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A MATHEMATICAL MODEL FOR SIMULATION OF THE APOLLO TELESCOPE MOUNT POINTING CONTROL SYSTEM

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ABSTRACT

In this report a mathematical model is presented which describes the dynamics of the Pointing Control System (PCS) for the Apollo Telescope Mount (ATM). The vehicle is modeled as two rigid bodies--a carrier and an experiment package--connected by a rigid, massless, two-degree-of-freedom set of gimbals. Attitude control of the carrier is provided by three two-degree-of-freedom Control Moment Gyroscopes (CMGs), and the experiment package is controlled by means of torquers located on the gimbal axes. Expressions are presented for three types of vehicle force and torque disturbances, namely, gravitational, aerodynamic, and crew motion.

In addition to the equations themselves, a digital computer program is presented which performs the operations necessary to obtain solutions to specific problems. This document is intended to present the mathematical model and the computer program in a form sufficiently general that they may be applied to either the ATM or to advanced or alternative vehicles employing similar attitude control systems. Both the model and the program have been used for the analysis of the ATM PCS and related attitude control systems for advanced stellar astronomy missions; the results of these investigations will be presented separately.

ACKNOWLEDGEMENT

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1.0 INTRODUCTION

1.1 Background

Orbiting astronomical telescopes, being essentially above the earth's atmosphere, have distinct advantages over their ground-based counterparts [1].* However, they also present a unique set of problems, one of these being the requirement to keep the telescope pointing precisely in the direction of interest. This pointing accuracy** requirement has been one of the major considerations in the design of the Pointing Control System (PCS) for the Apollo Telescope Mount (ATM).***

It was concluded early in the ATM program that if the experiment package were rigidly attached to the manned portion of an orbiting facility, the pointing accuracy required by the experiments could be maintained only if crew motion were severely restricted [2]. It is therefore desirable to have some means of isolating the effects of crew motion from the attitude dynamics of the experiments themselves. Several schemes have been suggested for doing this: allowing the experiment package to orbit independently but in close proximity to the manned craft [3], tether arrangements [4], optical transfer lens techniques [5], magnetic suspension systems [6], and gimbal mounting of the experiment package at its mass center. This report is concerned with the latter scheme.

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* Numbers in brackets designate references at the end of the report.

** The term "pointing accuracy" in this report refers to the accuracy of maintaining a specific telescope attitude once this attitude has been acquired, in contrast to the accuracy of acquisition itself.

***The ATM, which includes several solar astronomy experiments, is scheduled to be launched among the first Apollo Applications Program (AAP) payloads. Other optical telescopes, for both solar and stellar astronomy, are being considered for future AAP missions.

1.2 Purpose

The ATM carrier and experiment package, which together comprise the spacecraft, are connected by means of a two-degree-of-freedom system of gimbals whose axes are nominally normal to the experiment optical axis. An Experiment Pointing Control (EPC) system drives torquers on the gimbals so as to keep the experiment optical axis aligned with the sun line. Three two-degree-of-freedom Control Moment Gyroscopes (CMGs) are employed to maintain the sun-pointing attitude of the carrier.*

In the design of interacting control systems such as the ones described above, it is often necessary to determine the behavior of a given design or to assess the influence of certain design parameters. For example, PCS behavior may be compared for several schemes of computing the CMG gimbal rate commands, or it may be desired to know how effectively the experiment gimbal mounting isolates the optics from crew motion disturbances. The influence of several design parameters on system behavior is also of importance; such parameters are, for example, CMG feedback gains and the experiment package mass center location. It is the purpose of this report to present a set of differential equations which describe the PCS and the vehicle attitude dynamics; design analyses may then be performed either by numerical integration of the full set of equations on a digital computer or by making simplifications in the equations appropriate to the problem at hand and obtaining the analytical expressions of interest.

In regard to future AAP astronomy experiments, additional questions may be answered by use of the mathematical model, the basic question being that of the pointing accuracy attainable with a system of the PCS type. In order to study the pointing accuracy limitations of the PCS for future AAP missions it will be necessary to include some details not presently incorporated in the mathematical model, such as the effects of noise and other nonlinear effects in the sensors, compliance in the structure, etc. In anticipation of future studies, the equations have been written and programmed as much as possible in modular form so as to render updating a relatively simple task.

1.3 Outline of the Analysis

A broad outline of the sequel is presented here so that the reader will have a clear idea of the objective and a better understanding of how all of the details fit together.

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*More detailed information on the ATM and the associated attitude control systems may be obtained from Reference 7.

The purpose of this report is to develop a set of differential equations representing the dynamics of the vehicle and the PCS; the equations are presented with a view toward numerical solution on a digital computer, and a computer program for this purpose is included.

An analytical description of the vehicle to be investigated is given in Section 2. Here the five generalized coordinates that specify the vehicle attitude are introduced. The object of Section 3 is to obtain first order differential equations of motion, five of these, the dynamical equations, being obtained by use of a method due to Kane and Wang.* Quantities such as velocities, accelerations, and inertia forces used in the generation of the equations of motion, are obtained in the first part of Section 3. Expressions for the active forces and torques, which also enter into the equations of motion, are developed in Sections 4, 5, and 6. The CMGs are described in Section 4.1, and a formula is obtained for the CMG control torque in terms of the gimbal angle rates. There are a variety of schemes for determining gimbal angle rates in terms of the state variables, and these so-called control laws are discussed in 4.2. Torques acting between the telescope and the carrier are investigated in Section 5, whereas Section 6 is devoted to gravitational, aerodynamic, and crew motion effects. Section 7 describes a computer program which integrates the equations of motion, derived in Section 3, under the influence of the active forces treated in Sections 4, 5, and 6; the principal computer programs are listed in the appendices.

2.0 COORDINATES AND THE DYNAMICAL MODEL

2.1 Axes and Coordinates

Suppose a satellite comprising a carrier and an experiment package to be in a circular orbit about the earth, and consider three sets of right-handed mutually orthogonal axes,** XYZ, $x_d y_d z_d$, and $x_2 y_2 z_2$, the first set being fixed in an inertial reference frame such that, at some time t =0, X is directed toward the satellite's zenith and Y in the direction of the orbital motion; $x_2 y_2 z_2$ are fixed in the experiment package, and $x_d y_d z_d$ are fixed in an inertial reference frame such that $x_2 y_2 z_2$ and $x_d y_d z_d$ are aligned when the

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*This method [11, 12], although relatively unknown, offers certain advantages for dynamical systems such as the one considered here.

**Note that these axes do not comprise a cartesian coordinate system, for the axes need not intersect at a common point (origin), and an axis may be translated without affecting its definition.

satellite is in the desired orientation.* The orientation of $x_d y_d z_d$ relative to XYZ is specified by three angles ϕ_d, θ_d and ψ_d, generated as follows: initially align $x_d y_d z_d$ with XYZ; rotate $x_d y_d z_d$ about z_d by an amount ψ_d; follow with a rotation about y_d of amount θ_d; then perform a rotation ϕ_d about x_d, bringing $x_d y_d z_d$ into its final position. The foregoing procedure is sometimes known as a 3,2,1 Euler angle sequence involving the angles ψ_d, θ_d, ϕ_d, respectively. The orientation of $x_2 y_2 z_2$ relative to $x_d y_d z_d$ is likewise specified in terms of a 3,2,1 Euler angle sequence employing ψ, θ, and ϕ, respectively. Note that under these definitions ϕ, θ, and ψ measure the telescope attitude relative to the desired telescope attitude.

2.2 Description of the Dynamical Model

The carrier, called B_1, and the experiment package, B_2, **are assumed to be connected by massless gimbals whose two axes of rotation are mutually orthogonal and intersect at 0, as shown in Figure 2.1.

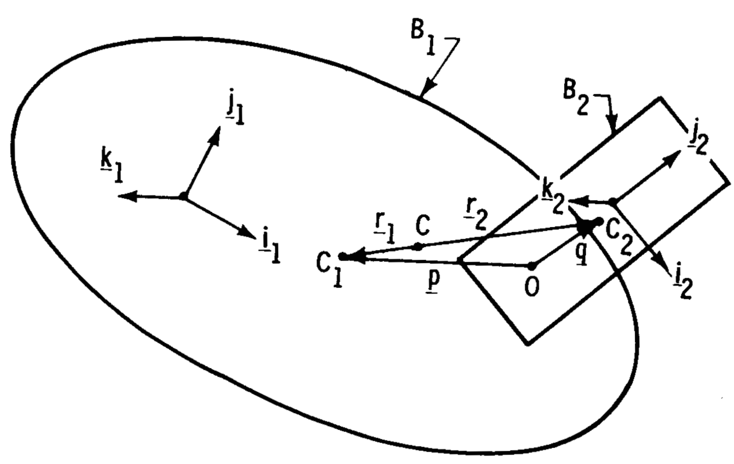

Figure 2.1

*In the case of the ATM, the desired orientation is one in which the experiment axis is directed toward the center of the sun and the Workshop axis is in the orbital plane.

**B_1 and B_2 are considered as rigid bodies.

Let γ_1 and γ_2 measure rotations about the gimbal axes. Introduce two sets of mutually orthogonal unit vectors, $\underline{i}_1$, $\underline{j}_1$, $\underline{k}_1$ and $\underline{i}_2$, $\underline{j}_2$, $\underline{k}_2$ such that $\underline{i}_2$, $\underline{j}_2$, $\underline{k}_2$ are respectively parallel to $x_2y_2z_2$ and such that the two sets of vectors are aligned when $\gamma_1=\gamma_2=0$. The unit vectors are oriented in the spacecraft such that $\underline{j}_2$ is parallel to the experiment optical axis and such that the gimbal axes corresponding to γ_1 and γ_2 are respectively parallel to $\underline{i}_1$ and $\underline{k}_2$. It follows that the two sets of unit vectors are related by the transformation

$$\begin{bmatrix} \underline{i}_1 \\ \underline{j}_1 \\ \underline{k}_1 \end{bmatrix} = \begin{bmatrix} c\gamma_2 & -s\gamma_2 & 0 \\ c\gamma_1 s\gamma_2 & c\gamma_1 c\gamma_2 & -s\gamma_1 \\ s\gamma_1 s\gamma_2 & s\gamma_1 c\gamma_2 & c\gamma_1 \end{bmatrix} \begin{bmatrix} \underline{i}_2 \\ \underline{j}_2 \\ \underline{k}_2 \end{bmatrix} \qquad (2.1)$$

where $c\gamma_1 = \cos\gamma_1$, $s\gamma_1 = \sin\gamma_1$, $c\gamma_2 = \cos\gamma_2$, and $s\gamma_2 = \sin\gamma_2$. Vectors $\underline{p}$ and $\underline{q}$ specify the position of C_1 and C_2, the respective mass centers of B_1 and B_2, relative to 0.

For the AAP Cluster,* $\underline{i}_1$ is assumed to be parallel to the Workshop axis and directed toward the CSM.

3.0 ATTITUDE DYNAMICS

3.1 Kinematics

Let m_1 and m_2 be the masses of B_1 and B_2; let C be the location of the mass center of the composite body B_1 and B_2, and introduce $\underline{r}_1$ and $\underline{r}_2$ as the position vectors of C_1 and C_2, respectively, relative to C:

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*The Cluster (See Figure 2.2) comprises an S-IVB Orbital Workshop, its solar arrays, an Airlock Module, and a Multiple Docking Adapter (MDA) with a Command and Service Module (CSM) docked to the end of the MDA and a Lunar Module Ascent Stage/ATM docked to the side of the MDA.

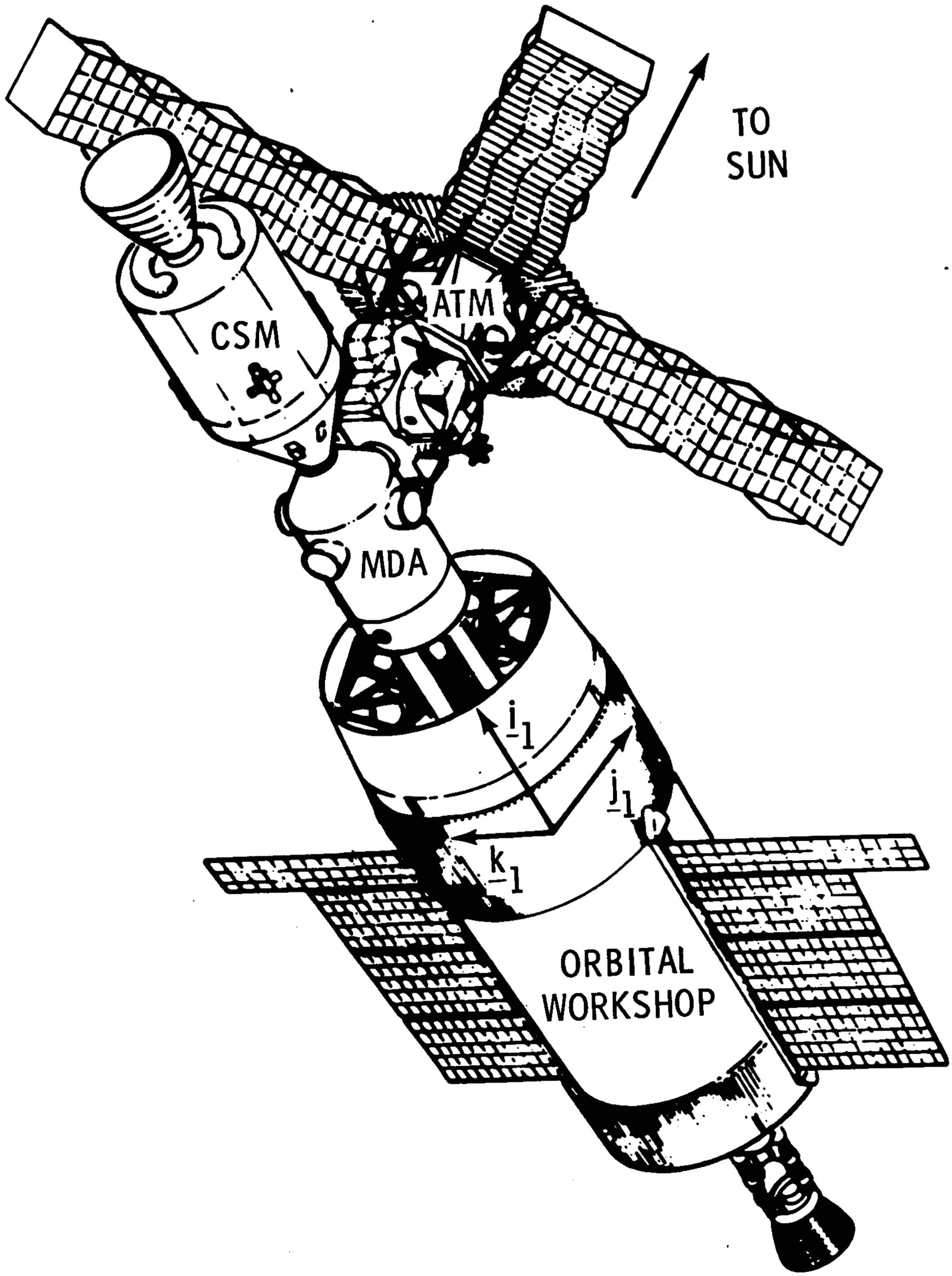
TO
SUN
CSM
ATM
MDA
$\underline{i}_1$
$\underline{j}_1$
$\underline{k}_1$
ORBITAL
WORKSHOP

FIGURE 2.2

$$\underline{r}_1 = \frac{m_2}{m_1+m_2} (\underline{p}-\underline{q}) \quad (3.1)$$

$$\underline{r}_2 = \frac{-m_1}{m_1+m_2} (\underline{p}-\underline{q}) \quad (3.2)$$

Velocities and accelerations in this analysis are taken relative to C rather than an inertially fixed point, since the former are easier to compute and the change of reference has no effect on the attitude dynamics.* By differentiating (3.1) and (3.2) in an inertial reference frame, we obtain the velocities of the bodies' mass centers:**

$$\underline{V}^{C_1} = \frac{d}{dt} \underline{r}_1 = \frac{m_2}{m_1+m_2} (\underline{\omega}^{B_1} \times \underline{p} - \underline{\omega}^{B_2} \times \underline{q}) \quad (3.3)$$

$$\underline{V}^{C_2} = \frac{d}{dt} \underline{r}_2 = \frac{-m_1}{m_1+m_2} (\underline{\omega}^{B_1} \times \underline{p} - \underline{\omega}^{B_2} \times \underline{q}) \quad (3.4)$$

where $\underline{\omega}^{B_1}$ and $\underline{\omega}^{B_2}$ are the angular velocities of B_1 and B_2, respectively. Accelerations follow by differentiation of the velocities:

$$\underline{a}^{C_1} = \frac{d}{dt} \underline{V}^{C_1}$$

$$= \frac{m_2}{m_1+m_2}\left[\dot{\underline{\omega}}^{B_1} \times \underline{p} - \dot{\underline{\omega}}^{B_2} \times \underline{q} + \underline{\omega}^{B_1} \times (\underline{\omega}^{B_1} \times \underline{p}) - \underline{\omega}^{B_2} \times (\underline{\omega}^{B_2} \times \underline{q})\right] \quad (3.5)$$

---

*Whittaker [8] shows that the attitude motion of a rigid body is the same as if the mass center were inertially fixed and the body subjected to the action of the same forces. This theorem may be generalized to include systems of connected rigid bodies such as those considered here.

