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TECHNICAL MEMORANDUM

INTERACTION OF THE SOLAR WIND WITH A PLANETARY IONOSPHERE

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ABSTRACT

An electrodynamic model for an ionosphere-solar wind interaction is developed based on the existence of a low β plasma below the anemopause. The currents for the interaction are driven by the solar wind motional electric field and induce a stagnation magnetic field at the anemopause. For Venus and Mars the lower region of the ionosphere near the electron density peak has the highest conductivity, and therefore the tangential component of the induction current flows substantially in this region. The current paths close in the anemopause, which is a solar wind current sheath analogous to the magnetopause. Both the fraction of the undisturbed solar wind motional electric field, which drives the induction current, and the required fraction of incident solar wind particles, crossing the anemopause to produce this current, are shown to be small.

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TECHNICAL MEMORANDUM

INTRODUCTION

Three distinct types of interactions of the solar wind with planetary bodies have been observed. The first, exemplified by the earth, is the interaction of a streaming plasma with an intrinsic magnetic field. In this case the geomagnetic field is strong enough to deflect the solar wind particles and produce a bow shock. The second type of interaction involves a streaming plasma impinging upon a solid, essentially non-conductive surface, as is the case for the moon. Here the solar wind is terminated at the lunar surface, where the particles are absorbed, but no strong disturbance has been observed in the solar wind on the sunlit side of the moon (Colburn, et al., 1967; Ness, et al., 1967). The third and most complex interaction is that between the solar wind plasma and an ionospheric plasma. Data from the Mariner V plasma and field and occultation experiments (Bridge, et al.,

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1967; Mariner Stanford Group, 1967) indicate that the observed bow shock at Venus is the result of this type of interaction. The presence of a bow shock at Venus implies that the solar wind-Venus interaction is more like the earth interaction than the moon interaction in that a substantial fraction of the incident solar wind particles are reflected, rather than absorbed, by the ionosphere.

Figure 1 shows schematically the daytime electron density profile for Venus obtained from the Mariner V occultation experiment (Mariner Stanford Group, 1967; Fjelbo and Eshleman, 1969). A sharp transition exists at an altitude of ∿500 km on the day side, where the electron density decreases abruptly from about 10^4 cm^{-3} to a value characteristic of the interplanetary plasma. Furthermore, the thickness of the transition region (anemopause or ionopause) is much less than any electron or ion mean free path, so the sharp density gradient is not a collisional phenomenon. Thus, it is plausible that it is a magnetic field that inhibits diffusion and maintains the observed density gradient.

Previous studies have been placed into three categories by Michel (1971): (1) a direct interaction between the solar wind and planetary ions (Cloutier, et al., 1969; Elco, 1969), (2) a tangential discontinuity separating the immiscible solar wind and ionospheric plasmas with a pressure balance across the interface (Bridge, et al., 1967; Bauer, et al., 1970; Herman, et al., 1971; Banks and Axford, 1970; Spreiter, et al., 1970), and (3) a magnetic barrier in which an induced magnetic field diverts the flow of solar wind plasma (Johnson and Midgley, 1969; Blank and Sill, 1970). The purpose of this paper is to investigate the electrodynamics of the third class of interactions in which the magnetic field pressure dominates, i.e., $\beta<1$ ($\beta = p/[B^2/2\mu]$ = particle pressure/magnetic pressure).

The mechanism which produces the induced magnetic field is the motional generator driven by the solar wind motional electric field (see e.g., Sonett and Colburn, 1968). Dessler (1968) has established that an ionosphere, such as exists on Venus or Mars, is easily conductive enough to allow the motional electric field to induce a magnetic field capable of standing off the solar wind. To gain an understanding of the ionosphere-solar wind current flow we propose a combination of two models. One is for the current through the ionosphere and is based on the electrical conductivity profile deduced from the Mariner V electron density measurements of Venus. The other is a simple sheath model for the solar wind return in the anemopause. The solutions for these two models are then matched to provide for a complete current path.

FORMULATION

The complexity of the interaction between the solar wind and planetary ionospheric plasmas precludes a complete self-consistent solution at this time. Such an approach would entail the simultaneous solution of the equations of motion and Maxwell's equations for both plasmas, together with the constitutive equations for a model ionosphere. However, it is possible to study certain simplified models which retain the salient features of the interaction.

Studies of the interaction of the solar wind and its electromagnetic field with a solid conducting body like the moon (Blank and Sill, 1969; Sill and Blank, 1970) indicate there are two basic modes of interaction. One of these modes (toroidal or transverse magnetic) has as its source the motional electric field which drives a current across the body. The current path then closes in a sheath in the solar wind plasma. In the second mode of interaction (poloidal or transverse electric) the time rate of change of the interplanetary magnetic field induces eddy currents which close within the planetary body. In both cases the induced current flow gives rise to an induced magnetic field which in turn can interact with the solar wind plasma. In principle, these two modes of interaction can also occur in the case of a conductive planetary ionosphere, but the nature of the interaction is bound to be more complex since currents can flow in a plasma not only in response to electric fields but also in response to magnetic field and particle pressure gradients. Additional complexities also arise since the plasma conductivity is sensitive to the magnetic field.

In the case of the poloidal interaction, driven by the time variations in the magnetic field, the boundary condition requires the continuity of the normal component of the magnetic field at the interface between the solar wind and ionospheric plasmas. Since most of the solar wind plasma and its magnetic field flows around Venus, the normal component of the field at the anemopause must be small; i.e., the normal component of the magnetic field is proportional to the plasma flow across the interface. Taking the normal component of the magnetic field at the interface to be about 0.1, the magnitude of the typical interplanetary magnetic field requires a poloidal (compressional) amplification factor of about 100 in order to produce an induced field with a magnetic pressure of the same order as the kinetic pressure of the solar wind plasma. An amplification factor of this magnitude would be produced if the induced field is compressed within a zone about 100 km thick. This compressional zone is a measure of the distance between the region where the eddy currents are flowing in the upper ionosphere and where the confining currents flow in the solar wind plasma. A thickness of 100 km is in accord with the estimate of the skin depth in the upper ionosphere, based on the maximum conductivity (magnetic field free) and time scales for the fluctuations of the order of 10^3 seconds. In this case the pressure of the shocked solar wind is transferred to the compressed, induced field in the region of the anemopause. As the magnetic field pressure decreases within a skin depth of this boundary, the decrease must be taken up by an increase in the charged particle pressure in the upper ionosphere. In a sense this model is like the second category of high β models, since the charged particle pressure will dominate the magnetic field pressure over most of the ionosphere except for a thin region near the anemopause. For this reason we will not consider this model further, but will concentrate on what follows on models based on the interaction driven by the motional electric field.

The driving force for the induction current in the case of the toroidal interaction is the motional electric field impressed across the planet by the solar wind flow. Since most of the solar wind flows around Venus, the magnitude of this electric field must be small compared to the free streaming motional electric field. In order to determine the current flow within the planetary ionosphere, we must establish the electrical conductivity profile of the ionosphere.

The electron electrical conductivity profile based on the Mariner V occultation experiment data is derived in Appendix A and the results are shown in Figure 3. The profiles are for an assumed 50γ magnetic field and an E-layer ionosphere. Here, $\sigma_{||}$ is the electron conductivity along the magnetic field, $\sigma_{\perp}$ the conductivity along the electric field component normal to B, and σ_H the Hall conductivity in the E × B direction. If $\omega_B = eB/m$ is the electron gyrofrqquency and τ the electron collision mean free time for momentum transfer, then the quantity $\omega_B\tau$ is a measure of the magnetic field strength. Since $\sigma_{||}$ does not vary significantly with altitude, it is apparent that

$\omega_B\tau$ is the dominant parameter in determining the shape of the profiles for $\sigma_\perp$ and σ_H.

