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Performance of Nondiversity Receivers for Spread Spectrum in Indoor Wireless Communications

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In this work we have considered direct-sequence spread-spectrum transmission for indoor wireless communications. We have modeled the indoor communications medium, which is a multipath fading channel, by a discrete set of Rayleigh faded paths. We have proposed new analytical techniques to evaluate the probability of error for the receiver terminals studied in this work. Numerical results reveal that, for the nondiversity receivers considered here, Rayleigh fading is very hostile to this form of modulation/multiple-access technique. The results also indicate that either some form of operation to prevent Rayleigh fading or diversity operation to counteract Rayleigh fading is required.

I. INTRODUCTION

A principal purpose of this paper is to evaluate the performance of a direct-sequence Spread-Spectrum Multiple-Access (SSMA) system using Binary Phase Shift Keying (BPSK) modulation for Indoor Wireless Communications (IWC).

In the past decade there has been increased interest in a class of multiple-access techniques known as Code Division Multiple Access (CDMA) in which the mode of access is due primarily to coding by spread-spectrum sequences. In contrast with traditional time- and

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frequency-division multiple access, these techniques do not require accurate time or frequency coordination among the transmitters in the system. This feature makes CDMA very attractive for IWC applications, because, as described later, an IWC takes place in a severe multipath fading environment. However, the cost of the ease of access is the increased channel bandwidth required by spread-spectrum codes. The bandwidth spreading leads to a low spectral energy density level, which is an advantage in military communications in hostile environments. It also offers the possibility of reusing overcrowded radio frequency bands, as recently studied by the Federal Communications Commission.[1]

SSMA is a common form of CDMA in which every user is assigned a code sequence modulated onto a carrier signal according to the user's digital information. A high-rate code, that is, many code chips per data bit, spreads the bandwidth of the information signal. Frequency-hopped[2-4] and phase-modulated SSMA[5] are two forms of SSMA. The latter, also known as direct-sequence spread-spectrum multiple access, is what we concern ourselves with in this work, for its multiple-access capability and ease of implementation.

Since in direct-sequence SSMA the entire channel bandwidth is available to all users of the system at all times, the code sequence applied by each user in spreading the information band must have low cross-correlation properties in order to achieve a low level of mutual interference among the users. Several classes of code sequences suitable for this application have been presented by Sarwate and Pursely.[6] A class of these codes that are employed in our work is the well-known Gold sequences, which possess the low cross-correlation properties required in multiple-access applications.

The chief purpose of this paper is to assess the communication performance of a direct-sequence SSMA system in the presence of multipath fading that is a characteristic of IWC. Our criterion of merit is average probability of error as a function of signal-to-noise ratio.

There is a sizable literature relating to the effects of multiple-access interference on the performance of a direct-sequence SSMA, among which are Refs. 7 through 12. All of these consider the communication channel to be a single path with no fading. In IWC applications, because of the existence of many reflectors and scatterers, multipath fading is severe. Preliminary impulse response measurements inside office buildings indicate the severity of multipath fades.[13,14] Hence, the attempt in this work is to incorporate multipath fading effects in the analysis of average probability of error of direct-sequence SSMA. Among the limited number of articles relating to the effects of multipath fading on the performance of direct-sequence SSMA is the work by G. Turin[15] that examines the structure of a number of receivers for mobile digital radio. However, the computer simulation results in Ref. 15 are restricted to the behavior of a single transmitter-receiver pair, and therefore, no multiple-access interference is taken into account. References 16 through 18 consider fading links, although Ref. 17 specifies single-tone, rather than multiple-access, interference. However, almost all studies have used measures other than average error rate in their evaluation. Reference 18 presents a simplified analysis by invoking the Gaussian assumption for the composite multiple-access interference distribution previously addressed in papers by Yao[9] and Mazo.[10] We avoid any argument about the validity and range of accuracy of the Gaussian assumption. In this work, unlike in Ref. 18, we make no assumption about the distribution of the composite multiple-access interference. By employing moment-generating techniques, we evaluate the average probability of error in the presence of multipath fading. In this evaluation we apply the numerical Gauss quadrature integration.[19] Specifically, our work extends the work in Ref. 12 to channels with multipath fading.