**Recall that $\underline{p}$ and $\underline{q}$ are known vectors fixed in B_1 and B_2, respectively. Derivatives here and in the equations that follow are obtained by use of standard formulas for relating derivatives taken in two different reference frames; see Reference 9 or 10.

$$\underline{a}^{C_2} = \frac{d}{dt}\underline{V}^{C_2}$$

$$= \frac{-m_1}{m_1+m_2}\left[\dot{\underline{\omega}}^{B_1} \times \underline{p} - \dot{\underline{\omega}}^{B_2} \times \underline{q} + \underline{\omega}^{B_1} \times (\underline{\omega}^{B_1} \times \underline{p}) - \underline{\omega}^{B_2} \times (\underline{\omega}^{B_2} \times \underline{q})\right] \qquad (3.6)$$

in which $\dot{\underline{\omega}}^{B_1}$ and $\dot{\underline{\omega}}^{B_2}$ are the angular accelerations of B_1 and B_2.

Three variables u_1, u_2, and u_3 are introduced such that

$$\underline{\omega}^{B_2} = u_1\underline{i}_2 + u_2\underline{j}_2 + u_3\underline{k}_2 \qquad (3.7)$$

Then according to the definitions of γ_1 and γ_2

$$\underline{\omega}^{B_1} = \underline{\omega}^{B_2} - \dot{\gamma}_1\underline{i}_1 - \dot{\gamma}_2\underline{k}_2 \qquad (3.8)$$

It follows that

$$\dot{\underline{\omega}}^{B_2} = \frac{d}{dt}\underline{\omega}^{B_2} = \dot{u}_1\underline{i}_2 + \dot{u}_2\underline{j}_2 + \dot{u}_3\underline{k}_2 \qquad (3.9)$$

$$\dot{\underline{\omega}}^{B_1} = \frac{d}{dt}\underline{\omega}^{B_1}$$

$$= \dot{\underline{\omega}}^{B_2} - \ddot{\gamma}_1\underline{i}_1 - \ddot{\gamma}_2\underline{k}_2 + (\dot{\gamma}_1\underline{i}_1 + \dot{\gamma}_2\underline{k}_2) \times \underline{\omega}^{B_2} - \dot{\gamma}_1\dot{\gamma}_2\underline{i}_1 \times \underline{k}_2$$

$$(3.10)$$

3.2 Forces and Torques

The D'Alembert inertia forces for B_1 and B_2 are

$$\underline{F}_1^* = -m_1 \underline{a}^{C_1} \tag{3.11}$$

$$\underline{F}_2^* = -m_2 \underline{a}^{C_2} \tag{3.12}$$

The inertia torque for B_1 is obtained by differentiating the angular momentum of this body:

$$\underline{T}_1^* = -\underset{\sim}{I}^{B_1} \cdot \dot{\underline{\omega}}^{B_1} - \underline{\omega}^{B_1} \times \underset{\sim}{I}^{B_1} \cdot \underline{\omega}^{B_1} \tag{3.13}$$

where $\underset{\sim}{I}^{B_1}$ is the inertia dyadic of B_1 for C_1, i.e.,

$$\begin{aligned} \underset{\sim}{I}^{B_1} = {} & I_1 \underline{i}_1 \underline{i}_1 + L_1 \underline{i}_1 \underline{j}_1 + M_1 \underline{i}_1 \underline{k}_1 \\ & + L_1 \underline{j}_1 \underline{i}_1 + J_1 \underline{j}_1 \underline{j}_1 + N_1 \underline{j}_1 \underline{k}_1 \\ & + M_1 \underline{k}_1 \underline{i}_1 + N_1 \underline{k}_1 \underline{j}_1 + K_1 \underline{k}_1 \underline{k}_1 \end{aligned} \tag{3.14}$$

I_1, J_1, K_1 are the (centroidal) moments of inertia of B_1, and L_1, M_1, N_1 are products of inertia, defined generically as

$$P_{\xi\eta} = -\int \xi\eta \, dm \tag{3.15}$$

As the optical axis of the ATM experiment package, and indeed of most astronomical telescopes, is nearly parallel to one of the principal axes, it is assumed that the principal axes of B_2 are parallel to $\underline{i}_2$, $\underline{j}_2$, and $\underline{k}_2$. Hence, an expression for the inertia torque for B_2 is

$$\underline{T}_2^* = \left[u_2u_3(J_2-K_2) - \dot{u}_1I_2\right]\underline{i}_2 + \left[u_3u_1(K_2-I_2) - \dot{u}_2J_2\right]\underline{j}_2 + \left[u_1u_2(I_2-J_2) - \dot{u}_3K_2\right]\underline{k}_2 \qquad (3.16)$$

In addition to the inertia forces and torques certain active forces and torques act on B_1 and B_2. These are

$\underline{T}_c$: the torque exerted on the three CMGs by B_1. (So far as vehicle dynamics is concerned, the CMGs are treated simply as devices capable of exerting torques on their mountings.) The computation of $\underline{T}_c$ is discussed in Section 4.0.

$\underline{T}_g = \underline{T}_{g1}\underline{i}_1 + \underline{T}_{g2}\underline{k}_2$: the sum of those torques exerted on B_2 by B_1 about the vernier gimbal axes, including torques contributed by the torque motors, the flexure pivots, and a wire bundle crossing the interface. $\underline{T}_g$ is discussed further in Section 5.0.

$\underline{F}_1$ and $\underline{F}_2$: the resultant of all forces acting on B_1 and B_2, respectively, except for forces exerted between the bodies. These include gravitational, aerodynamic, and crew motion disturbance effects, as discussed in Section 6.0.

$\underline{T}_1$ and $\underline{T}_2$: similarly, the resultant of all torques exerted on B_1 and B_2, respectively, when $\underline{F}_1$ acts at C_1 and $\underline{F}_2$ acts at C_2, exclusive of torques exerted between the bodies and torques exerted by the CMGs; refer to Section 6.0.

3.3 Equations of Motion

Five dynamical equations may be written in terms of the variables $u_1,\cdots,u_5$, the first three of these being defined by (3.7) and the last two by

$$u_4 = \dot{\gamma}_1 \tag{3.17}$$

$$u_5 = \dot{\gamma}_2 \tag{3.18}$$

The equations themselves are obtained by use of a method due to Kane and Wang [11,12]. To this end, two types of quantities, $\underline{V}^{C_i},_{u_r}$ and $\underline{\omega}^{B_i},_{u_r}$, are introduced, their definitions being such that

$$\underline{V}^{C_i} = \sum_{r=1}^{5} \underline{V}^{C_i},_{u_r} u_r \quad , \quad i=1,2 \tag{3.19}$$

$$\underline{\omega}^{B_i} = \sum_{r=1}^{r} \underline{\omega}^{B_i},_{u_r} u_r \quad , \quad i=1,2 \tag{3.20}$$

This is to say that $\underline{V}^{C_i}$ and $\underline{\omega}^{B_i}$ may be expressed as linear combinations of the u_r's and that the coefficients $\underline{V}^{C_i},_{u_r}$ and $\underline{\omega}^{B_i},_{u_r}$ may be obtained by inspection of (3.3), (3.4), (3.7), and (3.8). For example, from (3.7), (3.8), and (3.18),

$$\underline{\omega}^{B_1},_{u_5} = -\underline{k}_2 \quad , \quad \underline{\omega}^{B_2},_{u_5} = 0$$

The dynamical equations may be written compactly as

$$(\underline{F}_1^* + \underline{F}_1) \cdot \underline{V}^{C_1},_{u_r} + (\underline{F}_2^* + \underline{F}_2) \cdot \underline{V}^{C_2},_{u_r} + (\underline{T}_1^* - \underline{T}_c - \underline{T}_g + \underline{T}_1) \cdot \underline{\omega}^{B_1},_{u_r}$$

$$+ (\underline{T}_2^* + \underline{T}_g + \underline{T}_2) \cdot \underline{\omega}^{B_2},_{u_r} = 0 \quad , \quad r=1,\cdots,5 \tag{3.21}$$

These five equations may be written as explicit first-order differential equations in $u_1,\cdots,u_5$; but such an expression of (3.21) is so lengthy as to be of little value here, so the explicit equations will not be presented.* One may determine by examination, however, that $\dot{u}_1,\cdots,\dot{u}_5$ appear linearly in (3.21), and these equations may thus be written

$$P\dot{U} = Q \tag{3.22}$$

where P is a 5 x 5 matrix of functions of U and t, U is a 5 x 1 matrix composed of $u_1,\cdots,u_5$, and Q is a 5 x 1 matrix also composed of functions of U and t. One may obtain P and Q by expressing the accelerations (3.5), (3.6), (3.9), and (3.10) as linear combinations of $\dot{u}_1,\cdots,\dot{u}_5$, together with a term independent of $\dot{u}_i$, $i=1,\cdots,5$; the inertia forces and torques (3.11), (3.12), (3.13), and (3.16) are expressed in the same form. By performing the operations indicated in (3.21), the elements of P and Q are thus generated, respectively, from the coefficients in the linear combinations and from the terms independent of $\dot{u}_i$ (the latter terms also include contributions from the active forces and torques).

Five first-order equations in addition to (3.22) are necessary to solve for the motion. Three of these are the kinematical equations consistent with the 3,2,1 Euler angle sequence described in Section 2.1:

$$\begin{bmatrix} \dot{u}_6 \\ \dot{u}_7 \\ \dot{u}_8 \end{bmatrix} = \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi/\cos\theta & \cos\phi/\cos\theta \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{3.23}$$

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*Considerable simplification in (3.21) may be achieved by making certain approximations, and it then becomes reasonable to express the equations explicitly. (The author has obtained such a set of equations by linearizing (3.21).)

The remaining two equations are obtained from (3.17) and (3.18):

$$\dot{u}_9 = \dot{\gamma}_1 = u_4 \qquad (3.24)$$

$$\dot{u}_{10} = \dot{\gamma}_2 = u_5 \qquad (2.25)$$

4.0 CONTROL MOMENT GYROSCOPE SYSTEM

4.1 Control Torque

In this section, an expression for the control torque, $\underline{T}_c$, exerted on the CMGs by the carrier, B_1, is developed. Note that the torque exerted by the CMGs on B_1 is merely $-\underline{T}_c$.

The SIXPAC [13] CMG system is the system which is simulated. The system contains 3 gyros; parameters defined below that are associated with a specific gyro bear the subscript 1, 2, or 3. Figure 4.1 shows how the CMGs are mounted with respect to the coordinates of B_1. The CMGs are shown with their inner and outer gimbals at the zero position. The gyros are numbered according to the body axis along which the spin axis lies in this zero position.

Define three unit vectors $\underline{h}_1$, $\underline{h}_2$, and $\underline{h}_3$ that are parallel to each of the spin axes and let $\underline{h}_T$ be their sum. Let h designate the magnitude of the spin angular momentum for each gyro. Then, the total spin angular momentum vector of the CMG system is

$$h\,\underline{h}_T = h(\underline{h}_1 + \underline{h}_2 + \underline{h}_3) \qquad (4.1)$$

The control torque is merely the time rate of change with respect to inertial coordinates of the total spin angular momentum vector.* This can be expressed as

$$\underline{T}_c = h(\dot{\underline{h}}_T + \underline{\omega}^{B_1} \times \underline{h}_T) \qquad (4.2)$$

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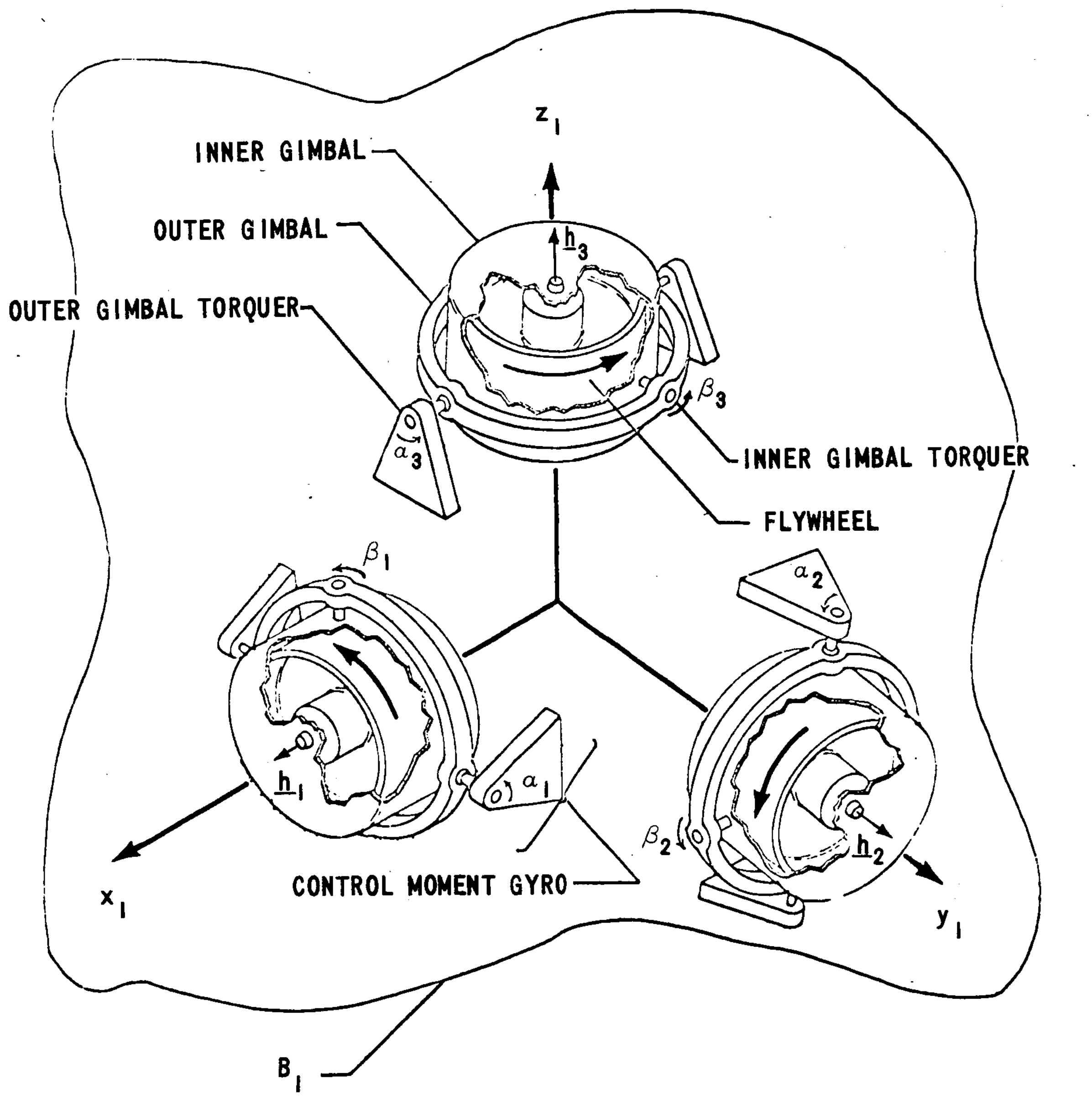
*The control torque $\underline{T}_c$ actually includes terms in addition to the time derivative of spin angular momentum; however, these other terms are small and may be neglected here.