To relate the current density to the field, we first note that in the vicinity of the electron density peak in the ionosphere the plasma is weakly ionized, i.e., the electrons collide predominantly with neutrals, and $\omega_B\tau < 1$. Here, the electron contribution to the current density driven by an electric field is much greater than the ion contribution, and conductivity is reasonably isotropic. Thus, we may use the simple form of Ohm's law with a constant conductivity to relate the electric field to the current density in the ionosphere. At higher altitudes in the induced magnetosphere the plasma is fully ionized and $\omega_B\tau >> 1$. Here, so long as the ion gyrofrequency is much greater than the ion collision frequency for momentum transfer, the ion contribution to the current density normal to B, driven by a dc electric field, exactly cancels the electron contribution. This condition exists everywhere but in the polar regions where B is reduced. Thus, not only is the plasma anisotropic at high altitudes, but electric *fields normal to B drive plasma convective motions rather than* currents. Steady-state electric currents normal to B are typically caused by pressure gradients (Spitzer, 1962). Thus, Ohm's law for the induced magnetosphere is intimately connected with the momentum equation, and an accurate description of the electrodynamics is quite involved.

The simplest model that reflects the important features of the conductivity profiles is the two layer model of Figure 2. The lower layer represents that part of the profile where the conductivity is high and isotropic, and since the electron density here is determined by the incoming ionizing radiation, we refer to this region as the ionosphere. The upper layer represents the anisotropic, low conductivity region and since the plasma is fully ionized and the magnetic field controls the plasma motion ($\beta < 1$), we term this region the induced magnetosphere in accord with Johnson and Midgley (1969).

IONOSPHERIC CURRENTS

Owing to the solar wind flow around the planet, the tangential component of the motional solar wind electric field at the anemopause, i.e., the driving field, is not known *a priori*. Furthermore, the flow results in a strong day-night asymmetry for the interaction. The proper procedure regarding the determination of the driving field is to match the tangential electric field at the anemopause with the self-consistent solar wind field. Instead, we shall assume that the motional electric field E_o available to drive the motional generator is uniform and some fraction of the undisturbed interplanetary field $E_m = -V \times B_o$, where V and B_o are the solar wind velocity and magnetic field. We may view this as the leading term in a spherical harmonic expansion of the driving field. The requirement that the energy density of the induced magnetic field B_ϕ at the anemopause must equal the solar wind energy density, i.e., $B_\phi^2/2\mu = MnV^2$, then suffices to determine E_o for a given ionosphere conductivity profile.

The model developed neglects time dependent effects, but model studies (Sill and Blank, 1970) show that the frequency responses are flat and equal to the dc response until the skin depth is of the order of thickness of the least conductive outer layer, at which point the response begins to decrease. The skin depth is of the order of the thickness of the outer layer for frequencies in the range from 10^{-3} to 10^{-4} Hz. However, the power spectrum of the interplanetary magnetic field peaks at much lower frequencies and it should suffice to treat the dc case. In any event, the ac case is not conceptually different.

The two-layer spherical shell model shown in Figure 2 is adopted for the ionosphere and the induced magnetosphere. The general solution to the azimuthally symmetric boundary value problem represented by Maxwell's equations with Ohm's law is presented in Appendix B. For a two-layer model with $h_1 = O(h_2) \ll b$ and $\sigma_2/\sigma_1 \ll 1$ the solution reduces to:

Ionosphere $a \leq r \leq a + h_1$

$$J_r = 2\sigma_1 (E_o/\alpha)(r/a-1) \cos\theta \tag{1}$$

$$E_\theta = -(E_o/\alpha) \sin\theta \tag{2}$$

$$B_\phi = \mu\sigma_1 b(E_o/\alpha)(r/a-1) \sin\theta \tag{3}$$

$$\alpha = 1 + 2(\sigma_1/\sigma_2)(h_1 h_2/ab) \tag{4}$$

Induced Magnetosphere $a + h_1 \le r \le b$

$$J_r = 2\sigma_1(E_o/\alpha)(h_1/a) \cos\theta \tag{5}$$

$$E_\theta = -(E_o/\alpha)(1 + [\alpha-1]\ [1-(b-r)/h_2]) \sin\theta \tag{6}$$

$$B_\phi = \mu\sigma_1 h_1(E_o/\alpha) \sin\theta \tag{7}$$

We note that the factor α is a function of the ratio of the layer resistances, i.e., $R_2/R_1 = (\sigma_1/\sigma_2)(h_1 h_2/ab)$ (Appendix A). When the resistance of the ionosphere limits the current $R_2/R_1 \ll 1$, $\alpha \sim 1$, and we see that the solution in the ionosphere as well as the magnetic field and radial component of the current in the induced magnetosphere are independent of the conductivity of the induced magnetosphere (Blank and Sill, 1970). The condition $R_2/R_1 \ll 1$ is satisfied, if the Hall conductivity is representative of the conductivity in the induced magnetosphere. Recall that the Hall conductivity is a representative measure of the <u>current</u> flow if the Hall <u>convection</u> of the plasma is <u>balanced</u> by a pressure gradient $\nabla p = neE$ which gives rise to a <u>current</u> $J = (B \times \nabla p)/B^2$ (Spitzer, 1962).

In the limit $R_2/R_1 >> 1(\alpha = 2R_2/R_1)$ one obtains a model compatible with that suggested by Johnson and Midgley (1969), in which the current and field is determined by the resistance (conductivity and thickness) of the induced magnetosphere. Appendix A shows that for Venus, $R_2/R_1 = O(1)$ if we take the lowest values of the perpendicular conductivity as representative of the induced magnetosphere. In this case the Hall convection (E × B drift) of the plasma must be allowed, since the quenching of this drift by a pressure gradient would give rise to a strong electron Hall current.

The electric field E_o which drives the current may now be computed by balancing the energy density of the streaming solar wind $Mn(V \sin\theta)^2$ against the induced magnetic field energy density at the anemopause, that is

$$B\ (r = b) = (2\mu Mn)^{1/2}\ V \sin\theta \tag{8}$$

Upon equating (7) and (8) we obtain

$$E_o = \frac{\alpha V}{\sigma_1 h_1}\left(\frac{2Mn}{\mu}\right)^{1/2} \tag{9}$$

This result is just Ohm's law for the circuit since E_o is proportional to the impressed voltage; $V(2M_m/\mu)^{1/2}$ is proportional to the current and $\alpha/\sigma_1 h_1$ is the resistance. For the case where the ionosphere limits the current $R_2/R_1 << 1$, $\alpha = 1$ and we find for Venus, $E_o = 2 \times 10^{-6}$ volt/m ($n = 10\ cm^{-3}$, $V = 400$ km/sec,

$\sigma_1 h_1 = 3 \times 10^4/\Omega$). This is a small fraction of the undisturbed solar wind motional field, which is typically a few millivolt/m. This result is consistent with the notion that for a strong interaction, i.e., a shock, most of the interplanetary field is convected with the solar wind plasma around the planet. In the other limit where the magnetosphere limits the current, $R_2/R_1 >> 1$, $\alpha = 2R_2/R_1$ and we see from (9) that the electric field required can be considerably larger than that estimated for the previous case. From Appendix A we have, however, the estimate for Venus, $R_2/R_1 = O(1)$ and in this case the electric field is of the same order as that given above.

