

## Methodology of Accelerated Aging

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(Manuscript received December 20, 1983)

Outlines are given for eight alternative black-box (i.e., input-output) methodologies that are appropriate for estimating, from external characteristics, the reliability of semiconductor lasers or other gradually degrading manufactured products with lifetimes too long to measure directly over practical time spans. These reliability estimates, which are essential for various components of such systems as submarine communication cables or satellites, are obtained from two classes of data. One class consists of the measured properties of statistically equivalent components, i.e., samples from the manufactured population, that have been operated to failure or at least to a significant degree of degradation. This degradation is often brought about in a shortened time span through the application of a temperature or other "accelerating stress" that is large compared to the operating temperature or other stress of the intended application. The other class of data is, for each component, comprised of the predeployment properties of that very component, including particularly its own predeployment degradation rate (which may also be measured under accelerating stresses). Brief consideration is given in passing to important special cases when only one of these two classes of data is available.

### I. INTRODUCTION

It is usually desirable to have reasonably accurate estimates of, or at least bounds on, the reliability (expected operating life, failure rate, etc.) of a semiconductor device or package or other manufactured component before it is put into service. This is particularly true in the

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case of submarine communication cables or satellites where the replacement of a failed component is enormously expensive or even impossible. Unfortunately, the lifetime of a component often cannot be directly measured. That is, most degradation mechanisms are irreversible, and so the lifetime of a particular component cannot be measured before that very component is put into service, because measuring the life wears out the component. Furthermore, the possibility of measuring the lifetimes of statistically equivalent components under operating conditions and then inferring, in a statistical or population sense, the distribution of lifetimes of the components going into service is also sometimes ruled out simply because the median component lifetime (>100 years) and the planned system life greatly exceed any acceptable delay in the deployment of the system. Nevertheless, there is a need to obtain predictions of the life-spans of even such long-lived components. For example, the development of a submarine cable or satellite that requires the survival of 90 percent or even 99+ percent of the components for years or decades of system life is undertaken only if the median component life can be persuasively shown to greatly exceed the system life.

As a consequence of the need for estimates of component lifetimes, the semiconductor industry continues to develop procedures for estimating lifetimes by extrapolation from data obtained over relatively short periods of time. These procedures are clearly flawed by the logical and practical limitations that are typical of schemes for predicting the future, but they have, nevertheless, often turned out to be sufficiently accurate to the extent that many semiconductor components now have a credible claim to *predictable* reliability. In the present paper we formalize some elements of an accelerated-aging methodology appropriate for semiconductor lasers and other components (devices, packaged assemblies, manufactured products, etc.) that are characterized by gradual degradation and by possibly temperature- or stress-dependent operating characteristics. The methodology concentrates on estimating, rather than bounding, the reliability. Bounding, which is often sufficient for the clearly acceptable and the clearly unacceptable components, usually takes the form of a simple limiting approximation to an estimate.

While microscopic studies of the degradation physics play an important complementary role, which is often specific to each component type, the present work is devoted to the macroscopic black-box (input-output) methodology, which is more or less applicable to every component. A black-box component is described by choosing a set of *independent variables* consisting, in the case of a laser, of such macroscopic observables as temperature and optical-output power (total power in all modes). Under test conditions these variables are pre-

scribed functions of time (often constant or piecewise-constant functions), where time is also an independent variable. Other macroscopic observables, such as current, voltage, and optical power in the fundamental transverse mode, are then the *dependent variables* or *dependent observables*. The changing values of the dependent observables are the measures of degradation. *Failure* or *wear-out* is the point at which the observables have so degraded that the component no longer meets the requirements of the particular application of interest. (Thus only part of this work is applicable to components which fail suddenly\* without prior influence on practically accessible observables.)

When a component goes into service, its life is, in general,<sup>†</sup> predicted using two classes of data: a statistical characterization of the population from which it is drawn (as inferred from measurements on samples) and observations of the predeployment rate of aging and other predeployment properties of that single component. The population data are usually “accelerated”; that is, a statistically equivalent group of components is worn out, or at least appreciably aged, in a short time period by the application of high temperature, high current, and/or other high stresses. The measured high-temperature degradation must then be extrapolated down to the system operating temperature. For the individual component’s own predeployment data, and sometimes for the population data, a further extrapolation in time of the initial rate of aging is also required in addition to any temperature extrapolation. Thus, the central issue of accelerated aging and of this work is the methodology of these temperature (stress) and time extrapolations.

For carrying out the extrapolations with respect to time and temperature (or other stresses) we discuss eight (i.e., 2<sup>3</sup>) alternative frameworks which are differentiated from each other by the following three dichotomies. First, a given framework uses either the (1) *activation* or the (2) *extrapolation* description. With the activation description, the degrading effects of various temperatures (or other accelerating stresses) are measured at a common reference temperature (or

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\* Often a failure is “sudden” only because relevant observables are not monitored. For example, a formerly sudden bond detachment may become predictable if the deterioration of the bond is inferred by adding the voltage to the monitored observables of a laser operated at constant current. Similarly, the onset of self-pulsation in a laser might become predictable if the ringing time or the peaks in the noise-power spectrum were monitored while the laser was still stable. To avoid the limitations of sudden-failure methodology, it is often desirable to make the effort to find a gradually degrading observable that is well correlated with failure.

<sup>†</sup> In particular cases only one class of data is available. If no degrading observable (precursor) is known (e.g., Ref. 1), or if cost considerations limit the measurements to samples, then only population data are available. If there are no spares to use as samples or if there is no time to degrade samples, then a component’s life is estimated only from its own predeployment characteristics.

stress level). With the extrapolation description, the effects of each temperature or other stress are measured at that temperature or stress level. Second, each framework considered uses either the (1) *replacement* or the (2) *high-reliability* strategy. For replacement components (expected component life  $\lesssim$  expected system life), such as automobile tires, where the component is likely to fail during the system (vehicle) life, the statistical distribution of lifetimes and, particularly, the mean component lifetime are of direct interest. On the other hand, for high-reliability components (as defined by the criterion that expected component life  $\gg$  expected system life), such as submarine-cable lasers or the springs and contacts under the keys of a cheap hand calculator that is discarded at the first failure, it is only the early-component-life reliability that affects the system performance, and it is thus sometimes more efficient to terminate measurements at the expected system life (as possibly contracted by temperature or other degradation-accelerating stresses), and to use a description based on the degree of degradation observed over the system life. In short, with the replacement strategy one measures the distribution of lifetimes at a given degree of degradation (failure), whereas with the high-reliability strategy one measures the distribution in the degree of degradation at the end of a given time (e.g., the system life or the guarantee period).<sup>\*</sup> Third, each framework that we discuss uses either the (1) *sampled-population* or the (2) *truncation* approach. With the sampled-population approach, every deployed component from a given manufactured population has the same statistical failure description as inferred from measurements on sample components drawn from that population. With the truncation approach, each deployed component has its own statistical description based not only on samples from the population but also on its own predeployment degradation rate and its other predeployment characteristics. This permits discarding those components likely to fail early (e.g., the components that initially degrade rapidly) and results in a distribution of deployed-component lifetimes that is a truncation of the distribution given by the sampled-population approach.<sup>†</sup>

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<sup>\*</sup> As a variation on the high-reliability strategy, the data are extrapolated past the system life to failure. The lifetime distribution is then analyzed using the replacement methodology with the understanding that the results are accepted with confidence only over the system life.

<sup>†</sup> We do not discuss explicitly the case where the absence of population data requires that a component's reliability be predicted only from its own predeployment characteristics. When the observed predeployment characteristic is the degradation of an observable, this is similar to the extensively treated problem of extrapolating the motion of a comet. Such an extrapolation is practical for a comet because of the existence of a known law of motion. Thus, extrapolation accuracy is limited only by measurement error. In contrast, for a given type of semiconductor component, there is typically no parameterized law of degradation-versus-time that is precisely applicable to all, or even

This paper is organized as follows:

- I. Introduction
- II. Lifetime prediction and improvement
- III. Activation factor
- IV. Activation energy
- V. Extrapolation factor and energy
- VI. An example
- VII. Comparison of activation and extrapolation methods
- VIII. Differential forms
- IX. Sampled-population methods
- X. Truncation
- XI. Summary.

Following a largely qualitative summary of typical procedures and underlying assumptions (Section II), Sections III through VIII describe primarily the deterministic degradation of a single black-box component as it depends on time, temperature, and other stresses. Populations of degrading components are then considered statistically in Sections IX and X.

## II. LIFETIME PREDICTION AND IMPROVEMENT

To improve the early lifetime, the median lifetime, or other reliability measures of shipped or deployed components, or to increase the confidence with which the relevant lifetime can be predicted, it is common to undertake certain reliability procedures during and/or after the manufacturing sequence. Specific procedures, or combinations of procedures, are known by such names as inspecting, performance evaluating, burning-in, accelerated burning-in, gettering, outgassing, tailored accelerated screening,<sup>2</sup> high-stress screening,<sup>2,3</sup> over-stressing, purging,<sup>4-6</sup> stabilization,<sup>4,6</sup> etc., and the reader is referred to the literature for their explicit descriptions. In the present section we simply outline a framework for characterizing such reliability procedures.

Following the remarks in Ref. 7 we note that many reliability procedures, including most of those we discuss, are premised on the tacit assumption of deterministic degradation. Determinism means here that, under given external conditions, each component degrades in a manner that is a unique consequence of its initial state; i.e., the component wears out. The observed component-to-component differences in operating life and in performance are thus understood as

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to any, components, even if the parameter values are chosen separately for each component. Thus, the inaccuracies of the "law," rather than measurement error, may be the main barrier to extrapolation. To evaluate this inaccuracy, population data on aged samples are required, as discussed in Section X, and thus we are back to the case of combined population and individual-component data.

consequences of the unintentional initial component-to-component differences that inevitably result from the limitations of any materials-preparation and manufacturing process. In contrast, the assumption of determinism is, for example, inapplicable to predictions about the subsequent imbalance between two initially equal radioactive radiation sources. Also, variations in lifetimes due to imprecisely anticipated variations in external conditions (e.g., a heat wave or a trawler attack on a submarine cable) are part of an overall system analysis, but are not considered in this work.

A given reliability procedure may either *extend* the life of the deployed components (i.e., yield a deployed population with a greater life than that of the original as-manufactured population), or *characterize* their life, or both. Thus aging samples to failure provides a characterization (e.g., estimates of the mean population lifetime, variance in lifetime, etc.), but it does not extend the lifetime of the population. On the other hand, a burn-in (predeployment operation) results in a survivor population that may have a greater lifetime than that of the original population. However, a burn-in does not, in itself, provide any verification that a lifetime extension has, in fact, occurred.

A given reliability procedure may be either *nonaccelerated* or *accelerated*. Thus, a burn-in may be carried out under the normal operating conditions of the deployed system, or it may be carried out to the same degree of degradation in a shortened time period through the application of stresses (temperature, current, humidity, etc.) exceeding those of normal operation.

A given reliability procedure is *nondestructive* or *destructive* according to whether or not there are survivors available for deployment. Aging samples to failure is destructive of the samples, whereas a burn-in or an inspection yields deployable survivors.

A nondestructive procedure is *benign* or *altering* depending on whether or not the component is changed by the procedure. There are *passive* benign procedures such as inspecting for a missing mirror coating or for a bad bond, and there are *active* benign procedures such as inspecting the voltage-current relation or other operating characteristics of a component. (The usual assumption is, of course, that the failure criterion associated with an inspection eliminates a subset of the components with a shorter expected life and leaves a population of survivors for deployment with a longer expected life.) In contrast to benign methods, procedures such as burning-in, annealing, out-gassing, gettering, or stabilizing typically alter the components, for example, by causing partial wear-out.

Most lifetime-affecting procedures are intended to produce a deployed population for which the relevant lifetime is longer than that of the original population. This can be achieved in any of three ways

according to whether the lifetime of the deployed components is (1) shorter than, (2) approximately the same as, or (3) longer than the lifetime of those very same components (same subset of the original population) before the procedure was carried out. For example, a successful burn-in yields deployed survivors with a longer relevant life than that of the original population. Even so, the lifetime of each survivor is shortened by the burn-in period. In contrast, active or passive inspection has little or no effect on the lifetime of the components that survive the inspection. Finally, certain procedures, such as the addition of a getter (agent for impurity removal) or a high-temperature controlled-environment outgassing, may actually increase the life of most or all survivors, whether or not the survivor yield is 100 percent. (Once the efficacy of such a procedure has been established, the procedure may well become just one more step in manufacturing; i.e., the lifetime distribution of the as-manufactured population would then be correspondingly redefined upward.)

Some of these distinctions can be quantified as we now show by considering an example that is hypothetical except for the real data points of Fig. 1. The example is based on the replacement strategy: the data represent the distribution of lifetimes for a fixed degree of degradation (failure), but an analogous example could equally well be constructed using the high-reliability strategy, i.e., the data would represent the distribution of the degradation\* for a fixed time (e.g., the system life or guarantee period). Figure 1 is the experiment of Ref. 7, as completed after the publication of Ref. 7. It shows the lifetime  $\tau$  (for producing a given level of optical output power) for GaAs lasers of a particular design and manufacturing sequence.

The data of Fig. 1 were obtained with aging at 70°C. To say that (increasing) temperature is an *accelerating stress* implies, in the broad sense of acceleration, that aging at a higher temperature yields a distribution that falls below that shown in Fig. 1. In the narrower sense that we will use, valid acceleration of a population implies that the two distributions differ from each other only in a vertical displacement in  $\ln\tau$ . Figure 2a shows a valid accelerating stress, whereas Figs. 2b and c show stresses that are accelerating only in the broad sense. In all three cases the medians  $\langle\tau\rangle$  and  $\langle\ln\tau\rangle$  are changed by the stress, but in Figs. 2b and c the shape of the distribution, and hence, such other properties as  $\sigma$ , the standard deviation in  $\ln\tau$ , are also changed. Once the existence of a valid accelerating stress has been established by demonstrating a vertical shift between the measured

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\* For example, if  $L_f$  were the failure (minimum useable) value of the optical output power and  $L_i$  were the initial value at a common current, then one possible candidate for the ordinate in Fig. 1 under the replacement strategy would be the lifetime estimate  $(L_f - L_i)t/\Delta L$  [or just  $(L_f - L_i)/\Delta L$ ] rather than  $\tau$ , where  $\Delta L$  is the output change in a fixed time  $t$ .