z_1
INNER GIMBAL
OUTER GIMBAL
OUTER GIMBAL TORQUER
$\underline{h}_3$
β_3
α_3
INNER GIMBAL TORQUER
FLYWHEEL
β_1
α_2
$\underline{h}_1$
α_1
β_2
$\underline{h}_2$
x_1
CONTROL MOMENT GYRO
y_1
B_1

FIGURE 4.1

where $\dot{\underline{h}}_T$ is a time derivative with respect to the body coordinates of B_1. It is most convenient to find $\dot{\underline{h}}_T$ from the relation

$$\dot{\underline{h}}_T = (\underline{\omega}_1 x \underline{h}_1) + (\underline{\omega}_2 x \underline{h}_2) + (\underline{\omega}_3 x \underline{h}_3) \qquad (4.3)$$

where $\underline{\omega}_1$, $\underline{\omega}_2$, and $\underline{\omega}_3$ are the relative angular velocities of the gyro inner gimbals with respect to the body coordinates of B_1.

At this point we define the column matrices

$$\dot{\underline{\alpha}} = \begin{bmatrix} \dot{\alpha}_1 \\ \dot{\alpha}_2 \\ \dot{\alpha}_3 \end{bmatrix} , \quad \dot{\underline{\beta}} = \begin{bmatrix} \dot{\beta}_1 \\ \dot{\beta}_2 \\ \dot{\beta}_3 \end{bmatrix} \qquad (4.4)$$

which give the time derivatives of the outer and inner gimbal angles, respectively.

Equation (4.3) can be organized in the form

$$\dot{\underline{h}}_T = G \, \dot{\underline{\alpha}} + F \, \dot{\underline{\beta}} \qquad (4.5)$$

where G and F are each 3 x 3 matrices whose elements are trigonometric functions of the outer and inner gimbal angles. Substituting (4.5) into (4.2) results in the equation which is programmed for the control torque:

$$\underline{T}_c = h(G \, \dot{\underline{\alpha}} + F \, \dot{\underline{\beta}} + \underline{\omega}^{B_1} \text{ x } \underline{h}_T) \qquad (4.6)$$

4.2 CMG Control Laws

A control law for the system is just an expression for $\dot{\underline{\alpha}}$ and $\dot{\underline{\beta}}$ in terms of the system state variables, namely, the attitude and the angular velocity of B_1 with respect to inertial coordinates and also the gimbal angles of each CMG.

To obtain a control law, a 3 x 1 column matrix $\underline{v}$ is first introduced in order to establish a constraint between $\dot{\underline{\alpha}}$ and $\dot{\underline{\beta}}$. That is, $\dot{\underline{\alpha}}$ and $\dot{\underline{\beta}}$ are expressed as

$$\dot{\underline{\alpha}} = A\,\underline{v} \quad , \quad \dot{\underline{\beta}} = B\,\underline{v} \tag{4.7}$$

where A and B are 3 x 3 matrices whose elements depend on the method chosen for introducing the constraints.

Matrices A and B may be determined in the following way. Imagine $\underline{v}$ to be a desired control torque per unit of spin angular momentum. Now express $\underline{v}$ in the inner gimbal coordinates of each gyro. Observe that the vector $\underline{\omega}_j \times \underline{h}_j$, j=1,2,3, is normal to $\underline{h}_j$, i.e., its component along $\underline{h}_j$ is zero. By equating the elements in $\underline{v}$ (in inner gimbal coordinates) to corresponding non-zero elements of $\underline{\omega}_j \times \underline{h}_j$, relations of the following form are obtained:

$$\begin{aligned} \dot{\alpha}_j &= (1 \times 3 \text{ row matrix})\underline{v} \\ \dot{\beta}_j &= (1 \times 3 \text{ row matrix})\underline{v} \end{aligned} \tag{4.8}$$

These relations when organized in matrix form yield the constraint equations (4.7).

If $\underline{v}$ is now specified as a function of the system state variables and substituted into (4.7) a control law for the CMGs is obtained. It is implied in such a control law that gimbal angle rates are commanded. Indeed this is the case for the CMG system on the ATM. The implementation of gimbal angle rate commands requires the use of a speed control servomechanism for each gimbal axis.

Four different control laws have been studied for possible use with the SIXPAC CMG system. These are generally known by the following names: Langley Control Law, Cross-Product Law, H-Vector Control, and Closed-Loop Torque Control.

Both the Langley Control Law and the Cross-Product Law use the same definition for $\underline{v}$. They differ in that certain terms and factors are deleted from the A and B matrices for the Langley Control Law. Consequently, there are only 3 different definitions for $\underline{v}$ that have been studied, and these will now be defined.

It is necessary to know the attitude of the carrier, B_1, with respect to inertial coordinates. Let ψ', θ', and ϕ' denote a 3,2,1 Euler angle sequence which defines the attitude of B_1 with respect to the axes $x_d y_d z_d$ (defined in Section 2.1). Since the attitude excursions of the experiment package, B_2, and B_1 are small we may write

$$\begin{aligned} \phi' &= \phi - \gamma_1 = u_6 - u_9 \\ \theta' &= \theta = u_7 \\ \psi' &= \psi - \gamma_2 = u_8 - u_{10} \end{aligned} \tag{4.9}$$

Define the attitude error column matrix

$$\underline{\varepsilon} = \begin{bmatrix} \phi' \\ \theta' \\ \psi' \end{bmatrix} \tag{4.10}$$

Basic to all the control laws is a linear combination of $\underline{\varepsilon}$ and $\underline{\omega}^{B_1}$:* That is, we define

$$\underline{e} = K_0 \underline{\varepsilon} + K_1 \underline{\omega}^{B_1} \tag{4.11}$$

---

*Instead of $\underline{\omega}^{B_1}$ it would be acceptable to use the vector $\begin{bmatrix} \dot{\phi}' \\ \dot{\theta}' \\ \dot{\psi}' \end{bmatrix}$.

where K_0 and K_1 are 3 x 3 diagonal matrices of attitude error gains and rate feedback gains, respectively.

The column matrix $\underline{v}$ for each of the control laws is defined as follows:

Langley and Cross-Product Laws $\underline{v} = \underline{e}$

H-Vector Control $\underline{v} = \int_0^t (\underline{e} - \dot{\underline{h}}_T)d\tau$ (4.12)

Closed Loop Torque Control $\underline{v} = \underline{e} - \dot{\underline{h}}_T$

By using (4.5) and (4.7) it is easy to show that with Closed Loop Control

$$\underline{v} = (I + G\ A + F\ B)^{-1}\underline{e} \qquad (4.13)$$

where I is the 3 x 3 identity matrix.

The CMG system for the ATM is currently being designed with H-Vector Control. The reason given is that this control law, while requiring a little more electronics, has minimal torque cross coupling and has a frequency response that is relatively insensitive to the orientation of gyro gimbals.

5.0 EXPERIMENT POINTING CONTROL SYSTEM

5.1 Description of the EPC

As described in Sections 1.2 and 2.2, the experiment package is attached to the carrier by means of a two degree of freedom system of gimbals. Certain torques act about each of the two associated gimbal axes, and it is the purpose of this section to provide an analytical description of these torques.

Attitude control of the experiment package about the two axes* normal to the optical axis is provided by a feedback control system that obtains error signals from fine sun sensors (or, in the case of stellar astronomy, star sensors in the telescope optics) and rate gyros aboard the experiment package and that provides commands to torque motors located on the gimbal axes. The present mathematical model provides for linear feedback of attitude error and attitude rate, except that the torque output is limited so that it never exceeds the capability of the torque motor.

5.2 Extraneous Torques

Other torques are exerted across the gimbals by the gimbal support mechanism itself and by cables crossing the interface for the transmission of electrical power and data. Flexure pivots are used to support the ATM experiment package within its gimbals; these devices may be treated analytically as linear springs, there being essentially no friction or deadband associated with them. Torques associated with the cable(s) present more of a problem: approximately 1000 wires are currently anticipated in the ATM cable, and measurements of the bending properties of such cables [14] indicate a substantial hysteresis effect.

The mechanical behavior of the wire harness is influenced highly by the construction of the harness and by its route as it passes from the carrier to the experiment package. Figures 5.1 a-c, taken from Reference 14, illustrate the force-deflection** properties of three different harnesses (1206, 1078, and 384 conductors, respectively) which are routed across the gimbals in three different ways. One may see that these curves differ in slope (stiffness), enclosed area (hysteresis effect), and symmetry about the origin. Rather than try to portray the harness behavior analytically in terms of a hysteresis loop, it is more convenient to represent it

---

*Attitude control about the optical axis is not as critical as that about the transverse axes, at least so far as the ATM experiments are concerned, and it is expected that the CMG system aboard the carrier can provide satisfactory attitude control about this axis. However, large aperture telescopes, such as those contemplated for future missions, will have more stringent attitude error requirements about this axis, and it may then be necessary to add a third axis of control to the EPC.

**In Reference 14 transverse forces at the ends of the harness are actually measured. However, it is the moments that these forces produce about the gimbal axes that are of importance here.

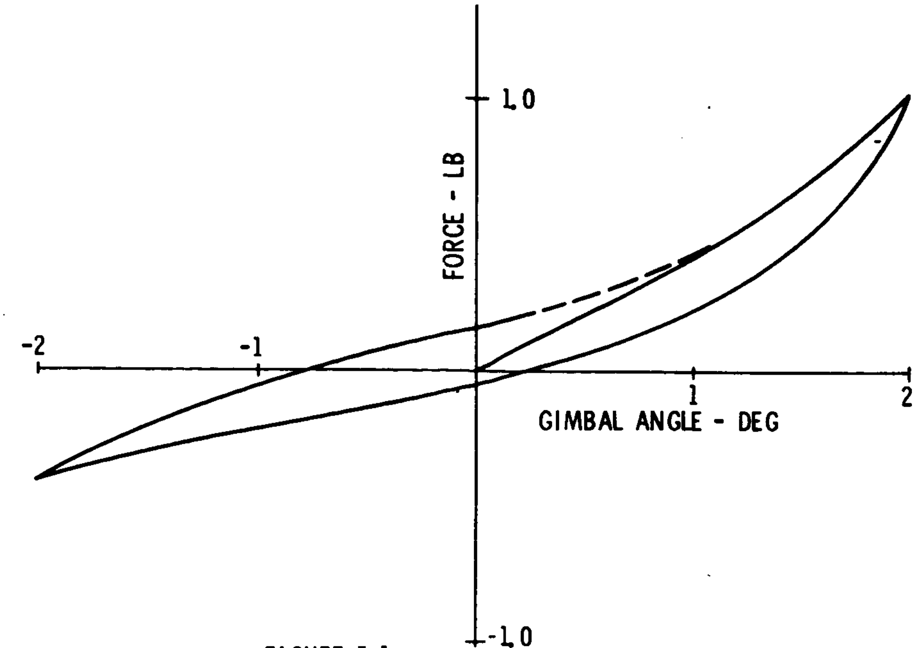


FIGURE 5.1a

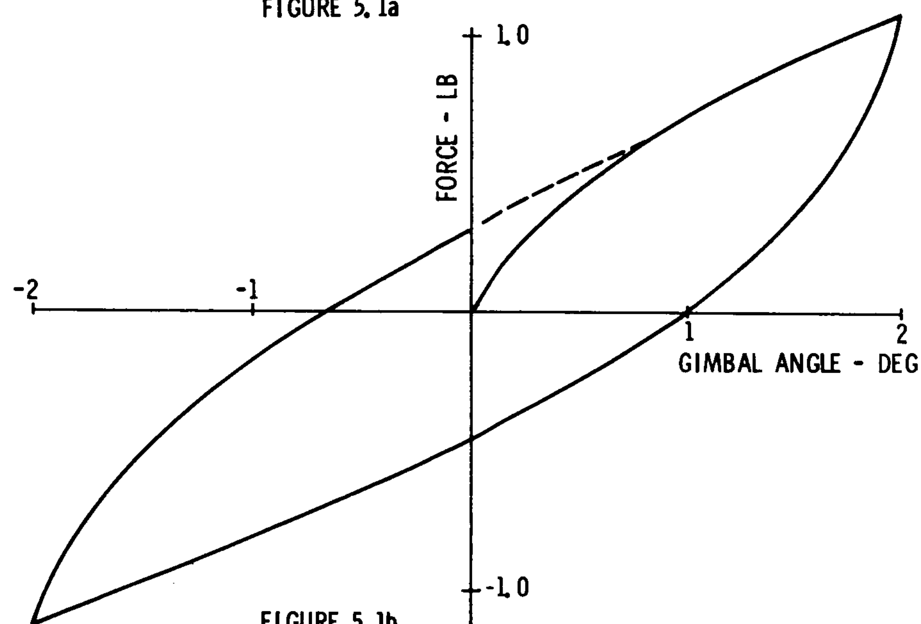


FIGURE 5.1b

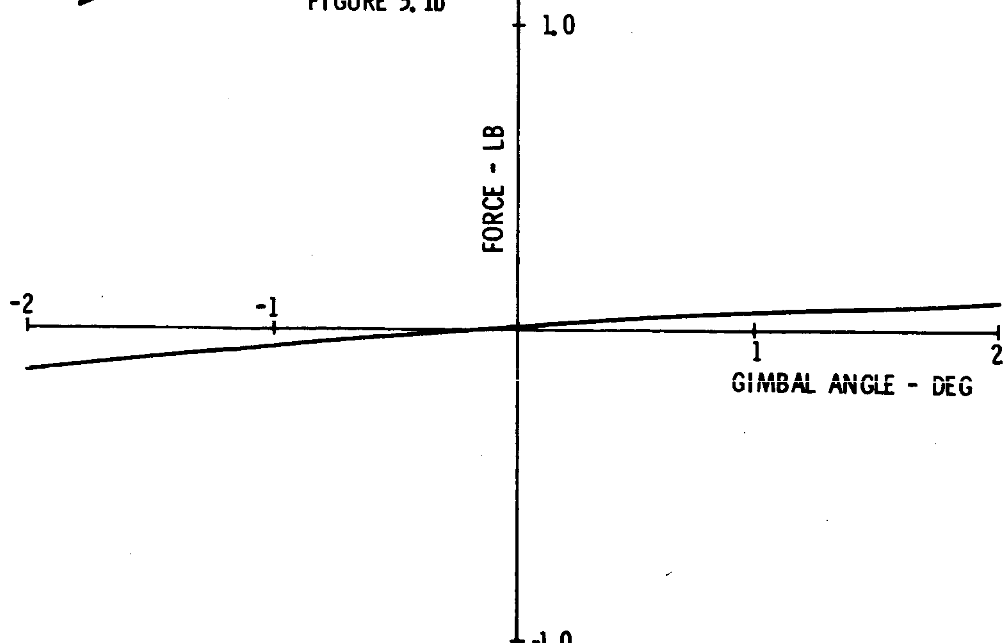


FIGURE 5.1c

as a sum of a constant torque (e.g., the force-intercept of the hysteresis loop) and a torque which is proportional to gimbal angle (i.e., a linear spring).

