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Beamwidth and Useable Bandwidth of Delay-Steered Microphone Arrays

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Automatic delay steering of microphone arrays improves sound pickup in large rooms. But steering to wave-arrival directions away from broadside degrades the acuity of the beam and diminishes the useable bandwidth of the array. This paper derives quantitative relations for the variation in beamwidth and bandwidth for one-dimensional, uniform, unweighted arrays composed of $(2N + 1)$ receivers spaced by distance d. The results show how the upper and lower useful frequencies and the beam acuity are conditioned not only by receiver spacing and frequency, but also by wave-arrival direction and steering direction. The relations developed permit detailed design of steerable arrays for specified frequency range and spatial coverage.

I. MICROPROCESSOR CONTROL OF MICROPHONE ARRAYS

Speech transduction or sound pickup in large rooms—such as auditoria or classrooms—traditionally is plagued by the distortions of room reverberation and noise interference from unwanted sources. The problem is minimized by obtaining, at the transducer, the greatest intensity possible for the signal travelling the direct path from source to receiver, and the least intensity possible for multipath room reflections of that signal and any additive interfering noise. Arrays of microphones, especially two-dimensional arrays designed for high directivity, are useful for achieving this high ratio of direct to distorted sound.

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But in an auditorium or conference room the desired sound source may shift position. The directional array must be steered, or pointed, to the shifting source to gain the full benefit. Rapid, automatic, electronic steering of the array is therefore attractive. Currently emerging microprocessors have the computational capability to perform beam forming and beam steering dynamically, and can be programmed to seek and track talkers in a room.[1]

Beam steering is implemented conveniently by digital delay adjustment of the signals received at each microphone of the array. But forming a beam and steering it to angles away from the normal to the array (i.e., away from "broadside") have two effects that must be taken into account when designing the system. One is that the useful bandwidth diminishes as the steering direction is made acute to the axis of the array (or to the plane of the array for a two-dimensional system). Another is that the beam pattern becomes asymmetric and less sharp. This report considers the variation in beamwidth and useable bandwidth under conditions of delay steering.

II. TIME, FREQUENCY, AND SPATIAL RESPONSES FOR THE UNIFORM DELAY-STEERED ARRAY

The most convenient expository vehicle is the one-dimensional uniform array, unweighted in amplitude. Related analyses apply for two-dimensional and three-dimensional arrays.

Figure 1 shows a line of $(2N + 1)$ receivers, spaced uniformly by distance d. All receivers have the same sensitivity, taken to be real and equal to unity. For beam forming and steering, the individual receiver outputs are passed through controllable delays and are summed to produce the array output.

For a plane, impulsive sound wave of unit amplitude, arriving from the direction of polar angle ϕ, the time-domain and frequency-domain responses of the array are, respectively,

$$h(t) = \sum_{n=-N}^{N} \delta(t + nT),$$

and

$$H(j\omega) = \sum_{n=-N}^{N} e^{j\omega nT}, \tag{1}$$

where

t is time,

$\omega = 2\pi f$ is the radian frequency,

$\delta(\cdot)$ is the delta function, and

$T = (\tau - \tau')$ is the time difference between the interelement wave transit delay and the interelement steering delay, in which

$$\tau = \frac{d}{c}(\cos \phi) \qquad \text{and} \qquad \tau' = \frac{d}{c}(\cos \phi'),$$

where ϕ' is the steering angle, and c is the sound velocity (taken here as 3.4×10^4 cm/s). For notational convenience, a constant delay term that ensures formal causality has been omitted from (1). Further deserving of emphasis, the impulse response $h(t)$ and its Fourier transform $H(j\omega)$ are both implicit functions of the wave-arrival direction ϕ and of the steering angle ϕ'. For specific frequencies ω, the spatial directivity is conveniently revealed in $H(j\omega)$. An elaboration of the expression for $H(j\omega)$ is therefore useful for our purposes.