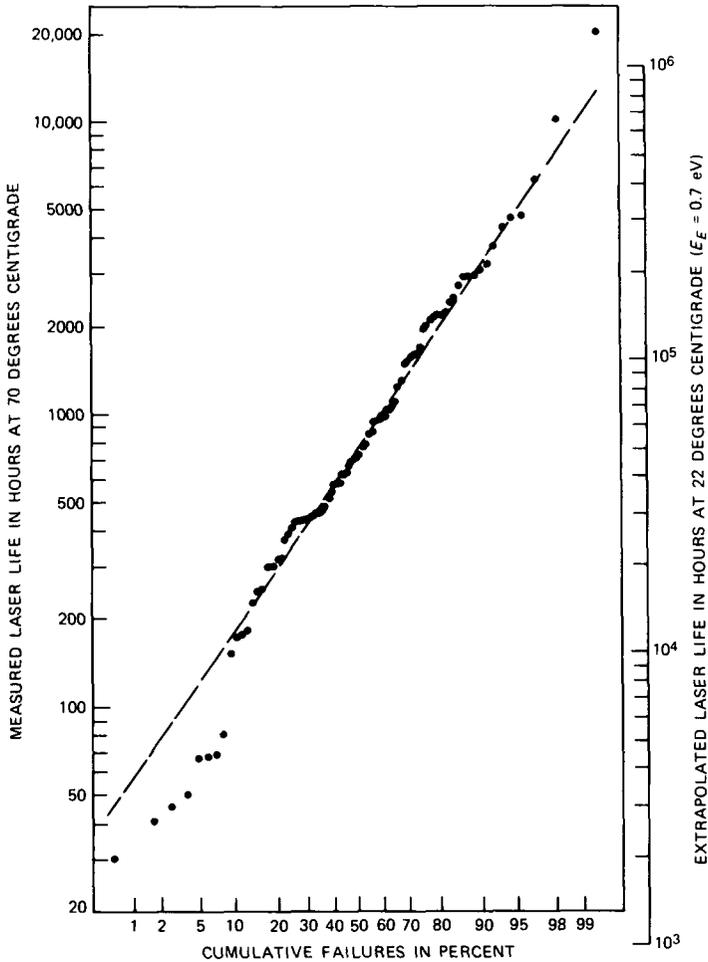


Fig. 1—Cumulative distribution on lognormal paper of AlGaAs laser lifetimes as measured at 70°C (left ordinate) and as extrapolated to 22°C with the Arrhenius relation (right ordinate).

distributions at different stress levels (e.g., two temperatures), then further data need only be taken at the higher level, and then the distribution at the lower level can be inferred by relabeling the ordinate in the manner of the right-hand scale in Fig. 1. An additional possible implication of a valid acceleration is that the lifetime has been shortened by the same factor for every component; i.e., the clock runs faster by a universal acceleration factor. This further implication should, however, be verified by step stressing; that is, by aging samples individually at two or more stress levels to see that they have, to within experimental error, a common acceleration factor, rather than

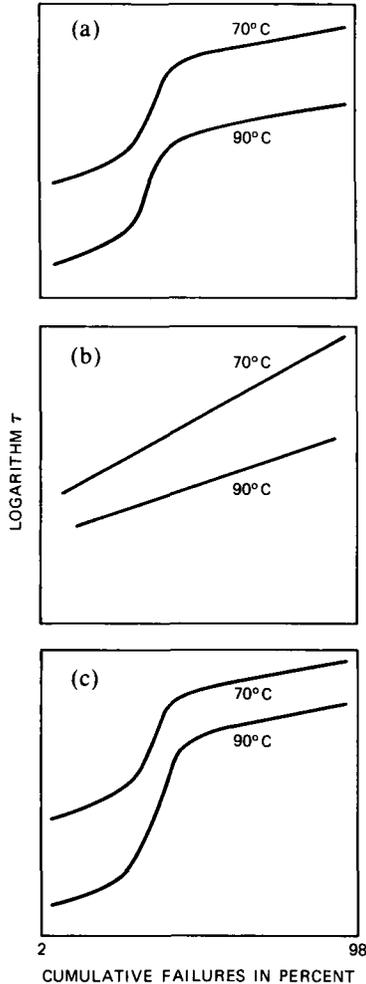


Fig. 2—Cumulative distribution of the lifetime  $\tau$  with isothermal aging at 70°C and at 90°C. The ordinate is the logarithm of  $\tau$  and the abscissa is an unspecified distribution. (a) Temperature is a valid accelerant; i.e., the clock runs faster by the same acceleration factor for all components, and the curves are vertically displaced. (b) Temperature is an invalid accelerant that accelerates different components differently, but conformance to the distribution (linearity of the curve) is preserved (see Fig. 5). (c) Temperature is a separately valid accelerant for each of the two subsets of the population.

different acceleration factors that are simply poorly correlated with the lifetimes.

Occasionally, accelerating stresses that yield the results of Figs. 2b or c can be utilized by invoking an acceleration factor that varies with time during aging or varies from component to component. For example, there may be two failure mechanisms (infant and normal), each with its own constant acceleration factor as in Fig. 2c, where the

curves become parallel on both sides of the bend. Often, the data are insufficient to justify high confidence in predictions based on a variable-acceleration-factor strategy. However, for example, it may be possible to screen out the components with the infant mechanism and leave a survivor population with a common acceleration factor. Alternately, as in Fig. 2b, one can sometimes scale both the median and the standard deviation. To give a physical example of how such a situation could arise we will later show, in conjunction with Fig. 5, that in some cases a single degradation mechanism can have an acceleration factor that varies from component to component.

After the first 10 percent or so of the lasers in Fig. 1 have failed, the dashed-line fit shows that remaining failures are well approximated by the lognormal distribution (the logarithm of the lifetime is normally distributed). Thus, in a replacement application where the mean and standard deviation of the lifetime are of paramount significance, it might be sufficient to use lognormal statistics to predict component reliability.

On the other hand, as is often observed in component lifetime distributions, the first few percent of the data fall significantly below the lognormal line, and lognormal statistics would be much too optimistic in a high-reliability application where only the first few percent of the failures were of importance. One high-reliability strategy would then be the use of a more realistic distribution, and indeed the data of Fig. 1 have been well fit by a double lognormal distribution.<sup>8</sup> This is a self-consistent approach, but not necessarily an appealing approach because of the high initial failure rate  $-N^{-1}dN/dt$  ( $N$  = number of surviving lasers) arising from the characteristic lower-than-lognormal lifetimes of the early failures.

An alternative approach, assuming that all known practical methods for improving the lifetime of the lasers have already been incorporated into the manufacture, is to accept a yield reduction and to try to somehow pick out the longer-lived lasers. Suppose for definiteness that the best 40 percent ( $y = 0.4$ ) of the lasers can be used for a particular application. What method should be used to select the 40 percent? The brute-force approach is a burn-in. Using the dash-line lognormal fit (median life =  $\tau_m = 750$  hours; standard deviation in  $\ln\tau = \sigma = 1.1$ ) shows that the 40-percent yield, 60-percent cumulative-failure point ( $c = 0.6$ ) corresponds to  $\tau = 991$  hours, i.e., a 40-percent yield would occur after about 991 hours of burn-in. To construct the post-burn-in distribution (the  $B_{y=0.4}$  curve in Fig. 3), we note, for example, that the 80-percent point on the original distribution (point  $i$  in Fig. 3) becomes the 50-percent point of the survivor distribution ( $i \rightarrow j$ ) and that its life is reduced by the 991-hours burn-in ( $j \rightarrow k = 991$  hours). By this means the original lognormal curve (LN) is mapped into the burn-in curve ( $B_{y=0.4}$ ). (Because the first 60 percent of the

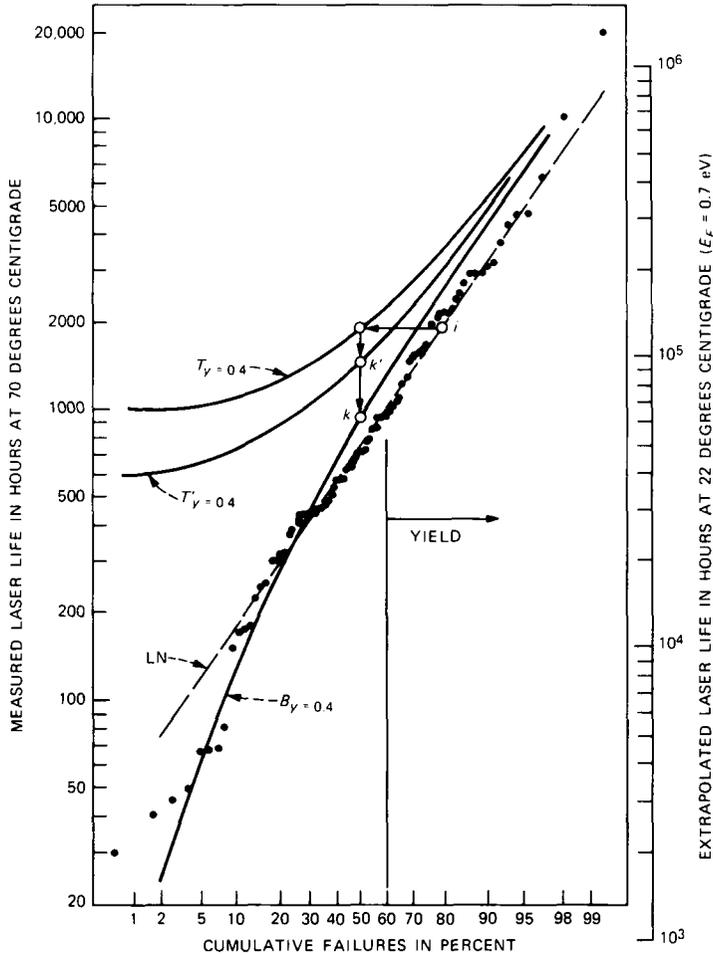


Fig. 3—LN is the lognormal distribution of Fig. 1.  $B_{y=0.4}$  is the 40-percent yield burned-in survivors from LN (991-hour burn-in).  $T_{y=0.4}$  is the 40-percent yield truncation of LN.  $T'_{y=0.4}$  is the 400-hour burn-in (stabilization plus monitor times) 40-percent yield truncation of LN.

original lasers do not affect the  $B_{y=0.4}$  curve, it does not matter that the first 10 percent or so of them fall below the LN curve.)

Several conclusions can be drawn from a comparison of the B and LN curves: After about 25-percent failure, the B curve is longer-lived than the original data, e.g., the median is raised from 750 to 900 hours. Thus the burn-in is of some value for a replacement application. Below  $c = 25$  percent, the B curve is marginally distinct from the original data; i.e., the burn-in strategy costs time and money, wastes 60 percent of the population, and offers no early-life improvement for a high-reliability application.

The lognormal distribution is not invariant to a burn-in; that is, the straight LN line results in a curved B line. The lack of invariance of the lognormal distribution under many such yield-reducing procedures and the common presence of partial yield at each stage of manufacture imply that the lognormal distribution may often be only an approximation to the actual lifetime distribution. In particular, the fall-off of the B curve in Fig. 3 at low cumulative failure comes not from any physical mechanism that would cause additional early (infant) failures, but rather from a burn-in or from any manufacturing stress that subtracts a common number of hours from all devices in an otherwise lognormal distribution. Thus in particular cases other distributions, such as the double lognormal, the exponential, or the Weibull distribution, or burned-in or truncated modifications of these distributions, may prove more accurate.

If the burn-in strategy were pursued, the large number of hours required (991) means that an accelerated burn-in would likely be sought. Measurement of the associated acceleration factor is not normally a difficulty; that is, the accelerating stress (current, humidity, vibration, increased temperature, etc.) is simply applied until 60 percent of the lasers have failed. The problem is verification that a burn-in has, in fact, occurred (as distinguished, for example, from the precipitation of a new otherwise irrelevant failure mechanism which might yield a better or worse distribution than a burn-in). Evidence for or against the occurrence of an accelerated burn-in comes from aging to failure a sample of the survivors of the tentative accelerated burn-in and comparing their lifetime distribution to the B curve.

If it were decided not to use a burn-in to eliminate *all* of the 60 percent, it still might prove desirable to eliminate some of the 60 percent by using a shorter burn-in. The merits of such an approach could be assessed by similarly constructing the  $B_y$  curves for smaller values of  $y$ . In practice a burn-in more often proves useful for eliminating only a small fraction of the components.

Ideally it would somehow be possible to identify the first 60 percent to fail without, however, using up any of the life of the other 40 percent. The surviving distribution (curve  $T_{y=0.4}$  in Fig. 3) is, in mathematical terms, a truncation of the original distribution, i.e., point  $i$  becomes point  $j$ , etc., as before, except that now there is no burn-in-time penalty ( $j \rightarrow k = 0$ ). Clearly, the truncated curve is greatly superior to both the original and the burned-in curves, particularly for high-reliability applications where the early failures are important. (Incidentally, the curvature in the truncated distribution shows that the lognormal distribution is not invariant to a truncation procedure.)

Inspection of the static or dynamic (operating) properties of the components is an appealing method for eliminating early failing de-

vices because inspections do not use up any of the life of the survivors. Also, no life is consumed for those components that fail the inspection, and thus they are available for other, less demanding applications. On the other hand, in contrast to a burn-in that automatically eliminates the early failing components, an inspection criterion may be imperfectly correlated with device life. The quality of a given inspection criterion is judged by comparing the lifetime distribution of the inspection survivors with the  $T_y$  curve constructed for the yield of that inspection. To the extent that the survivor curve falls below the  $T_y$  curve, the inspection criterion is rejecting at least some longer-lived components and passing some shorter-lived ones. As with burn-ins, known inspections often prove useful only for eliminating a modest fraction of the population.

