A Statistical Approach to Laser Certification

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This paper proposes a statistical methodology for certifying the longevity of individual lasers to be used in an undersea communications cable. In general, the strategy is to extrapolate the slope of a degradation parameter such as i (the current necessary to maintain a given level of optical-output power) when the laser is subjected to elevated temperatures. This extrapolation is made from time t_1 (when the test terminates) to time t_p (the system life, as contracted in time by an Arrhenius relationship). An important part of the process is the calculation of the variability of the estimated current value at time t_p, used to estimate the probability that it will exceed a critical failure level i_f; lasers that exceed a predetermined probability will be rejected. The methodology presented here must, in the absence of actual data on lasers to be used in an undersea communications cable, be tentative with respect to details; modifications may well be necessary to accommodate idiosyncrasies of the manufacturing process and the testing equipment.

I. INTRODUCTION

In fabricating a system such as an undersea communication cable, where the cost of replacing a failed laser package (or other gradually degrading component) is very great, it is not enough to use lasers from a production run that have normal operating characteristics. Rather, each laser's longevity must be certified by placing it on an accelerated life test for a time t_p and then using these data (operating characteristics at a sequence of observation times) to estimate its longevity. This laser is used in the cable only if its estimated longevity exceeds

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a certain criterion (expressed as a probability that a degradation parameter i, such as the current necessary to maintain 5 mW of optical-output power, will not exceed some failure value i_f before the end of the planned system life).

To simplify the formalism, we choose a degradation parameter I that has, to the extent possible, the following two properties:[1] (1) I should increase linearly in time, (2) the ranking of the lasers in order of increasing initial degradation rate dI/dt should also be the ranking in order of decreasing estimated lifetime. For example, suppose that i typically increases approximately exponentially in time. Let i_f be the failure value of i, which may well vary from laser to laser. (For a hypothetical example, a laser with a high initial value i_o fails when i reaches the maximum output of the power supply i_m, while a laser with an i_o less than $(1/2)$ i_m fails at $i_f = 2i_o$ due to the limitation of a compensating circuit.) Then an appropriate choice for the reduced current I is[1]

$$I = \frac{\ln(i/i_o)}{\ln(i_f/i_o)}, \tag{1}$$

and every laser current degrades from its initial value $I = 0$ to its failure value $I_f = 1$.

To obtain a significant amount of degradation in the time span available for certification, the aging process is speeded up by operation at an elevated temperature T_1 rather than by operation at the system temperature T_s. If aging for a time t_1 at T_1 causes the same degradation as aging for a time t_s at T_s, and if the Arrhenius relation is applicable, then

$$t_1/t_s = e^{(T_1^{-1} - T_s^{-1})E/k}, \tag{2}$$

where E is a parameter called the activation energy and K is Boltzmann's constant. In the following, two cases are considered according to whether E varies significantly from laser to laser (as determined from step stressing), or whether a common value for E may be used for all lasers.

Let $t_1 - t_p$ be the planned system life, as contracted by temperature [eq. (2)] at T_1. The predicted behavior of the purged[2,3] and stabilized[3] lasers vying for incorporation in the cable is based on a linear extrapolation of the data collected during time t_p at T_1. To establish a correction factor to the predicted behavior, one places a number (m) of expendable calibration lasers (not to be used in the cable) from the same population on accelerated life test for a time t_1 to establish a correction factor for the extrapolation of the vying lasers' behavior. In the following, we consider in detail the evaluation of this correction factor.

II. THE STANDARD MODEL

It is assumed that measurements are taken of I at various times between $t = 0$ and $t = t_p$ for the n lasers vying for certification, and between $t = 0$ and $t = t_1$ for the m calibration lasers; a sequence of times, $t, t + \Delta t, t + 2\Delta t, \cdots$, is the easiest to analyze. Although the lasers are aged at T_1, the measured values of I are obtained at T_s by momentarily cooling the lasers to T_s for each measurement. Alternately, the values of I at T_s may be inferred from measurements of i at T_1 if there is a known algorithm for converting these i values to their corresponding values at T_s. (For example, for certain types of lasers it has been shown that the temperature dependence of i is well modeled by the empirical relationship $i \approx \exp(T/T_o)$ over a usefully wide temperature range, where T_o is a parameter characterizing the temperature dependence.[3])