In Section II we begin with a description of the SSMA system and the channel model. We then derive the conditional error probability. Evaluation of average error probability by the moment-generating approach is described in Section III. Numerical results are discussed in Section IV. Finally, a summary and conclusions are presented in the last section of the paper.

II. THE MODEL AND THEORETICAL DEVELOPMENTS

2.1 System model

Consider the direct-sequence SSMA transmission system model for K users depicted in Fig. 1. The kth user's information signal $b_k(t)$ is a sequence of rectangular pulses taking on values from the set $\{\pm 1\}$ over a T-seconds time interval. Hence,

$$b_k(t) = \sum_{j=-\infty}^{\infty} b_j^k P_T(t - jT), \tag{1}$$

where b_j^k represents the kth user data at the jth timing interval and $P_T(\cdot)$ is a rectangular waveform of T-seconds duration. The kth user is assigned a code waveform $a_k(t)$ that consists of a periodic sequence of rectangular chips taking on values from the set $\{\pm 1\}$ each of duration T_c seconds. If a_i^k represents the ith-chip value of the kth user, then,

$$a_k(t) = \sum_{i=-\infty}^{\infty} a_i^k P_{T_c}(t - iT_c). \tag{2}$$

We assume each user code sequence has a period of $N = T/T_c$. That is, there is one period of code sequence per data bit.

After spreading the information bandwidth to N times its original value, by modulo-2 adding the direct-sequence code to the data signal and biphase modulating the result onto the carrier signal, $A\cos(\omega_c t + \theta_k)$— where A is the carrier level, ω_c is the nominal carrier frequency, and θ_k is the carrier phase that is assumed to be uniformly distributed between 0 and 2π—the transmitted signal of the kth user becomes

$$S_k(t) = Aa_k(t)b_k(t)\cos(\omega_c t + \theta_k), \qquad k = 1, 2, \cdots, K. \tag{3}$$

2.2 Transmission channel model

In spread-spectrum transmission over multipath fading channels, if the spread bandwidth of the transmitted signal exceeds the coherence bandwidth of the channel, where the latter is proportional to the inverse of the channel maximum multipath delay spread, the multipath components can be resolved into a discrete number of Rayleigh faded paths. The number of resolved paths depends on the channel multipath spread and the spreading bandwidth of the signal, as discussed by Proakis.[20] We assume that the IWC channel for the desired transmitter and receiver ($k = 1$) depicted in Fig. 1 can be represented by an L-paths Rayleigh fading model where a single transmitted pulse is received via L-paths at the random time instants t_l, $l = 1, \cdots, L$. We assume t_l is uniformly distributed over one bit period, i.e., over $(0, T)$. This is ensured by signaling at baseband at a rate less than the channel coherence bandwidth. Hence, intersymbol interference is negligible

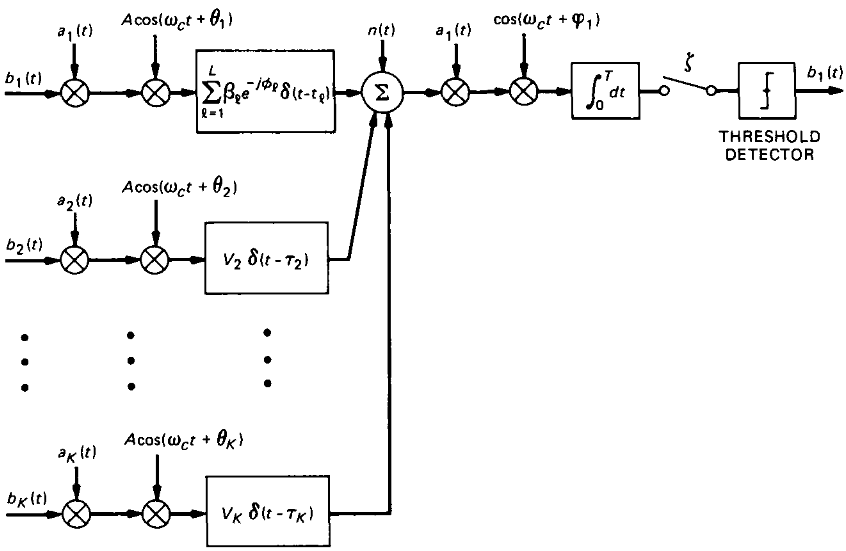

Fig. 1—System model.