Another truncation strategy, which is incorporated, for example, in the purge<sup>4-6</sup> and the tailored-accelerating-screen<sup>2</sup> procedures, is premised on the following physical model: All components eventually fail because all possess a common wear-out mechanism. In addition, certain flawed components are also afflicted with a second failure mechanism having a shorter life under normal operating conditions. Evidence suggesting this two-mechanism model is the so-called S-shape of the data in Fig. 1 that corresponds to a high failure rate in the early failing branch of the S. (However, as noted above in conjunction with the B curve, a fall-off of the early failing end of the distribution is no proof of a second mechanism because a fall-off can occur for other reasons.) In the case of the data of Fig. 1 there is physical evidence for two mechanisms; that is, postmortem inspections of failed lasers of this type show that early failing lasers typically exhibit dark-line defects<sup>9</sup> in their active layers whereas longer-lived lasers usually fail without developing inhomogeneities in their spontaneous emission patterns viewed through windows in their contacts.<sup>10</sup> Figure 2c shows another form of evidence for two mechanisms. The two distributions are vertically displaced (same slope) on either branch of the S curve, but the acceleration factor (vertical displacement) is different for the right branch (longer-lived mechanism) than for the left branch (second mechanism).

Where there is a shorter-life mechanism present in some components it may be possible to find a *selective* stress which accelerates this mechanism to component failure without, however, appreciably accelerating the common mechanism.<sup>2,4-6</sup> For example, a briefly applied reverse-bias current may be virtually harmless to a uniform laser, but the current may be so concentrated into the defect of a laser with a dark spot or line that the current-voltage characteristic of the laser with the defect is altered or destroyed almost instantly. The justification for a claim that selectivity has permitted an improvement over a

cumulative distribution obtainable from a burn-in is, again, a comparison with the  $T_y$  and  $B_y$  curves for the relevant yield, i.e., the data should at least fall above the  $B_y$  curve. (If the data were to fall above the  $T_y$  curve as well, then the selective stress is also affecting the unafflicted survivors in some way that increases their life.) In contrast to a burn-in, the selective-stress method is relatively inflexible with respect to yield; that is, if only 10 percent of the lasers are flawed with a second mechanism, then a selective stress cannot go very far toward identifying the 60 percent with the shortest lives. In common with a burn-in and in contrast to an inspection, a selective stress often destroys all but the surviving components. However, it is certainly possible in principle to develop a selective test (inspection) which identifies the second mechanism nondestructively.

Another important approximation to the truncation ideal is the *projection method* or rate monitoring in which each component is aged for a short while during which its degradation is monitored. Then the observed degradation is projected into an estimated life for that component. (Alternatively, with the high-reliability strategy it is the degree of degradation at the end of the system life that is obtained by projection.) The deployed survivors are, in the case of our example, the 40 percent with the longest projected lives. As with other procedures, the monitor-period degradation may be conducted on an accelerated basis.

A particularly convincing demonstration of the effectiveness of the projection method was given by English et al., who predicted the life of titanium-gold thin-film conductors from the predeployment rate of increase of each thin film's resistance.<sup>11</sup> The method has also proved useful for semiconductor lasers since the early days of reliability studies on such devices. In the case of Ref. 7, the projection of a laser's life, defined as the ability to emit some minimum optical-output power, was based on the early rate at which the drive current had to be increased in order to maintain that power. Ninety lasers were then aged to death and a comparison of their projected and actual lifetimes justified the conclusion that "a typical laser degrades fairly steadily toward a time of death which can be reasonably accurately estimated from the laser's early degradation rate."<sup>7</sup>

The confidence in the projection method can be assessed, and the accuracy of the method can be improved, if some samples from the population are aged, under normal or accelerating stresses, through the projection period (i.e., aged to failure with the replacement strategy or aged through the system life with the high-reliability strategy). This is discussed in Section X.

The accuracy of the projection method can also be improved if the population is first made more homogeneous, for example, by selectively

eliminating components with a second, shorter-lived failure mechanism or by inspecting for manufacturing defects.

Finally, the projection method is sometimes improved if the components are *stabilized*<sup>4,6</sup> in advance of the monitoring period on which the projection is based. The main idea of stabilization is to drive to saturation or to anneal any initial transients in the degradation rate. For example, a degrading surface current that is in parallel with the primary drive current of a device will distort the measurement of the degradation of the primary current. This surface current might reach a saturation value in a relatively short period of time under normal operation, or it might be possible to selectively accelerate the surface current to saturation through the temporary introduction of a selective stress (e.g., a particular atmosphere) that has little effect on the mechanism of the primary-current degradation. A difference between stabilization and the selective acceleration of a second mechanism of a flawed device is that in only the former case is the mechanism of a type that can be driven to saturation without causing device failure.\*

The projection method may be regarded as only a variation on the inspection method because both methods are based on a correlation between some predeployment measurement and the subsequent life of the same component. The projection method differs from the inspection method because it uses up some of the component's life. For example, if stabilization plus the monitor period were equivalent to four hundred hours of operation, then, for a 40-percent yield, the expected distribution in the case of perfect lifetime prediction is the  $T'_{y=0.4}$  curve in Fig. 3 ( $j \rightarrow k' = 400$  hours). Unless the stabilization process happened to actually improve the individual component's life, lifetime experiments on samples from the 40 percent with the longest predicted lives would be expected to yield a distribution which fell somewhere between the  $T'_{y=0.4}$  curve (perfect correlation between the projected and actual lifetime) and the burn-in curve  $B_{y=0.716}$  (no correlation between the initial degradation rate and the actual lifetimes). (A yield  $y = 71.6$ -percent corresponds to a 400-hour burn-in; this curve is not shown in Fig. 3.) Failure to exceed the  $B_y$  curve suggests that the monitored degrading observable is uncorrelated with the actual failure mechanism and that any improvement over the original distribution arises simply from the burn-in aspect of the stabilization and monitor period.

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\* The term stabilization is also sometimes used to mean simply waiting until the degradation rate has become sufficiently low. This arises, for example, when the degradation rate is described by a bounding function (such a linear time dependence extending the last observed tangent to a sublinear curve) rather than by an actual fit; i.e., in the context of bounding the life rather than of estimating it.

Certain procedures that we have outlined involve extrapolations with respect to time and temperature or other stresses. In the remainder of this paper we discuss these extrapolations in more detail. Sometimes we will assume, for example, that the acceleration factor for a given component does not change over its life, or that all components in the population have the same acceleration factor, etc. In each such case it is understood that appropriate screening procedures, such as inspection, stabilization, or selective acceleration, have already been identified and carried out as necessary in order to ensure the applicability of the assumption in question to the survivors.

### III. ACTIVATION FACTOR

Following the discussion in footnote 7 of Ref. 7, we develop in this section the first of two procedures for quantifying the acceleration that results from operation at enhanced stress levels. The treatment is based on four temperatures, some of which may be identical in particular cases:

$T_a$  = the higher aging temperature

$T_a'$  = the lower aging temperature

$T_r$  = the reference temperature where measurements are made

$T_s$  = the system temperature for deployed components.

In conventional terminology a fundamental assumption underlying typical predictions about a given type of device is that an *accelerating stress* (temperature, humidity, radiation, current, electric field, mechanical vibration, etc.) or combination of stresses can be found for that component which, when applied at levels in excess of the intended application, brings about, in a shortened length of time, the *same* process and degree of degradation that occurs in the intended application. In a macroscopic theory the degradation that a device or package undergoes is measured by the changing value of at least one macroscopic dependent observable  $I$ . For example,  $I$  could be chosen as the threshold current  $i$  of a semiconductor laser (or the current at some other value of the optical power), and  $I$  is then observed to degrade (increase) more rapidly as the temperature, or current, or other stress is increased. As another example,  $I$  is the coupling efficiency of a transmitter (fraction of the optical power from the laser that enters the fiber mode).

A macroscopic observable is, however, a potentially ambiguous measure of degradation if, for a given amount of degradation, its value also depends on the temperature (or on whatever stress is used for acceleration). For example, the threshold current of a semiconductor laser is typically strongly temperature dependent. Similarly, in addition to the irreversible degradation in time of the coupling efficiency

due, say, to solder creep, there may also be a separate reversible dependence of the coupling efficiency on temperature due to a mismatch of the thermal-expansion coefficients of the components of the structure supporting the laser and the fiber. The simplest way to remove this additional dependence, and to achieve thereby a unique relation between the amount of degradation and the value of the observable, is to make all measurements at one reference temperature even though the aging is carried out at two or more higher temperatures. Thus, we make the notion of acceleration precise as follows. As shown in Fig. 4, for the case where the stress is temperature  $T$ , a device is raised to the higher aging temperature  $T_a$  (path 1-2), aged for time  $\Delta t_a$  (path 2-3), and then brought back\* to the lower aging temperature  $T_{a'}$  (path 3-4) where it is found (horizontal-line comparison 4-5 in Fig. 4b) to have experienced the same degradation (same change in  $I$ ) that would have occurred (solid-line path 1-5 in Fig. 4b) in time  $\Delta t_{a'}$  at  $T_{a'}$ . The acceleration factor or *activation factor*  $m$  between  $T_a$  and  $T_{a'}$  is then defined by<sup>7</sup>

$$m = \Delta t_{a'} / \Delta t_a, \quad (1)$$

i.e., aging at  $T_a$  is equivalent, in this particular sense, to making the clock run  $m$  times faster than it runs at  $T_{a'}$ .

Equation (1) should be contrasted with other commonly advanced definitions of acceleration. In eq. (2) of Peck and Zierdt,<sup>3</sup> for example, acceleration is defined in terms of a scaling of the dependent observable  $I$  rather than of time. Thus their activation factor, which they represent as the exponential of an activation energy, would change if  $I$  were redefined as the logarithm of, or as some other function of, the threshold current rather than as the threshold current  $i$ . Since  $i$  and, say,  $\ln i$  or  $i^2 \ln T$  are equally good observables, it is not easy to say which, if any, of their corresponding activation energies is more closely associated with any energy associated with a microscopic degradation process, nor is it otherwise easy to establish any unique value for this Peck-Zierdt activation factor or activation energy. With the null-technique definition of eq. (1) the activation factor does not change with these redefinitions of  $I$ . Furthermore, in eq. (1) the effect of aging at both  $T_a$  and  $T_{a'}$  is expressed in terms of measurements made only at  $T_{a'}$ ; thus, we also contrast eq. (1) with those definitions of  $m$  (or of the activation energy) in which the effect of aging at each temperature is expressed in terms of measurements made at that temperature. In

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\* In practice the component may, or may not, actually be brought back to the lower stress level. For example, when eq. (22) is known to apply, the consequences of temperature lowering are inferred theoretically as in eqs. (28) through (31). Alternately, one may find experimentally a condition on  $I$  or  $T_a$  that corresponds to the condition on  $I$  which defines  $\Delta t_r$  at  $T_r$ .

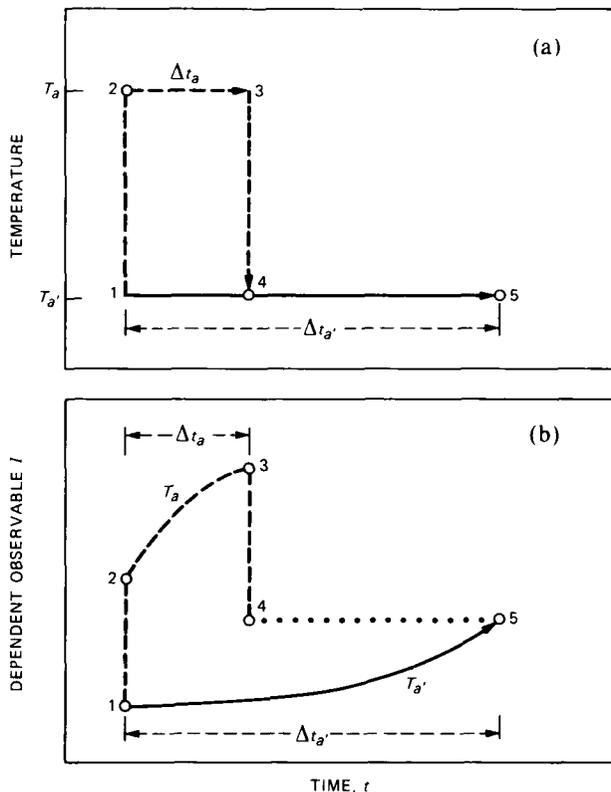


Fig. 4—The change (degradation) of an observable  $I$  (e.g., threshold current of a laser) as a function of time ( $t$ ) and temperature ( $T$ ). (a) The paths in time-temperature space. (b) The time dependences of the observable  $I$ .

addition to yielding an  $m$  that varies with the definition of  $I$ , these two-temperature-data definitions yield an  $m$  that almost always varies from device to device because of device-to-device differences in the temperature dependence of  $I$  or in the end-of-life criterion. (See Section V for an example based on a power-supply limitation.) By avoiding the additional arbitrariness associated with measurements at two temperatures, the definition of eq. (1) seeks to narrow the device-to-device differences in  $m$  and, hence, to uncover a single population value for  $m$  if such a single value exists. A final advantage of eq. (1) arises from the fact that  $m$  exists even when  $I$  is undefined at  $T_a$ , as, for example, when  $I$  is the threshold current (or the current at some higher value of the optical-output power) and  $T_a$  exceeds the maximum temperature for lasing.

It is useful to extend eq. (1) in two ways. Firstly, for graphical simplicity, Fig. 4 compares aging at  $T_a$  and  $T_{a'}$  on the basis of measurements made at a reference temperature  $T_r$  that is equal to  $T_{a'}$ .

Clearly the equality of aging at  $T_a$  and  $T_{a'}$  (path 4–5 in Fig. 4b) can be shown from measurements made at any reference temperature  $T_r$ . Thus, if  $T_a$  and  $T_{a'}$  are the aging temperatures, then eq. (1) and the value of  $m$  are independent of the reference (measurement) temperature  $T_r$ . Secondly, it is often convenient to think in terms of rates rather than times. The reciprocal of the elapsed time at a given aging temperature is proportional to the degradation rate (clock rate)  $R$  at that temperature. Thus, eq. (1) becomes

$$m = \Delta t_{a'}/\Delta t_a = R_a/R_{a'}. \quad (2)$$

The last equality of eq. (2) expresses  $m$  (a two-temperature function) as the ratio of two one-temperature degradation rates. Because the  $R$ 's are defined in terms of a ratio, they are indeterminate with respect to a common multiplicate constant; that is, we have defined relative rates at two aging temperatures, not an absolute rate at each temperature. For an example see eqs. (28) through (30).