We may also compute the fraction f of incident solar wind particles which must flow across the anemopause to provide the induction current. The quantity f is defined by:

$$f = \frac{J_r(b,\theta)}{neV \sin\theta} = \left[\frac{8M}{\mu n e^2 b^2}\right]^{1/2} \qquad (10)$$

where

$$J_r(b,\theta) = ([8MnV^2/\mu]^{1/2}/b) \cos\theta \qquad (11)$$

is the radial component of the current density at the anemopause, given by (5) and (9), which is independent of the conductivity model. We note that the expression (11) for J_r is merely Ampere's law for the current required to induce a stagnation magnetic field at the anemopause. Substituting the nominal values $n = 10\ cm^{-3}$, $b = 6000$ km for Venus, we find $f = 0.034$, while for Mars with $n = 2$, $b = 3400$ we have $f = 0.13$. Thus, $f << 1$ for both Venus and Mars, and most of the incident solar wind particles are reflected and flow around the planet. This result, $f << 1$, is compatible with the use, in gasdynamic calculations of the solar wind flow (Spreiter, et al., 1969), of the boundary condition that the component of the flow velocity normal to the anemopause vanish.

From (1), (2), (5) and (6) we have $J_r/J_\theta = O(h_1/a) << 1$, in the ionosphere, and in the magnetosphere, except for $R_2/R_1 < h_2/b$, $J_r > J_\theta$. Thus, the predominant current flow pattern is in the radial direction across the magnetosphere and tangential (θ - direction) in the ionosphere. The radial currents in the magnetosphere (5) and the ionosphere (1) are of the same order and, for the model of Venus presented in Appendix A, the radial current density is typically two orders of magnitude smaller than the tangential current density in the ionosphere. From (8) and (9) we see that for a given stagnation magnetic field, $E_o \sigma_1 h_1/\alpha$ is a constant. Therefore, for a given ionosphere resistance $R_1 = 1/\sigma_1 h_1$, the tangential current in the ionosphere and the radial currents in the magnetosphere and ionosphere are independent of the ratio R_2/R_1. That is, the ionospheric resistance plays a crucial role even if the resistance over the total current path is dominated by the magnetospheric resistance.

We have previously noted that in the induced magnetosphere the plasma is fully ionized and, except in the polar regions, $\omega_B\tau >> 1$. Hence, the electric field cannot drive a current normal to B. However, such currents can be driven by pressure gradients and magnetic field inhomogeneities. If we assume J_θ is driven by a pressure gradient, we estimate:

$$J_\theta \sim \sigma_H \frac{1}{Ne} \frac{\Delta p}{L} \qquad (12)$$

where Δp is the pressure difference and L is the length scale for the pressure gradient. If the pressure difference is of the order of the solar wind dynamic pressure or the ionosphere pressure and the length scale is of the order of the planetary radius, then the current is of the same order as the radial current given in (11). Thus, the required current could be produced, but an explicit demonstration necessitates the solution of the vector momentum equations for the particles, and therefore transcends the capabilities of the present model.

SOLAR WIND CURRENT SHEATH

Up to this point we have concerned ourselves only with the current flow below the anemopause. It was necessary merely to specify the boundary conditions at the anemopause and solve Maxwell's equations for the model ionosphere and induced magnetosphere. These results are mathematically independent of how the current paths close in the solar wind. By analogy with the case of the earth's magnetopause, we now adopt the viewpoint that the solar wind return current flows in a Ferraro-Rosenbouth type sheath (see, e.g., Ferraro, 1952; Rosenbluth, 1963; Sestero, 1965) adjacent to the induced magnetosphere, and identify this sheath as the observed anemopause at Venus.

The usual one-dimensional sheath model provides for a tangential current density J_θ which confines the planetary magnetic field. The physical mechanism for the current is the deflection of the solar wind particles by the self-consistent electromagnetic field in the sheath. There is no radial current density J_r for the one-dimensional model, as all the particles are reflected. The radial current density is introduced by the finite curvature of the sheath. Denoting the sheath thickness by λ, which we shall estimate below, we are interested in the case of a thin sheath; i.e., λ/b, << 1. We can therefore use the one-dimensional sheath model to compute J_θ to the lowest order in the parameter λ/b, and then solve $\nabla \cdot J = 0$ for J_r correct to lowest significant order in our small parameter.

A simple physical model for the one-dimensional sheath (see, e.g., Ferraro, 1952) pictures the electrons, owing to their smaller mass, as being turned before the protons in an electron sheath of thickness λ_1 due to an $E \times B$ drift. The protons are reflected electrostatically by the self-consistent field in a proton sheath whose thickness is on the order of a proton Debye length. Thus, an electric potential exists across the sheath of magnitude $e\phi \sim \frac{1}{2} MV^2$. Upon turning, the electrons acquire an E × B velocity v_θ given by $\frac{1}{2} mv_\theta^2 \sim e\phi$. Therefore, $v_\theta \sim (M/m)^{1/2} V \sin\theta$, and the θ-component of the current density in the sheath is

$$J_\theta \sim (M/m)^{1/2} \, ne \, V \sin\theta \tag{13}$$

Upon applying Ampere's law to the sheath, we estimate $B_\phi/\lambda_1 \sim \mu J_\theta$ where B_ϕ is the stagnation field given by (7). Solving for the sheath thickness λ_1, we find

$$\lambda_1 = \left(\frac{2\varepsilon_o mc^2}{ne^2} \right)^{1/2} = 2 \frac{c}{\omega_p} = \sqrt{2} \frac{mv_\theta}{eB_\phi} \tag{14}$$

where ω_p is the electron plasma frequency. Note that the quantity c/ω_p is the usual expression for the skin depth of a plasma at frequencies $<< \omega_p$, and mv_θ/eB_ϕ is the electron gyroradius in the sheath. Thus, we estimate $\lambda_1 = 3$ km for $n = 6 \text{ cm}^{-3}$ during the Mariner V occulation experiment, and the condition $\lambda/b << 1$ is well satisfied for typical solar wind conditions.

The radial current density $J_r = O(J_\theta \, \lambda/b)$ in the sheath, necessary for the motional generator, must exist to satisfy the condition $\nabla \cdot J = 0$. Thus

$$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 J_r) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} (\sin\theta \, J_\theta) = 0 \tag{15}$$

subject to the boundary condition that J_r vanish at the sheath edge $r = b + \lambda_1$. Substituting (13) for J we arrive at

$$\int_b^{b+\lambda_1} d(r^2 J_r) + 2\left(\frac{M}{m}\right)^{1/2} neV \cos\theta \int_b^{b+\lambda_1} rdr = 0$$

Integrating and substituting (14) for λ_1, we find

$$J_r\ (b,\theta) = ([8\ MnV^2/\mu]^{1/2}/b) \cos\theta \qquad (16)$$

correct to lowest significant order in the small parameter λ_1/b. This is identical to the required radial current density at the anemopause given by (10).

This simple sheath model is for a vacuum magnetic field-plasma boundary. Here the current is almost entirely due to the electrons by virtue of their gaining energy from the solar wind protons via the self-consistent electric field. In this case we find the thinnest region in which the current can flow and thus the expression (14) for λ_1 is properly viewed as a minimum sheath thickness. The presence of a background neutralizing plasma, such as exists in the induced magnetosphere, may serve to reduce the magnitude of the self-consistent electric field, and thereby cause the solar wind protons to carry a non-negligible fraction of the current. In this event the sheath thickness is larger than λ_1. The maximum possible sheath thickness occurs for the case where the background plasma provides total charge neutrality in the sheath. Here the current is carried predominantly by the solar wind protons owing to their larger energy.

