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Cross-Polarization Interference Cancellation and Nonminimum Phase Fades

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In this paper we examine the effects of nonminimum phase fades on dual-polarized M-state quadrature amplitude-modulated signals. Performance is evaluated in the presence of a baseband cross-polarization interference canceler described in another paper. Results indicate that the canceler performance is practically transparent to the fade type.

I. INTRODUCTION

This paper reports on a continuation of the work presented in Ref. 1, that is, cancellation of cross-polarization interference by a transversal structure at baseband when two M-state quadrature amplitude-modulated (M-QAM) signals are transmitted over the same channel using orthogonal field polarizations. In this paper we extend the scope of the previous study, which only dealt with minimum phase fades, to include evaluation of nonminimum phase fade effects encountered in transmission over multipath fading channels such as those experienced in line-of-sight terrestrial radio applications.

Recent studies[2-4] have examined IF and baseband equalizers in the transmission of a single QAM signal under nonminimum phase fades. For example, Ref. 4 examines transition distortions when the fade phase changes from minimum to nonminimum. In particular, Ref. 2 concludes that, in mitigating intersymbol interference, baseband equalizers outperform IF structures used for this purpose in the

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presence of nonminimum phase fades. Since, as pointed out in Ref. 2, on the average 50 percent of deep selective fades take on a nonminimum phase, we examine the proposed baseband interference canceler performance[1] in nonminimum phase fades. We use the static modeling of the dual-polarized channels introduced in Ref. 5 to emulate snapshots of fadings of the main (reference) and the cross-coupled paths. This simple model follows the results of recent measurements cited in Ref. 5.

In this study, we first show that some of the nonminimum phase fade channel models available in the literature that were meant to be used in single-signal transmission may not be appropriate for application in multicarrier transmission systems such as dual-polarized or space-diversity systems. Then, employing what seems to be the proper static model for nonminimum phase fades, we evaluate dual-polarized system performance with and without the proposed canceler.[1] It is demonstrated that, for a proper model, the type of fading has no impact on the performance of the baseband cross-polarized interference canceler, as long as the fadings of the reference (main) copolarized path signal and the cross-coupled path signal are of the same type, i.e., both minimum or both nonminimum phase. As in Ref. 1, dual-polarized system performance signature (M-curve) is used as a measure in the performance evaluation.

In the following section, we describe and compare the nonminimum phase-fading static models and select what seems to be the appropriate one to carry on the numerical evaluation in Section III.

II. ANALYTICAL CHANNEL MODEL

The underlying assumptions in modeling and cross-polarization cancellation are described.

2.1 Nonminimum phase fading model

In this section we compare three commonly used static models of nonminimum phase fades and demonstrate that only one of them seems to be appropriate for application in dual-polarized systems. The first model, which has been used in Ref. 4 and some other studies, states that if for minimum phase fades we employ a two-ray transfer function shown by

$$H(\omega) = a[1 - b \exp(-j(\omega - \omega_0)\tau)], \qquad 0 \le b \le 1, \tag{1}$$

in which b represents the fade notch depth and ω_0 is its displacement from the midband frequency, τ is the delay between the arrival times of the two rays, and a is the flat fade level, then for nonminimum phase fades we have to use

$$H(\omega) = a[1 - b' \exp(-j(\omega - \omega_0)\tau)], \qquad 1 \le b' \le 2, \tag{2}$$

in which b' is the fade notch depth. The second model suggests using

$$H(\omega) = a[b - \exp(-j(\omega - \omega_0)\tau)], \qquad 0 \le b \le 1 \tag{3}$$

for nonminimum phase fades, i.e., replacing the magnitude of the reference by that of the delayed ray. Finally, the third model, introduced in Ref. 6, prescribes the following transfer function under nonminimum phase fades:

$$H(\omega) = a[1 - b \exp(+j(\omega - \omega_0)\tau)], \qquad 0 \le b \le 1. \tag{4}$$