When the degradation is reversible (e.g., can be annealed out) the measurement of  $m$  is conceptually straightforward. Otherwise,  $m$  can be measured on a population basis by comparing the degradation times or rates for two statistically equivalent populations aged at two temperatures (*isothermal aging*)<sup>2,3,6,12</sup> or, on an individual-device basis, by partially aging a single device alternately at two stress levels (*step stressing*)<sup>3,6,12,13,14</sup> and then scaling the time intervals at  $T_a$  by that  $m$  that yields a smooth  $I(t)$  curve, all measurements of  $I$  being made at  $T_r$ . (Any transients associated with the temperature change are often small enough to be confidently removed in the analysis with short backward extrapolations of the post-transient data.)

With either approach for measuring  $m$  it is necessarily assumed that the lower aging temperature  $T_{a'}$  is chosen to be high enough so that a measurably large amount of degradation occurs at  $T_{a'}$  in the time available. This means that in some applications it is possible to choose  $T_{a'}$  as the system operating temperature  $T_s$ , while in other cases one is forced to choose  $T_{a'}$  above  $T_s$ , and determination of  $m$  between  $T_a$  and  $T_s$  also requires an extrapolation (see Section IV).

Most of the parameters in this work can be similarly measured on a population basis or on an individual-device basis, and an extrapolation may, or may not, be required. Because we do not treat the optimization of experimental procedures, we will not discuss the measurement of each parameter in detail.

In practice the methodology often reduces to fitting the data as well as possible with a linear degradation model ( $I$  changes linearly in time at constant  $T$ ), i.e.,

$$m = \frac{R_a}{R_{a'}} = \frac{\partial I_r / \partial t_a}{\partial I_r / \partial t_{a'}}. \quad (3)$$

In eq. (3),  $\partial I_r/\partial t_a$  is the isothermal degradation rate when  $dt_a$  is the elapsed time at  $T_a$  and  $\partial I_r$  is the resulting change in  $I$  as measured at  $T_r$ , that is, for small  $\Delta t_a$  and for  $T_r = T_{a'}$ ,

$$\frac{\partial I_r}{\partial t_a} \approx \frac{I_4 - I_1}{\Delta t_a} \neq \frac{I_3 - I_2}{\Delta t_a}, \quad (4)$$

with  $I_4$  and  $I_1$  as in Fig. 4b. Thus, eq. (3) becomes

$$m = \frac{(I_4 - I_1)/\Delta t_a}{(I_5 - I_1)/\Delta t_{a'}} = \frac{\Delta t_{a'}}{\Delta t_a}, \quad (5)$$

which agrees with eq. (2) and thereby justifies eq. (3). The invariance of  $m$ , when  $I$  is changed from  $i$  to some  $g(i, T)$  such as  $\ln i$  or  $i^2 \ln T$  can also be seen as follows. If  $I$  is replaced by  $g(I, T)$  in eq. (3), i.e., if  $\partial I$  is replaced by  $(\partial g/\partial I)\partial I$ , then the  $\partial g/\partial I$  of the numerator cancels that of the denominator because both are evaluated at the same  $I$  and at the same temperature (Fig. 4b).

Within a population of devices that are presumably degrading by a common mechanism, there invariably appear the expected device-to-device differences in the degradation rate that result from initially inequivalent devices. In addition there also sometimes appear less welcome device-to-device differences in the activation factor (as measured on an individual-device basis by step stressing). There thus arises the question of whether these differences in the activation factor should be assigned to measurement error (in which case a common population description of the activation factor should be used for each device) or whether real measurable device-to-device differences in  $m$  are to be expected (in which case reliability prediction for each device could sometimes be improved by measuring that device's own activation factor before putting it into service). Most of the literature tacitly assumes that any such differences are only experimental error. Our position on this question is that either case is possible, the choice depending upon the specifics of the mechanism. To justify this position we cite a mechanism of each type. On the one hand, for example, the temperature dependence of the degradation could be dominated by an energy level associated with the migration of an impurity or defect, and this level might well be negligibly affected by growth and manufacturing variations; i.e., to well within experimental error the level is essentially a material constant. On the other hand, consider, as in Fig. 5, a double-heterostructure device where the migration of an impurity or defect  $D$ , which crosses a p-type ternary-composition confining layer of  $\text{Al}_x\text{Ga}_{1-x}\text{As}$ , is driven by the energy of hole-electron recombination (recombination-aided diffusion).<sup>15</sup> The number of electrons which escape from the active layer and become available for recombination as minority carriers in the confining layer has a Boltzmann-factor dependence upon the conduction-band energy step  $E_1$  and hence

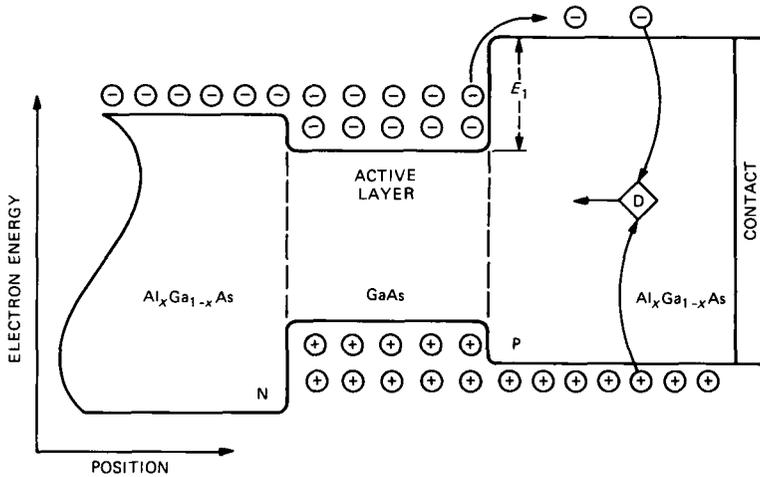


Fig. 5—Recombination-aided diffusion of a defect or impurity  $D$ , as controlled by the number of electrons which surmount the conduction-band barrier  $E_1$ . The magnitude of  $E_1$  is determined by the composition  $x$  of the p-type  $\text{Al}_x\text{Ga}_{1-x}\text{As}$  confining layer.

upon the composition  $x$  of the confining layer.<sup>16</sup> Because  $x$  can vary inadvertently from wafer to wafer, and even across a given wafer, apparently meaningful wafer-to-wafer<sup>12,17</sup> and device-to-device differences in the activation factor do not necessarily rule out the possibility of a single degradation mechanism.

For a concrete and simple hypothetical example of Fig. 5, assume that under lasing conditions the active-layer carrier concentration is the same for every laser and for every value of the temperature  $T$  and optical power  $P$ . Then the carrier concentration just inside the confining layer is  $n = \exp(-E_1/kT)$ . Assume that there is some observable  $I$  for which the recombination-aided degradation rate is  $R \sim n \sim \exp(-E_1/kT)$  and the lifetime is  $\tau \sim 1/R \sim \exp(E_1/kT)$ . Then if the laser-to-laser differences in  $E_1$  are the main causes of the lifetime differences and if  $E_1$  is normally distributed among the lasers, it follows that  $\tau$  is lognormally distributed except at the shortest of lifetimes (small  $E_1$ ) where Boltzmann statistics and small-signal modeling break down. Further, Arrhenius' rule is obeyed separately for each laser with  $E_1$  the activation energy. Finally, if  $\delta E_1$  is the standard deviation in the step heights, then  $\sigma$  in  $\ln \tau$  is given by a  $\sigma = \delta E_1/kT$  that is temperature dependent as shown in Fig. 2b. This sort of temperature-dependent  $\sigma$  is likely to occur often. Thus, we justify the common usage of a temperature-independent  $\sigma$  as follows:  $\delta E_1$  is sufficiently small that the dependence is weak over the relevant temperature range (which is small compared to the Kelvin temperature). Alternatively, certain empirical procedures automatically deduce "the" activation energy or factor over the relevant part of the distribution, e.g., the replacement strategy normally infers "the" activation energy at the

median of the distribution, and the high-replacement strategy automatically finds "the" activation energy at the early part of the degradation history.

For a more elaborate example let the degradation rate  $R$  be proportional to the product  $ns$  with  $n$  as before and  $s$  the concentration of the impurities  $D$  in an excited state with an excitation energy  $E_2$ ; i.e.,  $s \sim \exp(-E_2/kT)$ . Then  $\ln\tau \sim E/kT$  where  $E = E_1 + E_2$ , and the variance  $\sigma$  in  $\ln\tau$  is now given by  $\sigma = \delta E/kT = (\delta E_1 + \delta E_2)/kT$ . If the variance  $\delta E_2$  arises from weak impurity-impurity interactions of a low-concentration impurity, then  $\delta E$  might be negligibly small and thus further justify the usual temperature-independent- $\sigma$  approximation which underlies the valid-acceleration methodology.

It may, of course, turn out that aging at  $T_a$  yields a different form of degradation than aging at the system temperature  $T_s$ . For example, the device or its bond may melt at  $T_a$ , or a strongly temperature-dependent degradation mechanism,<sup>5,18</sup> which is of secondary significance at  $T_s$ , may become dominant at  $T_a$ . Then temperature, or at least the value  $T_a$ , is simply not a valid accelerating stress because it does not induce the *same*, or even approximately the same, physical degradation process as aging at  $T_s$ . For such reasons, the maximum available activation factor may be less than desirable, and the available time may not be sufficient for aging all the way to failure even at the highest valid stress level. Similarly, excessive current<sup>19</sup> or optical power<sup>20</sup> is known to introduce new degradation mechanisms in lasers. Incidentally, the suggestion of a different form of aging (invalid acceleration) occurs when two dependent observables are measured on the same device, and they yield unequal activation factors.

It may also turn out in some cases that the value of  $m$  depends upon the time interval  $\Delta t_a$  and the starting time. Then  $m$  is appropriately measured over the time interval of interest, which is typically the system life or the device life. Alternately, as in eq. (3), one considers the instantaneous activation factor  $m = dt_a'/dt_a$ , as approximated by measurements over short intervals.

It is common and preferable for the shape of the aging curve  $I(t)$  at  $T_a$  (line 2-3 in Fig. 4b) to be similar to the shape at  $T_a'$  (line 1-5 in Fig. 4b). However, Fig. 4b emphasizes the fact that the two shapes are not necessarily similar. For example, a physically remote, and possibly even aging, shunt path might conduct part of the current at  $T_a$ , but none at  $T_a'$ ,  $T_r$ , and  $T_s$ . Temperature might still be a valid accelerating stress for the degradation process of interest, although not necessarily an easy one to extrapolate, if the condition of the shunt path does not affect the observables at  $T_r$  and  $T_s$ . For another example of dissimilar shapes at different temperatures see the discussion in the paragraph following that with eq. (14).

In order to obtain a measurably large amount of degradation in the available time, it is often necessary that the lowest aging temperature  $T_{a'}$  be chosen above the system temperature  $T_s$ . Consequently, the activation factor  $m$  is used primarily as basis for extrapolation to  $T_s$  (see Section IV). However, we emphasize that the measured  $m$  is a directly useful quantity when  $T_{a'}$  and  $T_r$  can be taken equal to  $T_s$ . For example, consider the commonly encountered case where device-to-device and wafer-to-wafer differences in  $m$ , if any, are negligibly small in their effects compared to the corresponding differences in the device lifetimes. Then it would be appropriate to use a common value for the  $m$  of every device, with  $m$  estimated from a protracted measurement of  $\Delta t_{a'}$  or  $R_{a'}$  on samples, plus a brief corresponding measurement of  $\Delta t_a$  or  $R_a$  on a statistically equivalent sample. Thereafter, the population degradation rate or lifetime with operation at  $T_r = T_{a'}$  might well be usefully estimated for devices from each successively grown wafer or successively processed batch by using eq. (2) and only a (brief) measurement of  $\Delta t_a$  or  $R_a$  for sample devices from that wafer or batch.

Needless to say,  $m$  depends upon the operating conditions at  $T_a$ . For example, if a laser is operated at  $T_a$  with the same current as at  $T_{a'}$ , the value of  $m$  will, in general, differ from the value when the operation at  $T_a$  is at the same optical power as at  $T_{a'}$ . The point is that in both cases  $m$  may well turn out to be meaningfully, and even usefully, defined. For example, temperature  $T$ , current  $i$ , and optical power  $P$  are all potential accelerants for laser degradation, and it is not usually possible to change  $T$  while holding both  $i$  and  $P$  constant (unless, for example, one goes to such extremes as changing the mirror reflectivity in the case of a bulk-degradation mechanism). For instance, if  $P$  is held constant, then  $m(T_{a'}, T_a)$  tacitly includes some acceleration due to a changing  $i$ . The choice among holding  $i$ , or  $P$ , or some function of  $i$ ,  $P$ , and  $T$  constant while  $T$  changes is usually made on some practical basis, as exemplified in the following three paragraphs.

Aging at constant  $P$  is commonly used when the cause of degradation is unknown. Because  $i$  increases with  $T$  at constant  $P$ , a comparatively larger activation factor results, and neither  $i$  nor  $P$  loses influence by becoming negligibly small. In contrast, aging at constant  $i$  causes  $P$  to decrease with increasing  $T$ . This yields a comparatively smaller activation factor for given temperatures. Further, if  $P$  were the dominant cause, the activation factor could be small, or negative, or virtually impossible to extrapolate.

On the other hand, if  $i$  rather than  $P$  is known to be the dominant cause of degradation, then constant- $i$  aging has the advantage of permitting very large activation factors through the use of temperatures well above the maximum temperature for lasing.

Choices other than constant  $i$  and constant  $P$  have advantages. For

example, both constant- $P$  and constant- $i$  data are often taken, and it is then difficult to compare the results because of the differing activation factors at common temperature differences. Also it may be inefficient to generate two sets of data (constant  $i$  and constant  $P$ ) at temperatures  $T_a$  below the maximum temperature for lasing. If the current at fixed optical power follows an empirical rule  $i(T)$ —e.g.,  $i = i_0 e^{T/T_0}$ , where  $i_0 = i(T = T_0)$ —over much of the lasing temperature range of interest, then aging at  $i(T)$  at each  $T$  ages at constant  $P$  over much of the lasing range, and then attempts the extrapolation beyond the lasing range using a method that is as rational as the constant- $i$  method.

Constant- $T$  aging is also a possibility in principle since increased  $i$  and  $P$  both accelerated degradation to some extent.