The finite geometric series of (1) can be reformulated as

$$\begin{aligned} H(j\omega) &= \sum_{n=-N}^{N} e^{j\omega nT} \\ &= \frac{(e^{j\omega(N+1)T} - e^{-j\omega NT})}{(e^{j\omega T} - 1)} \\ &= \frac{\sin[(2N+1)(\omega T/2)]}{\sin(\omega T/2)}. \end{aligned} \tag{2}$$

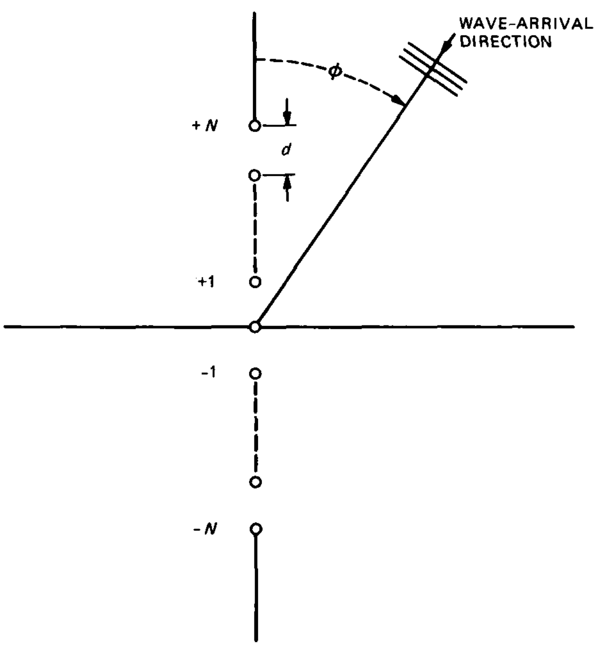

Fig. 1—Geometry for a one-dimensional microphone array of $(2N + 1)$ elements. The microphone spacing is d, and the polar coordinate for wave-arrival direction is ϕ. The array output is a summation of the outputs of individual microphones after delay adjustment.

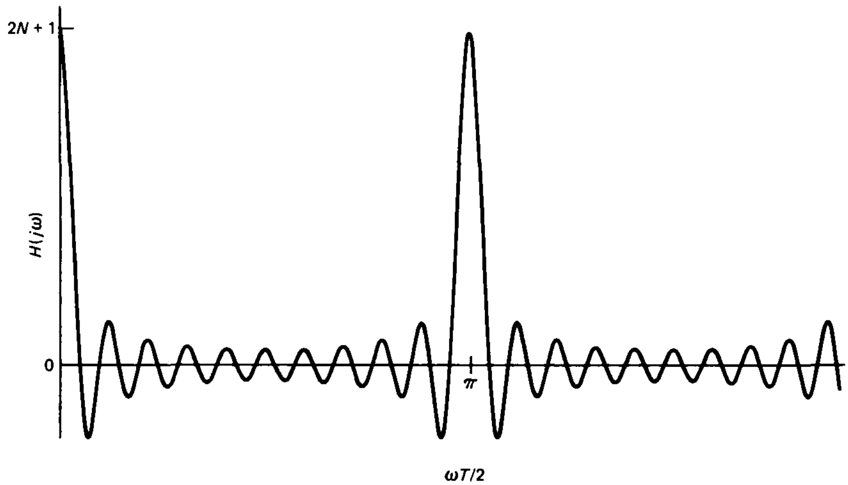

Fig. 2—Output of the uniform array characterized by $\sin[(2N + 1)(\omega T/2)]/\sin(\omega T/2)$.

Given values of the spatial coordinate ϕ and the steering parameter ϕ', one notices that $H(j\omega)$ is periodic in ω with period equal to $2\pi/T$. This relation is illustrated in Fig. 2. Low amplitude values of $H(j\omega)$ represent conditions where the array is discriminating against sources in direction ϕ and for frequency ω. At very low frequencies, by virtue of the characteristics of (2), the array cannot discriminate as a function of wave-arrival direction. At high frequencies, near $1/T$ and multiples thereof, a similar lack of spatial discrimination must occur. This "spatial aliasing" of the discriminatory capabilities fixes an upper limit to the useful frequency bandwidth of the array. This upper limit is

$$f_{\text{upper}} = 1/|T|_{\max},$$

where

$$|T|_{\max} = \frac{d}{c}\,|\cos\phi - \cos\phi'|_{\max}. \tag{3}$$

(Unlike time-domain sampling, where signal and impulse train are multiplied and the signal spectrum translated to sidebands about the sampling frequency and its multiples, this spatial aliasing acts only as a filter for the original frequency components of the source signal.)