Comparing equations (1) through (4), it is obvious that the magnitudes of the transfer functions are the same at the fade notch frequency in all three different models and are all equal to

$$|H(\omega)| = a[1 + b^2 - 2b\cos(\omega - \omega_0)\tau]^{1/2}. \tag{5}$$

For the magnitude to be the same at the fade notch frequency in eq. (2), the value of b' has to be set to $(2 - b)$ so one can compare minimum and nonminimum phase fades of equal magnitude. This stems from the fact that the minimum phase fade depth here is defined as

$$B = 10\log(1 - b)^2, \qquad 0 \le b \le 1,$$

and for the nonminimum phase fades of eq. (2) to have comparable magnitudes with minimum phase fades, b' has to be equal to $(2 - b)$. Although the fade magnitudes are all the same, the envelope delay responses of the different transfer functions cited above are not. That is exactly why one has to be careful in choosing a model to form static nonminimum phase fades in multicarrier channels. The envelope delay response as a function of frequency for the minimum phase transfer characteristic in eq. (1) can be found by using

$$T(\omega) = -\frac{d}{d\omega}\Phi(\omega),$$

where $\Phi(\omega)$ is the transfer function phase response; hence,

$$T_{\mathrm{MPF}}(\omega) = \frac{b\tau[b - \cos(\omega - \omega_0)\tau]}{1 + b^2 - 2b\cos(\omega - \omega_0)\tau}, \qquad 0 \le b \le 1, \tag{6}$$

where the subscript MPF stands for Minimum Phase Fade. In the following, for simplicity we use midband fades to prove our point, i.e., when ω in eq. (6) is equal to ω_0. The conclusions arrived at, however, are felt to be general. Hence, the envelope delay of a midband minimum phase fade from eq. (1) is

$$T_{\mathrm{MPF}}(\omega_0) = \frac{b\tau}{b - 1}, \qquad 0 \le b \le 1. \tag{7}$$

For the first, second, and third nonminimum phase fade transfer functions the envelope delay response is

$$T_{\mathrm{NMPF}}(\omega_0) = -T_{\mathrm{MPF}}(\omega_0) + 2\tau, \tag{8}$$

$$T_{\mathrm{NMPF}}(\omega_0) = -T_{\mathrm{MPF}}(\omega_0) + \tau, \tag{9}$$

and

$$T_{\mathrm{NMPF}}(\omega_0) = -T_{\mathrm{MPF}}(\omega_0), \tag{10}$$

respectively, where the subscript NMPF stands for nonminimum phase fade. Notice that the first and the second models introduce constant delays of 2τ and τ in the envelope delay response, respectively, and these delays have nothing to do with the nonminimum phase fade nature. To prove the point, in Fig. 1 we show the overall impulse response of a single minimum phase faded channel using a 7.5-dB midband fade notch. As seen, the optimum timing reference, t_{opt}, is to the left of the unfaded signal optimum timing reference, i.e., the time origin. This is of course due to the negative delay characteristic. Using a 7.5-dB midband nonminimum phase faded transfer function of the third model, the overall impulse response of the system for 45-percent roll-off Nyquist-shaped transmit/receive filters is shown in Fig. 2. Note that the optimum time reference is now to the right of the time origin owing to the positive delay character, and the impulse response is time reversed about the timing reference when compared to the corresponding one for the minimum phase fade of Fig. 1. The first or the second model will translate to the right the

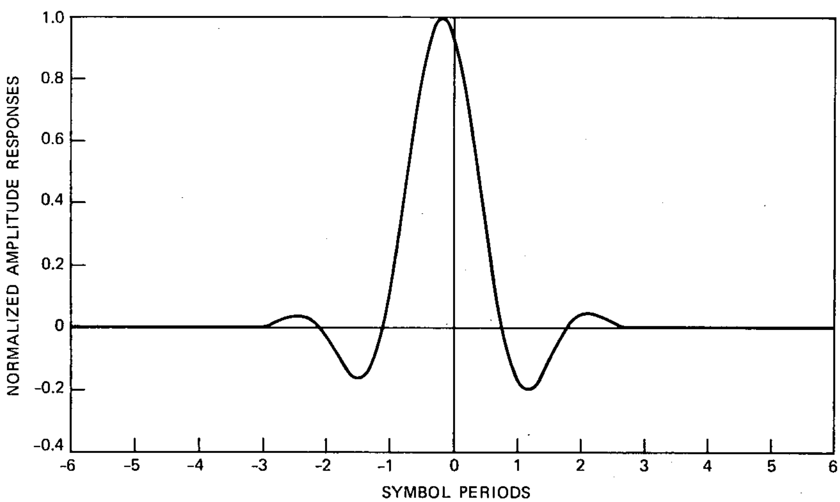

Fig. 1—In-phase received impulse response of a single-carrier system for a minimum phase fade.