One might try to apportion the cause of degradation between  $i$  and  $P$ . For example, below threshold, the degradation rate is modeled as a function of  $i$ , and this dependence is then extrapolated above threshold as a function of that part of the current that does not appear as optical power. Any observed above-threshold degradation that exceeds the extrapolated value is then attributed to, and modeled as a function of, the optical power  $P$ . This is experimentally difficult to carry out, and there is no assurance of success. For example, if the degradation were actually controlled by the active-layer carrier concentration, or by the current in one shunt path driven by the active-layer carrier concentration, then the apportionment between  $i$  and  $P$  would be misleading at best.

As a practical matter, confidence in a particular acceleration scheme over a particular temperature (stress) range comes largely from an observed smooth steadily increasing dependence of  $m$  on  $T_a$  over that range.

In summary, we have considered  $I = f(T_r, T_a, t_a)$ , which is the dependence on time  $t_a$  of the degradation of an observable  $I$  for a device or package which is isothermally aged at  $T_a$ , but which is periodically and briefly cooled for a series of measurements at a reference temperature  $T_r$ . Here  $f$  is an interpolated function that passes through the  $I$  values measured at  $T_r$ . If  $T$  (temperature or other stress) is a valid *accelerating stress*, then  $I$  is of the particular form

$$I = f(T_r, mt_a), \quad (6)$$

with an *activation factor*  $m$  given by

$$m = R(T_a)/R(T_r) = t_r/t_a. \quad (7)$$

In eqs. (6) and (7),  $t_a$  is the elapsed time (at  $T_a$ ) and  $R(T)$  is the so-called clock rate or degradation rate at  $T$ . A realistic degradation model of the type of eq. (6), where there is both a benign transient

and a wear-out mechanism, appears in Ref. 21. Equation (14) is a simple hypothetical example of eq. (6).

#### IV. ACTIVATION ENERGY

A reliability prediction for a device drawn from a population is often based, at least partially, on measurements of the degradation of sample devices from that population. For each sample device one would typically like to measure the isothermal degradation at the system temperature of at least one observable  $I$ . That is, one would like to observe the function

$$I = f_s(t_s) \quad (8)$$

at certain times or at all times  $t_s$  over the interval  $0 \leq t_s \leq t'$ , where  $t_s$  is the elapsed time at the system temperature  $T_s$ , and  $t'$  is at least as long as the lesser of the system life and the device life. In the usual case, however, the available measurement time is much less than  $t'$ . Then  $f$  must be inferred by an extrapolation that consists, in general, of two parts.

First the shape of  $f$  at  $T_s$  is found by the method of Fig. 6. That is, if temperature is a valid accelerating stress, the device is aged isothermally along the solid line labeled  $T_a$  in Fig. 6. Periodically and momentarily the device is cooled (dotted line) to  $T_s = T_r$ , where  $I$  is

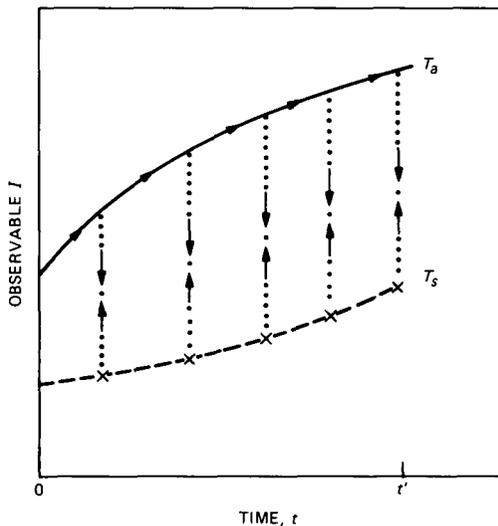


Fig. 6—The change (degradation) of an observable  $I$  which is aged isothermally (solid line) at  $T_a$ . Periodically and momentarily the component is dropped to temperature  $T_r$ , and returned to  $T_a$  (down and up each dotted line) to yield an interpolated curve (dashed line) between the data points (x's) at  $T_s$ .

measured ( $\times$ 's in Fig. 6). Interpolation between the data points yields the dashed line of Fig. 6, which can be written in the form

$$I = F_s(t_a). \quad (9)$$

$F_s$  in eq. (9) is an empirical function which gives the shape of  $I(t)$  for measurements at  $T_r = T_s$  and aging at  $T_a$ . Often  $F_s$  is then fitted with an analytical function.

Secondly, the inferred rate at which the shape would have been traced out, had the aging temperature been  $T_s$ , is found from eq. (7) by extrapolating the activation factor. Often it is shown empirically (at a series of temperatures  $T_a, T_{a'}, T_{a''}, \dots$  that are all high enough to induce appreciable degradation in the available time) or else it is just assumed that, to within the measurement error,  $R(T)$  is of the Arrhenius form

$$R(T) = R_\infty e^{-E_A/kT}, \quad (10)$$

where  $E_A$  (the *activation energy*) and  $R_\infty = R(T = \infty)$  are parameters. As indicated in the example associated with Fig. 5, the physical origin of the Arrhenius form is normally the Boltzmann distribution (which dominates the weaker algebraic dependence that usually also enters the degradation rate as prefactors to the Boltzmann exponential). Extrapolating, i.e., assuming that eq. (10) applies down to  $T_s$ , causes eq. (7) to become

$$m = e^{-E_A(1/kT_a - 1/kT_s)} = t_s/t_a. \quad (11)$$

Thus  $f_s$  of eq. (8) is given by

$$I = f_s(t_s) = F_s(t_s/m) \quad (12)$$

with  $m$  from eq. (11). Specific properties then follow immediately from eq. (12); for example, the time  $t_s^*$  when  $I$  reaches a particular value  $I^*$  (e.g., failure to meet system requirements) is

$$t_s^* = mF^{-1}(I^*). \quad (13)$$

The procedure of eqs. (9) to (13) assumes that measurements can be made at the system temperature ( $T_r = T_s$ ) even though there may not be enough time for perceptible aging at  $T_s$ . However, when it is undesirable (e.g., thermal-cycling-induced strain and degradation effects occur) or inconvenient to periodically drop the device temperature all the way down to  $T_s$ , a theoretical extrapolation of a parameterized degradation model is used instead of the empirical procedure of eq. (9). As a simple example, let  $I$  be the threshold current  $i$  of a hypothetical laser. Suppose that  $i$  depends exponentially on temperature at  $t = 0$  and suppose that at elevated temperatures  $i$  increases linearly in time, doubling in a period that depends upon the aging

temperature but not upon the reference (measurement) temperature. Then, if the Arrhenius rule is valid, eq. (6) takes the specific form of eq. (14),

$$\begin{aligned} I &= Ce^{T_r/T_0}(1 + R_\infty e^{-E_A/kT_a t_a}) \\ &= Ce^{T_r/T_0}(1 + R_\infty e^{-E_A/kT_r m t_a}), \end{aligned} \quad (14)$$

where  $C$ ,  $T_0$ ,  $R_\infty$ , and  $E_A$  are parameters, the parameter  $T_0$  being the temperature range over which  $I$  changes by a factor of  $e$  at  $t = 0$ . If  $E_A$  is determined from population measurements, then the parameter  $R_\infty$  can be found from the initial degradation rate at some elevated  $T_a$ . (See Section VIII if  $E_A$  must be found on an individual-device basis.) Extrapolation consists of invoking eq. (11) and assuming that eq. (14) is valid for  $T_r$  equal to  $T_s$ . Equation (8) is finally obtained by setting  $T_r$  equal to  $T_s$  and  $t_a$  equal to  $t_s$  in eq. (14).

In summary, if measurements can be made at the system temperature  $T_s$ , then the shape of  $I(t)$  can be measured at  $T_s$  and we need only extrapolate the rate at which the  $I(t)$  curve is traced out. If measurements at  $T_s$  are not possible, then the shape must also be inferred by extrapolation. Sometimes the extrapolation of the shape presents no problem, as when  $I$  degrades linearly in time. In other applications such an extrapolation is virtually impossible, and measurements must be made at the system temperature. For example, if  $I$  is the optical-power coupling efficiency<sup>22</sup> between a laser and a fiber, then  $I$  may or may not pass through a maximum according to whether the long-term creep of the alignment does or does not pass through a point of better alignment than the initial position.<sup>23</sup> Because of the changed alignment caused by differential thermal expansion, a maximum in  $I(t)$  under isothermal aging may occur only over a limited temperature range, i.e., the temperature dependence of the function  $I(t)$  can be too complicated to extrapolate with confidence.

Most of the comments on the activation factor that were given in the previous section are, of course, equally applicable to the activation energy. Thus, if the activation factor varies from device to device, so does the activation energy. Where other authors have defined their  $R$  or  $m$  differently than we have, so will the value of their  $E_A$  or  $m$  differ from ours.

The Arrhenius rule is not unique; for example, variations on the Eyring rate such as<sup>24,25</sup>

$$R(T) \sim T^{-1} e^{-E_A/kT} \quad (15)$$

or eq. (35) are sometimes endorsed. If alignment degradation is caused by material creep, then the Arrhenius rule is replaced by the empirical temperature dependence of creep for that material, as moderated by

any temperature dependence of the driving stress that may arise, for example, from imperfectly compensated differential thermal expansion. Also temperature is just an example here. For a different stress, or for a combination of stresses, a similar discussion applies except that  $R$  may have a different functional form. As an arbitrary example, recent experiments<sup>26</sup> suggest that under constant optical power the dark-line-defect mechanism of InGaAsP laser degradation may approximately scale according to the following two-parameter ( $E_A$  and  $J_0$ ) dependence on the two stresses  $T$  and  $J$ ,

$$R(T, J) \sim e^{J/J_0 - E_A/kT}, \quad (16)$$

where  $J$  is the junction current density. As another example the degradation of Ti-Au thin films by electromigration scales approximately as  $R \sim J^4$  where the stress  $J$  is the current density.<sup>11</sup>

## V. EXTRAPOLATION FACTOR AND ENERGY

In the preceding two sections the effect of a valid accelerating stress (or stresses) is characterized in terms of an activation factor and an activation energy. Following footnote 7 of Ref. 7 again, we give here an alternative characterization of the same accelerating stress (say, temperature) in terms of an *extrapolation factor* and an *extrapolation energy*. Both characterizations involve a comparison based on aging at two temperatures ( $T_a$  and  $T_{a'}$ ), but with the extrapolation characterization the effect of each temperature is represented in terms of measurements made *at that temperature*. Specifically, at least one observable rate  $\mathcal{R}$  is defined. The main criterion for the choice of  $\mathcal{R}$  is that it act as some measure of a device's life, and thus the units of  $1/\mathcal{R}$  normally contain time to the first power. For example,  $\mathcal{R}$  might be the reciprocal of the lifetime  $t^*$  when an observable  $I$  no longer meets a system specification—cf. eq. (13)—or  $\mathcal{R}$  might be the pre-deployment isothermal degradation rate  $\partial I/\partial t$ , where  $I$  is the current of a laser at threshold or the current at some other value of the optical power.

In analogy to eq. (2), an *extrapolation factor*  $m_E$  is defined by

$$m_E(T_a, T_{a'}) = \mathcal{R}_a/\mathcal{R}_{a'}, \quad (17)$$

where, in contrast to eq. (2),  $\mathcal{R}_a = \mathcal{R}(T_a)$  is the rate when both aging and measurements are at  $T_a$ . If  $\mathcal{R}$  is the initial or instantaneous rate, the form of eq. (3) would again be applicable

$$m_E = \frac{\mathcal{R}_a}{\mathcal{R}_{a'}} = \frac{\partial I_a/\partial t_a}{\partial I_{a'}/\partial t_{a'}} \quad (18)$$

except now, in contrast to eq. (3), it is understood that  $\partial I_a/\partial t_a$  is the

isothermal degradation rate with both aging and measurement at  $T_a$ . That is, for the case  $T_r = T_a$  of Fig. 2, one has

$$\frac{\partial I_a}{\partial t_a} \approx \frac{I_3 - I_2}{\Delta t_a}, \quad (19)$$

which contrasts with eq. (4).

For extrapolation one shows empirically, over some higher-temperature range, or theoretically that  $\mathcal{R}$  is of the Arrhenius,

$$\mathcal{R}(T) = \mathcal{R}_\infty e^{-E_E/kT}, \quad (20)$$

Eyring, or other (e.g., eq. [35]) form where the parameter  $E_E$  is called the *extrapolation energy*.<sup>7</sup> If, for example,  $1/\mathcal{R}$  is the lifetime  $t^*$ , and  $t^*$  is the quantity of interest, then setting  $T$  equal to  $T_s$  in eq. (20) completes the extrapolation. Alternatively,  $\mathcal{R}$  may appear in some analytic isothermal model

$$I = g(T, \mathcal{R}t), \quad (21)$$

for which  $E_E$  and any other parameters are determined by data fitting at higher temperatures. Equation (21) is then extrapolated by assuming validity for  $T = T_s$  and  $t = t_s$ .

We emphasize that  $m_E$  and  $E_E$  are rather arbitrary quantities. For example, suppose that the operational lifetime of each device from a homogeneous population with a range of initial currents corresponds to the time  $t^*$  when that device draws, at the system temperature  $T_s$ , a current  $i$  equal to the maximum output (100 mA) of its power supply, and suppose that a particular device has an initial current  $i_0$  of 50 mA. For that device the failure criterion at  $T_s$  can be expressed in various ways including  $i = 100$ ,  $i - i_0 = 50$ , or  $i = 2i_0$ . Reversible (no aging) heating of the device to the accelerating temperature  $T_a$  raises its initial current to 70 mA. Do we define the device's lifetime at  $T_a$  by the condition  $i = 100$ ,  $i = 120$  (i.e.,  $i - i_0 = 50$ ), or  $i = 140$  (i.e.,  $i = 2i_0$ )? Perhaps each of these definitions would yield a smooth temperature dependence for  $1/\mathcal{R}(T) = t^*(T)$  which could be extrapolated, but clearly each definition would have its own  $E_E$ . Furthermore, if two devices (with unequal initial currents) had the same  $E_A$ , they would typically have unequal  $E_E$ 's with most of these definitions. Thus one cannot speak of "the" extrapolation energy as if it were on an equal footing with the activation energy. Rather, there is, in general, a separate extrapolation energy for each definition of each observable. Even with a given definition of the observable and a common degradation mechanism, one typically expects each device to have its own extrapolation energy.