The certification of an individual laser is carried out as follows. Let I denote the estimate of the current at time t_s if the laser has been continuously operated at temperature T_s. It is estimated by the certification equation

$$I = I_o + t_1 b_{T_1} + K, \tag{3}$$

where t_1 is a constant derived from the assumed value of E, I_o is the value of the current measured at time $t = 0$ and temperature T_s, b_{T_1} is the estimated slope of a least-squares straight-line fit to the certification measurements on the laser (from $t = 0$ to $t = t_p$), and K is a correction factor obtained from the calibration lasers that have been operated at temperature T_1 up to time t_1 (see Fig. 1). In the discussion below, it is assumed that I_o, b_{T_1}, and K are uncorrelated random variables. The validity of this assumption can be tested by looking at the three-dimensional scatter plot of these variables for the set of calibration lasers. If correlation exists, it may be necessary to rewrite the equation above in a form in which the variables are uncorrelated, such as $I = I_o + b_{T_1}(t_1 + K)$ or $I = I_o[1 + b_{T_1}(t_1 + K)]$.

To estimate K, (1) calculate for each calibration laser the quantity d_i, the difference between the actual current I_t measured at time t_1, and an extrapolated current I_e determined by a least-squares straight-line fit to the calibration measurements on the laser from $t = 0$ to $t = t_p$ only; and (2) calculate $K = \sum d_i/m$ over all the m calibration lasers.

Not all of the calibration lasers will survive until time t_1; some will catastrophically fail at earlier times. In the case of eq. (1), *catastrophic failure* means that no I in the experimentally accessible range $0 \leqslant I \leqslant I_m > 1$ will yield a 5 mW output. These times should be recorded, as it will be necessary to use them to calculate the probability of catastrophic failure as a function of the variability of E. (This variability
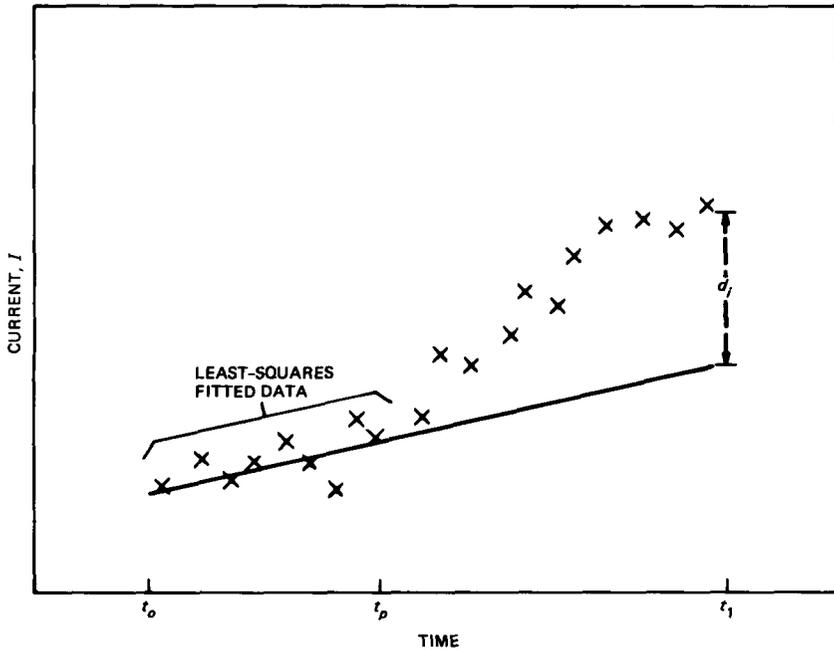
Fig. 1—Determination of correction factor d_i for an individual laser (run at elevated temperature T_1).

may be caused either by measurement error, or by inherent differences in E from laser to laser.)