The first zero of the response $H(j\omega)$ can be taken as a measure of the lowest frequency below which the array cannot provide spatial discrimination. This first zero, as seen from (2), corresponds to

$$(2N + 1)(\omega T/2) = \pi,$$

or

$$f_{\text{lower}} = \frac{f_{\text{upper}}}{(2N + 1)}. \tag{4}$$

As apparent from (3), given the directions ϕ, ϕ', the upper useful frequency of the array is conditioned by the receiver spacing d. Similarly from (4), the lowest useful frequency for spatial discrimination is dictated by the array size N. Alternatively, (4) gives the required N as

$$N = \left[\frac{1}{2}\left(\frac{f_{\text{upper}}}{f_{\text{lower}}} - 1\right)\right]_{\text{nhi}},$$

where nhi equals the next highest integer.

Further, using the relevant zeros of (2) (i.e., the first, and the first less than f_{upper}), an approximation of the useful discrimination bandwidth of the array is

$$BW = (f_{\text{upper}} - 2f_{\text{lower}}). \tag{5}$$

Additionally, one notes that the value of $|T|$ is conditioned by the values ϕ, ϕ' and has the limits

$$0 \leqslant |T| \leqslant 2d/c,$$

corresponding, respectively, to

$$\phi = \phi' \text{ (i.e., beam steered to the wave-arrival direction),}$$

and to

$$\begin{aligned} \phi &= 0^\circ \quad \text{for} \quad \phi' = 180^\circ, \text{ or} \\ \phi &= 180^\circ \quad \text{for} \quad \phi' = 0^\circ \end{aligned} \tag{6}$$

(i.e., the beam steered to one axial direction with the wave arrival from the opposite axial direction). As a consequence of the trigonometric relations, the useful upper and lower frequencies do not vary linearly with ϕ'. This variation is plotted in Fig. 3 for a value of $N =$ 10.

Finally, if ω and ϕ' are prescribed as parameters, then (2) gives the amplitude response of the array as a function of the wave-arrival direction ϕ. The response can then be considered an explicit function of the spatial coordinate, or $H(\phi)$. Wave arrival from the steering direction, $\phi = \phi'$, gives an infinite period to (2). Also for this condition, $H(\phi) = |H(\phi)|_{\max} = (2N + 1)$, and $f_{\text{upper}} \rightarrow \infty$, and the array passes all frequencies ideally. Wave arrival from a direction away from the steering direction diminishes the period of (2), as well as the output amplitude of $H(\phi)$ for frequencies between the upper and lower values. This diminution in output reflects the beam discrimination, or beamwidth, of the array. The half-power output (i.e., $|H(\phi)|_{-3\text{dB}} =$