The simple physical model for this one-dimensional sheath pictures the protons as turning in the magnetic field gradient and, since there is no energy gain from the field, producing a current density in the sheath of order

$$J_\theta \sim neV \sin\theta \tag{17}$$

The solar wind electrons turn before the protons in an electron sheath whose thickness is of the order of an electron gyroradius and whose location is determined by a balance between the electron and magnetic field energy densities. The proton sheath thickness λ_2 is estimated from Ampere's law, $B_\phi/\lambda_2 \sim \mu J_\theta$, where B_ϕ is the stagnation field given by (7) and J_θ is given by (17). Thus, we estimate the maximum sheath thickness to be

$$\lambda_2 = \left(\frac{2\varepsilon_o Mc^2}{ne^2}\right)^{1/2} = 2\,\frac{MV\sin\theta}{eB_\phi} \tag{18}$$

where $MV\sin\theta/eB_\phi$ is the proton gyroradius in the sheath. Comparing (14) with (18), we note that $\lambda_1/\lambda_2 = (m/M)^{1/2}$. For $n = 6\ \text{cm}^{-3}$ at the time of the Mariner V occultation experiment we estimate $\lambda_2 = 130$ km. Thus, the thin sheath criterion $\lambda_2/b \ll 1$ is also satisfied for the maximum sheath thickness.

It is straightforward to proceed as before in calculating the radial current density J_r by re-solving (15) ($\nabla \cdot J = 0$) with (17) substituted for J_θ. The result is again (16) which is independent of the sheath thickness; that is, the radial current density required by Ampere's law to induce the stagnation magnetic field is independent of the detailed structure of the sheath. Consequently, the models for the sheath and ionosphere currents match properly and we have determined a complete, consistent current path.

CONCLUDING REMARKS

The electrodynamic model proposed for the ionosphere solar wind interaction is based on the existence of a low β plasma just below the anemopause. For $\beta < 1$, the solar wind pressure is balanced at the anemopause predominantly by the induced planetary magnetic field pressure rather than the plasma pressure. The planetary magnetic field arises from an induction current driven by the motional solar wind electric field in the rest frame of the planet. The current flows predominantly in a radial direction across the poorly conductive magnetosphere and tangentially in the highly conductive region near the electron density peak. The current paths close in the anemopause which is a solar wind current sheath analogous to the magnetopause.

From the solution we calculate both the fraction E_o/E_m of the undisturbed solar wind electric field which drives the induction current and the fraction f of incident solar wind particles, required to cross the anemopause to produce this current, and show both fractions to be small. Thus, the model is consistent with the Mariner V Venus observation of a bow shock which implies that most of the incident solar wind particles flow around the planet. The model should be equally applicable to Mars. Thus, we would predict the existence of a Martian anemopause on the sunlit hemisphere, which is similar to that observed at Venus. Another observational check on the model is to obtain a magnetic field profile which should be similar to the schematic profile shown in Figure 5.

The simple two-layer model has two limits of interest in this paper. In the first case the resistance of the ionosphere limits the current and from our conductivity model of Venus this case is appropriate, if the Hall conductivity is representative of the magnetosphere conductivity. This seems likely if the tendency for E × B convection is balanced by a pressure gradient, which in turn drives a current. In the second case the resistance of the magnetosphere dominates as in the model of Johnson and Midgely (1969). This condition will tend to prevail if the perpendicular conductivity is appropriate for the magnetosphere conductivity. Here pressure differentials of the order of the solar wind dynamic pressure are not permitted as they would drive currents considerably larger than the current due the electric field and the perpendicular conductivity. As the resistance of the magnetosphere increases, the electric field (and the convection of particles across the anemopause) needed to produce a stagnation magnetic field increases above that needed in the first case (ionosphere limits current). Estimates of the magnetospheric resistance for Venus, due to the perpendicular conductivity alone, indicate that it is of the same order as the ionospheric resistance.

It is apparent that future studies should investigate the detailed pressure balance since, at least in the induced magnetosphere, the current density is strongly affected by the pressure gradient, which necessitates solution of the momentum equations. The present study includes only a gross pressure balance in that the magnetic pressure dominates in the induced magnetosphere and the plasma pressure increases with decreasing altitude until it dominates in the ionosphere. Furthermore, if the motional electric field E_o drives a current density J_θ which flows predominantly in the vicinity of the electron density peak, the driving field must map into the ionospheric peak region. This suggests that there could be significant convective motion in the induced magnetosphere whose characteristics can be ascertained only by solving the vector momentum equations of the constituent species.

The model proposed here is partly an observational one, since it utilizes the occulation experiment results to estimate particle densities and pressures, instead of simultaneously solving the equations of motion and Maxwell's equations for these quantities. Thus, *a priori* the model is incapable of determining the size, density, and pressure profiles of the induced magnetosphere. The motivation for the approach adopted here is to elucidate the electrodynamic features of the interaction of the solar wind with an ionosphere. The addition of the equations of motion to the model should serve to further our understanding of the dynamics of the induced magnetosphere.

W. R. Sill for J. L. Blank

J. L. Blank

W. R. Sill

W. R. Sill

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Attachments

APPENDIX A

ELECTRICAL CONDUCTIVITY MODEL

In the presence of a magnetic field, the electron conductivity of a plasma is anisotropic with the following components

$$\sigma_{||} = Ne^2\tau/m \qquad \text{parallel to B} \tag{A1}$$

$$\sigma_H = \sigma_{||} \frac{\omega_B\tau}{1+(\omega_B\tau)^2} \qquad \text{in the E} \times \text{B direction} \tag{A2}$$

$$\sigma_\perp = \sigma_{||} \frac{1}{1+(\omega_B\tau)^2} \qquad \text{in the B} \times \text{(E} \times \text{B) direction} \tag{A3}$$

where N is the electron number density, e and m the electron charge and mass, τ the electron mean free collision time for momentum transfer, and $\omega_B = eB/m$ is the electron gyrofrequency. The number density profile in the Venus ionosphere has been deduced from the Mariner V occulation experiment (Mariner Stanford Group, 1967). To determine the conductivity we also must know the magnetic field and collision frequency.

In anticipation of finding that the dominant contribution to the planetary magnetic field comes from currents in the vicinity of the electron density peak, and that $\omega_B\tau < 1$ there, we assume $B = B_\phi = (2\mu MnV^2)^{1/2} \sin\theta$, the stagnation field, everywhere in the ionosphere and induced magnetosphere. For typical solar wind parameters at the orbit of Venus, $B_\phi = 80\gamma$ at the subsolar point. Electron collision frequency profiles for the ionosphere electrons have been calculated by Sill (1968) and are shown in Figure 6. The corresponding conductivity profiles based on these curves and Eqs. (A1) - (A3) are presented in Figure 3 for the case of an E-layer ionosphere and an assumed 50γ magnetic field.

For our azimuthally symmetric model, $E \cdot B = 0$ and we need not concern ourselves with $\sigma_{||}$. If the plasma is isotropic, we should find $\sigma_H/\sigma_{||} << 1$ with $\sigma_{||} = \sigma_\perp$. From Figures 1 and 3 we note that this condition is satisfied at the electron density peak, which is the most conductive portion of the ionosphere. Furthermore, in the upper portion of the atmosphere both $\sigma_\perp$ and σ_H decrease rapidly with increasing height.

This suggests that the least resistive current path has the bulk of the tangential current density J_θ flowing near the electron density peak. If we view the ionosphere as composed of spherical layers of thickness h_i and parallel conductivity σ_i, the ionospheric resistance R_1 appropriate for the J_θ portion of the circuit, where the layers are parallel, is

$$\frac{1}{R_1} = \sum_i \frac{1}{R_i} \cong \sum_i \sigma_i h_i \quad \text{(A4)}$$

From Figure 3 we estimate $R_1 \simeq 3 \times 10^{-5}$ Ω for this portion of the circuit, and an equivalent single layer would have a thickness of 30 km for a conductivity of 1 mho/m.