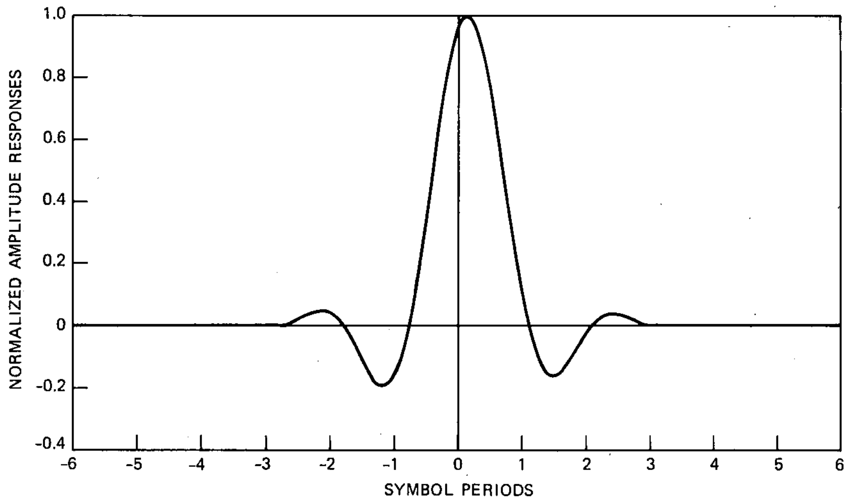

Fig. 2—In-phase received impulse response of a single-carrier system for a nonminimum phase fade.

position of the optimum timing reference of the nonminimum phase faded impulse response of Fig. 2 by 2τ or τ, respectively. Such a translation is harmless in single-carrier systems because once the optimum timing reference is established, taking samples of the impulse response at every baud period results in the same set of samples for minimum and nonminimum phase fades regardless of the actual position of the timing reference. However, in dual-polarized, or generally in multicarrier systems, the position of the timing reference can be as important as the sampling points in time. To demonstrate this, we use the dual-polarized channel model shown in Fig. 1 of Ref. 1, and illustrate the received in-phase impulse responses formed by the main copolarized and the cross-coupled paths in Fig. 3. These responses are composed of the real part of the main-path signal $U_{i,\mathrm{I}}$, cross-coupling of the main-path quadrature phase part $U_{q,\mathrm{I}}$, cross-coupled in-phase part of the cross-coupled path $U_{i,\mathrm{II}}$, and cross-coupled quadrature phase part of the cross-coupled path $U_{q,\mathrm{II}}$. Using notations of Refs. 1 and 5, the in-phase received waveforms shown in Fig. 3 correspond to 7.5-dB midband minimum phase fading of the main-polarization path and a cross-coupled path fading of (−20., 0., 0.),* that is, an interfering cross-coupled signal with a flat fading level of 20 dB below the main-polarized signal level. Hence, the main-path impulse response timing

* This triplet describes the fading status of the cross-coupled path. The first number shows the flat fading level of the interfering signal compared to that of the main-path signal. The second number is the dispersive fade notch depth, and the third one is the fade notch position.