The above arbitrariness can be resolved if there exists a criterion at  $T_a$  that corresponds, for every device (every initial current), to 100 mA

at  $T_s$ . (See Section VI for an example.) Often, however, such a criterion cannot be found, except, possibly, in a rather approximate sense. Alternately one can, of course, periodically cool the device to see when it draws 100 mA at  $T_s$ , but then one has simply abandoned the extrapolation method in favor of the activation method.

In the case where the threshold current  $i$  of a laser is the dependent observable, and  $\mathcal{R}$  is the initial isothermal degradation rate  $\partial I/\partial t$ , arbitrariness results from the fact that  $I$  could be chosen as  $i$ , or as  $i^2$ , or as  $A(T) + B(T)\ln i$ , etc. Sometimes, this ambiguity can be partially resolved with the requirement that  $\mathcal{R}$  scale time rather than  $i$ , i.e., by finding an  $I = f(i)$  which changes linearly in time for all devices in the population. Thus, if isothermal degradation is of the form  $i \sim \exp(\mathcal{R}t)$ , then  $I$  is chosen to be a linear function of  $\ln i$ , as in the third choice, above. In practice, however, experimental error, limited observation time, temperature dependence of  $f$ , device-to-device differences, and small differences between initial and final values often make it difficult to show that such an  $f(i)$  exists. Even when  $f(i)$  can be found,  $E_E$  may vary, in the case of a laser, according to whether  $i$  is chosen as the threshold current ( $P \approx 0$ ) or the current at some larger value of the stimulated optical power  $P$ .

In summary,  $m_E$  and the extrapolation energy  $E_E$  are model- and observable-dependent parameters which typically have no recognizable physical significance. Nevertheless, they often form the basis of useful extrapolations when used self-consistently with their defining models. In contrast to the activation method, where multiple dependent observables on a common device are expected to have a common activation energy for a valid accelerating stress, the extrapolation method typically yields a different extrapolation energy for each dependent observable.

## VI. AN EXAMPLE

It is instructive to take a simple example and contrast the activation and acceleration energies. Assume that the threshold current  $i$ , or the current at some other optical power, of an idealized laser conforms to the isochronal relation

$$i(t, T) = i(t, T_r)e^{(T-T_r)/T_0}. \quad (22)$$

That is, at any time,  $i$  depends exponentially on  $T$  with a characteristic temperature interval  $T_0$  that is unaffected by the aging history. (The accuracy of eq. [22] over a useful temperature range and the approximate invariance of  $T_0$  during aging have been reported for certain types of lasers.<sup>6,12</sup>)

Without specifying the time dependence of the isothermal aging law, we define a possibly-time-dependent  $b_a$  by the requirement that

isothermal aging along the solid-line path 1-5 in Fig. 4 causes  $i_1 = i(t = 0, T_{a'})$  to evolve into  $i_5$ , i.e.,  $b_{a'}i_1$  is defined as the isothermal increase in  $i_1$ ,

$$i_5 = i_1 + b_{a'}i_1. \quad (23)$$

If  $b_a i_2$  is similarly defined as  $i_3 - i_2$ , i.e.,

$$i_3 = i_2 + b_a i_2, \quad (24)$$

then it follows from eqs. (22) and (24) that aging around the dashed-line path 1-2-3-4 evolves  $i_1$  into  $i_4$  given by

$$i_4 = i_1 + b_a i_1. \quad (25)$$

But  $i_4 = i_5$  (Fig. 4b), and hence from eqs. (23) and (25)

$$b_{a'} = b_a. \quad (26)$$

That is, regardless of the form of the isothermal degradation law, equal amounts of degradation at  $T_a$  and  $T_{a'}$  correspond to equal percent changes in the isothermal currents.

To find the instantaneous activation factor  $m$ , write eqs. (23), (26), and (24), respectively, as

$$\begin{aligned} b_{a'} &= \frac{1}{i_{a'}} di_{a'} = \frac{\partial \ln i_{a'}}{\partial t_{a'}} dt_{a'} \\ &= b_a = \frac{\partial \ln i_a}{\partial t_a} dt_a, \end{aligned} \quad (27)$$

where  $\partial \ln i_a / \partial t_a$  is the isothermal degradation rate of  $\ln i_a$ , with aging and measurement at  $T_a$ . From eqs. (1) and (27)

$$m = dt_{a'} / dt_a = \frac{\partial \ln i_a \partial t_a}{\partial \ln i_{a'} / \partial t_{a'}} = \frac{e^{-T_a/T_0} \partial i_a / \partial t_a}{e^{-T_{a'}/T_0} \partial i_{a'} / \partial t_{a'}}, \quad (28)$$

where eq. (22) was used for the last equality. Following eq. (2) we interpret  $m$  as the ratio of two one-temperature rates  $R$ . Thus from eqs. (2) and (28), two of the many correct expressions for  $R$  are the isothermal rates

$$R(T) = \partial \ln i / \partial t \quad (29)$$

and

$$R(T) = e^{-T/T_0} \partial i / \partial t, \quad (30)$$

where aging and measurement are both at temperature  $T$ . If  $R$  obeys the Arrhenius rule, eq. (10), then

$$E_A = k \frac{\ln(R_a/R_{a'})}{T_{a'}^{-1} - T_a^{-1}}, \quad (31)$$

where  $R_a = R(T_a)$ . An important point here is that eq. (22) has been used to avoid the necessity for the periodic temperature lowering of Fig. 2 even though  $T_0$  may vary appreciably from laser to laser; that is,  $m$ ,  $E_A$ , and  $R$  are expressed in terms of rates measured at the aging temperatures.

This example may be equivalently interpreted as follows. Equations (29) and (30) show that if the observable  $I$  is chosen as  $\ln i$  or as  $e^{-T/T_0}i$  (where  $i$  is the threshold current at  $T$ ), then the extrapolation energy associated with the isothermal rate  $\mathcal{R} = \partial I/\partial t$  is equal to the activation energy.

If, however, the observable  $I$  had been arbitrarily chosen, say as  $i$ , then measurement of the isothermal degradation rate

$$\mathcal{R} = \partial i/\partial t \quad (32)$$

at the two temperatures  $T_a$  and  $T_{a'}$  and application of the Arrhenius rule would have led to the following value for the extrapolation energy

$$\begin{aligned} E_E &= k \frac{\ln(\mathcal{R}_a/\mathcal{R}_{a'})}{T_{a'}^{-1} - T_a^{-1}} \\ &= E_A + kT_a T_{a'}/T_0 \end{aligned} \quad (33)$$

$$= E_A + 0.14 \text{ eV}, \quad (34)$$

where eqs. (29), (31), and (32) were used. The example of eq. (34) is based on the values  $T_{a'} = 333\text{K}$ ,  $T_a = 343\text{K}$ , and  $T_0 = 70\text{K}$ . Equation (34) shows that the quoted value of an extrapolation energy can depend significantly on the choice of  $I$ . Also, if  $T_0$  varies appreciably from laser to laser, as it often does, then eq. (33) shows that at least one of the energies  $E_A$  and  $E_E$  must also vary appreciably from laser to laser.

Although it would not be apparent from aging at only two temperatures, it is clearly true that  $R(T)$  and  $\mathcal{R}(T) = \partial i/\partial t$  cannot both obey the Arrhenius rule. If, for example,  $R(T)$  conforms to Arrhenius, then the corresponding temperature scaling rule for  $\mathcal{R}$  is

$$\mathcal{R}(T) = \partial i/\partial t \sim e^{-E_A/kT+T/T_0}. \quad (35)$$

This suggests strongly that the application of the Arrhenius rule should be justified in each context by measurement of  $\mathcal{R}(T)$  or  $R(T)$  at a number of temperatures. Alternately, the definition of  $I$  (and hence of  $\mathcal{R} = \partial I/\partial t$ ) could be determined by the requirement that the Arrhenius (or other) rule be applicable to  $\mathcal{R}$ .

Parenthetically we note from eq. (22) that a useful alternative form of eq. (28) is

$$m = \left( \frac{1}{i_a} \frac{\partial i_a}{\partial t_a} \right) / \left( \frac{1}{i_{a'}} \frac{\partial i_{a'}}{\partial t_{a'}} \right), \quad (36)$$

where  $i'_a$  and  $i'_a'$  are the currents evaluated at a common  $t'$  (any convenient time such as the moment of temperature change in a step stress) while  $\partial i_a/\partial t_a$  and  $\partial i'_a/\partial t'_a$  are evaluated at one or more other common times  $t$ .

## VII. COMPARISON OF ACTIVATION AND EXTRAPOLATION METHODS

We compare the activation method (Sections III and IV) and the extrapolation method (Section V) by noting that each has advantages over the other. For example, if the quantity of interest is the lifetime  $\tau$  defined as the time  $t^*$  when a laser will no longer put out a given amount of stimulated optical power  $P$  despite arbitrary adjustments of the drive current, and if the stimulated power plays an important role in the degradation process, then temperature acceleration of  $t^*$  is not possible by the activation method. The reason is that  $t_r^*$  at the reference temperature  $T_r = T_s$  corresponds to an aging time  $t_a^*$  beyond the time when the laser will lase at the accelerating temperature  $T_a$ . (Typically, a laser that has died at  $T_a$  will still lase when lowered to  $T_r$ .) But, by assumption, operation without optical power is not relevant, and so the activation method cannot provide acceleration of  $t^*$ . (On the other hand, the extrapolation method may well yield a useful extrapolation of that lifetime  $\tau$  defined as the time  $t^+$  ( $\neq t^*$ ) at each temperature when power  $P$  can no longer be obtained at that temperature.<sup>27</sup>) The activation method can also be ruled out if temperature cycling is unacceptable (because of transients or unwanted cycling degradation) or inconvenient.

By contrast, the coupling efficiency or a low-optical-power laser for which the current density,  $J$ , but not the stimulated optical power  $P$ , is significant to degradation can undergo accelerated aging above the maximum temperature for lasing by the activation method. Typically, however, there is no practical degrading observable to monitor above the maximum temperature for lasing, and thus the extrapolation method is inapplicable. Also, as suggested by the dissimilar shapes of Fig. 4b or of Fig. 6, the extrapolation method is of limited value if processes (e.g., shunt-path effects) are operative at  $T_a$  but not at the system temperature  $T_s$ . Finally, the arbitrariness of the numerical values of an extrapolation energy is objectionable to investigators seeking connections with microscopic degradation mechanisms.

Although the activation energy is less arbitrary than the extrapolation energy, neither energy can claim to be any sort of fundamental constant. For example if  $I$  is the threshold current  $i$ , and if degradation consists of dark-line-defect formation,<sup>9</sup> then  $I$  increases because of additional nonradiative recombination in the dark line. Furthermore, to overcome the optical loss caused by the dark line the quasi-Fermi level separation must be increased, and this increases the radiative

and nonradiative recombination even in the line-free regions of the laser. The increased separation also increases both the minority carrier leakage over the double-heterostructure barrier<sup>16</sup> and the shunt current around a buried active layer.<sup>6,28</sup> Each of these four contributions to the increase in  $i$  may have its own temperature dependence. Thus, except for fortunate cases where a single response dominates, any macroscopic black-box activation energy necessarily reflects multiple microscopic processes. In so far as these processes are not multiplicative in their effect on  $I$ , they also induce curvature in an Arrhenius plot ( $\ln R$  versus  $1/T$ ).

A further arbitrariness in any black-box activation or extrapolation energy arises from imprecision in the value of "the" temperature. In the case of dark-line laser degradation<sup>9</sup> a line causes pronounced, and typically unmeasured, local heating. Even with a uniform mode of degradation<sup>10</sup> it is believed that large temperature gradients exist within the active layer.<sup>29</sup>

If temperature  $T$  and some other stress such as the current density  $J$  are both accelerants, and if the acceleration rate is not a product function [ $R(T, J) \neq f(T)g(J)$ ], then when only  $T$  is varied the deduced value of  $E_E$  may depend implicitly upon the fixed value of  $J$ .

In summary, despite their limitations the activation method and the extrapolation method are each useful under conditions when the other is inapplicable. Thus, we conclude that both methods will survive indefinitely.

## VIII. DIFFERENTIAL FORMS

In the reliability literature theoretical models of device degradation are usually presented in terms of isothermal-aging expressions of the form

$$I = f(t, T), \quad (37)$$

where  $I$  is an observable measured at  $T$  after aging at  $T$  from  $t = 0$  to  $t = t$ . (If  $T_a = T_r$ , then eq. [14] becomes an example of eq. [37].) In this section we emphasize the limitations of eq. (37) and advance alternatives.

The main problem with eq. (37) is its incompleteness. For example, knowledge of  $f(t, T)$  for every  $t$  and  $T$  is not sufficient information to calculate even the activation factor  $m$ . This can be seen as follows. At  $t = 0$  one can move up the path 1-2 in Fig. 4a ( $I_1 \rightarrow I_2$  in Fig. 4b) by raising the value of  $T$  in eq. (37) to  $T_a$ . Path 2-3 ( $I_2 \rightarrow I_3$ ) can then be followed by increasing  $t$  from  $t = 0$  to  $t = t_a$  in eq. (37). But eq. (37) is insufficient to follow path 3-4, i.e., insufficient to deduce  $I_4$ . That is, if the values  $t = t_a$  and  $T = T_a'$  corresponding to  $I_4$  are inserted into eq. (37), this (isothermal) equation yields that  $I_4'$  that results from tra-

versing the isothermal solid-line path 1-4 in Fig. 4a, not the  $I_4$  that results from the dashed path 1-2-3-4. Alternately, we could try to validate eq. (37) down the path 3-4 by asserting that  $I_4' = I_4$ . But we are then saying that the value of  $I_4$  does not depend upon its history (i.e., does not depend upon the route in  $t - T$  space leading to a given end point). In such a case "accelerated" aging (route 1-2-3-4) actually accomplishes no acceleration because it yields the same result as normal aging (solid-line route 1-4 in Fig. 4a). Such path-independent (nonaccelerating) stresses may well exist, but they are not the stresses of interest in accelerated-aging theory. Thus, while eq. (37) is useful for the special case of isothermal aging from a particular starting time (i.e., a particular starting physical state of degradation), we abandon eq. (37) as a basis for formulating degradation models.