It is possible with this representation to determine the maximum force-intercept and spring rate consistent with a given level of performance, the maximum value of these two parameters thus being harness design constraints.

5.3 Control Torques

The torques T_{g1} and T_{g2} about the gimbal axes (see Sections 3.2 and 3.3) are computed as described in the two previous sections. They are

$$T_{g1} = -K_{ga}\phi - K_{gb}u_1 - K_{g1}\gamma_1 + T_{h1} \qquad (5.1)$$

$$T_{g2} = -K_{gc}\psi - K_{gd}u_3 - K_{g2}\gamma_2 + T_{h2} \qquad (5.2)$$

where $K_{ga},\cdots,K_{gd}$ are feedback gain constants associated with the EPC, u_1 and u_3 are defined by (3.7), K_{g1} and K_{g2} are spring constants for the flexure pivots and the wire harness, and T_{h1} and T_{h2} are the constant torques associated with the harness.

6.0 EXTERNAL TORQUES AND FORCES

6.1 Introduction

In addition to those discussed in the previous two sections, certain forces and torques act to influence the attitude of the carrier and the experiment package. These include gravitational, magnetic, and aerodynamic effects and also solar pressure. The latter may be shown to be negligible in comparison with the others for near-earth orbits,* and magnetic effects are also relatively small for the AAP Cluster Configuration [16].

*For the AAP Cluster, a solar radiation pressure of 9×10^{-5} dyne/cm^2 [15] may be shown to give rise to a moment whose magnitude is on the order of 0.01 ft lb. This is two orders of magnitude less than the peak aerodynamic torque at 230 NM.

On the other hand, in addition to gravitational and aerodynamic effects it is convenient to treat motions of the crew within the carrier as forces that act on the carrier. Strictly speaking, the crew acts as distinct masses moving about within the carrier. However, the results of experimental work done on astronaut motion is reported in terms of forces, which makes it a more straightforward matter to consider the crew as massless force generators.

6.2 Gravitational Effects

Gravitational forces acting on the carrier and the experiment package, respectively, are given by*

$$\underline{F}_{Gi} = \frac{-GMm_i}{R_i^3}\underline{R}_i \quad , \quad i=1,2 \tag{6.1}$$

where G is the gravitational constant, M is the mass of the earth, and $\underline{R}_i$ is the position vector of C_i relative to the center of the earth, R_i being the length of $\underline{R}_i$. Equation (6.1) is unsatisfactory for numerical computation, however, because $\underline{F}_{G1}$ and $\underline{F}_{G2}$ are comprised almost entirely of components which have no effect on the attitude dynamics of the vehicle. Thus, a formula for the gravitational force is sought which avoids the loss of significance associated with (6.1). To this end, let $\underline{R}$ be the position vector of C relative to the center of the earth so that

$$\underline{R}_i = \underline{R} + \underline{r}_i \tag{6.2}$$

The total gravitational force acting on the spacecraft is

$$\underline{F}_G = \underline{F}_{G1} + \underline{F}_{G2} = \frac{-GM(m_1+m_2)}{R^3}\underline{R} \tag{6.3}$$

---

*Since B_i is not a particle this formula is only approximate; the approximation is maintained to the same order as the most significant term of the gravitational torque expression. Similar approximations are used throughout Section 6.2.

Forces $\underline{F}'_{G1}$ and $\underline{F}'_{G2}$ are introduced as

$$\underline{F}'_{Gi} = \frac{m_i}{m_1+m_2} \underline{F}_G \qquad (6.4)$$

and these in turn are used to define $\underline{f}_1$ and $\underline{f}_2$:

$$\underline{F}_{Gi} = \underline{F}'_{Gi} + \underline{f}_i \qquad (6.5)$$

Now, $\underline{F}'_{G1}$ and $\underline{F}'_{G2}$ satisfy the hypotheses of the theorem in Appendix I, so they can be neglected without affecting the vehicle attitude dynamics. The remaining gravitational forces, $\underline{f}_1$ and $\underline{f}_2$, are evaluated by using (6.1), (6.3)-(6.5):

$$\underline{f}_i = -GMm_i\left(\frac{\underline{R}_i}{R_i^3} - \frac{\underline{R}}{R^3}\right) \qquad (6.6)$$

Equation (6.2) is used to eliminate $\underline{R}_i$ and R_i from (6.6).

$$R_i^2 = (\underline{R}+\underline{r}_i)^2 = R^2 + r_i^2 + 2\underline{R} \cdot \underline{r}_i \qquad (6.7)$$

$$R_i^{-3} = \frac{1}{R^3}\left[1 + 2\left(\frac{\underline{R}\cdot\underline{r}_i}{R\ r_i}\right)\frac{r_i}{R} + \left(\frac{r_i}{R}\right)^2\right]^{-3/2}$$

$$= \frac{1}{R^3}\left[1 - 3\left(\frac{\underline{R}\cdot\underline{r}_i}{R\ r_i}\right)\frac{r_i}{R} + \cdots\right] \qquad (6.8)$$

Within the order of approximation noted earlier, $\underline{f}_i$, obtained from (6.2), (6.6), and (6.8), is given as

$$\underline{f}_i = \frac{-GMm_i}{R^3}\left\{(\underline{R}+\underline{r}_i)\left[1 - 3\,\frac{\underline{R}\cdot\underline{r}_i}{R\,r_i}\,\frac{r_i}{R}\right] - \underline{R}\right\} \qquad (6.9)$$

Simplification and further approximation yields the expression

$$\underline{f}_i = \frac{-GMm_i}{R^3}\left[\underline{r}_i - 3(\underline{r}_i\cdot\underline{n})\underline{n}\right] \qquad (6.10)$$

where

$$\underline{n} = \underline{R}/R \qquad (6.11)$$

The gravitational torques acting on B_1 and B_2 are computed from the formula

$$\underline{T}_{Gi} = \frac{3GM}{R^3}\,\underline{n} \times \underset{\sim}{I}^{B_i} \cdot \underline{n} \qquad (6.12)$$

where $\underset{\sim}{I}^{B_i}$ is the inertia dyadic of B_i, as defined by (3.14). Hence, formulas (6.10) and (6.12) may be used to compute those portions of the gravitational attraction that influence the vehicle attitude dynamics.

6.3 Aerodynamic Effects

As in the case of gravitational forces, there will in general be an aerodynamic force and an aerodynamic torque associated with each body. However, in view of the difficulties involved in assessing the magnitudes of these four quantities, some assumptions are made to facilitate the calculations. This approach has the advantage that the magnitude of the aerodynamic effect may be determined in terms of just one parameter, rather than the four parameters that would otherwise be required.

We first assume that the moment of the aerodynamic forces acting on the experiment package about its mass center is zero. This is a reasonable assumption for the ATM experiment package since it is relatively small and is symmetric and reasonably homogeneous. Next assume that the resultant aerodynamic forces on B_1 and B_2 are proportional to their respective masses and directed parallel to the vehicle orbital velocity vector. According to the theorem in Appendix I these forces do not influence the attitude motions of the vehicle. It is therefore necessary to consider only the aerodynamic torques exerted on the carrier, and the problem of computing aerodynamic effects on this two-body vehicle reduces to the simpler problem of computing the aerodynamic torque on a single body. Another set of assumptions can be used to bring about essentially the same simplification: the ATM experiment package will quite likely be cooled by a cold sleeve attached to the carrier which covers all but the ends of the experiment package; under these conditions the experiment package is shielded from the airstream, and only the aerodynamic effect on the carrier need be considered.*

For a 230 nautical mile circular orbit the aerodynamic torque on the AAP Cluster is small in comparison with the gravitational torque [17], and it therefore seems that a simple model of the aerodynamic loads is quite sufficient. More elaborate models of the aerodynamic loads [18] on the AAP Cluster are available if more precision is needed or if the orbital height is lowered to the point that aerodynamic loads become appreciable.

The aerodynamic drag force $\underline{F}_A$ acting on a body of projected area A_p traveling at speed V in a direction parallel to a unit vector $\underline{m}$ is given by

$$\underline{F} = -\frac{1}{2} \rho V^2 A_p C_D \underline{m} \tag{6.13}$$

where ρ is the atmospheric density and C_D is the drag coefficient, which is determined experimentally for a particular shape. For circular cylindrical bodies of large length-to-diameter ratio A_p may be approximated in terms of the broadside area A (length times diameter) and a unit vector $\underline{i}$ which is parallel to the axis of the cylinder

*The aerodynamic force on the carrier does affect the attitude dynamics in this case, since the result of Appendix I cannot be invoked.

$$A_p = A|\underline{m} \times \underline{i}| \tag{6.14}$$

A large portion of the broadside area of the AAP Cluster is due to the solar arrays (see Figure 2.2), and although such a configuration is not axisymmetric as is a cylinder, one may develop a formula for the worst-case aerodynamic loads by using (6.14) if A is taken to include the solar array area. Let r_{cp} be the distance from C_1 to the center of pressure of B_1, and assume that the center of pressure is located such that its position vector relative to C_1 is $r_{cp}\underline{i}$. The aerodynamic torque on B_1 is then simply

$$\underline{T}_{A_1} = r_{cp}\underline{i} \times \underline{F} = \frac{1}{2} \rho V^2 A\ C_D\ r_{cp}|\underline{m} \times \underline{i}|(\underline{m} \times \underline{i}) \tag{6.15}$$

Rather than use this formula, it is more convenient to express the aerodynamic torque in terms of a parameter α such that

$$\underline{T}_{A_1} = \alpha \frac{3}{2} \frac{GM}{R^3} (J-I)|\underline{m} \times \underline{i}|(\underline{m} \times \underline{i}) \tag{6.16}$$

where I and J are, respectively, the minimum and maximum moments of inertia of B_1. Then α is the ratio of the maximum aerodynamic torque acting on B_1 to the maximum gravity-gradient torque acting on this body; typical values of α run from 0.05 to 0.20 for the AAP Cluster in a 230 nautical mile circular orbit [17].

Large variations in aerodynamic torque are also caused by changes in atmospheric density over a circular orbit. This so-called diurnal bulge effect may be approximated by introducing α', a modified value of α.

$$\alpha' = \frac{\alpha}{1+\beta} [1 - \beta \cos(\Omega t + \gamma)] \tag{6.17}$$

In this formula Ω is the orbital rate, and β and γ are parameters related to the diurnal bulge; β typically has a value of 0.6 for a 230 nautical mile orbit, and γ is used to locate the bulge on the orbit (normally $\gamma = 60°$). Replace α in (6.16) by α' to account for variations in density.

6.4 Crew Motion

Crew motion generally produces much larger vehicle attitude errors than either gravitational or aerodynamic effects. Not only are the forces and torques produced by the crew larger than those due to gravity and drag, but they also occur faster and may thereby tax the PCS's ability to respond fast enough.

Studies have been performed at the Martin Marietta Corporation concerning the motions of and forces excerted by an astronaut while in a state of free fall (so-called zero g) as he performs tasks typical of AAP missions [19]. For purposes of ATM PCS design, four crew motions have been selected by MSFC as standard crew disturbances [20]. Descriptively, the four motions are called bounce walk, wall push off, arm motion-CSM, and arm motion-LM; the details of these motions are given in Appendix II.

7.0 DIGITAL COMPUTER PROGRAM

7.1 Description of the Program

A digital computer program has been written to perform operations indicated in the foregoing. Basically, the program reads input parameters relating to vehicle dynamics and the control system; it solves equations (3.21), (3.23)-(3.25), and (4.7) by means of step-by-step numerical integration; and lastly it prints the independent variable (time), specified state variables, and certain indicators at specified time intervals. In the process of solving the differential equations, forces and torques due to gravitational effects, aerodynamic drag (subject to the parameter α), and crew motion (one of the eight disturbances described in Appendix II) are computed.

The program, written in Fortran V, is comprised of a main program, which handles input/output and provides control over the integration process, and several subroutines which perform specific functions. The principal parts of the program are included herewith as indicated below, the remainder being available from the author on request.

Program Name	Appendix	Function
Main	III	input/output, control over integration
Subroutine F	IV	define the differential equations, perform all control system computations
Subroutine TF	V	compute grav. and aero. effects, combine these with crew disturbance

7.2 Symbol Correspondence List

The program listings themselves are well documented to provide specific information about the program. However, since the symbols used in the foregoing sometimes differ from the corresponding symbols used in the program, the following list should be useful for correlation purposes.

Symbol in the Text	First Occurrence in the Text (Section No.)	Symbol in the Program
ϕ_d, θ_d, ψ_d	2.1	AD
ϕ, θ, ψ	2.1	A(=U(6),···,U(8))
γ_1, γ_2	2.2	GAMMA 1(=U(9)),GAMMA 2(=U(10))
$\underline{p}, \underline{q}$	2.2	P,Q
m_1, m_2	3.1	M1,M2
$\underline{V}^{C_1}, \underline{V}^{C_2}$	3.1	——
$\underline{a}^{C_1}, \underline{a}^{C_2}$	3.1	AC1,AC2
$\underline{\omega}^{B_1}, \underline{\omega}^{B_2}$	3.1	OMB1,OMB2

Symbol in the Text	First Occurrence in the Text (Section No.)	Symbol in the Program
$\underline{\dot{\omega}}^{B_1}, \underline{\dot{\omega}}^{B_2}$	3.1	DOMB1,——
$u_1, u_2, \cdots$	3.1	U(1),U(2),···
$\underline{F}_1^*, \underline{F}_2^*$	3.2	F1STAR,F2STAR
$\underline{T}_1^*, \underline{T}_2^*$	3.2	T1STAR,T2STAR
$\underset{\sim}{I}^{B_1}$	3.2	EYE1
I_1, J_1, K_1	3.2	I1,J1,K1
L_1, M_1, N_1	3.2	IJ1,IK1,JK1
I_2, J_2, K_2	3.2	I2,J2,K2
$\underline{T}_c, \underline{T}_g$	3.2	TC,TG
$\underline{F}_1, \underline{F}_2$	3.2	F1,F2
$\underline{T}_1, \underline{T}_2$	3.2	T1,T2
$\underline{V}^{C_1}{}_{,u_r}, \underline{V}^{C_2}{}_{,u_r}$	3.3	VC1,VC2
$\underline{\omega}^{B_1}{}_{,u_r}, \underline{\omega}^{B_2}{}_{,u_r}$	3.3	OMBIM,____
h	4.1	H
$\underline{\dot{\alpha}}, \underline{\dot{\beta}}$	4.1	DALPHA,DBETA
α_1, β_1	4.1	U(11),U(14)
G, F	4.1	GG,FF
$\underline{v}$	4.2	V
A, B	4.2	AM,BM
$\underline{\varepsilon}, \underline{e}$	4.2	AMAD,E

Symbol in the text	First Occurrence in the Text (Section No.)	Symbol in the Program
T_{g_1}, T_{g_2}	5.3	TG1,TG2
T_{h_1}, T_{h_2}	5.3	TWIRE1,TWIRE2
$\underline{n}, \underline{n}$	6.2	N1,N2
$\underline{f}_1, \underline{f}_2$	6.2	SF1,SF2
$\underline{T}_{G_1}, \underline{T}_{G_2}$	6.2	TG1,TG2*
α, β, γ	6.3	ALP,BET,GAMMA
α'	6.3	ALPP
$\underline{m}$	6.3	M
$\underline{T}_{A_1}$	6.3	TA1

1022-PGS-mef

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Attachments
Appendices I through V
References

*Not the same as Tg1 and TG2 associated with Section 5.3.