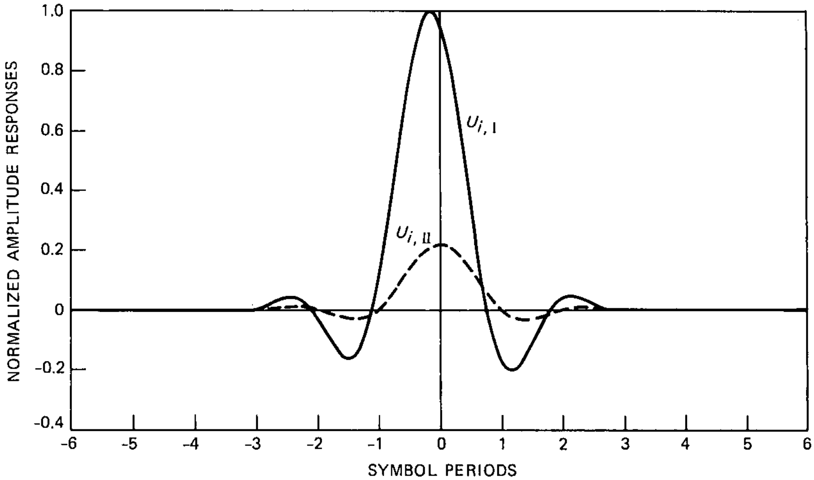

Fig. 3—In-phase received impulse responses of dual-polarized system for a midband minimum phase fade on the main path and a flat fade on the cross-coupled path.

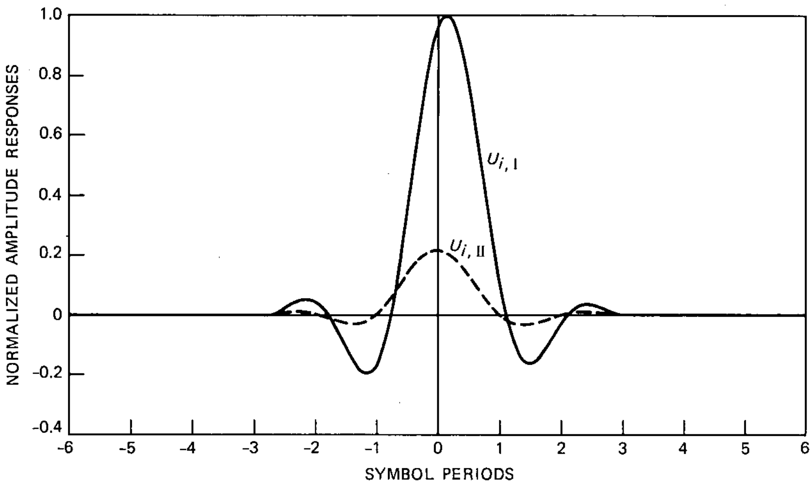

Fig. 4—In-phase received impulse responses of dual-polarized system for a midband nonminimum phase fade on the main path and a flat fade on the cross-coupled path.

reference is to the left of the time origin because of its minimum phase fading, and the cross-polarized path impulse response is an attenuated Nyquist pulse centered at the origin because of its flat fading, as expected. The corresponding shapes for an equivalent nonminimum phase fade of the main path, using the third model, are shown in Fig. 4. As seen, the interferer impulse-response shape does not change; however, the main-path impulse-response timing reference moves to the right of the time origin, and the impulse response shape itself is time reversed. Now in the dual-polarized system, once the receiver timing recovery circuit locates the optimum time reference by minimum-peak-distortion or minimum-mean-square error criterion, the main and the interfering impulse responses are sampled using the same reference timing, as explained in detail in Ref. 5; hence, the unnecessary time shifts inherent to the first and the second nonminimum phase fade models will result in establishing an incorrect reference point and a false set of samples of the cross-coupled interfering impulse responses. For further illustration, the displacement of the optimum timing reference from the origin as a function of the fade notch offset for different fade notch depths is shown in Fig. 5. Clearly, the upper half-plane represents nonminimum phase and the lower half corresponds to minimum phase fades. We employed the second nonminimum phase fade transfer function model in this case. As seen, the two sets of curves are vertically symmetrical except for a constant time shift of 6.3 ns which happens to be the value of τ used in this example. The constant time shift is what the transfer function model