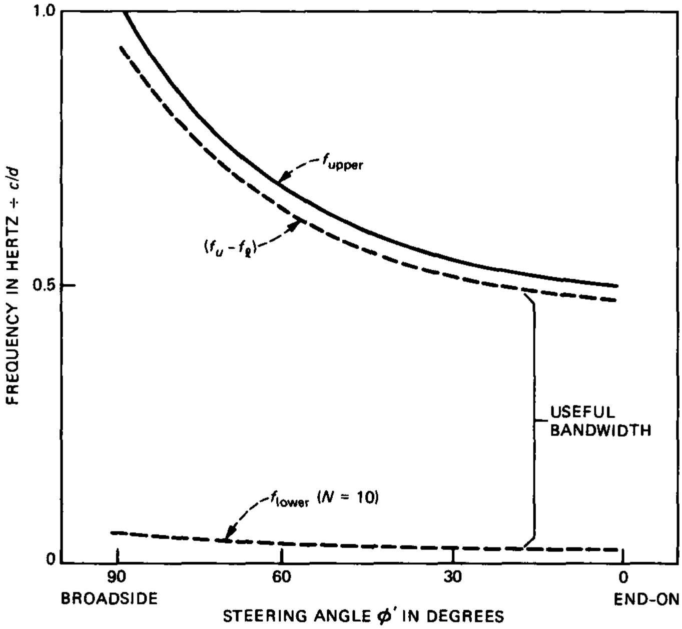

Fig. 3—Variation of the useable array bandwidth as a function of the steering direction ϕ'.

$0.707|H(\phi = \phi')|$ is frequently used to define the beamwidth. Typically, two such half-power directions exist, ϕ_1 and ϕ_2, where $\phi_2 > \phi_1$. Their difference, $\Delta = (\phi_2 - \phi_1)$, is taken as the beamwidth.

III. BEAMWIDTH RELATIONS

Returning to the response function (2) and viewing its dependence upon the spatial coordinate, we can examine the behavior of $H(\phi)$ in the vicinity of $\phi = \phi'$. This corresponds to a value $T = 0$, and hence to $(\omega T/2) = 0$ in the argument of (2). For small magnitudes of the argument we may approximate the denominator term of (2) as

$$\sin(\omega T/2) \cong (\omega T/2), \qquad |\omega T/2| \ll \pi/2. \tag{7}$$

As a consequence,

$$H(\phi) \cong (2N + 1)\left\{\frac{\sin[(2N + 1)(\omega T/2)]}{[(2N + 1)(\omega T/2)]}\right\}. \tag{8}$$

The half-power output occurs for a value of the argument of (8) that makes the $(\sin x/x)$ term equal to 0.707, which requires

$$[(2N + 1)(\omega T/2)] = \pm 1.4 \text{ radians}. \tag{9}$$

Equation (9) therefore implies the half-power wave-arrival direction as

$$\phi_{-3\text{dB}} = \cos^{-1}\left[\cos\phi' \pm \frac{2.8c}{(2N+1)\omega d}\right]. \qquad (10)$$

Typically, there will be two half-power directions, ϕ_1 and ϕ_2, corresponding respectively to the + and − signs in (10). It is clear from (10) that the bracketed term must have a magnitude no greater than unity to produce a valid value for $\phi_{-3\text{dB}}$. As the steering direction is made acute to the axis of the array, the value of $|\cos\phi'| \rightarrow 1$, which indicates a limiting steering direction beyond which (for a given frequency and array design) the array will cease to exhibit two directions (in the hemisphere) where its output drops to half power. This limiting direction, $\phi' = \phi'_{\text{lim}}$, also means that one value of $\phi_{-3\text{dB}}$ must be the axial direction (for example, $\phi_1 = 0°$, or $\phi_2 = 180°$), which implies

$$\phi'_{\text{lim}} = \cos^{-1} \pm\left[1 - \frac{2.8c}{(2N+1)\omega d}\right]. \qquad (11)$$

The positive sign for the bracketed term corresponds to $\phi_1 = 0°$ with ϕ'_{lim} in the first quadrant, and the negative sign corresponds to $\phi_2 = 180°$ with ϕ'_{lim} in the second quadrant.

Furthermore, the beamwidth for steering to these limits is simply

$$\Delta_{\text{lim}} = \cos^{-1}\left[1 - \frac{2(2.8c)}{(2N+1)\omega d}\right]. \qquad (12)$$

Recall that the original approximation requires $|(\omega T)/2| \ll \pi/2$, which holds for ϕ near ϕ', and for frequencies substantially lower than $(f_{\text{upper}}/2)$ for any ϕ, ϕ'.

The spatial directivity pattern for the one-dimensional array is a figure of revolution about the array axis. (In contrast, a two-dimensional array produces a cigar-shaped beam, confined in two dimensions.) Equations (10), (11), and (12) describe the conditions for finding two half-power beamwidth directions (in the hemisphere $0 \leqslant \phi \leqslant 180°$) for steering directions no more acute to the axis than ϕ'_{lim}. The complete (figure of revolution) spatial directivity pattern is therefore cone shaped, with the interior of the cone remaining hollow. These limits may be particularly appropriate to the case of the array mounted on an infinite baffle, such as the wall of a room.