To complete the induction circuit the current must flow across the induced magnetosphere from the electron density peak region to the solar wind current sheath. This portion of the current pattern is a series circuit and the corresponding resistance R_2 of the induced magnetosphere is

$$R_2 = \sum_j R \cong \sum_j \frac{h_j}{\sigma_j b^2} \qquad \text{(A5)}$$

where σ_j is the appropriate conductivity of the j^{th} layer. The dominant contribution to this resistance comes from the upper layers. Since the plasma in the induced magnetosphere is fully ionized and, except for the polar regions, $\omega_B \tau << 1$, the radial current is driven by the magnetic field inhomogeneity and/or by the pressure gradient. If, in the absence of a detailed momentum balance for computation of this current, we make the conservative estimate for R_2 by substituting the Hall conductivity for σ_j in (A5), we then estimate $R_2 \approx 10^{-7}$ Ω and $R_2/R_1 << 1$. An equivalent single layer has a thickness of 350 km and a conductivity of about 3×10^{-2} mho/m.

In the alternative case, where the perpendicular conductivity is taken as the appropriate one, we find that $R_2 \sim 10^{-5}$ Ω and $R_2/R_1 = O(1)$. Here the equivalent single layer has a thickness of 350 km and a conductivity of about 3×10^{-4} mho/m.

APPENDIX B

SOLUTION OF BOUNDARY VALUE PROBLEM

We wish to solve Maxwell's equations with Ohm's law ($\nabla \cdot B = 0$, $\nabla \times B = \mu J$, $J = \sigma_i E$, $\nabla \times E = 0$, $\nabla \cdot J = 0$) in the i^{th} layer for the spherically symmetric, layered model depicted in Figure 2. If we introduce the potential ϕ via $E = -\nabla\phi$, then in the i^{th} layer

$$\phi_i = -\sum_{n=1}^{\infty} \left[\alpha_{in} r^n + \beta_{in} r^{-(n+1)}\right] P_n (\cos \theta)$$

where $P_n (\cos \theta)$ are the Legendre polynomials. Thus, in each layer of constant conductivity the general solution of the boundary value problem is

$$J_r = \sigma_i \sum_{n=1}^{\infty} \left[n\alpha_{in} r^{n-1} - (n+1) \beta_{in} r^{-(n+2)}\right] P_n (\cos \theta) \qquad \text{(B1)}$$

$$E_\theta = \sum_{n=1}^{\infty} \left[\alpha_{in} r^{n-1} + \beta_{in} r^{-(n+2)}\right] \frac{dPn}{d\theta} \qquad \text{(B2)}$$

$$B_\phi = -\mu\sigma_i \sum_{n=1}^{\infty} \left[\frac{\alpha_{in}}{(n+1)} r^n - \frac{\beta_{in}}{n} r^{-(n+1)}\right] \frac{dPn}{d\theta} \qquad \text{(B3)}$$

The boundary conditions are the continuity of J_r (or B_ϕ) and E_θ at the surfaces r=a, r=a+h, and r=b. The planet and neutral atmosphere are represented by a sphere of radius a and conductivity σ=0.

Upon evaluating the coefficients, the solution in the ionosphere and induced magnetosphere takes the form Ionosphere $a \leq r \leq a + h_1$

$$J_r = \sigma_1 E_o \frac{3}{2} \zeta \ (1 - [a/r]^3) \cos\theta \qquad \text{(B4)}$$

$$E_\theta = -E_o \frac{3}{2} \zeta \ (1 + \frac{1}{2} [a/r]^3) \sin\theta \qquad \text{(B5)}$$

$$B_\phi = \frac{3}{4} \mu\sigma_1 E_o r \ \zeta \ (1 - [a/r]^3) \sin\theta \qquad \text{(B6)}$$

Induced magnetosphere $a + h_1 \leq r \leq b$

$$J_r = \sigma_2 E_o \ \xi \ \zeta \left[1 - \frac{1}{\xi} \left\{ \left[1 + \frac{1}{2} \left(\frac{a}{a+h_1} \right)^3 \right] - \frac{\sigma_1}{\sigma_2} \left[1 - \left(\frac{a}{a+h_1} \right)^3 \right] \right\} \left(\frac{a+h_1}{r} \right)^3 \right] \cos\theta \qquad \text{(B7)}$$

$$E_\theta = - E_o \ \xi \ \zeta \left[1 + \frac{1}{2\xi} \left\{ \left[1 + \frac{1}{2} \left(\frac{a}{a+h_1} \right)^3 \right] - \frac{\sigma_1}{\sigma_2} \left[1 - \left(\frac{a}{a+h_1} \right)^3 \right] \right\} \left(\frac{a+h_1}{r} \right)^3 \right] \sin\theta \qquad \text{(B8)}$$

$$B_\phi = \frac{1}{2} \mu \sigma_2 E_o r \; \xi \; \zeta \left[1 - \frac{1}{\xi} \left\{ \left[1 + \frac{1}{2} \left(\frac{a}{a + h_1} \right)^3 \right] - \frac{\sigma_1}{\sigma_2} \left[1 - \left(\frac{a}{a + h_1} \right)^3 \right] \right\} \left(\frac{a + h_1}{r} \right)^3 \right] \sin \theta \qquad \text{(B9)}$$

where

$$\zeta = \left\{ \left[1 + \frac{1}{2} \left(\frac{a + h_1}{b} \right)^3 \right] \left[1 + \frac{1}{2} \left(\frac{a}{a + h_1} \right)^3 \right] + \frac{1}{2} \frac{\sigma_1}{\sigma_2} \left[1 - \left(\frac{a + h_1}{b} \right)^3 \right] \left[1 - \left(\frac{a}{a + h_1} \right)^3 \right] \right\}^{-1} \qquad \text{(B10)}$$

and

$$\xi = \left[1 + \frac{1}{2} \left(\frac{a}{a + h_1} \right)^3 \right] + \frac{1}{2} \frac{\sigma_1}{\sigma_2} \left[1 - \left(\frac{a}{a + h_1} \right)^3 \right] \qquad \text{(B11)}$$