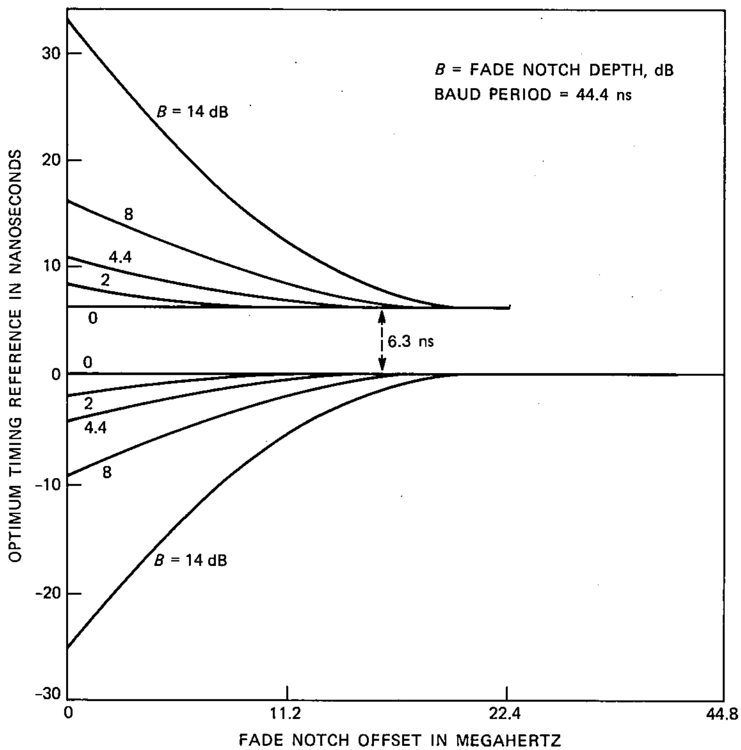

Fig. 5—Constant time shift in optimum timing reference by the second nonminimum phase fade model.

imposes on the overall impulse response. The method of establishing the optimum reference timing point in all the figures shown is the minimum-peak-distortion, trial-and-error method, described in Ref. 5. To avoid the unnecessary time shifts introduced by the first and second nonminimum phase transfer function models, we will use the third model in the canceler performance evaluation of Section III.

2.2 System model

The system model and cross-coupled interference canceler are the same as those introduced in Figs. 1 and 6 of Ref. 1, respectively. The channel model allows for a single frequency-selective fade notch, both in the reference copolarized and the cross-coupled paths; therefore, it requires the two-ray model parameters of both paths.

The canceler structure shown in Fig. 6 of Ref. 1 operates on the baseband received digitized signals. The main-lobe estimators and decision circuits together make preliminary estimates of the main lobe of the impulse response of the reference copolarized path. These estimators can be made of two to three tap transversal equalizers, and decisions made by detection circuits are provided to the cross-polarization canceler transversal filter to form and then eliminate the interfering main lobe from the opposite polarization received signal that has been properly delayed. Then, the cross-coupled interference canceled signals are equalized for mitigating intersymbol interference and cross-rail interference. The tap coefficients of the main-lobe estimators can either be derived from the preliminary decision circuit error signals or, to get a better performance, they can be set by using the error signals of the final decision circuits, as shown in Fig. 6 of Ref. 1. The slow channel time variations allow the use of the final error signals in estimating the tap coefficients of the estimators.

III. NUMERICAL CALCULATIONS OF CANCELER PERFORMANCE IN NONMINIMUM PHASE FADES

In this section we assume two 16-QAM signals are dual-polarized and transmitted over the channel model shown in Fig. 1 of Ref. 1 assuming *nonminimum phase fading of the main path.* The cross-coupled path is assumed to be flat *or minimum phase faded.* For nonminimum phase fades we use the third model described in Section II. The transmit/receive filters are the Nyquist type of 45-percent roll-off. The symbol rate is chosen to be 22.5 Mbaud and a signal-to-noise ratio of 60 dB is assumed. Performance signature (M-curve) for a 10^{-3} symbol error rate is used in the numerical evaluation, and the Gauss quadrature method[5] is used to derive the probability of error, accurately.

Before presenting the calculated results, it might be helpful to consider the following two examples. In the first example the main path is nonminimum phase faded with a fade notch depth of 7.5 dB and a notch offset of 11.2 MHz from the band center. The cross-coupled path in this case is assumed to be flat faded by 20 dB. Clearly, the cross-coupled path signal has a perfect Nyquist shape and is centered at the time origin. The main-path signal timing reference is to the left or the right of the origin depending on whether the main-path fade is minimum or nonminimum phase. However, since the amount by which the timing reference is translated has the same absolute value, as seen in Figs. 6 and 7, once the timing reference is found, the main and the cross-coupled signals are sampled using the same reference; hence, the same set of interferer samples results, regardless of the type of the fade. Therefore, the resulting end performance should not change. However, this is no longer true when the cross-coupled path is faded dispersively because the interfering impulse response is no longer symmetrical about the vertical axis going through the origin. This is shown in Figs. 8 and 9 where, in addition to a 7.5-dB depth and an 11.2-MHz offset fading of the main path, the cross-coupled path is faded by a 5-dB midband fade. Hence, depending on whether the main-path fading is a minimum or nonminimum phase, different sets of samples of the interfering impulse response are involved and the end performance may not necessarily remain the same.