APPENDIX I

Theorem

Consider a dynamical system comprised of two connected bodies, each acted upon by a given set of forces. The attitude motions of such a system are unaffected by the addition of two more forces, provided these forces have the following properties:

1. One force, $\underline{F}_1$, acts at the mass center of body 1.
2. The other, $\underline{F}_2$, acts at the mass center of body 2.
3. $\underline{F}_1$ and $\underline{F}_2$ are parallel and have the same sense.
4. The magnitudes of $\underline{F}_1$ and $\underline{F}_2$ are proportional to the respective masses m_1 and m_2, i.e.,

$$m_2^2\underline{F}_1^2 = m_1^2\underline{F}_2^2$$

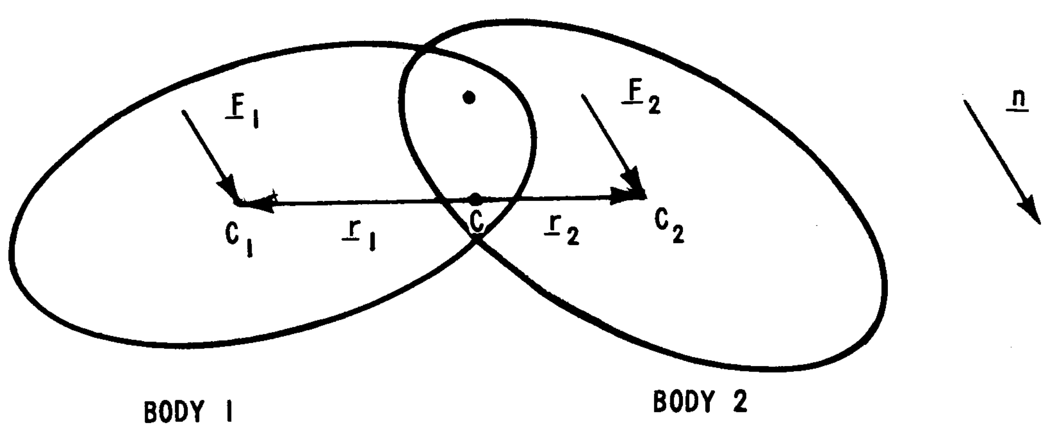


FIGURE I-1

Appendix I (contd.)

Proof:* $\underline{F}_1$ and $\underline{F}_2$ affect the motion of the system only insofar as they contribute to the generalized active force F_r, defined in this case as

$$F_r = \underline{F}_1 \cdot \underline{V}^{C_1},_{u_r} + \underline{F}_2 \cdot \underline{V}^{C_2},_{u_r} \quad , \quad r=1,\cdots,n$$

According to hypotheses 3 and 4, $\underline{F}_1$ and $\underline{F}_2$ may be written in terms of the unit vector $\underline{n}$ and the constant λ as

$$\underline{F}_1 = \lambda m_1 \underline{n} \quad , \quad \underline{F}_2 = \lambda m_2 \underline{n}$$

and so

$$F_r = \lambda \underline{n} \cdot \left(m_1 \underline{V}^{C_1},_{u_r} + m_2 \underline{V}^{C_2},_{u_r} \right)$$

Since C is the mass center of the system,

$$m_1 \underline{r}_1 + m_2 \underline{r}_2 = 0$$

which may be differentiated, yielding

$$m_1 \frac{d\underline{r}_1}{dt} + m_2 \frac{d\underline{r}_2}{dt} = m_1 \underline{V}^{C_1/C} + m_2 \underline{V}^{C_2/C} = 0$$

---

*The proof employs concepts used in Section 3.3 and in particular is based on the same approach as that outlined in Reference 11 (p. 575).

Appendix I (contd.)

The last equation must hold regardless of the values of $u_1, \cdots, u_n$, and this leads, by means of equation (3.19), to

$$m_1 \underline{V}^{C_1/C},_{u_r} + m_2 \underline{V}^{C_2/C},_{u_r} = 0$$

The velocities are related by

$$\underline{V}^{C_1} = \underline{V}^{C_1/C} + \underline{V}^{C} \quad , \quad \underline{V}^{C_2} = \underline{V}^{C_2/C} + \underline{V}^{C}$$

and it follows that

$$\underline{V}^{C_1},_{u_r} = \underline{V}^{C_1/C},_{u_r} + V^{C},_{u_r} \quad , \quad \underline{V}^{C_2},_{u_r} = \underline{V}^{C_2/C},_{u_r} + \underline{V}^{C},_{u_r}$$

The generalized inertia force is thus

$$F_r = \lambda \underline{n} \cdot \left[m_1 \underline{V}^{C_1/C},_{u_r} + m_2 \underline{V}^{C_2/C},_{u_r} + (m_1+m_2)\underline{V}^{C},_{u_r} \right]$$

The sum of the first two terms in brackets is zero, as established above, F_r being reduced to

$$F_r = \lambda(m_1 + m_2)\underline{n} \cdot \underline{V}^{C},_{u_r}$$

$\underline{V}^{C},_{u_r}$ is clearly connected with the motion of the mass center of the system and not with attitude motions about the mass center. Because of the independence of the motion of the mass center and motions about the mass center, the last equation shows that F_r has no influence on the attitude motions of the system.

APPENDIX II

STANDARD CREW DISTURBANCES USED FOR ATM PCS DESIGN

The motion of an astronaut within the carrier may be thought of as producing a force (on the carrier) whose measure numbers are F_{Dx}, F_{Dy}, F_{Dz} and a moment having measure numbers T_{Dx}, T_{Dy}, T_{Dz} when the force acts at the mass center of the carrier. For the standard crew disturbances [20] the measure numbers are defined in terms of three time dependent functions D_1, D_2, D_3 as follows:

$$T_{Dx} = T_{Dy} = F_{Dz} = 0$$

$$T_{Dz} = x_D D_2 - y_D D_1 + D_3$$

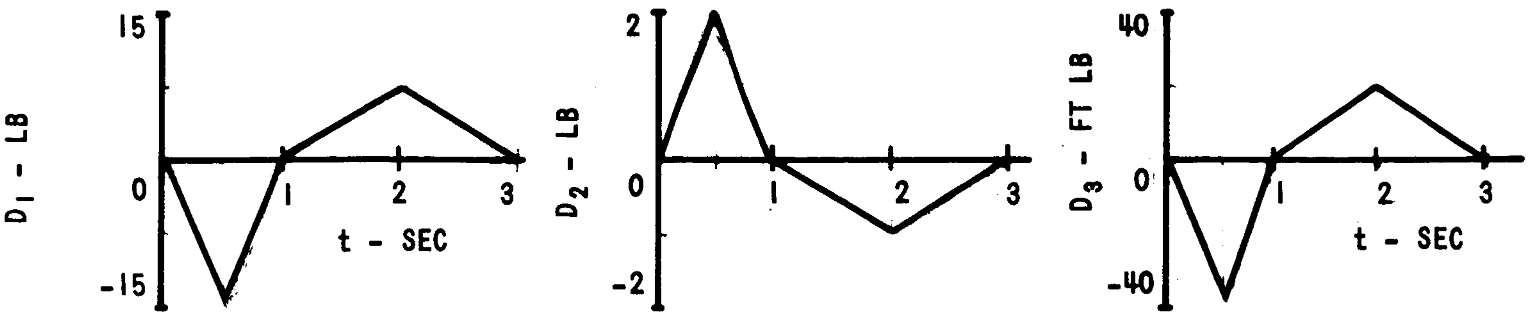
$$F_{Dx} = D_1 \quad , \quad F_{Dy} = D_2$$

where x_D, y_D, z_D determine the position of the astronaut relative to the carrier mass center. Definitions of the functions D_1, D_2, D_3 for the standard crew disturbances are given in Figure II-1,* and the corresponding astronaut locations are given in the following table.

Name	x_D(ft)	y_D(ft)	z_D(ft)
bounce walk**	-38.1	-9.8	-9.8
walk push off**	-35.1	0	0
arm motion-CSM	23.0	0	0
arm motion-LM	14.1	9.8	0

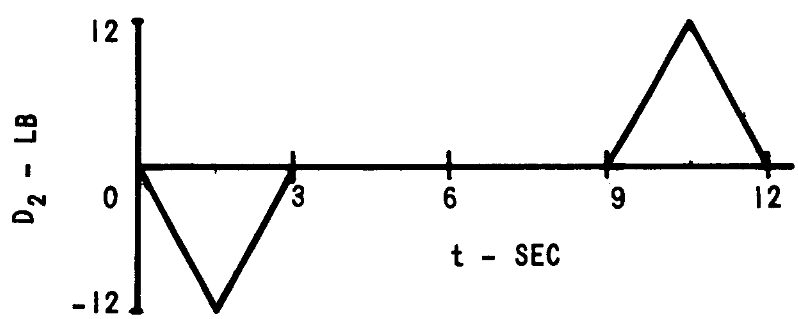
*D_1 and D_2 actually correspond to forces and D_3 corresponds to a torque in an astronaut-fixed reference frame. The expressions above may be obtained by properly rotating and translating the astronaut relative to the vehicle and by letting z_D=0 so as to avoid cross coupling.

**At the base of the Orbital Workshop.



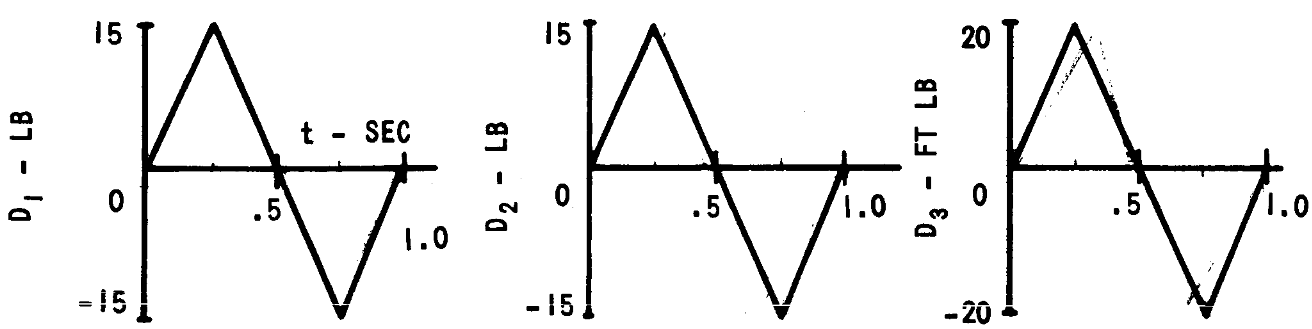
BOUNCE WALK

$D_1 \equiv 0$



$D_3 \equiv 0$

WALL PUSH OFF



ARM MOTION - CSM OR LM

FIGURE II-1

Because of spacecraft symmetry relative to the xy plane, the four standard crew disturbances defined above only influence attitude motions about the z axis. The other axis perpendicular to the experiment package optical axis, namely the x axis, is also of interest so far as crew motion is concerned since attitude error about telescope transverse axes is more critical than error about the optical axis itself. For this reason we introduce four additional crew disturbances, corresponding to the four above, which affect attitude motions about the x and y axes (attitude motions about these two axes are coupled due to lack of symmetry). The table of astronaut locations and Figure II-1 hold for both the standard and the additional crew disturbances, but the torques and forces for the additional disturbances are computed as follows:

$$T_{Dz} = F_{Dy} = 0$$

$$T_{Dx} = y_D D_2$$

$$T_{Dy} = z_D D_1 - x_D D_2 + D_3$$

$$F_{Dx} = D_1 \quad , \quad F_{Dz} = D_2$$

APPENDIX III

MAIN PROGRAM

```
C TITLE          DYNAMICS OF A SPACECRAFT WITH GIMBAL MOUNTED TELESCOPE
C
C AUTHOR         C. F. BANICK
C
C DATE           7-26-67
C
C PURPOSE        TO STUDY THE DYNAMIC BEHAVIOR OF A TELESCOPE ATTACHED
C                TO A CMG-CONTROLLED SPACECRAFT BY MEANS OF A VERNIER
C                GIMBAL CONTROL SYSTEM
C
C METHOD         DIFFERENTIAL EQUATIONS REPRESENTING THE VEHICLE DYNAMICS
C                AND THE TWO CONTROL SYSTEMS ARE SOLVED NUMERICALLY AS
C                AN INITIAL VALUE PROBLEM
C
C INPUT          THROUGH A NAMELIST CALLED -INPUT-
C                GAMMA1    X-AXIS VERNIER GIMBAL ROTATION (DEG)
C                GAMMA2    Z-AXIS VERNIER GIMBAL ROTATION (DEG)
C                U         THE VECTOR (U(1),U(2),U(3)) IS THE INITIAL
C                          ANGULAR VELOCITY OF THE TELESCOPE (DEG/SEC)
C                U(4)      INITIAL RATE OF GAMMA1 (DEG/SEC)
C                U(5)      INITIAL RATE OF GAMMA2 (DEG/SEC)
C                A         A THREE-DIMENSIONAL ARRAY WHICH DESCRIBES
C                          THE INITIAL ATTITUDE OF THE TELESCOPE RELATIVE
C                          TO THE DESIRED ATTITUDE, EXPRESSED IN EULER
C                          ANGLES (DEG)
C                AD        A THREE-DIMENSIONAL ARRAY WHICH DESCRIBES THE
C                          DESIRED ATTITUDE OF THE TELESCOPE WITH RESPECT
C                          TO INERTIAL COORDINATES (DEG)
C                ALPHA(I)  OUTER GIMBAL ANGLE OF THE ITH GYROSCOPE OF
C                          THE CARRIER AT TIME TO (DEG)
C                BETA(I)   INNER GIMBAL ANGLE OF THE ITH GYROSCOPE OF
C                          THE CARRIER AT TIME TO (DEG)
C                GM0       CMG ATTITUDE ERROR GAIN CONSTANT MATRIX
C                          ((RAD/SEC)/RAD ERROR)
C                GM1       CMG ATTITUDE RATE GAIN CONSTANT MATRIX
C                          ((RAD/SEC)/(RAD/SEC)ERROR)
C                P         POSITION VECTOR OF THE MASS CENTER OF THE
C                          CARRIER RELATIVE TO THE CENTER OF ROTATION
C                          OF THE VERNIER GIMBALS (FEET)
C                Q         POSITION VECTOR OF THE MASS CENTER OF THE
C                          TELESCOPE RELATIVE TO THE CENTER OF ROTATION
C                          OF THE VERNIER GIMBALS (FEET)
C                M1,M2     MASSES OF THE CARRIER AND TELESCOPE,
C                          RESPECTIVELY (SLUGS)
C                I1,J1,K1  CENTROIDAL MOMENTS OF INERTIA OF THE CARRIER
C                          (SLUG/FT**2)
C                IJ1,IK1,  CENTROIDAL PRODUCTS OF INERTIA OF THE CARRIER
C                JK1       (SLUG/FT**2)
C                I2,J2,K2  CENTROIDAL PRINCIPAL MOMENTS OF INERTIA OF
C                          THE TELESCOPE (SLUG/FT**2)
C                KGA       ATTITUDE ERROR GAIN ASSOCIATED WITH GAMMA1
C                          (FT-LB/RAD ERROR)
C                KGB       ATTITUDE RATE GAIN ASSOCIATED WITH GAMMA1
```

```
C                          (FT-LB/(RAD/SEC)ERROR)
C         KGC              ATTITUDE ERROR GAIN ASSOCIATED WITH GAMMA2
C                          (FT-LB/RAD ERROR)
C         KGD              ATTITUDE RATE GAIN ASSOCIATED WITH GAMMA2
C                          (FT-LB/(RAD/SEC)ERROR)
C         KG1,KG2          SPRING CONSTANTS CORRESPONDING TO GAMMA1 AND
C                          GAMMA2 RESPECTIVELY (FT-LB/RAD)
C         TWIRE1,          CONSTANT TORQUES EXERTED ON THE TELESCOPE BY
C         TWIRE2           WIRE BUNDLES CROSSING GIMBALS 1 AND 2,
C                          RESPECTIVELY
C         TMAX1,            MAXIMUM VERNIER GIMBAL MOTOR TORQUE FOR
C         TMAX2            GIMBALS 1 AND 2
C         CRUDIS           INDICATES THE TYPE OF CREW DISTURBANCE
C                          ABS(CRUDIS)  DISTURBANCE
C                             1         BOUNCE WALK
C                             2         WALL PUSH OFF
C                             3         ARM MOTION - CSM
C                             4         ARM MOTION - LM
C                          CRUDIS IS POSITIVE FOR DISTURBANCES ABOUT THE
C                          Z AXIS AND NEGATIVE FOR DISTURBANCES ABOUT THE
C                          X-Y AXES
C         H                MAGNITUDE OF THE MOMENTUM OF EACH GYROSCOPE
C                          (FT-LB-SEC)
C         ALP              VARIABLE BETWEEN 0. AND 1. WHICH SPECIFIES
C                          THE MAXIMUM VALUE OF THE AERODYNAMIC TORQUE
C                          IN TERMS OF THE MAXIMUM VALUE OF THE GRAVITY
C                          GRADIENT TORQUE
C         GAMA             ORBITAL LOCATION OF DIURNAL BULGE (DEGREES)
C         BET              DETERMINES THE MAGNITUDE OF THE DIURNAL BULGE
C         OH               ORBITAL HEIGHT (N. MILES)
C         INDG             IF INDG=1, THE OUTER AND INNER GIMBAL ANGLES
C                          OF THE CARRIER ARE REQUIRED TO REMAIN WITHIN
C                          GIVEN BOUNDS. IF INDG=0, NO BOUNDS ARE GIVEN,
C                          AND THE PART OF THE PROGRAM WHICH TESTS THE
C                          VALUES OF THESE ANGLES IS SKIPPED
C         ALPHLM           BOUND FOR THE OUTER GIMBAL ANGLES.
C                          ABS(ALPHA(I)) MUST REMAIN .LE. ALPHLM (DEG)
C         BETALM           BOUND FOR THE INNER GIMBAL ANGLES.
C                          ABS(BETA(I)) MUST REMAIN .LE. BETALM (DEG)
C         DOTMAX           BOUND FOR THE RATE OF CHANGE IN THE GIMBAL
C                          ANGLES.  ABS(RATE) MUST BE .LE. DOTMAX. IF NOT,
C                          THE RATE IS SET EQUAL TO DOTMAX OR -DOTMAX,
C                          DEPENDING UPON ITS SIGN (DEG/SEC)
C         DOTMIN           BOUND FOR THE RATE OF CHANGE IN THE GIMBAL
C                          ANGLES.  ABS(RATE) MUST BE .GE. DOTMIN. IF NOT,
C                          THE RATE IS SET EQUAL TO 0. (DEG/SEC)
C         KSL              SPEED GAIN CONSTANT
C         TO               BEGINNING TIME (SEC)
C         ORBITS           NUMBER OF ORBITS FOR WHICH THE SYSTEM OF
C                          DIFFERENTIAL EQUATIONS WILL BE SOLVED
C         DTPRNT           TIME INCREMENT AT WHICH OUTPUT IS DESIRED (SEC)
C         DTMAX            MAXIMUM STEP SIZE TO BE USED IN INTEGRATING
C                          THE DIFFERENTIAL EQUATIONS (SEC)
C         ERBND            AN ARRAY OF VALUES EACH OF WHICH REPRESENTS
C                          A CONVERGENCE CRITERION FOR ONE OF THE
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```
C                            DIFFERENTIAL EQUATIONS
C               GMBL         A THREE DIMENSIONAL ARRAY WHICH INDICATES
C                            WHICH OF THE CARRIER GYROSCOPES ARE OPERATIVE.
C                            GMBL(I)=0. OR 1. IF GMBL(I)=0., THE GIMBAL
C                            RATES OF THE ITH GYROSCOPE ARE SET TO ZERO
C                            AND THE MAGNITUDE OF THE MOMENTUM OF THE ITH
C                            GYROSCOPE IS SET TO ZERO.
C               ICNTRL       INDICATES WHICH CONTROL LAW IS TO BE USED.
C                            IF ICNTRL=1, THE LANGLEY CONTROL LAW IS USED
C                            IF ICNTRL=2, THE H VECTOR CONTROL LAW IS USED
C                            IF ICNTRL=3, THE CROSS PRODUCT CONTROL IS USED
C                            IF ICNTRL=4, THE CLOSED LOOP TORQUE CONTROL
C                            IS USED
C               NOTEL        IF NOTEL=0, THE TELESCOPE IS GIMBAL MOUNTED.
C                            OTHERWISE, IT IS RIGIDLY MOUNTED
C               IBSNSR       IF IBSNSR=0, INERTIAL SENSORS ARE USED. IF
C                            IBSNSR=1, BODY SENSORS ARE USED
C               INDCON       IF INDCON=1, -MIDPT2- BUILDS AN ARRAY -ICON-
C                            WHICH INDICATES THE CONVERGENCE PATTERN OF
C                            EACH EQUATION. THIS ARRAY IS PRINTED OUT
C                            WHENEVER NON CONVERGENCE OCCURS.
C                            ICON(2)=10100 INDICATES THAT THE CONVERGENCE
C                            CRITERION FOR THE SECOND EQUATION WAS NOT
C                            MET ON THE FIRST AND THIRD ITERATIONS.
C                            (-MIDPT2- MAKES AT MOST 5 ITERATIONS)
C                                 IF INDCON=0, THE PART OF THE PROGRAM
C                            WHICH BUILDS THIS ARRAY WILL BE SKIPPED, AND
C                            THE CONVERGENCE OF THE EQUATIONS WILL BE
C                            TESTED ONLY UNTIL ONE EQUATION FAILS.
C                            THEREFORE, IF THE INFORMATION OBTAINED FROM
C                            THE -ICON- ARRAY IS NOT NEEDED, SET INDCON=0
C                            TO SAVE TIME.
C
C OUTPUT        INPUT IS PRINTED OUT. VALUES OF THE SOLUTION OF THE
C               SYSTEM ARE PRINTED OUT EVERY DTPRNT SECONDS
C
C               TIME         CURRENT TIME (SEC)
C               GAMMA1       X-AXIS VERNIER GIMBAL ROTATION AT TIME TIME
C                            (ARCSEC)
C               GAMMA2       Z-AXIS VERNIER GIMBAL ROTATION AT TIME TIME
C                            (ARCSEC)
C               PHI,         DESCRIBE THE ATTITUDE OF THE TELESCOPE AT
C               THETA,       TIME TIME RELATIVE TO THE DESIRED
C               PSI          ATTITUDE  (ARCSEC)
C
C               U            THE VECTOR (U(1),U(2),U(3)) DESCRIBES THE
C                            ANGULAR VELOCITY OF THE TELESCOPE AT TIME
C                            TIME(ARCSEC/SEC)
C               DT           CURRENT STEP SIZE
C               INDOT        INDICATES THE BEHAVIOR OF THE RATE OF CHANGE
C                            OF THE GIMBAL ANGLES.  INDOT=102011 INDICATES
C                            THAT THE RATE OF CHANGE OF ALPHA(1),BETA(2),
C                            BETA(3), WAS LESS THAN DOTMIN, THAT OF ALPHA(3)
C                            WAS MORE THAN DOTMAX, AND THAT OF ALPHA(2) AND
C                            BETA(1) WAS WITHIN THE PRESCRIBED LIMITS.
```

```
C
C SUBROUTINES  MIDPT2    COMPUTES THE SOLUTION OF A SYSTEM OF
C USED                  SIMULTANEOUS FIRST ORDER DIFFERENTIAL
C                       EQUATIONS BY THE METHOD OF ROMBERG APPLIED
C                       TO THE MIDPOINT RULE
C
C             F         A SUBROUTINE USED BY -MIDPT2-. IT
C                       DEFINES THE DIFFERENTIAL EQUATIONS USED TO
C                       DESCRIBE THE INITIAL VALUE CONTROL PROBLEM.
C
C             TF        A ROUTINE WHICH SUPPLIES VALUES OF THE
C                       VECTORS F1,F2,T1,T2. F1 AND F2 ARE THE
C                       FORCES EXERTED ON THE CARRIER AND TELESCOPE
C                       RESPECTIVELY, DUE TO GRAVITY GRADIENT,
C                       AERODYNAMIC AND CREW DISTURBANCE EFFECTS.
C                       T1 AND T2 ARE THE CORRESPONDING TORQUES.
C
C             CREW      A ROUTINE USED BY TF TO DESCRIBE THE CREW
C                       DISTURBANCE EFFECTS
C
C             INTERP    A FUNCTION SUBPROGRAM USED BY THE SUBROUTINE
C                       -CREW- FOR LINEAR INTERPOLATION
C
C             GAUSS     A ROUTINE USED BY THE F SUBROUTINE TO SOLVE
C                       A SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS
C
C             INPR      A ROUTINE USED BY -GAUSS- TO COMPUTE INNER
C                       PRODUCTS
C
C             VECTOR    A PACKAGE OF FUNCTION SUBPROGRAMS USED TO
C                       PERFORM ELEMENTARY VECTOR AND MATRIX
C                       OPERATIONS FOR THREE-DIMENSIONAL VECTORS AND
C                       3X3 MATRICES
C
      REAL M1,M2,M1C1,M2C2,I1,J1,K1,I2,J2,K2,KG1,KG2,KGA,KGB,
     . KGC,KGD,KSL,IJ1,IK1,JK1
      INTEGER CRUDIS
C
      DIMENSION U(22),ALPHA(3),BETA(3),ERBND(22),WU(22),DX(22),
     . TITLE(20),TEMPU(22),A(3),AD(3),CAD(3),SAD(3)
C
      COMMON/ALL/C1,C2,P(3),Q(3),M1,M2,I1,J1,K1,I2,J2,K2,
     . GM0(3,3),GM1(3,3),H,IBSNSR,TD(3,3),TMAX1,TMAX2,
     . IND(6),NCALL,KSL,DOTMAX,DOTMIN,INDOT,ALP,CRUDIS,
     . OMEGA0,GMBL(3),ICNTRL,HV(3),KG1,KG2,KGA,KGB,
     . BET,GAM,
     . KGC,KGD,M1C1,M2C2,NOTEL,T00,EYE1(3,3),TWIRE1,TWIRE2
C
      EQUIVALENCE (U(9),GAMMA1),(U(10),GAMMA2),(U(11),ALPHA),
     . (U(14),BETA),(U(6),A(1))
C
      DATA TITLE/30HLANGLEY CONTROL LAW             ,
     .           30HH VECTOR CONTROL LAW            ,
     .           30HCROSS PRODUCT CONTROL LAW       ,
     .           30HCLOSED LOOP TORQUE CONTROL LAW/
```

```
C
              NAMELIST/INPUT/U,A,AD,ALPHA,BETA,GM0,GM1,P,Q,GMBL,ERBND,
     .        GAMMA1,GAMMA2,M1,M2,I1,J1,K1,IJ1,IK1,JK1,I2,J2,K2,KGA,
     .        KGB,KGC,KGD,KG1,KG2,H,T0,ORBITS,DTPRNT,INDCON,ALP,OH,
     .        INDG,BETALM,ALPHLM,DOTMAX,DOTMIN,KSL,ICNTRL,IBSNSR,DTMAX,
     .        TWIRE1,TWIRE2,NOTEL,CRUDIS,TMAX1,TMAX2,BET,GAMA
C
C      READ AND PRINT INPUT
C
 21           READ(5,INPUT)
              WRITE(6,50)
              WRITE(6,INPUT)
C
C      DEFINE CONVERSION FACTORS USED WITHIN THE PROGRAM
C
              DGTORD=3.1415927/180.
              RDTODG=180./3.1415927
              RDTOSC=RDTODG*3600.
C
C      DEFINE CONSTANTS USED WITHIN THE PROGRAM
C
              C1=1./(1.+M1/M2)
              C2=-1./(1.+M2/M1)
C
              M1C1=-M1*C1
              M2C2=-M2*C2
C
              EYE1=AMDEF(I1,IJ1,IK1,IJ1,J1,JK1,IK1,JK1,K1)
C
C      CONVERT THE ORBITAL HEIGHT TO FEET
C
              OH=OH*6076.1155+20925738.
              OMEGA0=SQRT(.14076E17/OH)/OH
C
C      DETERMINE THE ENDING TIME
C
              TEND=2.*3.1415927/OMEGA0*ORBITS+T0
C
C      PRINT TITLES
C
              IR=(ICNTRL-1)*5+1
              IS=IR+4
              WRITE(6,10)(TITLE(I),I=IR,IS)
C
              DO 1 I=1,3
              AD(I)=AD(I)*DGTORD
              SAD(I)=SIN(AD(I))
              CAD(I)=COS(AD(I))
 1            CONTINUE
C
C
C      TD TRANSFORMS INERTIAL INTO DESIRED TELESCOPE COORDINATES
C
              TD(1,1)=CAD(3)*CAD(2)
              TD(2,1)=CAD(3)*SAD(2)*SAD(1)-SAD(3)*CAD(1)
```

```
              TD(3,1)=CAD(3)*SAD(2)*CAD(1)+SAD(3)*SAD(1)
              TD(1,2)=SAD(3)*CAD(2)
              TD(2,2)=SAD(3)*SAD(2)*SAD(1)+CAD(3)*CAD(1)
              TD(3,2)=SAD(3)*SAD(2)*CAD(1)-CAD(3)*SAD(1)
              TD(1,3)=-SAD(2)
              TD(2,3)=CAD(2)*SAD(1)
              TD(3,3)=CAD(2)*CAD(1)
C
C     PRINT INITIAL CONDITIONS
C
              WRITE(6,30)TO,U(9),U(10),(A(I),I=1,3),(U(I),I=1,3)
C
C     CONVERT INPUT TO RADIANS
C
              GAM=GAMA*DGTORD
              ALPHLM=ALPHLM*DGTORD
              BETALM=BETALM*DGTORD
              DOTMAX=DOTMAX*DGTORD
              DOTMIN=DOTMIN*DGTORD
C
              DO 2 I=1,16
  2           U(I)=U(I)*DGTORD
C
C     DETERMINE THE NUMBER OF DIFFERENTIAL EQUATIONS TO BE SOLVED
C
              NDIFEQ=16
              IF(ICNTRL.EQ.2)NDIFEQ=19
              IF(IBSNSR.EQ.1)NDIFEQ=NDIFEQ+3
C
C     IF ICNTRL=2, THE H VECTOR CONTROL LAW IS USED AND INITIAL VALUES
C     ARE NEEDED FOR THE COMPONENTS OF THE COMMANDED ANGULAR
C     MOMENTUM OF THE VEHICLE
C
              IF(ICNTRL.NE.2)GO TO 3
              CALL F(TO,U,DX,NDIFEQ)
              U(17)=VEQPL(HV)
C
  3           TOO=TO
              TIME=TO
              DT=DTMAX
C
              KTIME=1
              PTIME=FLOAT(KTIME)*DTPRNT+TOO
C
  4           TO=TIME
              ITKCNT=0
  5           TIME=TO+DT
C
C     IF THE TIME IS GREATER THAN THE NEXT PRINTING TIME, SET IT
C     EQUAL TO THAT PRINTING TIME
C
              INDP=0
              IF(TIME.LT.PTIME)GO TO 6
              TIME=PTIME
              INDP=1
```

```
C
C      CALL -MIDPT2- TO SOLVE THE SYSTEM OF DIFFERENTIAL EQUATIONS
C
 6            CALL MIDPT2(NDIFEQ,T0,TIME,U,ERBND,INDCON,TEMPU,ITK)
C
C      IF NECESSARY, ADJUST THE STEP SIZE 'DT'
C
              GO TO (8,11,9,9,7),ITK
C
 7            DT=DT/2.0
              IF(ITKCNT.EQ.10)CALL EXIT
              ITKCNT=ITKCNT+1
              GO TO 5
C
 8            KN2=KN2+1
              IF(KN2.LT.5)GO TO 11
              DT=AMIN1(DTMAX,2.*DT)
              KN2=0
              GO TO 11
C
 9            DT=DT/2.
C
 11           DO 12 I=1,NDIFEQ
 12           U(I)=TEMPU(I)
C
C      IF INDG=0, THE GIMBAL ANGLES NEED NOT BE TESTED
C
              IF (INDG.EQ.0) GO TO 18
              IND(1)=VDEF(0.,0.,0.)
              IND(4)=VDEF(0.,0.,0.)
C
              DO 14 I=1,3
              UTEMP=U(I+10)
              IF(ABS(UTEMP).LT.ALPHLM)GO TO 14
              IF(UTEMP.GE.ALPHLM)GO TO 13
C
              U(I+10)=-ALPHLM
              IND(I)=-1
              GO TO 14
C
 13           U(I+10)=ALPHLM
              IND(I)=1
 14           CONTINUE
C
              DO 16 I=4,6
              UTEMP=U(I+10)
              IF(ABS(UTEMP).LT.BETALM)GO TO 16
              IF(UTEMP.GE.BETALM)GO TO 15
C
              U(I+10)=-BETALM
              IND(I)=-1
              GO TO 16
C
 15           U(I+10)=BETALM
              IND(I)=1
```

```
 16             CONTINUE
C
                NCALL=0
                CALL F(TIME,U,DX,NDIFEQ)
                NCALL=1
C
                DO 17 I=1,6
                IF(FLOAT(IND(I))*DX(I+10).LE.0.)IND(I)=0
 17             CONTINUE
C
C        IF INDP=1, IT IS TIME TO PRINT
C
 18             IF(INDP.EQ.0)GO TO 4
C
C        CONVERT OUTPUT TO ARCSECONDS
C
                DO 19 I=1,16
 19             WU(I)=U(I)*RDTOSC
C
                WRITE(6,30)TIME,WU(9),WU(10),WU(6),WU(7),WU(8),(WU(I),
     .          I=1,3),DT,INDOT
C
                IF(TIME.GT.TEND)GO TO 21
C
C        DETERMINE THE NEXT PRINT TIME
C
                KTIME=KTIME+1
                PTIME=FLOAT(KTIME)*DTPRNT+T00
                GO TO 4
C
 10             FORMAT(1H0,5A6/1H0,3X,4HTIME,5X,6HGAMMA1,4X,6HGAMMA2,
     .          6X,3HPHI,5X,5HTHETA,7X,3HPSI,7X,2HU1,8X,2HU2,8X,
     .          2HU3,6X,2HDT,6X,5HINDOT/1H ,3X,'(SEC)',1X,5(2X,'(ARCSEC)'
     .          ),3(1X,'(ARCS/SC)'))
 20             FORMAT(1H1,5A6/1H0,3X,4HTIME,5X,6HALPHA1,4X,6HALPHA2,4X,
     .          6HALPHA3,5X,5HBETA1,5X,5HBETA2,5X,5HBETA3,5X,2HDT,6X,
     .          5HINDOT/1H ,3X,5H(SEC),6(5X,5H(DEG)),4X,5H(SEC))
C
C
 30             FORMAT(1H ,F9.2,1X,9(1PE9.2,1X),I6)
 40             FORMAT(1H ,F9.2,1X,7(1PE9.2,1X),I6)
 50             FORMAT(1H1)
         END
```