If the array is unbaffled and freely suspended, steering to angles more acute than ϕ'_{lim} may be useful. In the limit ϕ' equals 0° or 180°, which is the end-fire condition. For all steering directions more acute than ϕ'_{lim}, only one half-power direction exists in the hemisphere, and the interior of the cone-shaped directivity pattern "fills in" until the directivity pattern merges into a single main lobe that is axially directed. In this case the half-power beamwidth is simply

$$\Delta = 2\phi_2, \qquad 0^\circ \leq \phi \leq 90^\circ, \qquad \phi' < \phi'_{\text{lim}},$$

or

$$\Delta = 2(180^\circ - \phi_1), \qquad 90^\circ \leq \phi \leq 180^\circ, \qquad \phi' > \phi'_{\text{lim}}. \tag{13}$$

Additionally, the end-fire beamwidth ($\phi' = 0^\circ$ or 180°) is

$$\Delta_{\text{end}} = 2\left\{\cos^{-1}\left|1 - \frac{2.8c}{(2N+1)\omega d}\right|\right\}. \tag{14}$$

IV. REPRESENTATIVE DESIGNS

To fix the utility of these relations, let us consider a practical set of conditions. Suppose the bandwidth of interest is approximately the telephone bandwidth. This bandwidth is included within a range which, for example, we take as 170 to 3500 Hz. Further, suppose the desired range of steering is $60^\circ \leq \phi' \leq 120^\circ$, or $\pm 30^\circ$ from broadside. For this condition $|\cos\phi - \cos\phi'|_{\max} = 1.5$ and the relation

$$f_{\text{upper}} = 1/|T|_{\max} = 3500 \text{ Hz}$$

suggests

$$d = 6.5 \text{ cm}.$$

Similarly, the relation

$$N = \left[\frac{1}{2}\left(\frac{f_{\text{upper}}}{f_{\text{lower}}} - 1\right)\right]_{\text{nhi}}$$

requires

$$N = 10.$$

These values can be used in eqs. (2) and (10) to examine the spatial discrimination and beamwidth under conditions of steering, both inside and outside the prescribed range of ϕ'. Figures 4 and 5 illustrate, on linear plots, the values of $|H(\phi)|$ versus ϕ for $\phi' = 90$, 60, 30, and 0 degrees, and for frequencies two octaves apart, namely, 500 and 2000 Hz. As mentioned previously, the complete pattern of spatial selectivity for the one-dimensional array is a figure of revolution about the array axis.

Consistent with (10), Figs. 4 and 5 show that the acuity of the steered beam diminishes with frequency, and with steering directions away from the normal to the array axis. In addition, using (11) through (14) provides a summary characterization of Δ versus ϕ', with fre-