The system performance signatures for two different fading conditions of the cross-coupled path with and without the canceler of Ref. 1 are shown in Figs. 10 and 11. In Fig. 10 it is assumed that the cross-

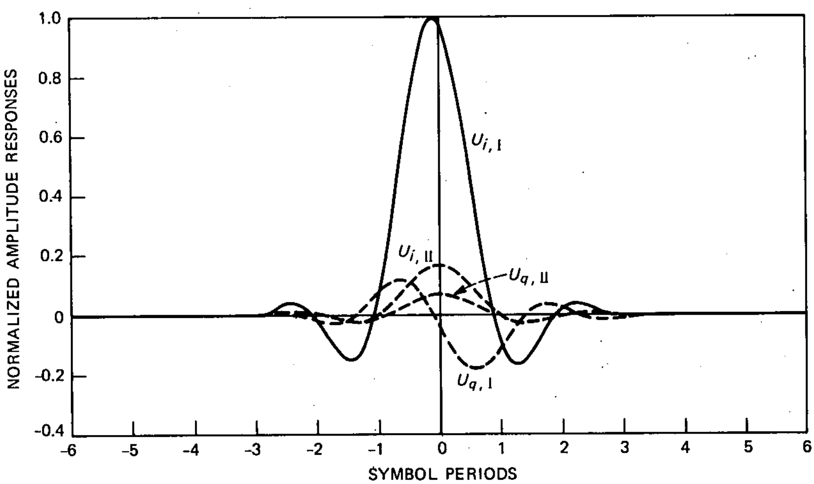

Fig. 6—In-phase received impulse responses of dual-polarized system for an offset minimum phase fade on the main path and a flat fade on the cross-coupled path.

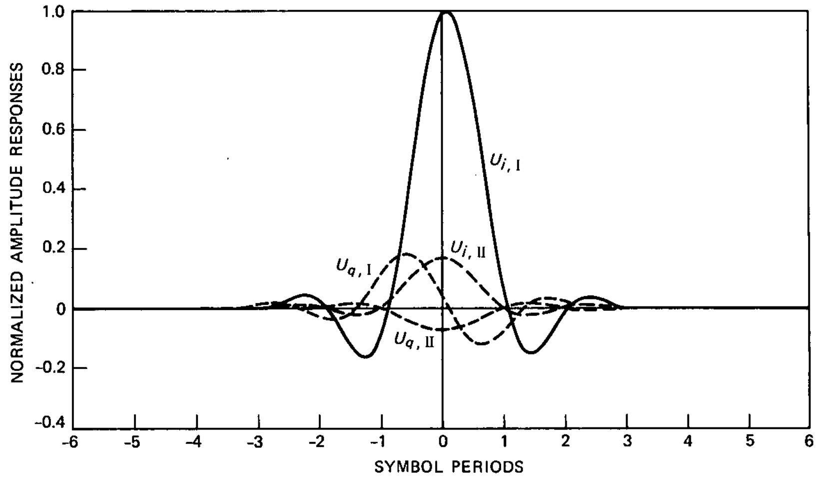

Fig. 7—In-phase impulse responses of dual-polarized system for an offset nonminimum phase fade on the main path and a flat fade on the cross-coupled path.