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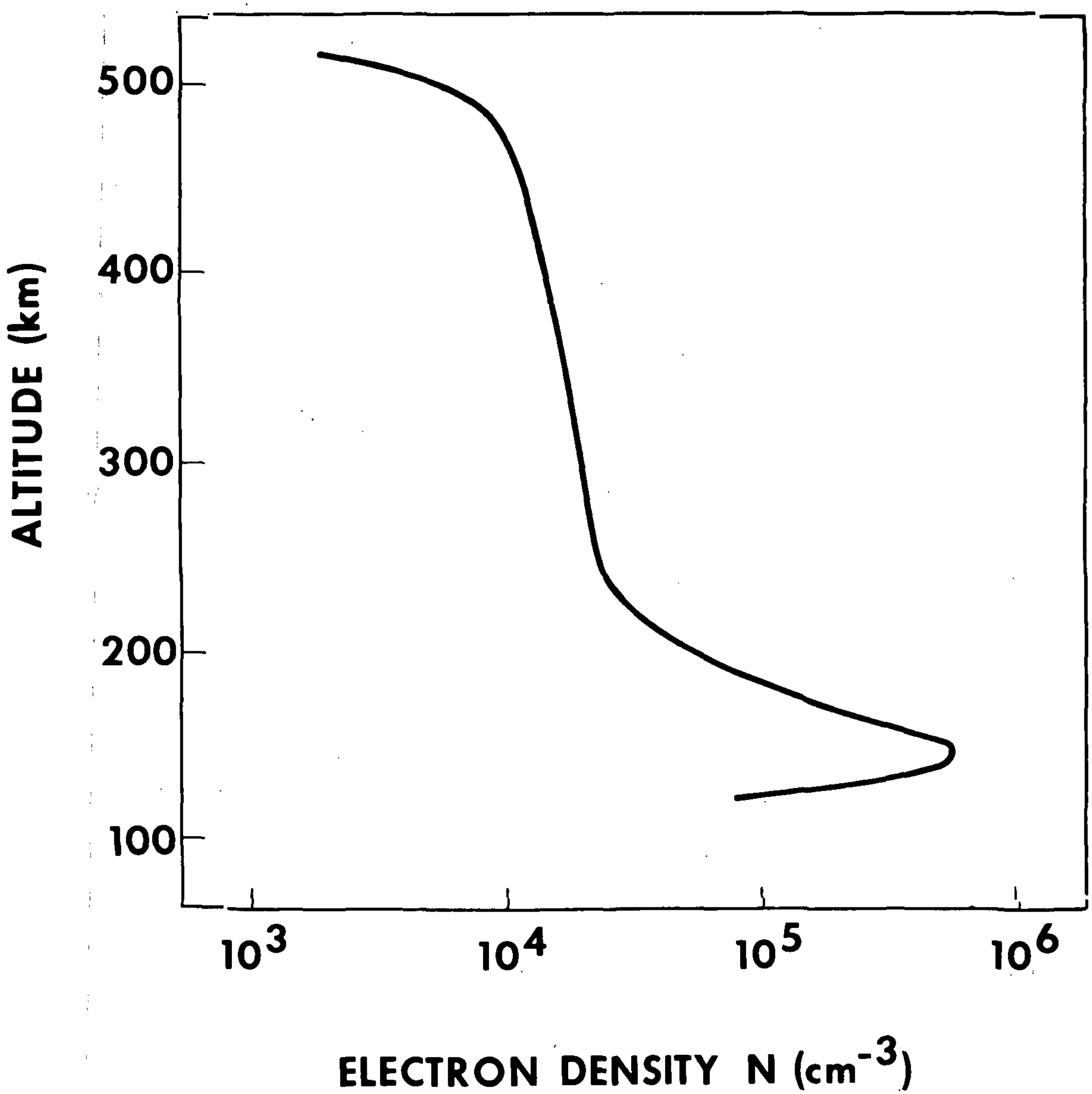

FIGURE 1 - ELECTRON DENSITY PROFILE FOR THE DAYTIME VENUS IONOSPHERE AS DEDUCED FROM THE MARINER V DUAL FREQUENCY OCCULATION EXPERIMENT