here. Therefore, the low-pass equivalent impulse response of the passband channel, $h(t)$, can be represented by

$$h(t) = \sum_{l=1}^{L} \beta_l \delta(t - t_l) e^{j\phi_l}, \tag{4}$$

where $\delta(\cdot)$ is the Kronecker delta, β_l is a Rayleigh distributed random path gain, and ϕ_l is the random path phase, uniformly distributed between 0 and 2π. It is further assumed that all the parameters of all paths are identically distributed over their specified range. These assumptions are also related to G. Turin's[15] description of a discrete multipath fading environment. As stated earlier, the L-paths model stems from the fact that spread-spectrum signaling with a transmitted signal bandwidth much wider than the coherence bandwidth of the multipath fading channel enables the multipath components to be resolved. Therefore, the channel seems to be fading-frequency selective to the signal. In eq. (4) all the variables are time invariant.

To keep the analysis tractable we further assume that the kth interfering user of the multiple-access system is linked to the receiver of Fig. 1 via a single Rayleigh fading path with a uniformly distributed random delay τ_k ranging from zero to one bit period, T. The conclusions will reveal that there is no loss in generality in making such an assumption. Since our chief aim is to show what is not possible, more elaborate models incorporating more noise sources could only strengthen our conclusions.

Since τ_k and t_l are independent, the model results in a Rayleigh faded interfering user sequence being received asynchronously compared with the desired user sequence at the receiver in Fig. 1. In our formulation we specify the Rayleigh distributed path gain of the interfering users by V_k, $k = 2, \cdots, K$. Therefore, as depicted in Fig. 1, the received signal for the fading model described above is given by

$$r(t) = A \sum_{l=1}^{L} \beta_l a_1(t - t_l) b_1(t - t_l) \cos(\omega_c t - \omega_c t_l + \phi_l + \theta_1)$$
$$+ A \sum_{k=2}^{K} V_k a_k(t - \tau_k) b_k(t - \tau_k) \cos(\omega_c t - \omega_c \tau_k + \theta_k) + n(t), \tag{5}$$

where $n(t)$ is white Gaussian noise with double-sided spectral density of $N_0/2$ level and θ_1 can be assumed to be zero with no loss of generality.

The desired receiver is assumed to coherently recover the carrier phase and delay lock to the first arriving desired signal. Therefore, after (1) the correlation operation that collapses the wideband coded signal into a narrowband modulated signal and (2) the demodulation process, a signal sample at the receiver low-pass filter output can be expressed as

$$\xi = \int_0^T r(t)a_1(t)\cos(\omega_c t)dt; \tag{6}$$

or, using eq. (5) we have

$$\xi = \frac{A}{2}\sum_{l=1}^{L}\beta_l \int_0^T a_1(t-t_l)b_1(t-t_l)a_1(t)\cos(\psi_l)dt$$
$$+ \frac{A}{2}\sum_{k=2}^{K} V_k \int_0^T a_k(t-\tau_k)b_k(t-\tau_k)a_1(t)\cos(\Theta_k)dt + \eta, \tag{7}$$

where η is a sample of the Gaussian noise with zero mean and $(N_0T)/4$ variance, $\psi_l = \phi_l - \omega_c t_l$ and $\Theta_k = \theta_k - \omega_c\tau_k$.

2.3 Detection problem and average error probability

The assumption of phase and delay locking of the receiver to the first desired modulated signal that is received enables one to rewrite eq. (7) as

$$\xi = \beta_1 \frac{A}{2}\int_0^T a_1^2(t)b_1(t)dt$$
$$+ \frac{A}{2}\sum_{l=2}^{L}\beta_l \int_0^T a_1(t-t_l)b_1(t-t_l)a_1(t)\cos(\psi_l)dt$$
$$+ \frac{A}{2}\sum_{k=2}^{K} V_k \int_0^T a_k(t-\tau_k)b_k(t-\tau_k)a_1(t)\cos(\Theta_k)dt + \eta. \tag{8}$$

We notice that from eq. (1),

$$b_1(t) = \sum_{j=-\infty}^{\infty} b_j^1 P_T(t-jT), \tag{9}$$

and therefore, eq. (8) can be expressed as

$$\xi = \beta_1 \frac{AT}{2} b_0^1 + \frac{A}{2}\sum_{l=2}^{L}\beta_l[b_{-1}^1 R_{1,1}(t_l) + b_0^1\hat{R}_{1,1}(t_l)]\cos(\psi_l)$$
$$+ \frac{A}{2}\sum_{k=2}^{K} V_k[b_{-1}^k R_{k,1}(\tau_k) + b_0^k\hat{R}_{k,1}(\tau_k)]\cos(\Theta_k) + \eta, \tag{10}$$

where b_0^1 represents the information bit being detected and b_{-1}^1 is the preceding bit, which, due to the channel delay spread, affects the detection of b_0^1 received on the first path between the desired transmitter and receiver.