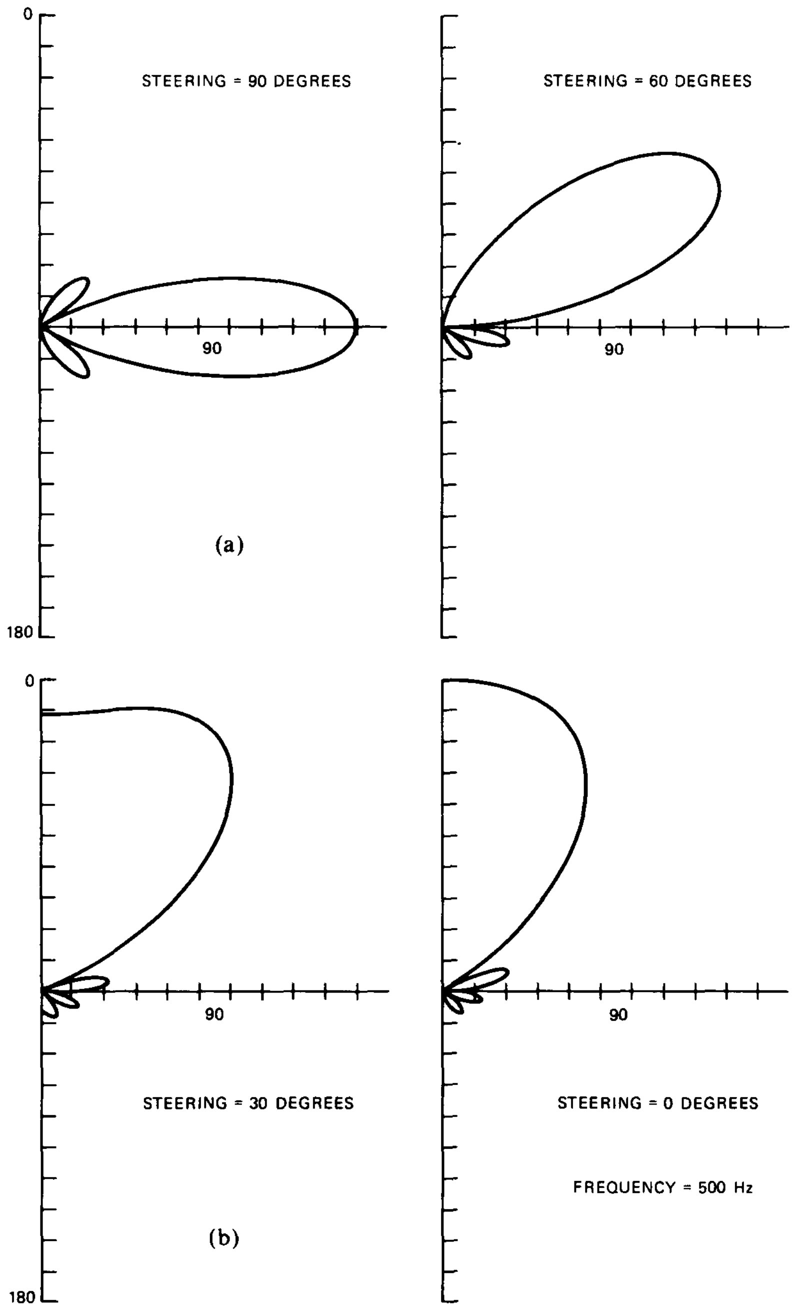

Fig. 4—Spatial response for the uniform array. The parameters are $N = 10$, $d = 6.5$ cm, and $f = 500$ Hz for (a) $\phi' = 90°$, $60°$ and (b) $\phi' = 30°$, $0°$. The polar plots show $|H(\phi)|$ versus ϕ on a linear amplitude scale. For the one-dimensional array, the complete spatial directivity pattern is a figure of revolution about the vertical axis.