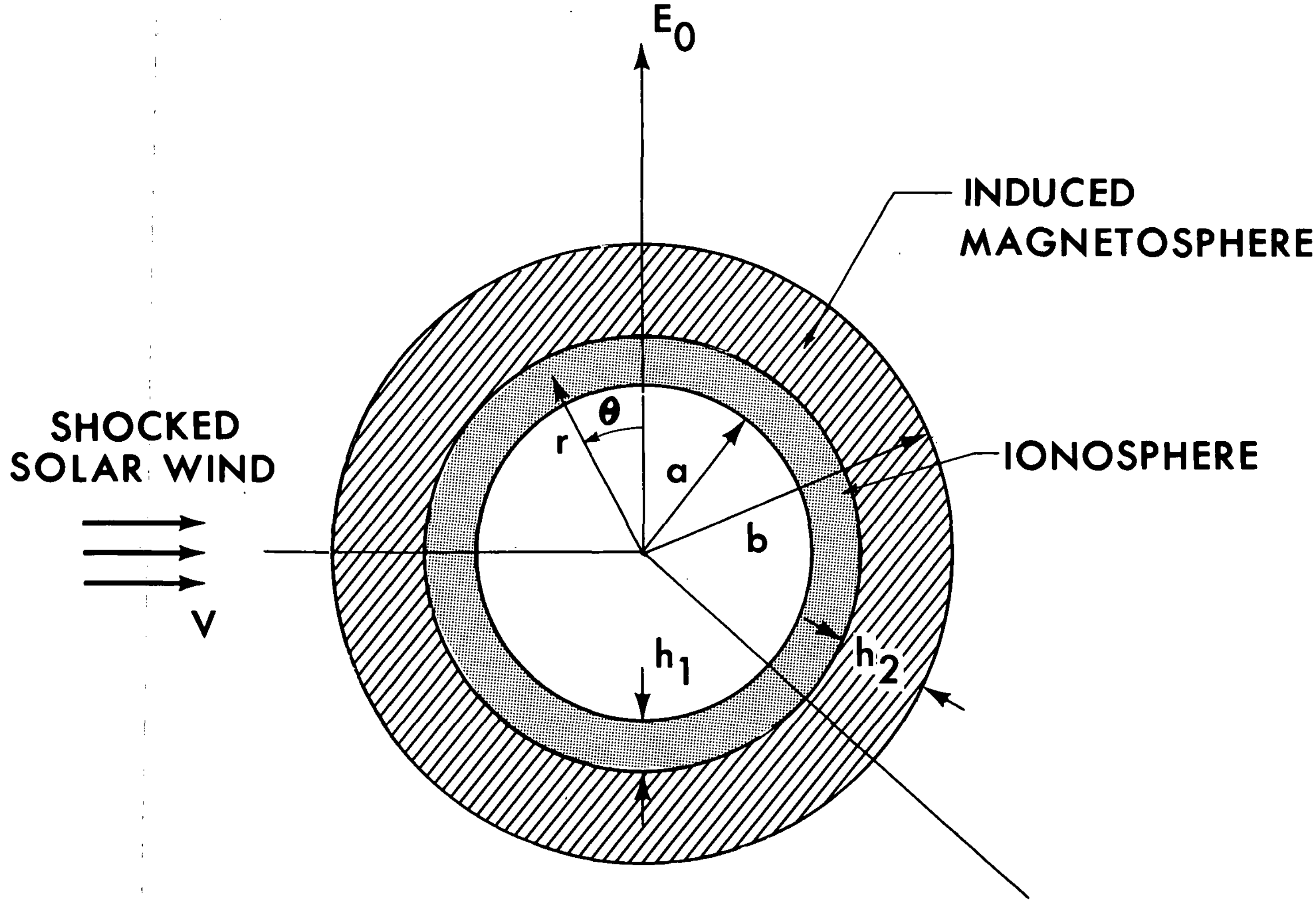

FIGURE 2 - SCHEMATIC ILLUSTRATION OF THE TWO-LAYER, IONOSPHERE-INDUCED MAGNETOSPHERE, MODEL

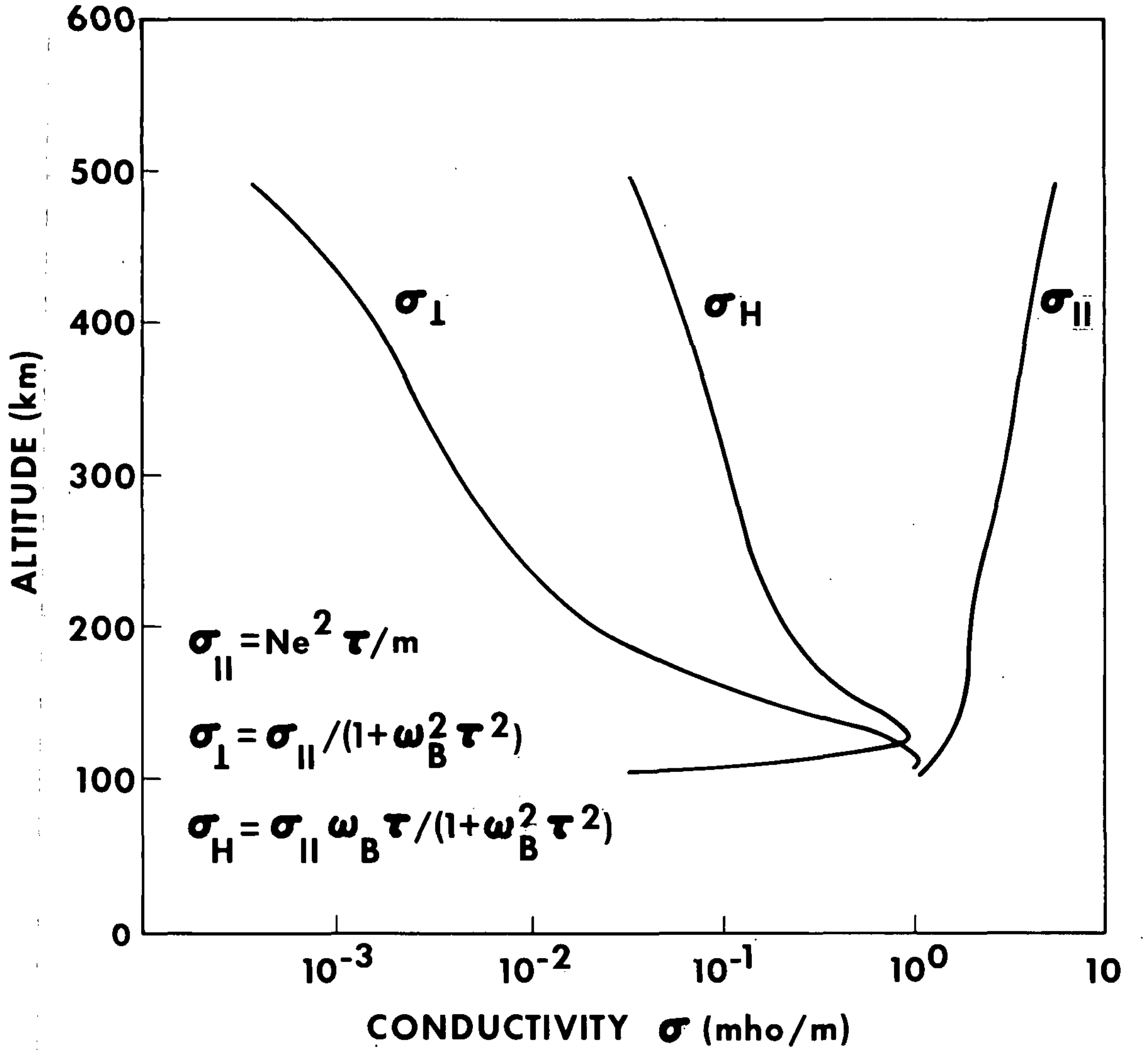

FIGURE 3 - ELECTRON ELECTRICAL CONDUCTIVITY PROFILES FOR AN E-LAYER DAYTIME VENUS IONOSPHERE BASED ON THE MARINER V DUAL FREQUENCY OCCULTATION EXPERIMENT DATA AND AN ASSUMED 50γ MAGNETIC FIELD

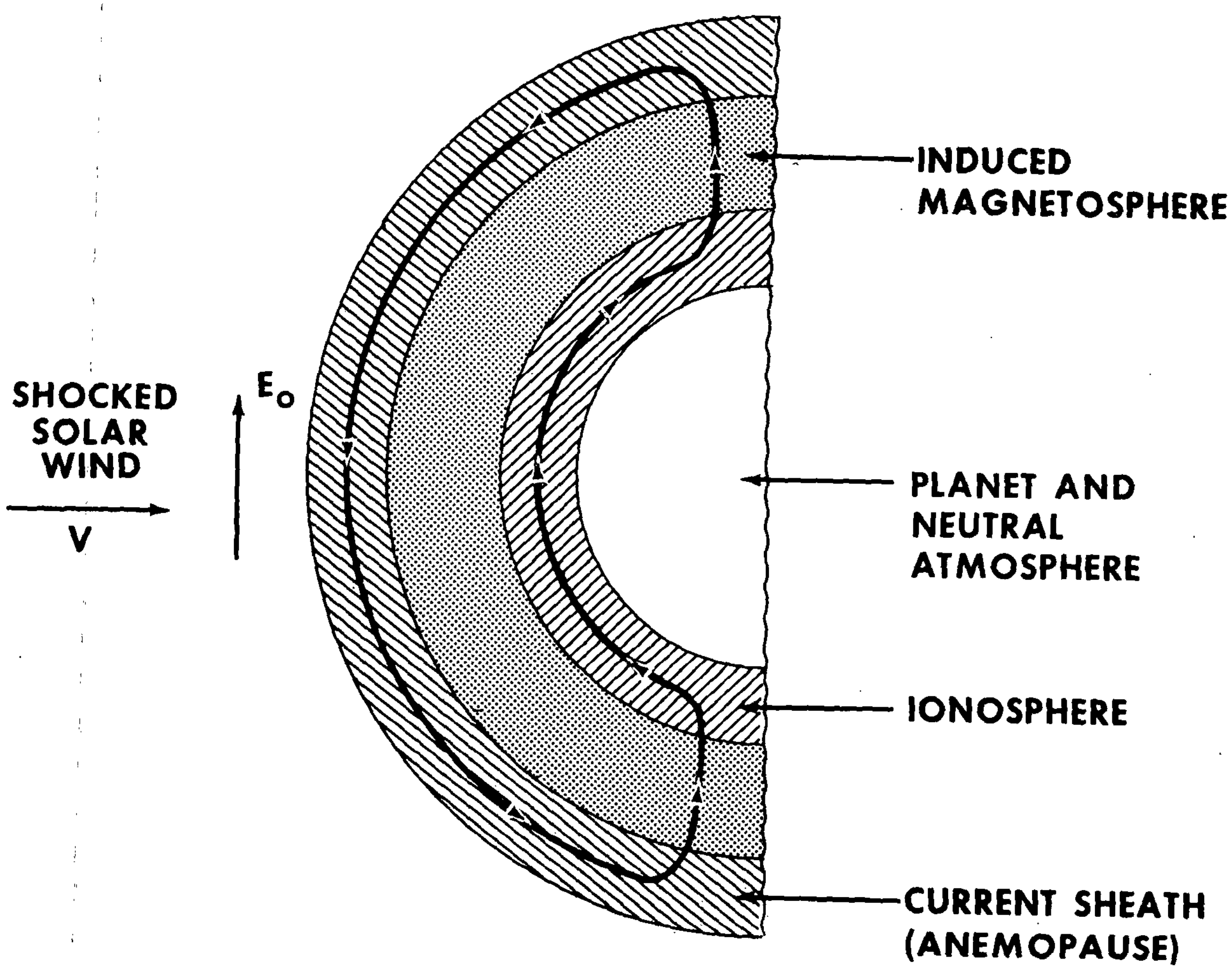

FIGURE 4 - SCHEMATIC REPRESENTATION OF THE CURRENT PATH THROUGH THE IONOSPHERE AND INDUCED MAGNETOSPHERE WITH CLOSURE IN THE SOLAR WIND CURRENT SHEATH (ANEMOPAUSE)