In eq. (10),

$$R_{1,1}(t_l) = \int_0^{t_l} a_1(t-t_l)a_1(t)dt \tag{11}$$

and

$$\hat{R}_{1,1}(t_l) = \int_{t_l}^{T} a_1(t - t_l)a_1(t)dt$$

are partial autocorrelation functions of the regenerated desired code at the receiver, that is, $a_1(t)$ with its delayed version received via the lth Rayleigh faded path, namely, $a_1(t - t_l)$. We note that the coded sequence received via the first path between the transmitter and receiver of Fig. 1 is fully correlated with the regenerated sequence $a_1(t)$, owing to the delay locking operation introduced at the receiver. Also, due to the asynchronous arrival of the interfering user's code, eq. (10) contains partial cross correlations of the regenerated sequence, $a_1(t)$, and a delayed version of the interfering codes defined by

$$R_{k,1}(\tau_k) = \int_{0}^{\tau_k} a_k(t - \tau_k)a_1(t)dt \tag{12}$$

and

$$\hat{R}_{k,1}(\tau_k) = \int_{\tau_k}^{T} a_k(t - \tau_k)a_1(t)dt.$$

In the second term of eq. (10), if the polarity of the two adjacent data bits b_{-1}^1 and b_0^1 happens to be the same, the sum of the two partial autocorrelations turns into a full autocorrelation with the same polarity as b_0^1. This could have been useful if the path phase were known. However, due to the random path phase, the second term in eq. (10) only adds to the channel noise. In general, signals delayed by amounts outside $\pm T_c$ seconds about a correlation peak in the correlation of $a_1(t)$ are attenuated by an amount determined by the processing gain of the code, that is, $N = T/T_c$. To assess the degree of degradation that is due to modulated partial correlation, in a separate case, we postulate having off periods of a T-second period between information bits, which forces to zero the undesired term containing b_{-1}^1 in eq. (10). Analysis of this case is presented in Appendix A, and associated numerical results are given in Section 4.2.

For now, we return to our normal transmission policy, where no off period is allowed between adjacent information bits.

The objective of the detector is to compare the received sample in eq. (10) with a preset threshold in order to make a decision on the polarity of the data bit being detected, that is, b_0^1.

Now the detector makes a wrong decision if ξ is negative while $b_0^1 = +1$, or if ξ is positive and $b_0^1 = -1$. We note that during the detection interval of b_0^1 the other three data bits in eq. (10), namely, b_{-1}^1, b_{-1}^k, and b_0^k, $k \neq 1$, can independently take on $\{\pm 1\}$. Hence, the conditional probability of error is expressed by

$$P_{e|\beta_1,x_1,x_2,z} = \frac{1}{2} P_r \left\{ \beta_1 \frac{AT}{2} + \frac{AT}{2}(x_1 + z) + \eta < 0 \,|\, b_0^1 = +1 \right\}$$

$$+ \frac{1}{2} P_r \left\{ -\beta_1 \frac{AT}{2} + \frac{AT}{2}(x_2 + z) + \eta > 0 \,|\, b_0^1 = -1 \right\}, \quad (13)$$

where

$$x_1 = \sum_{l=2}^{L} \frac{\beta_l}{T} \{ b_{-1}^1 R_{1,1}(t_l) + \hat{R}_{1,1}(t_l) \} \cos(\psi_l), \quad (14)$$

$$x_2 = \sum_{l=2}^{L} \frac{\beta_l}{T} \{ b_{-1}^1 R_{1,1}(t_l) - \hat{R}_{1,1}(t_l) \} \cos(\psi_l), \quad (15)$$

and

$$z = \sum_{k=2}^{K} \frac{V_k}{T} \{ b_{-1}^k R_{k,1}(\tau_k) + b_0^k \hat{R}_{k,1}(\tau_k) \} \cos(\Theta_k). \quad (16)$$