Having removed the catastrophic failures, it is now necessary to ascertain the variability of the value, I, calculated for a laser surviving to time t_1. One cannot necessarily assume that the Central Limit Theorem will apply, leading to a Gaussian distribution of I (and, therefore, requiring only that the mean and variance of I be known). Instead, it will be necessary to use the calibration data to actually determine the distribution function followed by I, taken as a function of the underlying b_{T_1} (rate of degradation) of a laser, the variability of E (if it is significant), and the variability of the estimated b_{T_1} (related to the number of observations of the current, the times of these observations, and the residual variability of a single observation). Once the distribution function of I is known, then it is easy to calculate the probability for each certification laser that its current at time t_1 is greater than the critical level I_f, given that I has been calculated from the certification equation.

Let us now look at the method of calculating a distribution function for I in more detail, on a term-by-term basis. It is impossible to calculate this distribution function analytically in the general case with E variable, and even in the unlikely case that E is known exactly,

it is possible only if the distribution of the d_i and the residual error of a single observation of current are both Gaussian. Instead, the distribution function must be simulated, and an empirical multiparameter distribution (say, a gamma) fitted to the simulated data. Specifically, we assume that the distribution function of E is known (or, perhaps, approximated by a Gaussian distribution with known mean and variance). The distribution of b_{T_1} is uniquely determined by the expression

$$\sum (x_i - \bar{x}) y_i / \sum (x_i - \bar{x})^2, \tag{4}$$

where $x_1, \cdots, x_n$ are the times of observation between t_o and t_p, and $y_1, y_2 \cdots y_n$ are the corresponding current measurements. If the distribution of the measurement errors of the y_i are known, then the distribution of b_{T_1} is uniquely specified; for example, if the y_i measurement errors are independent observations from a common Gaussian population, then b_{T_1} also has a Gaussian distribution with a certain mean and variance. In fact, as long as n is not too small (say, $n \geq 10$), b_{T_1} will approach a Gaussian distribution even if the individual measurement errors are independent random variables from a non-Gaussian population. The essential point is that the distribution of b_{T_1} is specifiable as soon as the mean and variance of the measurement errors are given.

Putting all this together, one draws a random variable from the E distribution, converts it to a value of t_1 by the acceleration equation given earlier, and multiplies this by a second random variable, which is the sum of (an assumed) b_{T_1} and a value drawn from a Gaussian distribution with a known variance. One then adds to this a third random variable, drawn from a distribution function that has been previously fitted to the d_i; the sum of these yields a single value of the distribution of $I - I_o$ (I_o is assumed to be known, or with a very small error, but if necessary this could be folded in as well). It is worth noting in passing that the distribution function of the d_i may depend upon the value of the slope b_{T_1}; if so, a family of d_i distributions corresponding to different slopes must be inferred from the calibration data. In practice, this family might be approximated by a simple empirical b_{T_1}-dependent scaling of the d distribution. For example, the empirical distribution might turn out to be only weakly correlated, if at all, with b_{T_1} or with t_1.

One repeats this sampling experiment many times (say, 100 or more) to build up enough data to make a distributional fit to the simulation data. Note that it is necessary to do this for various assumed values of three parameters—the variance of b_{T_1}, b_{T_1} itself, and the variance of E—obtaining a three-dimensional family of $I - I_o$ distributions. The appropriate member of the family is then used when

certifying each laser (which will have its own unique b_{T_1} and, possibly variance of b_{T_1}).

From the proper distribution function of $I - I_o$, the probability that the certified laser will have an I exceeding I_f is at once calculated. To this, of course, must be added the probability of catastrophic failure, obtained from an integration of the calibration laser failure data taken over the variability of t_1 (determined from the variability of E, as indicated above). If necessary, this probability can be readily obtained by another simulation procedure.

In setting up the calibration studies, note that one must run the lasers for a time t_1 corresponding to the upper 5-percent (say) value of E.

III. ACKNOWLEDGMENT

REFERENCES

1. W. B. Joyce et al., "Methodology of Accelerated Aging," AT&T Tech. J., this issue.
2. F. R. Nash et al., "Selection of a Laser Reliability Assurance Strategy for a Long-Life Application," AT&T Tech. J., this issue.
3. F. R. Nash et al., "Implementation of the Proposed Reliability Strategy for an InGaAsP/InP, Planar Mesa, Buried Heterostructure Laser Operating at 1.3 μm for Use in a Submarine Cable," AT&T Tech. J., this issue.

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