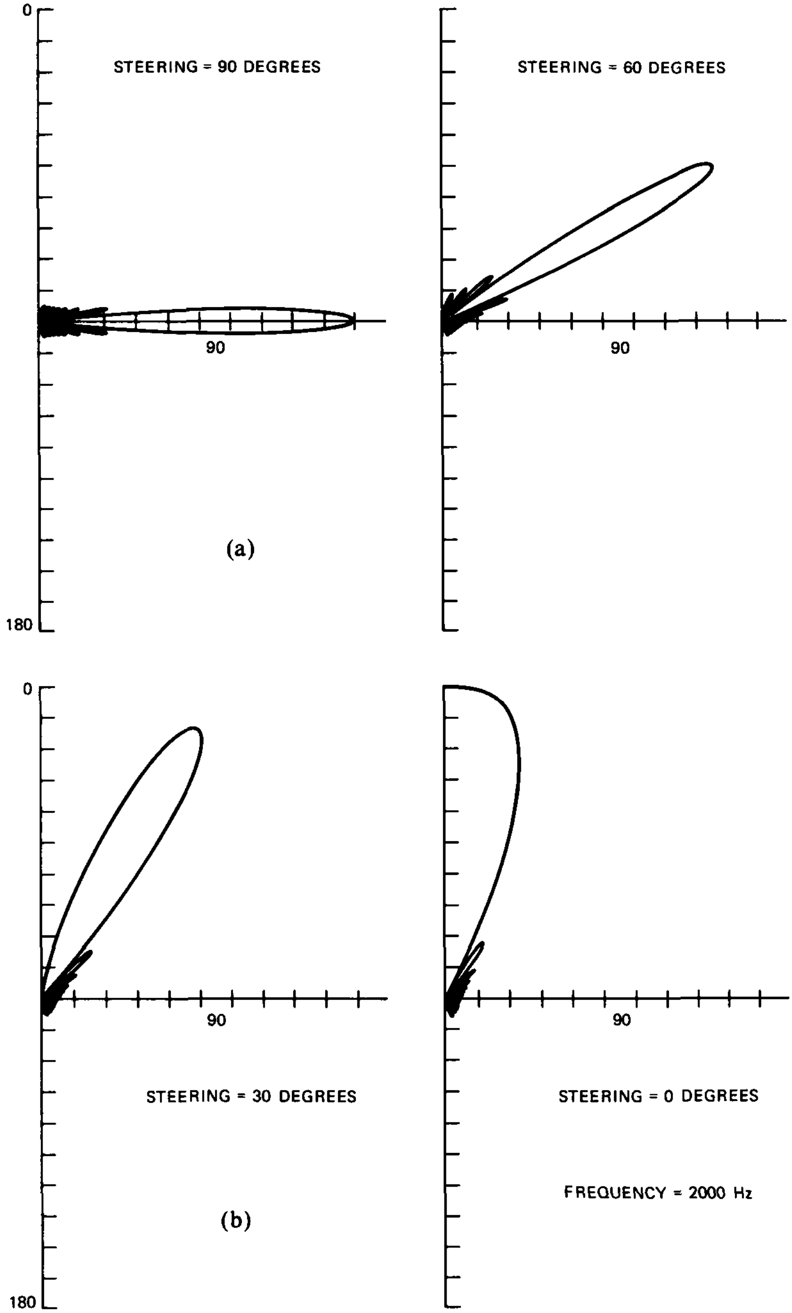

Fig. 5—Spatial response on a linear scale for the uniform array. The parameters are $N = 10$, $d = 6.5$ cm and $f = 2000$ Hz for (a) $\phi' = 90°$, $60°$ and (b) $\phi' = 30°$, $0°$.