We can rewrite eq. (13) as

$$P_{e|\beta_1,x_1,x_2,z} = \frac{1}{4} \left\{ \operatorname{erfc} \left[\frac{\beta_1 \frac{AT}{2} + \frac{AT}{2}(x_1 + z)}{\sigma\sqrt{2}} \right] + \operatorname{erfc} \left[\frac{\beta_1 \frac{AT}{2} - \frac{AT}{2}(x_2 + z)}{\sigma\sqrt{2}} \right] \right\}, \quad (17)$$

where

$$\operatorname{erfc}(\mu) = \frac{2}{\sqrt{\pi}} \int_{\mu}^{\infty} e^{-y^2} dy \quad (18)$$

and σ is the standard deviation of the Gaussian noise. We notice that variables x_1 and x_2 in eq. (17) are in a conjugate form and have identical even moments. This is because the data symbols have zero mean; therefore, all the odd moments of x_1 and x_2 are zero. As a result, all the cross-product terms in the derivation of the even moments become zero. It is then easy to see that the even moments of x_1 and x_2 are identical. Now, to evaluate the average error probability—as will be explained in the next section—we apply the moments of the interference terms along with the numerical Gauss quadrature integration.[19] It is not too difficult, then, to observe that instead of using eq. (17) we can use the following:

$$P_{e|\beta_1,x,z} = \frac{1}{2}\,\mathrm{erfc}\left\{\frac{\frac{AT}{2}\,[\beta_1 - (x + z)]}{\sigma\sqrt{2}}\right\}, \tag{19}$$

where x in this equation can be either x_1 or x_2. In other words, both eqs. (17) and (19) will result in the same average error probability under evaluation by moment-generating functions.

Also, recalling

$$\sigma = \frac{\sqrt{N_0T}}{2} \tag{20}$$

and observing the bit energy,

$$E_b = \frac{A^2T}{2}, \tag{21}$$

we can express eq. (19) as

$$P_{e|\beta_1,x,z} = \frac{1}{2}\,\mathrm{erfc}\left\{\sqrt{\frac{E_b}{N_0}}\,[\beta_1 - (x + z)]\right\}. \tag{22}$$

If instead of a sample value of a Rayleigh random variable in eq. (22) we had a constant value of d_0, then eq. (22) would become

$$P_{e|x,z} = \frac{1}{2}\,\mathrm{erfc}\left\{\sqrt{\frac{E_b}{N_0}}\,[d_0 - (x + z)]\right\}. \tag{23}$$

Now, in the absence of any multipath fading and multiple-access interference, $d_0 = 1$, and this equation becomes

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right), \tag{24}$$

which is the well-known[20] performance of a coherently demodulated BPSK signal.

In the Rayleigh fading case the actual received signal-to-noise ratio is

$$\gamma = \beta_1^2\,\frac{E_b}{N_0}, \tag{25}$$

and its average is

$$\gamma_0 = E\{\beta_1^2\}\,\frac{E_b}{N_0}, \tag{26}$$

where $E\{\cdot\}$ denotes the expected value. Since β_1 is Rayleigh distributed, γ has an exponential density. Hence,

$$P_{e|x,z} = \int_0^\infty P_{e|\beta_1,x,z} p(\gamma) d\gamma, \tag{27}$$

where

$$p(\gamma) = \frac{1}{\gamma_0} e^{-\gamma/\gamma_0} \tag{28}$$

and

$$P_{e|x,z} = \frac{1}{2\gamma_0} \int_0^\infty \operatorname{erfc}\left\{\sqrt{\gamma} - \sqrt{\frac{E_b}{N_0}}(x+z)\right\} e^{-\gamma/\gamma_0} d\gamma. \tag{29}$$

This may be integrated by parts if we apply the following change of variable:

$$t = \sqrt{\gamma} - \sqrt{\frac{E_b}{N_0}}(x+z);$$

and after some manipulations it results in

$$P_{e|x,z} = \frac{1}{2}\left\{\operatorname{erfc}\left[-\sqrt{\frac{E_b}{N_0}}(x+z)\right] - \frac{\sqrt{\gamma_0}}{\sqrt{\gamma_0+1}} \exp\left[\frac{-\frac{E_b}{N_0}(x+z)^2}{\gamma_0+1}\right] \cdot \operatorname{erfc}\left[-\frac{\sqrt{\gamma_0}}{\sqrt{\gamma_0+1}}(x+z)\sqrt{\frac{E_b}{N_0}}\right]\right\}. \tag{30}$$