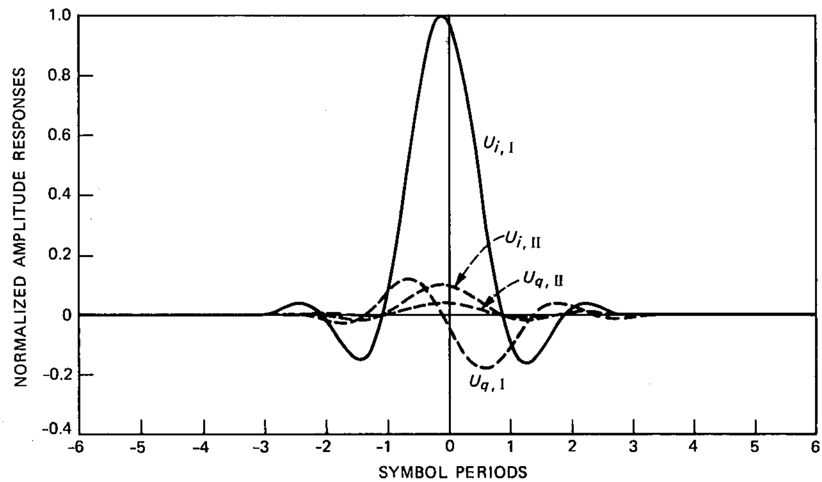

Fig. 8—In-phase impulse responses of dual-polarized system for an offset minimum phase fade on the main path and a midband minimum phase fade on the cross-coupled path.

coupled path is only flat faded; hence, the nonminimum phase performance signatures with and without the canceler are identical to the minimum phase faded signatures of Fig. 7 in Ref. 1. As seen in Fig. 10, the single-tap canceler[1] improves the dual-pol system performance to nearly that of a single-pol system shown by a dotted M-curve. For midband fades, as observed in Fig. 10, the timing reference of the main-path impulse response is offset from the peak of the main lobe

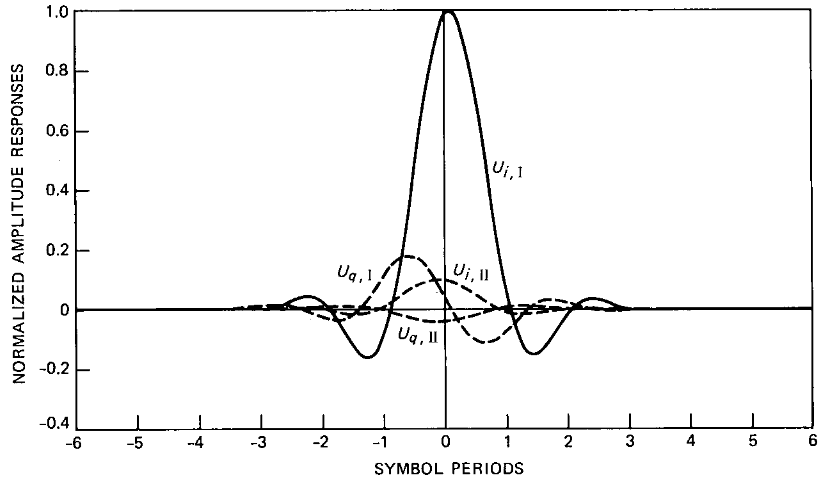

Fig. 9—In-phase impulse responses of dual-polarized system for an offset nonminimum phase fade on the main path and a midband minimum phase fade on the cross-coupled path.