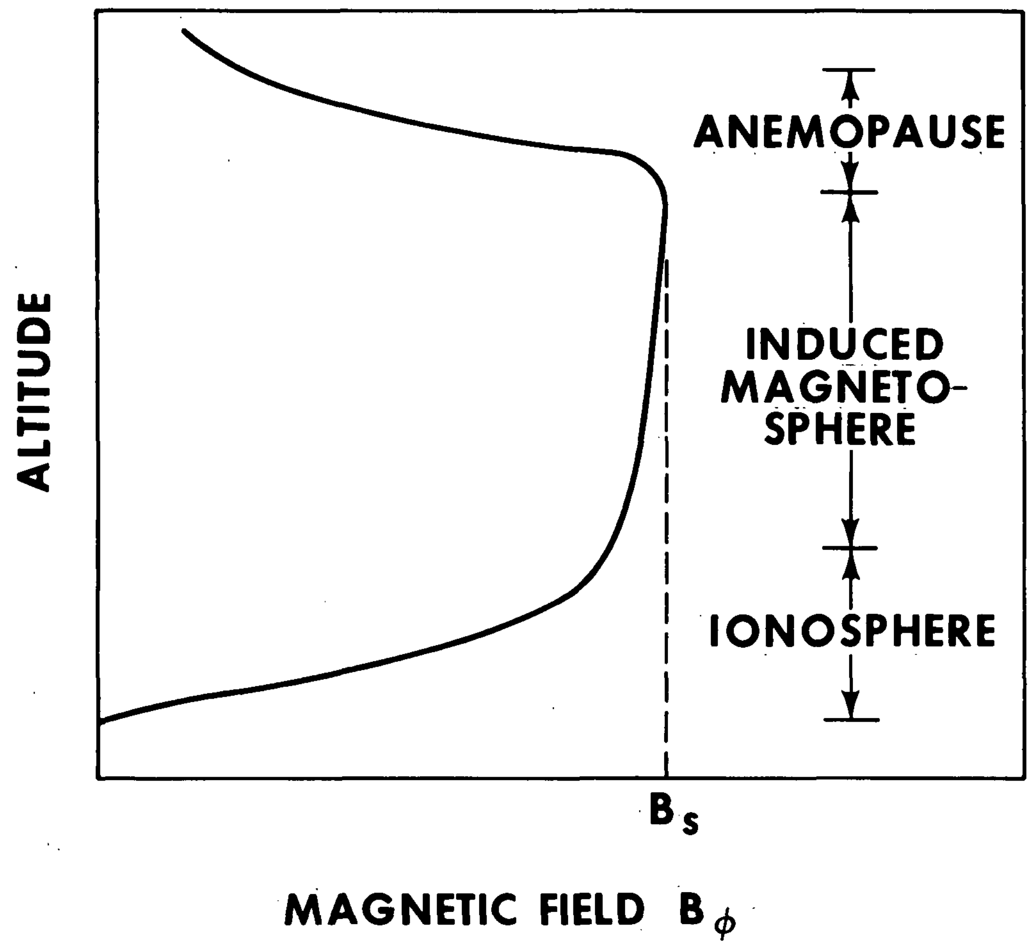

FIGURE 5 - SCHEMATIC ILLUSTRATION OF THE MAGNETIC FIELD PROFILE IN THE IONOSPHERE, INDUCED MAGNETOSPHERE, AND SOLAR WIND CURRENT SHEATH (ANEMOPAUSE) AT A FIXED VALUE OF θ (SEE FIGURE 2) ON THE DAYLIGHT HEMISPHERE. HERE $B_S = (2\mu Mn)^{1/2}\ V\ \sin\theta$ IS THE STAGNATION MAGNETIC FIELD AT THE ANEMOPAUSE

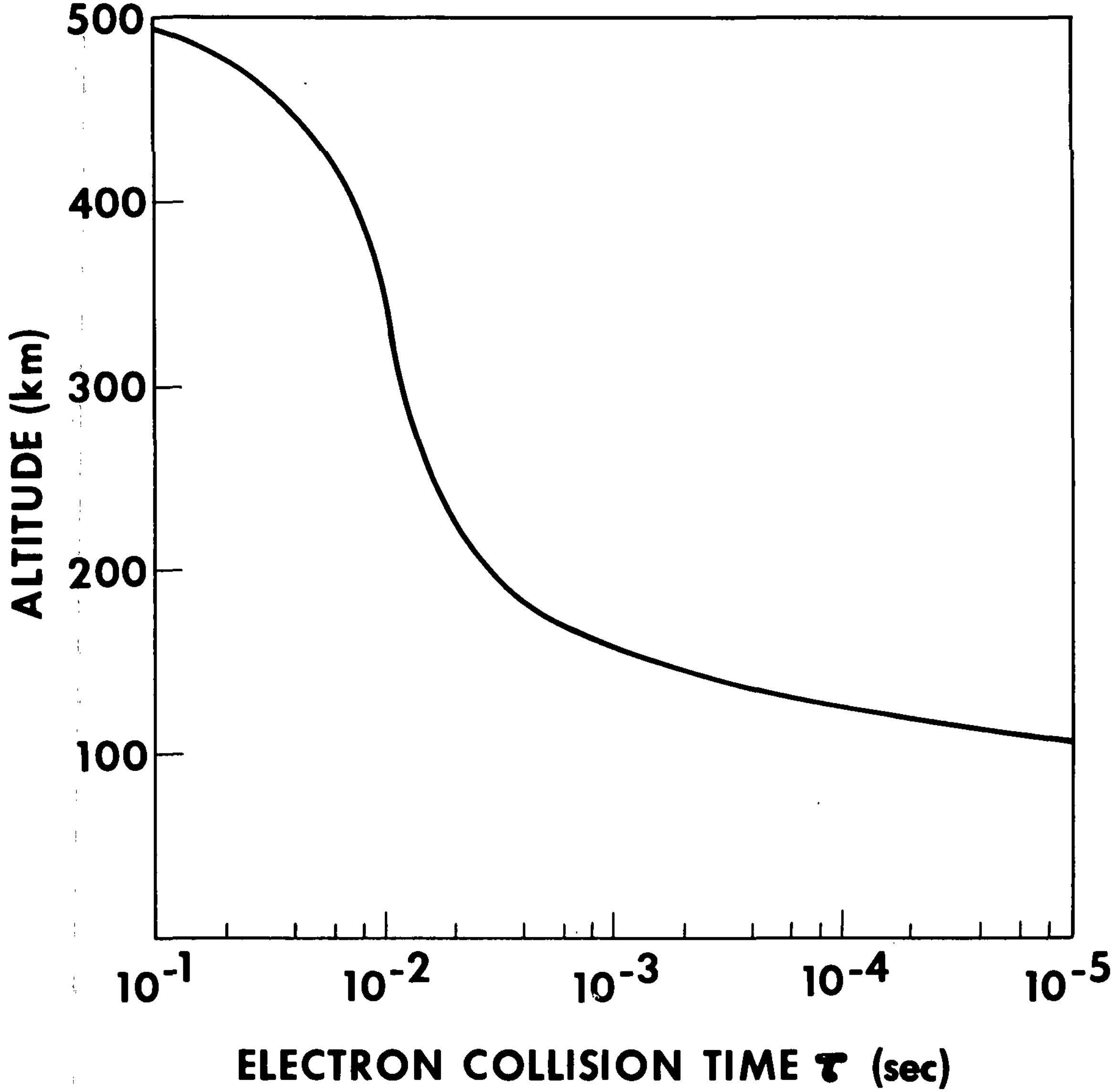

FIGURE 6 - ELECTRON MEAN FREE COLLISION TIME FOR MOMENTUM TRANSFER BASED ON AN E-LAYER IONOSPHERE (SILL, 1968)

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