Interested readers are referred to Appendix B for a detailed derivation of eq. (30) (also see Ref. 21). We notice that in the absence of multiple-access interference and a single-path fading of the desired signal, eq. (30) becomes

$$P_e = \frac{1}{2}\left[1 - \frac{\sqrt{\gamma_0}}{\sqrt{\gamma_0+1}}\right], \tag{31}$$

which is the ideal performance of a single-path Rayleigh fading channel.[20]

Now using Gauss Quadrature integration[19] the average probability of error can be obtained numerically, by averaging the conditional probability of eq. (30) over the interference term, $x + z$. This is accomplished by first evaluating the $N_m = 2N_c + 1$ moments of $x + z$, which are applied in evaluation of the N_c weights and nodes of the Quadrature Rule.[19] Hence, the average probability of error is

$$P_e = \frac{1}{2} \sum_{j=1}^{N_c} W_j \left\{ \operatorname{erfc}\left(- \sqrt{\frac{E_b}{N_0}} \zeta_j \right) \right.$$
$$\left. - \frac{\sqrt{\gamma_0}}{\sqrt{\gamma_0 + 1}} \exp\left[- \frac{\frac{E_b}{N_0} \zeta_j^2}{\gamma_0 + 1} \right] \operatorname{erfc}\left(- \zeta_j \sqrt{\frac{E_b}{N_0}} \frac{\sqrt{\gamma_0}}{\sqrt{\gamma_0 + 1}} \right) \right\}, \quad (32)$$

where W_j's and ζ_j's are the N_c weights and nodes of the Quadrature Rule.[19] A detailed formulation of this is given in Appendix C. By the same token, the average probability of error in eq. (23) becomes

$$P_e = \frac{1}{2} \sum_{j=1}^{N_c} W_j \operatorname{erfc} \left\{ \sqrt{\frac{E_b}{N_0}} (d_0 - \zeta_j) \right\}. \quad (33)$$

III. MOMENT-GENERATING APPROACH

The average probability of error expression in eqs. (32) and (33) assumes $2N_c + 1$ moments of random variable $(x + z)$, which is a function of independent random parameters: β_l, t_l, τ_k, ψ_l, Θ_k, and b_i^k.

Furthermore, x and z are independent and symmetrically distributed; hence the odd moments of $(x + z)$ are all zero. Therefore, having the even moments of x and z, one can determine the moments of $(x + z)$ using the simple binomial rule, that is,

$$E\{(x + z)^m\} = \sum_{i=0}^{m} \binom{m}{i} E\{x^i\} \cdot E\{z^{m-i}\}. \quad (34)$$

In this section we derive the moments of z, and by similarity we deduce the moments of x.

Since

$$z = \sum_{k=2}^{K} z_k,$$

where

$$z_k = \frac{V_k}{T} \{b_{-1}^k R_{k,1}(\tau_k) + b_0^k \hat{R}_{k,1}(\tau_k)\} \cos \Theta_k,$$

then,

$$E\{z_k^{2m}\} = \frac{1}{T^{2m}} E\{V_k^{2m}\} \cdot E([\cos \Theta_k]^{2m} \cdot [b_{-1}^k R_{k,1}(\tau_k) + b_0^k \hat{R}_{k,1}(\tau_k)]^{2m}). \quad (35)$$

To evaluate the second expectation of the right-hand side of eq. (35), since Θ_k is an independent random variable, we can deal with it separately.

That is, we first can evaluate

$$E\{[\cos\Theta_k]^{2m}\} = \frac{\binom{2m}{m}}{4^m}$$

and then find the second expectation of eq. (35) as

$$H = \frac{\binom{2m}{m}}{4^m} E\{[b_{-1}^k R_{k,1}(\tau_k) + b_0^k \hat{R}_{k,1}(\tau_k)]^{2m}\}$$

$$= \frac{\binom{2m}{m}}{4^m} \cdot \frac{1}{T} \cdot \int_0^T [b_{-1}^k R_{k,1}(\tau_k) + b_0^k \hat{R}_{k,1}(\tau_k)]^{2m} d\tau_k,$$

where the expectation in H is over the random delay τ_k.