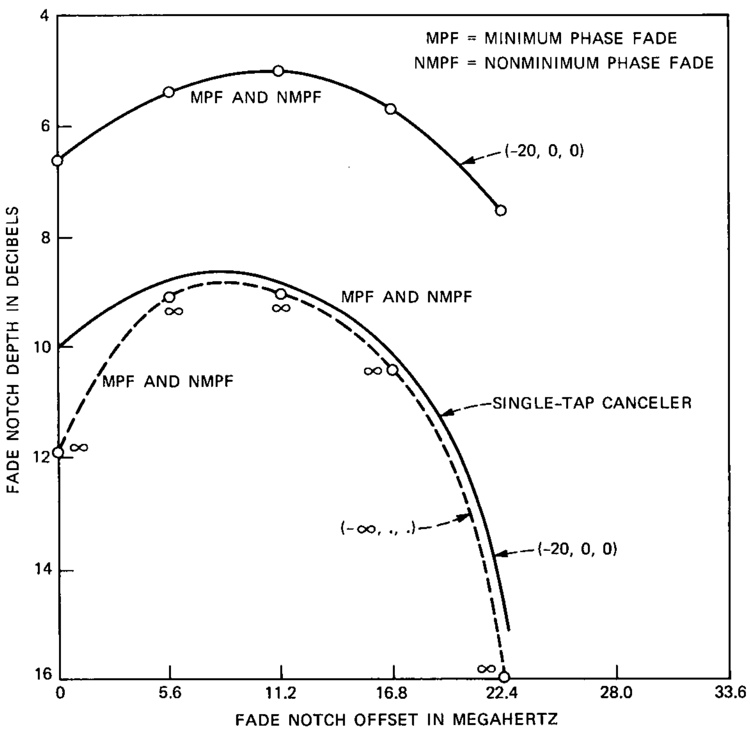

Fig. 10—Canceler performance in dual-polarized 16-QAM radio for a flat fade on the cross-coupled path.

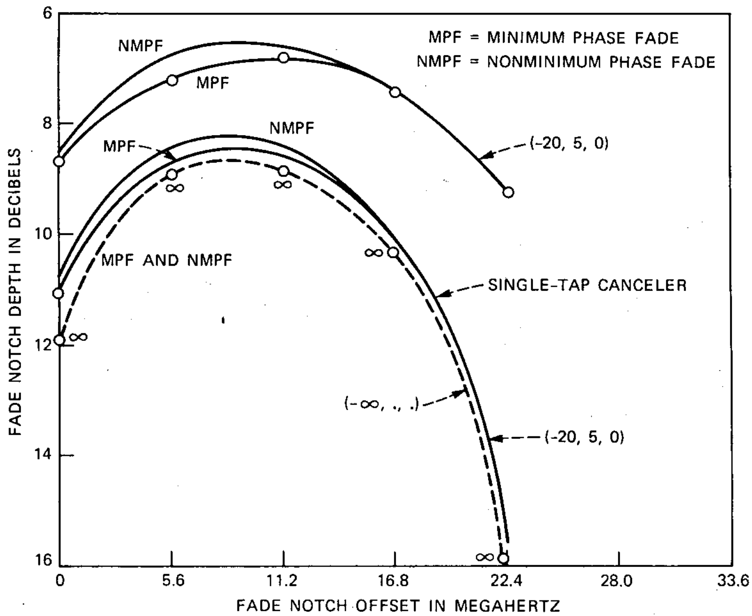

Fig. 11—Canceler performance in dual-polarized 16-QAM radio for a midband dispersive fade on the cross-coupled path.

of the interfering impulse response shape; hence, the canceler does not perform as well for midband fades as it would for offset fades of the main path because as the main-path fade notch moves toward the band edge, the timing reference of the overall impulse response moves toward the origin; hence, the two peaks tend to align. Of course, the remedy for this situation is to increase the number of canceler transversal filter taps. In Fig. 11, we consider the case where the *cross-coupled path is dispersively faded by a minimum phase midband fade of 5-dB notch depth.* In this case, since by asymmetry two different sets of samples of the interfering impulse responses are involved, the resulting signatures under minimum or nonminimum phase fades are not the same, and the nonminimum phase fade increases the outage slightly for the same cross-coupled path fading conditions. As the fade notch moves away from the band center, and since the reference timing of the main-path impulse response moves toward the time origin for both minimum and nonminimum phase fades, the two sets of samples taken of the interfering impulse responses become similar; hence, the minimum and nonminimum phase fade signatures approach one another and become approximately the same around the band edge.

A comparison of the signatures for minimum and nonminimum phase fadings of the main path indicates that the baseband canceler performance is relatively insensitive to the type of the fade as anticipated in Ref. 1.

Notice that the difference in the signatures for minimum and nonminimum phase fading of the copolarized channel is because we have assumed a minimum phase fade for the cross-coupled channel in both cases. Given that the copolarized and the cross-coupled channel fading have both minimum or both nonminimum phase, the signatures are indeed identical in both cases.

IV. CONCLUSIONS

In this paper performance of a previously proposed canceler[1] is evaluated under nonminimum phase fades, and it is shown that the canceler performance is practically transparent to the type of the fade. An investigation of different static models for nonminimum phase fades is also performed, and it is shown that some of the known fading models for single signal transmission may not be appropriate for multicarrier systems such as dual-polarized channels.

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