In principle the sequence of horizontal and vertical paths in  $t - T$  space that describe step stressing could be followed analytically if the isothermal relation, eq. (37), were supplemented with an isochronal relation like eq. (22). Both relations must, however, be valid for arbitrary starting time and temperature, i.e., after an arbitrary aging history. (Expressions of the class of eq. [37] in the literature are often valid only for aging from a particular state of degradation at  $t = 0$ .) Because of the complexity arising from the arbitrary starting time and because curved paths (e.g., eq. [47]) are of interest, we shall not follow this approach.

The simplest path-dependent model for the observable  $I$  is the *differential form*

$$dI = \mathcal{R}dt + \theta dT, \quad (38)$$

where all variables in eq. (38) are evaluated at the same time and temperature. Sometimes it is sufficient to assume that the partials  $\mathcal{R}$  and  $\theta$  are *state functions*, i.e., functions which depend only on the present state ( $T, I$ ) of the system. However, we allow for an explicit time dependence and write

$$\mathcal{R} = \mathcal{R}(t, T, I) = \partial I / \partial t \quad (39)$$

$$\theta = \theta(t, T, I) = \partial I / \partial T. \quad (40)$$

Such an explicit time dependence arises, for example, when moisture diffuses into a hermetically sealed can at a rate that is independent of the aging history  $T(t)$  of the enclosed device. (In some such cases the explicit time dependence of  $\theta$  and  $\mathcal{R}$  could be removed by adding humidity as an additional independent variable.)

As an example if  $I$  is the threshold current  $i$  and if

$$\theta = I/T_0, \quad (41)$$

then the isochronal ( $dt = 0$ ) integration of eq. (38) yields eq. (22). If

$$\mathcal{R} = B(T)I, \quad (42)$$

then the isothermal ( $dT = 0$ ) integration of eq. (38) yields  $I = I_i \exp(Bt)$ , where initially  $I_i = I(t = 0; T)$ . The linear isothermal law  $I = I_i + B(T)t$  (e.g., the  $T_r = T_s$  case of eq. [14]) can be represented by

$$\mathcal{R} = B(T), \quad (43)$$

in eq. (38).

To find  $I$  at any point  $(t, T)$  along the curved solid-line route  $T(t)$  in Fig. 7, express the route in differential form as  $dT = (dT/dt)dt$  so that eq. 38 becomes

$$dI = (\mathcal{R} + \theta dT/dt)dt. \quad (44)$$

Integration of eq. (44) yields the path-dependent generalization of eq. (37) that we seek. If the route is given as  $t(T)$ , then

$$dI = (\theta + \mathcal{R} dt/dT) dT. \quad (45)$$

In practice the path is often given implicitly. Consider, for example, the commonly occurring runaway curved path that results when, for convenience, a so-called "isothermal" aging experiment is actually performed at a constant heat-sink temperature rather than at a constant temperature  $T$  of the relevant region within a device (e.g., the active layer of a laser). If  $\mathcal{R}$  and  $\theta$  are state functions and if the temperature  $T$  of the active layer of a laser operated at constant optical power is a known function  $g(I)$  of the degrading (increasing) drive current  $I$  and of the constant heat-sink temperature,\* then eq. (38) can be written

$$dI = \mathcal{R}[g(I), I]dt + \theta[g(I), I](dg/dI)dI, \quad (46)$$

which separates into

$$\frac{1 - \theta dg/dI}{\mathcal{R}} dI = dt \quad (47)$$

and reduces evaluation of  $I(t)$  and of the curved-path  $T(t) = g[I(t)]$  to quadrature.

If eq. (44) or (45) yields a path-independent integral, i.e., eq. (37), then eq. (38) is an exact differential and  $\mathcal{R}(t, T, I)$  and  $\theta(t, T, I)$  satisfy the following test:

$$\theta \partial \mathcal{R} / \partial I + \partial \mathcal{R} / \partial t = \mathcal{R} \partial \theta / \partial I + \partial \theta / \partial t. \quad (48)$$

For example, insertion of eqs. (41) and (43) into eq. (48) and integra-

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\* In the usual case the thermal resistance has the constant value  $\Omega$ . Then  $T = g(I) = I^2 \Omega / 2 + T_{hs}$ , where  $T_{hs}$  is the heat-sink temperature.

tion of the resulting equation for  $B$  reveal that temperature is not an accelerating stress if

$$B(T) = \text{const.} \times e^{T/T_0}. \quad (49)$$

A change of variable from  $I$  to  $I'$ , i.e.,

$$I = g(t, T, I'), \quad (50)$$

is sometimes useful for simplifying  $\mathcal{R}$  or  $\theta$ . Then eq. (38) becomes

$$dI' = \mathcal{R}'dt + \theta'dT, \quad (51)$$

where

$$\mathcal{R}'(t, T, I') = \frac{\mathcal{R}(t, T, g) - \partial g / \partial t}{\partial g / \partial I'} \quad (52)$$

and

$$\theta'(t, T, I') = \frac{\theta(t, T, g) - \partial g / \partial T}{\partial g / \partial I'}. \quad (53)$$

As a trivial example of eq. (50), recall that in Section VI the variable change to  $I' = \ln i$  from  $I = i = \exp I' = g(I')$  transformed  $\theta = I/T_0$  into  $\theta' = T_0^{-1}$ . A more interesting example occurs in the case of Fig. 7,

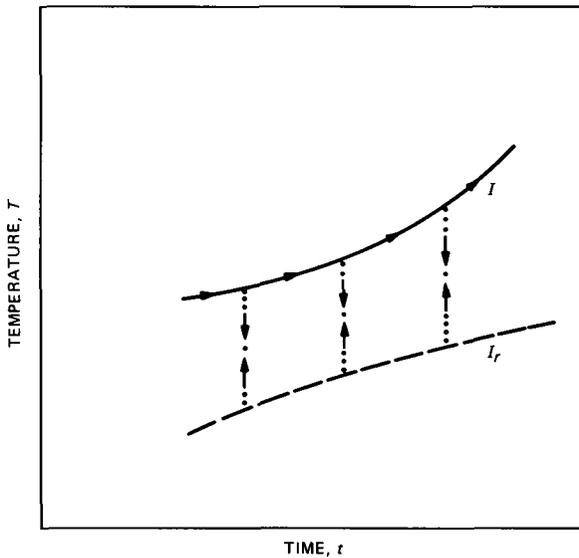


Fig. 7—Corresponding to the evolving value of an observable  $I$ , as a component is aged along the solid-line route in time-temperature ( $t - T$ ) space, is the evolving reduced observable  $I_r$ , which is measured if the component is periodically and momentarily dropped to the dashed-line path and returned to the solid-line path (down and up each dotted line).

where  $I$  is the degrading value of an observable which is aged along the solid-line path (e.g., an isothermal or eq. [47]) in  $t - T$  space and  $I' = I_r$  is the *reduced observable*, i.e., the value of  $I$  when the component is momentarily and periodically cooled to the dashed-line reference path in  $t - T$  space. In this case  $g$  of eq. (50) is, from eq. (38), the integral of

$$dI = \theta(t, T, I)dT, \quad (54)$$

with integration up the vertical dotted path (between the limits  $I_r$  and  $I$ ) in Fig. 7. For example, if  $I = i$  and  $\theta = I/T_0$ , then integration of eq. (54) shows that eq. (50) becomes eq. (22). If the dashed curve in Fig. 7 were a horizontal line (at  $T_r$ ), then  $\mathcal{R}'$  is the  $R$  of Section III, and eq. (51) reduces to

$$dI_r = Rdt.$$

Thus, if  $\theta = I/T_0$ , eq. (52) becomes

$$R = \mathcal{R}' = \mathcal{R}e^{-(T-T_r)/T_0} \quad (55)$$

in agreement with eqs. (30) and (32) (allowing for the arbitrary choice in eq. [30] of the multiplicative constant in  $R$ ).

For multiple dependent observables (e.g.,  $I_1 =$  current,  $I_2 =$  voltage,  $I_3 =$  peak wavelength,  $I_4 =$  thermal resistance,  $I_5 =$  strain,  $I_6 =$  optical power in the fundamental mode,  $I_7 = \dots$ , etc.) and multiple independent observables (e.g.,  $T_1 =$  temperature,  $T_2 =$  total optical power,  $T_3 =$  humidity, etc.), one understands  $\theta$  as a matrix while  $I$ ,  $\mathcal{R}$ , and  $T$  are vectors in eq. (38). In principle the differential form is completely general with  $\theta$  and  $\mathcal{R}$  as state functions, because if there are enough observables to completely specify the state of a classical system, then the laws of physics can be written as first-order differential equations with state-function coefficients. If, for a set of independent observables, one finds a set of dependent observables such that eq. (38) is an adequately accurate description with  $\theta$  and  $\mathcal{R}$  state functions, then the observables in question may be said to form a *complete macroscopic set*.<sup>30</sup>

Instead of increasing the number of dependent observables, one may seek a complete description by going to higher-order differential equations or by including an integral over the history  $T(t)$ . We will not consider such generalizations any further here except to note that when a complete theoretical model in two ( $t$  and  $T$ ) or more independent variables appears to be quite complex, then the empirical one-variable reduced-observable method of eqs. (6), (12), and (54) becomes particularly appealing.

## IX. SAMPLED-POPULATION METHODS

The essential idea of sampled-population methods is that the characterization of components going into service is based upon aging properties measured on a statistically equivalent sample rather than on any aging measurements on the very components going into service. We outline here two approaches that are differentiated from each other by the fact that one approach is based on aging to failure, while the other approach is based on aging only through the time period of interest (system life, guarantee period, etc.).

Aging to failure, which is particularly appropriate for components which are likely to be replaced (mean component life  $\leq$  system life), or which fail suddenly<sup>1</sup> (no continuously degrading observable can be found), has been treated exhaustively in the semiconductor reliability literature.<sup>2,3,13,31-33</sup> Typically a sample set of components from the population is aged isothermally to failure at an elevated value of the temperature or other stress, and the observed lifetimes  $\tau$  (as defined by either the activation or extrapolation method) are then used to estimate the parameters in an analytical function  $F(\tau)$  that is assumed to describe the distribution of lifetimes in the population. Good fits to the data are often obtained with the exponential,<sup>34</sup> Weibull,<sup>34</sup> or lognormal distributions<sup>34</sup> or with mixtures of these as in the double-lognormal distribution.<sup>8</sup> Also, we believe that the burned-in and truncated forms of these distributions will prove applicable, as discussed in conjunction with Fig. 3. Because it is so often used and because many of its properties cannot be expressed in terms of a finite number of elementary functions, the lognormal distribution ( $\ln \tau$  is normally distributed) has had its implications developed in particularly great detail.<sup>31-33</sup>

When the temperature (or other stress) is believed to be a valid accelerating stress, as after the selective elimination of flawed components having a secondary failure mechanism, it is customary to measure the isothermal mean life  $\langle \tau \rangle$  (or  $\langle \ln \tau \rangle$ ) at two or more temperatures (using separate statistically equivalent samples at each temperature). Then the rate  $1/\langle \tau \rangle$  is extrapolated by assuming that it follows the Arrhenius rule (eq. [10]) with a single population value for  $E_A$ . The standard deviation in  $\ln \tau$  is assumed to be temperature-independent, as follows from eq. (6) when  $E_A$  has a common population value. When  $\ln \tau$  is the ordinate, this amounts to a vertical displacement of the distribution (Fig. 2a) or to relabeling the ordinate, as shown on the right-hand side of Fig. 1. Validly accelerated distributions other than the lognormal are similarly scaled by noting from eq. (6) that temperature extrapolation is simply a scaling of time. If, as discussed in conjunction with Figs. 2b and 5, the activation energy varies from component to component, then the variance may also

depend upon the temperature. Analogous procedures are commonly used with the extrapolation-energy method, although the justifying rationales, if any, are necessarily based on specific models of degradation or specific observations (because eq. [6] is not applicable and because it is rarely easy to argue a priori that all components should have a common extrapolation energy).

The limitations of cost or of available time and the long life of some components, even under accelerated-aging conditions, often prevent aging more than a small fraction of the sample components to failure. Under such circumstances the method just outlined may still be feasible, in principle, if the lifetimes of the longer-lived components can be estimated on the basis of an established correlation with one or more early component properties. For example, significant correlation between lifetime and initial degradation rate has been reported for some types of lasers<sup>7</sup> and thin-film conductors.<sup>11</sup> Initial properties other than an aging rate may also exhibit correlation with lifetime, as in Refs. 35 and 36.

For a high-reliability component (mean component life  $\gg$  system life) the cost and time for aging a sample to even the mean life can easily exceed, by more than an order of magnitude, the cost and time for aging only through the *effective system life* (= the *temperature-contracted system life*, i.e., the system life divided by the activation or extrapolation factor). This expenditure of time and money for aging past the effective system life may not be justified if the consequences of practical interest depend only on the expected number of failures during the system life. In short, as an alternative to measuring the distribution of times for a given amount of degradation (i.e., failure), it is sometimes preferable to measure the distribution in the amount of degradation for a given time. (That time may be the effective system life, or a longer time with interpolation back to the effective system life if the degradation is still in the noise after only the effective system life, or that time may simply be the available time in a crash program with an extrapolation out to the effective system life.)

Figure 8 is an example which contrasts the two approaches of this section. Consider a population of hypothetical lasers. A laser has a predeployment threshold current  $i_0$ , and the isothermal threshold current  $i$  increases approximately linearly in time up to a failure value  $i_f$ , which is defined as the lesser of  $2i_0$  (a compensating-circuit limitation) and 100 mA (a power-supply limitation). Under these conditions the observable  $I$ , defined by

$$I = \frac{i - i_0}{i - i_f}, \quad (56)$$

has convenient properties; i.e.,  $I$  increases approximately linearly from

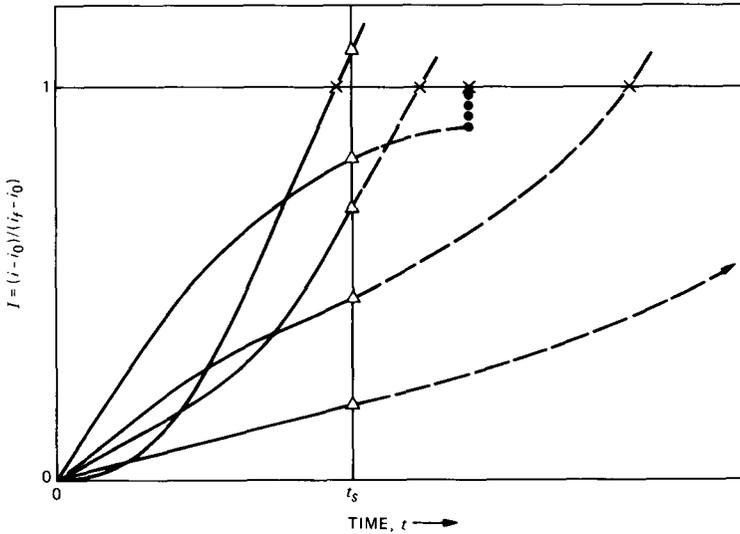


Fig. 8—Idealized isothermal degradation curves for semiconductor lasers. Here  $i_0$  and  $i_f$  are the initial and failure values of the threshold current (or the current at some other value of the optical power). The effective system life is  $t_s$ .