Now we can expand the latter integral over N chips period. Hence,

$$H = \frac{\binom{2m}{m}}{4^m} \cdot \frac{1}{T} \sum_{n=0}^{N-1} \int_{nT_c}^{(n+1)T_c} [b_{-1}^k R_{k,1}(\tau_k) + b_0^k \hat{R}_{k,1}(\tau_k)]^{2m} d\tau_k.$$

We can then use the standard notations of Pursley[7] to evaluate the correlation functions. This is accomplished by assuming rectangular chips and noting that for any $0 \le nT_c \le \tau \le (n+1)T_c \le T$, as shown by Pursley,[7]

$$\begin{cases} R_{k,1}(\tau) = A_{n_{k,1}} T_c + B_{n_{k,1}}(\tau - nT_c) \\ \hat{R}_{k,1}(\tau) = \hat{A}_{n_{k,1}} T_c + \hat{B}_{n_{k,1}}(\tau - nT_c) \end{cases}, \tag{36}$$

where

$$\begin{cases} A_{n_{k,1}} = C_{k,1}(n-N) \\ B_{n_{k,1}} = C_{k,1}(n+1-N) - C_{k,1}(n-N), \\ \hat{A}_{n_{k,1}} = C_{k,1}(n) \qquad k = 1, 2, \cdots, K \\ \hat{B}_{n_{k,1}} = C_{k,1}(n+1) - C_{k,1}(n) \end{cases} \tag{37}$$

where the discrete aperiodic cross-correlation term $C_{k,1}(\cdot)$ is related to chip sequences a_j^k and a_j^1 via

$$
C_{k,1}(n) = \begin{cases} \sum_{j=0}^{N-1-n} a_j^k a_{j+n}^1 & 0 \le n \le N-1 \\ \sum_{j=0}^{N-1+n} a_{j-n}^k a_j^1 & -(N-1) \le n \le 0. \\ 0 & \text{else} \end{cases} \tag{38}
$$

Therefore, H becomes

$$
H = \frac{1}{T} \frac{\binom{2m}{m}}{4^m} \sum_{n=0}^{N-1} \sum_{r=0}^{m} \binom{2m}{2r}
\cdot \int_{nT_c}^{(n+1)T_c} [\mathrm{A}_{n_{k,1}} \mathrm{T}_c + \mathrm{B}_{n_{k,1}} (\tau_k - \mathrm{nT}_c)]^{2r}
\cdot [\hat{A}_{n_{k,1}} T_c + \hat{B}_{n_{k,1}} (\tau_k - nT_c)]^{2(m-r)} d\tau_k. \tag{39}
$$

Notice that in deriving eq. (39), because of the even moments, b_{-1}^k and b_0^k are averaged to one. Now in eq. (39), by changing $\tau_k - nT_c$ to WT_c, H becomes

$$
H = \frac{T_c^{2m+1}}{T} \frac{\binom{2m}{m}}{4^m} \sum_{n=0}^{N-1} \sum_{r=0}^{m} \binom{2m}{2r}
\cdot \left\{ \int_0^1 [A_{n_{k,1}} + B_{n_{k,1}} W]^{2r} \cdot [\hat{A}_{n_{k,1}} + \hat{B}_{n_{k,1}} W]^{2(m-r)} dW \right\}. \tag{40}
$$

Therefore, to determine H we have to solve

$$
\Gamma_{m,r,n} = \int_0^1 [A_{n_{k,1}} + B_{n_{k,1}} W]^{2r} \cdot [\hat{A}_{n_{k,1}} + \hat{B}_{n_{k,1}} W]^{2(m-r)} dW. \tag{41}
$$

This can be done recursively, using integration by parts, and the result is

$$
\Gamma_{m,r,n} = \sum_{i=0}^{2r} (-1)^i \frac{(B_{n_{k,1}})^i}{(\hat{B}_{n_{k,1}})^{i+1}} \cdot \frac{1}{(i+1)} \cdot \frac{\binom{2r}{i}}{\binom{2(m-r)+i+1}{i+1}}
\cdot \{ (A_{n_{k,1}} + B_{n_{k,1}})^{2r-i} \cdot (\hat{A}_{n_{k,1}} + \hat{B}_{n_{k,1}})^{2(m-r)+i+1}
- (A_{n_{k,1}})^{2r-i} \cdot (\hat{A}_{n_{k,1}})^{2(m-r)+i+1} \}. \tag{42}
$$