APPENDIX IV

SUBROUTINE F

```
C AUTHOR        C.F. BANICK
C
C DATE          7-18-67
C
C
C PURPOSE       SUPPLY THE DIFFERENTIAL EQUATION SOLVING SUBROUTINE
C               -MIDPT2- WITH NUMERICAL VALUES OF THE DERIVATIVES USED
C               TO DEFINE THE INITIAL VALUE PROBLEM
C
C CALL          CALL F(X,U,DU,NDIFEQ)
C
C INPUT         CALL LIST...
C               X         VALUE OF TIME FOR WHICH THE DERIVATIVES ARE TO
C                         BE COMPUTED
C               U         ARRAY OF VALUES OF THE FUNCTIONS AT TIME X.
C                         SEE THE MAIN PROGRAM -GMT- FOR DEFINITION OF
C                         THIS ARRAY
C               NDIFEQ    NUMBER OF DIFFERENTIAL EQUATIONS TO BE SOLVED.
C                         NDIFEQ IS DETERMINED IN PROGRAM -GMT-.
C               COMMON...
C               SEE MAIN PROGRAM -GMT- FOR DESCRIPTION AND/OR ORIGIN OF
C               COMMON VARIABLES
C
C OUTPUT        CALL LIST...
C               DU        ARRAY OF VALUES OF DERIVATIVES OF THE FUNCTIONS
C                         AT TIME X. THE ORDER OF THE ELEMENTS OF THIS
C                         ARRAY CORRESPONDS TO THAT OF THE U ARRAY.
C               COMMON...
C               INDOT     INDICATES THE BEHAVIOR OF THE RATE OF CHANGE
C                         OF THE GIMBAL ANGLES. FOR COMPLETE DESCRIPTION
C                         SEE PROGRAM -GMT-.
C               HV        COMMANDED ANGULAR MOMENTUM OF THE VEHICLE
C                         VECTOR. NEEDED FOR INITIAL VALUES IF THE
C                         H VECTOR CONTROL LAW IS USED.
C
C SUBROUTINES   TF        A ROUTINE WHICH SUPPLIES VALUES OF THE
C USED                    VECTORS F1,F2,T1,T2. F1 AND F2 ARE THE
C                         FORCES EXERTED ON THE CARRIER AND TELESCOPE
C                         RESPECTIVELY, DUE TO GRAVITY GRADIENT,
C                         AERODYNAMIC AND CREW DISTURBANCE EFFECTS.
C
C               GAUSS     A ROUTINE USED TO SOLVE A SYSTEM OF
C                         SIMULTANEOUS LINEAR EQUATIONS
C
C               INPR      A ROUTINE USED BY -GAUSS- TO COMPUTE INNER
C                         PRODUCTS
C
C               VECTOR    A PACKAGE OF FUNCTION SUBPROGRAMS USED TO
C                         PERFORM ELEMENTARY VECTOR AND MATRIX
C                         OPERATIONS FOR THREE-DIMENSIONAL VECTORS AND
C                         3X3 MATRICES.
C
                SUBROUTINE F(X,U,DU,NDIFEQ)
```

```
C
      REAL M1,M2,M1C1,M2C2,I1,J1,K1,I2,J2,K2,KG1,KG2,KGA,KGB,
     .     KGC,KGD,KSL
      INTEGER CRUDIS
C
      DIMENSION U(22),DU(22),TM(3,3),TMI(3,3),OMB2(3),
     .          OMB1(3),OMB1M(3,5),DOMB1(3,6),
     .          U5K21(3),OMB21(3),U41C52(3),OMB2CQ(3,5),
     .          VC1(3,5),VC2(3,5),F1STAR(3,6),F2STAR(3,6),
     .          AC1(3,6),AC2(3,6),TEMPV(3),TEMPVV(3),
     .          T1STAR(3,6),T2STAR(3,6),TC(3),TG(3),F1(3)
      DIMENSION F2(3),T1(3),T2(3),S(5,6),AMTRX(3,3),
     .          AMAD(3),SALPHA(3),CALPHA(3),E(3),
     .          SBETA(3),CBETA(3),DALPHA(3),S3(3,3),
     .          S16(5),V(3),FF(3,3),GG(3,3),AN(3,3),BN(3,3),
     .          DBETA(3),AM(3,3),BM(3,3),DX(6),
     .          TETA(3,3),TDA(3,3),SA(3),CA(3),DUMMY(3)
C
C
      COMMON/ALL/C1,C2,P(3),Q(3),M1,M2,I1,J1,K1,I2,J2,K2,
     .GM0(3,3),GM1(3,3),H,IBSNSR,TD(3,3),TMAX1,TMAX2,
     .IND(6),NCALL,KSL,DOTMAX,DOTMIN,INDOT,ALP,CRUDIS,
     .OMEGA0,GMBL(3),ICNTRL,HV(3),KG1,KG2,KGA,KGB,
     .BET,GAM,
     .KGC,KGD,M1C1,M2C2,NOTEL,T0,EYE1(3,3),TWIRE1,TWIRE2
C
      EQUIVALENCE (DX(1),DALPHA),(DX(4),DBETA)
C
      CGAM1=COS(U(9))
      CGAM2=COS(U(10))
      SGAM1=SIN(U(9))
      SGAM2=SIN(U(10))
C
C     TMI ... TRANSFORMS TELESCOPE COORDINATES INTO SPACECRAFT
C     COORDINATES
C
      TMI=AMDEF1(CGAM2,CGAM1*SGAM2,SGAM1*SGAM2,-SGAM2,
     .           CGAM1*CGAM2,SGAM1*CGAM2,0.,-SGAM1,CGAM1)
C
C     TM ... TRANSFORMS SPACECRAFT COORDINATES INTO TELESCOPE
C     COORDINATES
C
      TM=AMTR(TMI)
C
C     OMB2 ... ANGULAR VELOCITY OF TELESCOPE
C
      OMB2=VDEF(U(1),U(2),U(3))
C
C     OMB1 ... ANGULAR VELOCITY OF CARRIER
C
      OMB1=VDEF(U(1),U(2),U(3)-U(5))
      OMB1=AMTV(TMI,OMB1)
      OMB1(1)=OMB1(1)-U(4)
C
C     OMB1M ... PARTIAL RATES OF CHANGE OF THE CARRIER ANGULAR VELOCITY
```

```
C
            OMB1M=AMEQPL(TMI)
            OMB1M(1,4)=-1.
            OMB1M(1,5)=VEQMN(TMI(1,3))
C
C     DOMB1 ... ANGULAR ACCELERATION OF THE CARRIER
C
            DOMB1=AMEQPL(TMI)
            DOMB1(1,4)=VDEF(-1.,0.,0.)
            DOMB1(1,5)=VEQMN(TMI(1,3))
C
            U5K21=VSCALR(TMI(1,3),U(5))
            OMB21=AMTV(TMI,OMB2)
            U41C52=VDEF(0.,U(4)*U5K21(3),U(4)*U5K21(2))
C
            DOMB1(1,6)=VDEF(U(4),0.,0.)
            DOMB1(1,6)=VADD(DOMB1(1,6),U5K21)
            DOMB1(1,6)=VCROSS(DOMB1(1,6),OMB21)
            DOMB1(1,6)=VSUB(DOMB1(1,6),U41C52)
C
            OMB2CQ=AMDEF1(0.,-Q(3),Q(2),Q(3),0.,-Q(1),-Q(2),Q(1),0.)
            OMB2CQ=AMTM(TMI,OMB2CQ)
C
C
C     VC1 ... PARTIAL RATES OF CHANGE OF THE VELOCITY OF THE MASS
C     CENTER OF THE CARRIER
C     VC2 ... PARTIAL RATES OF CHANGE OF THE VELOCITY OF THE MASS
C     CENTER OF THE TELESCOPE
C     AC1 ... ACCELERATION OF THE MASS CENTER OF THE CARRIER
C
            DO 1 I=1,5
            VC1(1,I)=VCROSS(OMB1M(1,I),P)
            VC1(1,I)=VSUB(VC1(1,I),OMB2CQ(1,I))
            VC2(1,I)=AMTV(TM,VC1(1,I))
            VC1(1,I)=VSCALR(VC1(1,I),C1)
            VC2(1,I)=VSCALR(VC2(1,I),C2)
C
            AC1(1,I)=VCROSS(DOMB1(1,I),P)
 1          CONTINUE
C
            AC1=AMSUB(AC1,OMB2CQ)
C
            TEMPV=VCROSS(OMB1,P)
            TEMPV=VCROSS(OMB1,TEMPV)
C
            TEMPVV=VCROSS(OMB2,Q)
            TEMPVV=VCROSS(OMB2,TEMPVV)
            TEMPVV=AMTV(TMI,TEMPVV)
C
            TEMPV=VSUB(TEMPV,TEMPVV)
C
            AC1(1,6)=VCROSS(DOMB1(1,6),P)
            AC1(1,6)=VADD(AC1(1,6),TEMPV)
C
C     AC2 ... ACCELERATION OF THE MASS CENTER OF THE TELESCOPE
```

```
C
                AC2(1,1)=AMTM(TM,AC1(1,1))
                AC2(1,4)=AMTM(TM,AC1(1,4))
C
C       F1STAR AND F2STAR ... NOW INERTIA FORCES FOR THE RESPECTIVE
C       BODIES
C       T1STAR AND T2STAR ... NOW INERTIA TORQUES FOR THE RESPECTIVE
C       BODIES
C
                DO 2 I=1,6
                F1STAR(1,I)=VSCALR(AC1(1,I),M1C1)
                F2STAR(1,I)=VSCALR(AC2(1,I),M2C2)
C
                T1STAR(1,I)=AMTV(EYE1,DOMB1(1,I))
 2              T1STAR(1,I)=VEQMN(T1STAR(1,I))
                DUMMY=AMTV(EYE1,OMB1)
                DUMMY=VCROSS(DUMMY,OMB1)
                T1STAR(1,6)=VADD(DUMMY,T1STAR(1,6))
C
                T2STAR(1,1)=-I2
                T2STAR(2,2)=-J2
                T2STAR(3,3)=-K2
                T2STAR(1,6)=U(2)*U(3)*(J2-K2)
                T2STAR(2,6)=U(3)*U(1)*(K2-I2)
                T2STAR(3,6)=U(1)*U(2)*(I2-J2)
C
C       COMPUTE FORCES AND TORQUES DUE TO CREW MOTION, GRAVITATIONAL, AND
C       AERODYNAMIC EFFECTS
C
                ETA=OMEGA0*X
                SETA=SIN(ETA)
                CETA=COS(ETA)
                TETA=AMDEF1(CETA,SETA,0.,-SETA,CETA,0.,0.,0.,1.)
                TETA=AMTM(TD,TETA)
                DO 25 I=1,3
                SA(I)=SIN(U(I+5))
 25             CA(I)=COS(U(I+5))
                TDA(1,1)=AMDEF1(CA(3)*CA(2),CA(3)*SA(2)*SA(1)-SA(3)*CA(1)
     .          ,CA(3)*SA(2)*CA(1)+SA(3)*SA(1),SA(3)*
     .          CA(2),SA(3)*SA(2)*SA(1)+CA(3)*CA(1),
     .          SA(3)*SA(2)*CA(1)-CA(3)*SA(1),-SA(2),
     .          CA(2)*SA(1),CA(2)*CA(1))
                TETA(1,1)=AMTM(TDA,TETA)
C
C       TETA IS NOW A TRANSFORMATION FROM LOCAL VERTICAL TO ACTUAL
C       TELESCOPE COORDINATES
C
                CALL TF(X,TETA(1,1),TETA(1,2),TMI,TM,F1,F2,T1,T2)
C
C
                F1STAR(1,6)=VADD(F1STAR(1,6),F1)
                F2STAR(1,6)=VADD(F2STAR(1,6),F2)
C
C       COMPUTE VERNIER GIMBAL CONTROL TORQUE, TG
C
```

```
          TG1=-KGA*U(6)-KGB*U(1)-KG1*U(9)+TWIRE1
          IF(ABS(TG1).GT.TMAX1)TG1=SIGN(TMAX1,TG1)
          TG2=-KGC*U(8)-KGD*U(3)-KG2*U(10)+TWIRE2
          IF(ABS(TG2).GT.TMAX2)TG2=SIGN(TMAX2,TG2)
          TG=VSCALR(TMI(1,3),TG2)
          TG(1)=TG(1)+TG1
C
C     START COMPUTATION OF CMG CONTROL TORQUE,TC (TO STATEMENT 18)
C
          IF(ABS(H).LT.(1.E-34))GO TO 18
C
          DO 3 I=1,3
          SALPHA(I)=SIN(U(I+10))
          CALPHA(I)=COS(U(I+10))
          SBETA(I)=SIN(U(I+13))
          CBETA(I)=COS(U(I+13))
 3        CONTINUE
C
          IF(IBSNSR.EQ.1)GO TO 4
          AMAD(1)=U(6)-U(9)
          AMAD(2)=U(7)
          AMAD(3)=U(8)-U(10)
          GO TO 5
C
C     IF IBSNSR=1, ATTITUDE IS OBTAINED BY INTEGRATING ATTITUDE RATE