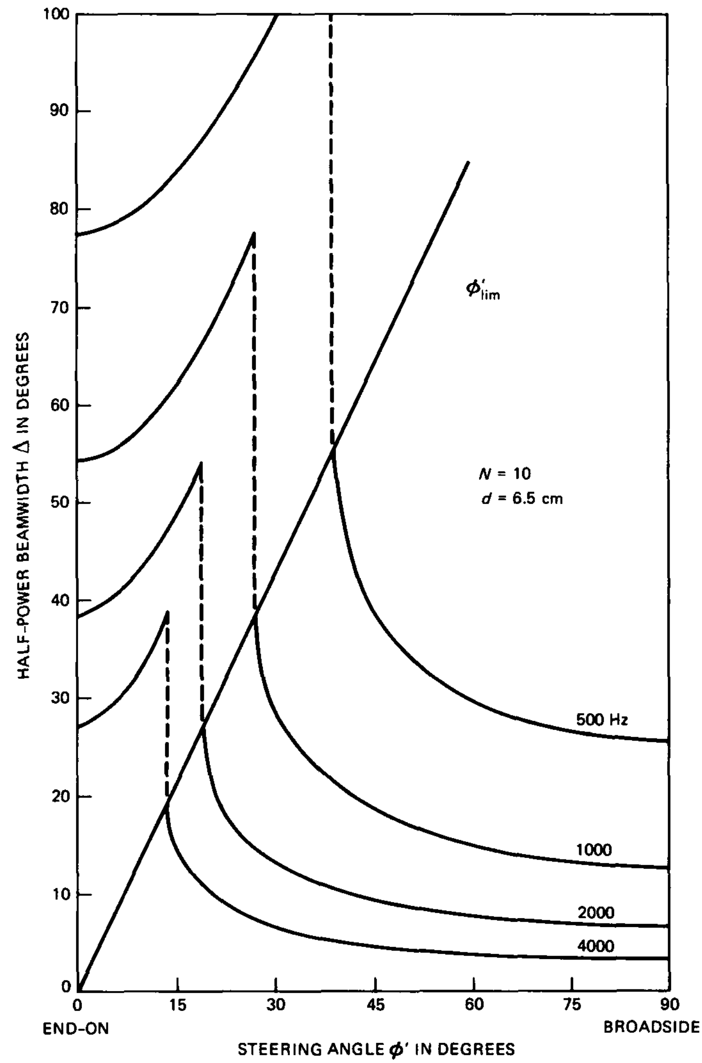

Fig. 6—Variation of half-power beamwidth Δ as a function of steering angle ϕ'. Frequency is the parameter. The limiting steering angle, ϕ'_{lim}, is calculated from eq. (11). For steering angles more acute to the array axis than ϕ_{lim}, only a single half-power response direction exists in the right hemisphere. The beam pattern merges to an axially directed lobe, as specified in eq. (14), and as indicated by the dashed portions of the curves.

quency the parameter, as shown in Fig. 6. The plot is shown for the first quadrant, $0° \leqslant \phi' \leqslant 90°$. The second quadrant, $90° \leqslant \phi' \leqslant 180°$, exhibits mirror-image curves, symmetric about $\phi' = 90°$.

Another aspect of interest is the asymmetry of the beam about the

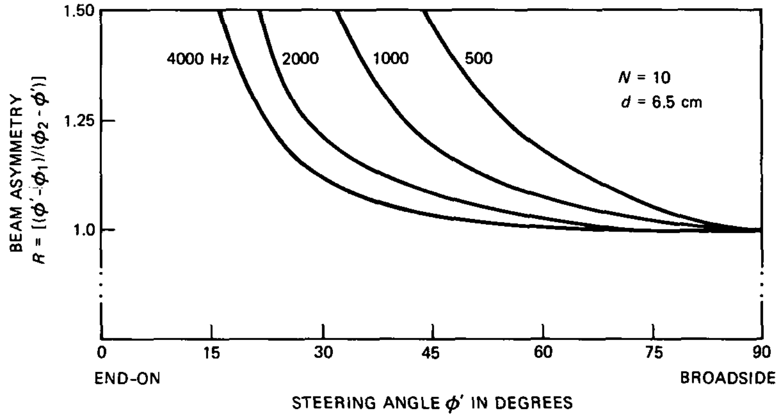

Fig. 7—Variation of beam asymmetry R as a function of steering angle ϕ', with frequency as the parameter.

steering direction ϕ'. This is reflected by the values $(\phi' - \phi_1)$ and $(\phi_2 - \phi')$, and by their ratios, R. This asymmetry, as a function of the steering angle and with frequency as a parameter, is plotted in Fig. 7.

V. CONCLUDING COMMENT

Automatic, electronic steering of microphone arrays is an attractive means for improving sound pickup in large rooms. This improvement makes more feasible interactive communication between sizable groups of people seated in auditoria or meeting rooms remote from one another. Recently emerging microprocessors have the computational capability for automatic detection of desired sound sources in the room, and for automatic beam forming and beam steering to the desired source.[1] But spatial acuity is affected by steering, typically diminishing as the beam is steered away from the broadside position. Therefore, in addition to the considerations of useful bandwidth (and, hence, array geometry), the design of a beam-steered system must take into account the variations in acuity with steering. The relations derived here provide quantitative means for designing uniform, unweighted arrays for specified conditions of room coverage.

VI. ACKNOWLEDGMENT

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REFERENCES

1. J. L. Flanagan et al., "Computer-Steered Microphone Arrays for Conference Telephony," J. Acoust. Soc. Am., Suppl. 1, *70* (Fall 1981), p. S79.

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