$I_0 = 0$  to  $I = 1$  (failure) for every laser regardless of its  $i_0$  and its isothermal degradation rate  $\partial i / \partial t$ . Also, as exploited in Section X,  $(\partial I_0 / \partial t)^{-1}$  is the initial estimate of the component life  $\tau$ ; thus  $\partial t / \partial I_0$  has statistical properties which are similar to those of  $\tau$ , and a similar methodology can be used. Each curve in Fig. 8 represents the observed degradation of one laser. For the lifetime-distribution method of Fig. 1, the relevant data points are the  $\times$ 's in Fig. 8 that fall along the failure line ( $I = 1$ ). For the high-reliability-component method the data points are the triangles which fall on the effective-system-life line ( $t = t_s$ ), i.e., the values  $I_s = I(t = t_s)$ . For the latter method it is not necessary to generate the dashed-line portion of the data. In a typical case the distribution of  $1/I_s$  values is roughly similar to the distribution of lifetime ( $\tau$ ) values and can be treated similarly to the treatment of the cumulative distribution  $F(\tau)$ . The similarity in the distributions of  $\tau$  and  $1/I_s$  need not be overly close, however, because the objective here is to estimate the probability of failure before  $t_s$  rather than to estimate later properties such as the mean lifetime  $\langle \tau \rangle$ . (Alternatively, if data are available at intermediate times and if the  $I(t)$  curves are fairly smooth, the dashed sections of the curves can be estimated by extrapolation from the solid sections. Then the high-reliability data can be analyzed as if they were replacement data. The inaccuracy in the  $\times$ 's at large  $t$  may not be serious if they prove to be a smooth extension of the early  $\times$ 's and if the implications of the analysis are restricted to the range  $0 < t < t_s$ ).

The two methods of this system have an important attribute that should be emphasized. They are useful methods whether or not the various components in the sample age with a common functional dependence on time. (Of course, when all components do have a common parameterized functional dependence on time—e.g., eq. [14]—and when the number of parameters is not too large for confident estimates, then standard statistical methods provide a third approach in which the observed degradation of the sample components is used to estimate the distribution of parameters over the population of components.)

## X. TRUNCATION

Sampled-population methodology is sometimes too crude because it ignores important predictors of a given component's life. As previously noted these predictors include the predeployment degradation rate<sup>7,11</sup> and other initial properties of that component.<sup>35,36</sup> Utilization of this additional class of data implies that each component has its own statistical description including, possibly, its own acceleration factor or activation energy. If the step-stress measurement error is larger than the component-to-component difference in the acceleration factor, then the best estimate of the acceleration factor or activation energy for each component is usually the common population value inferred from isothermal aging of sample populations at different temperatures. If the converse is true, then the best estimate for a component is usually its own value inferred from step stressing that component before deployment. Hereafter we discuss the truncation method in terms of data at the system temperature. Part or all of these data may, of course, actually be inferred, in the manner previously discussed, by extrapolation from accelerated measurements at elevated stress levels.

We consider first the case of replacement components characterized by a cumulative lifetime distribution  $F(\tau)$ . Ideally, the initial degradation rate is an exact predictor of a component's life, and one simply uses the initial rate to identify and reject all components with lifetimes less than some value  $\tau'$ . If  $F(\tau)$  is the cumulative distribution (failed fraction at time equal to  $\tau$ ) and if  $S(\tau) = 1 - F(\tau)$  is the surviving fraction (fraction of the components with lifetimes longer than  $\tau$ ) of the population, then after *truncation* (discarding of the short-lived components) the distribution  $F' = 1 - S'$  of the retained components is given by

$$\begin{aligned} S'(\tau) &= 1 - F'(\tau) = 1 & \tau \leq \tau' \\ &= S(\tau)/S(\tau'), & \tau \geq \tau', \end{aligned} \quad (57)$$

as exemplified in the  $T_{y=0.4}$  curve of Fig. 3.

Equation (57) is too idealized for most applications because it implies that 100-percent reliability can be achieved simply by choosing  $\tau'$  greater than the system life. To be more realistic one must first measure the correlation between the initial degradation rate and the lifetime, as has been done, for example, in an important experiment by English et al.,<sup>11</sup> who studied, for Ti-Au thin-film conductors, the correlation between the film lifetime  $\tau$  and the initial rate of degradation (increase) in the resistance of the conductor. From the data points in their Fig. 7 (~150 conductors), it appears that 16 percent of their conductors had failed by 400 hours, while the final 16 percent failed after 2500 hours. Thus as a very rough estimate we find that the standard deviation in  $\ln \tau$  would be  $\sigma \approx (1/2) \ln(2500/400) = 0.92$  if the conductors were deployed on a sampled-population basis. However, a similar analysis of their figure shows that  $\sigma$  of the quantity  $\ln \tau - \ln \tau_{\text{est}}$  is approximately 0.75 for a conductor with an estimated lifetime  $\tau_{\text{est}}$  inferred from its initial degradation rate. This means that the early-failing components could be identified and eliminated with some success. For example, for their data, the time of the earliest failures is raised by about three orders of magnitude (from ~11 hours to ~10<sup>3</sup> hours) if the 60 percent of the conductors of their actual population with the highest initial degradation rates are discarded (cf. Fig. 3).

To quantify the preceding let  $\rho(R_0, \tau)d\tau$  be the conditional probability of lifetime between  $\tau$  and  $\tau + d\tau$  for a component with initial (i.e., post-stabilization, predeployment) degradation rate  $R_0$  (or  $\mathcal{R}_0$ ), and let  $P(R_0)dR_0$  be the fraction of the components with initial rates between  $R_0$  and  $R_0 + dR_0$ . If components with initial rates greater than  $R'_0$  are discarded, then the (truncated) distribution  $F'(\tau)$  of the components retained for deployment is given by

$$dF'(\tau)/d\tau = \int_{-\infty}^{R'_0} \rho(R_0, \tau) P(R_0) dR_0 / \int_{-\infty}^{R'_0} P(R_0) dR_0. \quad (58)$$

For perfect correlation, i.e.,  $\tau = g(R_0)$  and  $\rho = \delta[g(R_0) - \tau]$ , eq. (58) reduces to eq. (57). Here  $\delta[\ ]$  is the delta function.

The success of predeployment inspections implies that it is sometimes possible to establish a correlation between lifetime and some initial property other than the degradation rate, such as the initial value of the degrading observable  $I$ , or the thermal resistance of a laser bond, or the thermally induced nonlinearity in the current-voltage characteristic of a nominally linear circuit element,<sup>35</sup> or the stripe width<sup>36</sup> (in the case of incipient formation of a nonlinear "kink" in the light-current characteristic of a laser), etc.<sup>37</sup> The form of eq. (58) is still applicable with  $R_0$  replaced by the parameter that is

correlated with lifetime. For components that fail suddenly<sup>1</sup> (no degrading observable) such other initial properties are, of course, the only predictors of individual-component life.

In principle,  $\rho$  is defined to take advantage of all observed correlations, e.g.,  $\rho(i_0, \partial i_0/\partial t, \tau) d\tau$  is conditional upon both the predeployment threshold current  $i_0$  and the predeployment isothermal degradation rate  $\partial i_0/\partial t$  of a laser. In some situations it is feasible to measure multiple correlations, as in Ref. 37. Often, however, limited data or small homogeneous populations do not permit the functional form of  $\rho$  nor the multiple correlation and cross-correlation parameters to be established with much confidence. As a compromise in such cases one may stay with eq. (58) while redefining  $R$  as any combination of  $i_0$  and  $\partial i_0/\partial t$  which maximizes the correlation. For example, the definition given by eq. (56) or the definition

$$I = \frac{\ln(i/i_0)}{\ln(i_f/i_0)} \quad (59)$$

may be superior to the definition  $I = i$  or  $I = i/i_0$  when maximizing the correlation between the lifetime  $\tau$  and the initial value of the isothermal degradation rate  $R = \partial I_r/\partial t_a$  or  $\mathcal{R} = \partial I_a/\partial t_a$ . (Equation [59] replaces eq. [56] when  $i$  increases approximately exponentially in time. Then  $(\partial I_0/\partial t)^{-1}$  is still the initial estimate of component life.)

In a high-reliability application, where it is inefficient or even impossible to age beyond the effective system life, similar considerations apply. In a typical procedure, samples are aged through the system life, perhaps on an accelerated basis, and a best-fit analytical model is then sought for the functional form of  $i(t)$ . Because measurement error typically leads to low confidence when many parameters appear, a fairly simple function is normally chosen; for example,  $i = be^{ct}$  with the parameters  $b$  and  $c$  fitted separately for each component. Usually no such simple function will precisely model the behavior of every, or even any, component, and thus extrapolations based on the model function (e.g., on  $i = be^{ct}$ ) would be somewhat inaccurate even if there were no measurement errors.\* It is convenient, although not necessary, to choose an  $I$  which increases linearly in time, e.g., eq. (59) in the case of  $i = be^{ct}$ . The behavior of  $I$  during the monitor period  $t_0$  is then extrapolated (projected) through the effective system life (dashed line in Fig. 9). This is compared with the actual behavior

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\* Even if all devices degrade by a common mechanism (i.e., after perfect purging) with a common activation energy, the functional form of the isothermal aging curve,  $I(t)$ , could be very complicated and difficult to parameterize for the whole population. For example, if clusters of impurities or defects exist initially at different surface regions of a laser,  $I(t)$  might be expected to exhibit a staircase character as the successive diffusion fronts from the clusters reach the active layer.

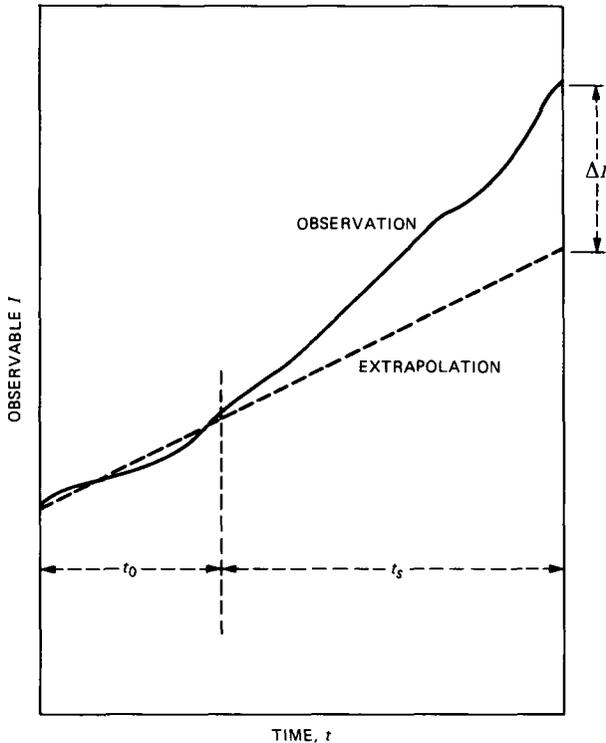


Fig. 9—After an observation or monitor period  $t_0$ , the observable  $I$  is extrapolated (dashed line) through the effective system life  $t_s$ . Then  $\Delta I$  is the error between the subsequently observed value (solid line) and the extrapolated (projected) value.

through the system life  $t_s$  (solid line) to determine the error  $\Delta I$  between the actual and the expected values of  $I$ . An analytical  $\rho(R, I)$  or  $\rho(R, I, t)$  is fitted to the  $\Delta I$  data, where  $\rho(R_0, I)dI$  is the probability that  $I$  will fall between  $I$  and  $I + dI$  at the end of  $t_s$  given a mean rate  $R_0$  during  $t_0$ . A scatter plot of the expected  $I$  versus the observed  $I$  is often useful for choosing the functional form of  $\rho$ . Then, in analogy to eq. (58),  $\rho$  is used to generate the distribution  $F'(I)$  at the end of the system life. ( $I$  replaces  $\tau$  in eq. [58].) In this case, however,  $\rho$  may include a finite fraction at  $I = \infty$ , (or, alternatively, the triangles are extrapolated to produce surrogate  $\times$ 's for  $t > t_s$ , and an  $F(\tau)$ , valid primarily for  $t \leq t_s$ , is fitted to the  $\times$ 's which fall on both sides of  $t_s$ .) In contrast to familiar procedures for extrapolating parameterized functions (e.g., extrapolation of an elliptical orbit), the empirical distribution  $\rho(R_0, I)$  has the important property that it accounts not only for the implications of measurement error but also for the fact that there may not exist any parameterized function  $i(t)$  that is precisely applicable to all, or even to any, of the components in the

population. If the measurement error can be separately evaluated, its contribution can be subtracted from  $\rho(R, I)$  to reveal that part of  $\rho$  which arises from the imperfect applicability of the empirical model function. This is of interest for such purposes as assessing the potential benefits of more accurate instrumentation. This projection procedure is further developed in Ref. 38. (Of course, in the fortunate case that a single parameterized function  $i(t)$  accurately describes the entire population, then, as an alternate approach, one may use standard methods to estimate from the data the distributions of the parameters.)

## XI. SUMMARY

A conceptual framework and quantitative procedures were presented for the accelerated aging and reliability predictions applicable to continuously degrading components.

## XII. ACKNOWLEDGMENTS

We would like to acknowledge many stimulating discussions with E. I. Gordon, B. W. Hakki, J. H. Rowen, and M. Tortorella.

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