRY AGAIN.')
               GO TO 1
  52           I=MXNP
               WRITE(6,119)I
  119          FORMAT(' NUMBER OF VARIABLES MUST BE LESS THAN',I5,
          .    ' TRY AGAIN.')
               GO TO 2
  53           WRITE(6,120)
  120          FORMAT(' LSRP TERMINATES.')
               CALL EXIT
  54           WRITE(6,121)
  121          FORMAT(/' *** ATA MATRIX OF REBSOM SINGULAR ***')
               GO TO 53
  55           WRITE(6,122)
  122          FORMAT(/' *** NO VARIABLES LEFT IN REGRESSION ***')
               GO TO 53
          END

END OF COMPILATION:        NO DIAGNOSTICS.
```

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