C
 4        KBS=NDIFEQ-2
          AMAD=VEQPL(U(KBS))
          DU(KBS)=VEQPL(OMB1)
C
C     COMPUTE COMPOSITE ERROR VECTOR E
C
 5        TEMPV=AMTV(GMO,AMAD)
          TEMPVV=AMTV(GM1,OMB1)
          E=VADD(TEMPV,TEMPVV)
C
          GKSL=8./9.*KSL
C
          DO 9 I=1,3
          J=3*(1/I)+(I-1)
          K=I+1-3*(I/3)
C     I=1,2,3
C     J=3,1,2
C     K=2,3,1
          GG(I,I)=-CBETA(I)*SALPHA(I)
          GG(J,I)=-CBETA(I)*CALPHA(I)
          FF(I,I)=-SBETA(I)*CALPHA(I)
          FF(J,I)=SBETA(I)*SALPHA(I)
          FF(K,I)=CBETA(I)
C
C     COMPUTE TOTAL ANGULAR MOMENTUM
C
          HV(I)=-GMBL(I)*GG(J,I)+GMBL(J)*SBETA(J)-GMBL(K)*CBETA(K)
     •         *SALPHA(K)
C
```

```
C       THE DEFINITION OF VECTOR V DEPENDS ON WHICH CONTROL LAW IS USED
C
              GO TO (7,6,7,8),ICNTRL
 6            V(I)=U(I+16)-HV(I)
              DU(I+16)=E(I)
              GO TO 8
 7            V(I)=E(I)
C
 8            AM(I,I)=-KSL*SALPHA(I)/CBETA(I)
              AM(K,I)=-KSL*CALPHA(K)/CBETA(K)
              BM(I,I)=GKSL*FF(I,I)
              BM(J,I)=GKSL*CBETA(J)
              BM(K,I)=GKSL*SBETA(K)*SALPHA(K)
C
 9            CONTINUE
C
              IF(ICNTRL.NE.4)GO TO 12
C
C       COMPUTE V FOR CLOSED LOOP CONTROL
C
              AN(1,1)=AMTM(GG,AM)
              BN(1,1)=AMTM(FF,BM)
              AN(1,1)=AMADD(AN,BN)
              DO 10 I=1,3
 10           AN(I,I)=AN(I,I)+1.
              BN(1,1)=AMINV(AN)
              V=AMTV(BN,E)
C
C       COMPUTE DERIVATIVES OF ALPHA AND BETA
C
 12           DALPHA(1)=AMTV(AM,V)
              DBETA(1)=AMTV(BM,V)
              IF(ICNTRL.NE.1)GO TO 14
C
C       SPECIAL COMPUTATION OF DALPHA AND DBETA FOR LANGLEY CONTROL LAW
C
              DO 13 I=1,3
              K=I+1-3*(I/3)
              DALPHA(I)=DALPHA(I)*CBETA(I)
 13           DBETA(I)=GKSL*CBETA(I)*V(K)
C
C       ADJUST DALPHA AND DBETA TO ACCOUNT FOR LIMITING AND DEADBAND
C       INDOT IS PRINTED TO INDICATE GIMBAL ANGLE RATE BEHAVIOR
C
 14           INDOT=1
              DO 17 I=1,6
              IJ=I-3*(I/4)
              INDOT=INDOT*10
              IF(IND(I)*NCALL.EQ.0)GO TO 15
              DX(I)=0.
              GO TO 17
 15           IF(ABS(DX(I)).LE.DOTMAX)GO TO 16
              DX(I)=SIGN(DOTMAX,DX(I))
              INDOT=INDOT+2
              GO TO 17
```

```
 16             IF(ABS(DX(I)).GE.DOTMIN)GO TO 17
                DX(I)=0.
                INDOT=INDOT+1
 17             DX(I)=DX(I)*GMBL(IJ)
                INDOT=INDOT-1000000
C
C       COMPUTE CONTROL TORQUE TC FROM DALPHA AND DBETA
C
                TEMPV=AMTV(GG,DALPHA)
                TEMPVV=AMTV(FF,DBETA)
                TEMPV=VADD(TEMPV,TEMPVV)
                TEMPVV=VCROSS(OMB1,HV)
                TEMPV=VADD(TEMPV,TEMPVV)
                TC=VSCALR(TEMPV,H)
C
C       COMPUTE THE ACTIVE TORQUE ACTING ON BODY 1
C
                T1=VSUB(T1,TC)
 18             T1=VSUB(T1,TG)
                T1STAR(1,6)=VADD(T1STAR(1,6),T1)
C
                TG=VSCALR(TM(1,1),TG1)
                TG(3)=TG(3)+TG2
                T2=VADD(TG,T2)
                T2STAR(1,6)=VADD(T2STAR(1,6),T2)
C
C       GENERATE THE COEFFICIENT MATRIX, S, TO REPRESENT THE DYNAMICAL
C       EQUATIONS AS A SIMULTANEOUS SYSTEM OF LINEAR ALGEBRAIC EQUATIONS
C       S*DU=S16
C
 81             DO 19 I=1,5
                DO 19 J=1,6
                SS=VDOT(F1STAR(1,J),VC1(1,I))
                SS=SS+VDOT(F2STAR(1,J),VC2(1,I))
                S(I,J)=SS+VDOT(T1STAR(1,J),OMB1M(1,I))
                IF(I.GT.3)GO TO 19
                S(I,J)=S(I,J)+T2STAR(I,J)
 19             CONTINUE
C
                DO 21 I=1,5
 21             S16(I)=-S(I,6)
C
C       IF NOTEL.NE.0, DU(4)=DU(5)=0 AND ONLY THREE DYNAMICAL EQUATIONS
C       REMAIN
C
                IF(NOTEL.EQ.0)GO TO 23
                DU(4)=0.
                DU(5)=0.
                DO 22 IJ=1,3
 22             S3(1,IJ)=VEQPL(S(1,IJ))
                CALL GAUSS(3,1,S3,S16,DU,DET,ERR)
                GO TO 24
C
 23             CALL GAUSS(5,1,S,S16,DU,DET,ERR)
C
```

```
C      KINEMATICAL EQUATIONS RELATING U(1),U(2),U(3) TO THE EULER ANGLES
C
 24            SPHI=SIN(U(6))
               CPHI=COS(U(6))
               TTHETA=TAN(U(7))
               CTHETA=COS(U(7))
C
               AMTRX=AMDEF1(1.,0.,0.,SPHI*TTHETA,CPHI,SPHI/CTHETA,
     .                   CPHI*TTHETA,-SPHI,CPHI/CTHETA)
               DU(6)=AMTV(AMTRX,U)
C
C      DERIVATIVES OF THE VERNIER GIMBAL ANGLES
C
               DU(9)=U(4)
               DU(10)=U(5)
C
C      DERIVATIVES OF THE CMG GIMBAL ANGLES
C
               DU(11)=VEQPL(DALPHA)
               DU(14)=VEQPL(DBETA)
C
               RETURN
       END
```

APPENDIX V

SUBROUTINE TF

```
C AUTHOR          C.F. BANICK
C
C DATE            7-20-67
C
C PURPOSE         SUPPLY VALUES OF FORCES AND TORQUES DUE TO GRAVITY
C                 GRADIENT, AERODYNAMIC, AND CREW DISTURBANCE EFFECTS
C
C CALL            CALL TF(TIME,N2,M,TMI,TM,F1,F2,T1,T2)
C
C INPUT           CALL LIST...
C                 TIME      VALUE OF TIME FOR WHICH THE FORCES AND
C                           TORQUES ARE TO BE COMPUTED
C                 N2        LOCAL VERTICAL VECTOR
C                 M         UNIT VECTOR PARALLEL TO SPACECRAFT VELOCITY
C                 TMI       MATRIX USED TO TRANSFORM TELESCOPE COORDINATES
C                           INTO SPACECRAFT COORDINATES
C                 TM        MATRIX USED TO TRANSFORM SPACECRAFT
C                           COORDINATES INTO TELESCOPE COORDINATES
C                 COMMON...
C                 SEE MAIN PROGRAM -GMT- AND CALLING PROGRAM -FGMT- FOR
C                 DESCRIPTION AND/OR ORIGIN OF COMMON VARIABLES
C
C OUTPUT          CALL LIST...
C                 F1,F2     FORCES EXERTED ON THE CARRIER AND TELESCOPE
C                           RESPECTIVELY
C                 T1,T2     TORQUES EXERTED ON THE CARRIER AND TELESCOPE
C                           RESPECTIVELY
C
C SUBROUTINES     CREW      A ROUTINE USED TO DESCRIBE THE CREW
C USED                      DISTURBANCE EFFECTS
C
C                 VECTOR    A PACKAGE OF FUNCTION SUBPROGRAMS USED TO
C                           PERFORM ELEMENTARY VECTOR AND MATRIX OPERATIONS
C                           FOR THREE-DIMENSIONAL VECTORS AND 3X3 MATRICES
C
                  SUBROUTINE TF(TIME,N2,M,TMI,TM,F1,F2,T1,T2)
C
                  REAL N1,N2,IX,IY,M,M1,M2,I1,J1,K1,I2,J2,K2,M1C1,M2C2,KSL
                  INTEGER CRUDIS
                  DIMENSION TMI(3,3),F1(3),F2(3),T1(3),T2(3),N1(3),
     .                      N2(3),R1(3),R2(3),SF1(3),SF2(3),TG1(3),TG2(3),
     .                      Q1(3),M(3),TA1(3),FD1(3),TD1(3),
     .                      CI1M(3),TM(3,3)
C
C
                  COMMON/ALL/C1,C2,P(3),Q(3),M1,M2,I1,J1,K1,I2,J2,K2,
     .            GM0(3,3),GM1(3,3),H,IBSNSR,TD(3,3),TMAX1,TMAX2,
     .            IND(6),NCALL,KSL,DOTMAX,DOTMIN,INDOT,ALP,CRUDIS,
     .            OMEGA0,GMBL(3),ICNTRL,HV(3),KG1,KG2,KGA,KGB,
     .            BET,GAM,
     .            KGC,KGD,M1C1,M2C2,NOTEL,T0,EYE1(3,3),TWIRE1,TWIRE2
C
C       COMPUTATION OF GRAVITATIONAL FORCES AND TORQUES
C
                  ALPP=ALP/(1.+BET)*(1.-BET*COS(OMEGA0*TIME+GAM))
```

```
            SQOMEG=OMEGA0**2
C
            N1=AMTV(TMI,N2)
            Q1=AMTV(TMI,Q)
            R1=VSUB(P,Q1)
            R2=AMTV(TM,R1)
            R1=VSCALR(R1,C1)
            R2=VSCALR(R2,C2)
C
            S=-3.*VDOT(N1,R1)
            SF1=VSCALR(N1,S)
            SF1=VADD(R1,SF1)
            SF1=VSCALR(SF1,-SQOMEG*M1)
C
            S=-3.*VDOT(N2,R2)
            SF2=VSCALR(N2,S)
            SF2=VADD(R2,SF2)
C
            TG1(1)=N1(2)*N1(3)*(K1-J1)
            TG1(2)=N1(3)*N1(1)*(I1-K1)
            TG1(3)=N1(1)*N1(2)*(J1-I1)
C
            TG1=VSCALR(TG1,3.*SQOMEG)
C
            TG2(1)=N2(2)*N2(3)*(K2-J2)
            TG2(2)=N2(3)*N2(1)*(I2-K2)
            TG2(3)=N2(1)*N2(2)*(J2-I2)
C
C
C     COMPUTATION OF AERODYNAMIC TORQUES
C
            IX=I1+I2+M1*R1(2)**2+M2*R2(2)**2
            IY=J1+J2+M1*R1(1)**2+M2*R2(1)**2
C
            M=AMTV(TMI,M)
C
            CI1M=VDFF(0.,-M(3),M(2))
            CON=-1.5*ALPP*SQOMEG*(IY-IX)*VMAG(CI1M)
C
            TA1=VSCALR(CI1M,CON)
C
C     COMPUTATION OF CREW DISTURBANCE EFFECTS
C
            TTIME=TIME-T0
            CALL CREW(TTIME,CRUDIS,FD1,TD1)
            F1=VADD(SF1,FD1)
            F2=VSCALR(SF2,-SQOMEG*M2)
C
            T1=VADD(TG1,TD1)
            T1=VADD(T1,TA1)
            T2=VSCALR(TG2,3.*SQOMEG)
C
            RETURN
      END
```

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