A detailed derivation of $\Gamma_{m,r,n}$ is included in Appendix D. For H in eq. (40) we now have

$$H = \frac{T_c^{2m+1}}{T} \frac{\binom{2m}{m}}{4^m} \sum_{n=0}^{N-1} \sum_{r=0}^{m} \binom{2m}{2r} \Gamma_{m,r,n},$$

and for $E\{z_k^{2m}\}$ in eq. (35),

$$E\{z_k^{2m}\} = \frac{\binom{2m}{m}}{4^m} E\{V_k^{2m}\} \cdot \frac{1}{N^{2m+1}} \sum_{n=0}^{N-1} \sum_{r=0}^{m} \binom{2m}{2r} \Gamma_{m,r,n}. \quad (43)$$

We notice that for a Rayleigh distributed V_k,[22]

$$E\{V_k^{2m}\} = 2^m \cdot \nu_{0_k}^m (m!), \quad (44)$$

where $\nu_{0_k} = E\{V_k^2/2\}$ is the average strength of the Rayleigh faded path associated with the kth interfering user. Note that assuming equal average strength cochannel interferers under a fixed total interference power corresponds to a maximum average probability of error in digital communications. Therefore, the results are conservative with respect to this assumption.

To find the moments of

$$z = \sum_{k=2}^{K} z_k,$$

we can use a three-step method prescribed in Ref. 12, where from the cumulants of random variable z_k, $\gamma_m(z_k)$, the moments of z are arrived at. This simple algorithm is outlined below.

The first step is to find

$$M_{2m} = E\{z_k^{2m}\}$$

and then

$$\gamma_{m+1}(z_k) = M_{m+1}(z_k) - \sum_{r=0}^{m-1} \binom{m}{r} \gamma_{r+1}(z_k) \cdot M_{m-r}(z_k)$$

with

$$\gamma_1(z_k) = M_1(z_k) = 0. \quad (45)$$

The second step is to find

$$\gamma_m(z) = \sum_{k=2}^{K} \gamma_m(z_k). \quad (46)$$

The third step is to find

$$M_{m+1}(z) = \gamma_{m+1}(z) + \sum_{r=0}^{m-1} \binom{m}{r} \gamma_{r+1}(z) \cdot M_{m-r}(z)$$

with

$$M_1(z) = \gamma_1(z). \tag{47}$$

As stated earlier, we can use a similar method to find the moments of x. Recall that

$$x = \sum_{l=2}^{L} x_l,$$

where

$$x_l = \frac{\beta_l}{T}\{b_{-1}^1 R_{1,1}(\tau_l) + b_0^1 \hat{R}_{1,1}(\tau_l)\}\cos(\psi_l)$$

and that use of the technique given above to find the moments of z yields

$$E\{x_l^{2m}\} = \frac{\binom{2m}{m}}{4^m} E\{\beta_l^{2m}\} \cdot \frac{1}{N^{2m+1}} \sum_{n=0}^{N-1} \sum_{r=0}^{m} \binom{2m}{2r} \Delta_{m,r,n}, \tag{48}$$

where

$$\Delta_{m,r,n} = \sum_{i=0}^{2r} (-1)^i \frac{(B_{n_{1,1}})^i}{(\hat{B}_{n_{1,1}})^{i+1}} \cdot \frac{1}{(i+1)} \cdot \frac{\binom{2r}{i}}{\binom{2(m-r)+i+1}{i+1}} \cdot \{(A_{n_{1,1}} + B_{n_{1,1}})^{2r-i} \cdot (\hat{A}_{n_{1,1}} + \hat{B}_{n_{1,1}})^{2(m-r)+i+1} - (A_{n_{1,1}})^{2r-i} \cdot (\hat{A}_{n_{1,1}})^{2(m-r)+i+1}\} \tag{49}$$

and

$$E\{\beta_l^{2m}\} = 2^m \rho_{0l}^m \cdot (m!),$$

where ρ_{0l} is the average strength of the lth path. Again, as stated earlier, equal partitioning of interferers' strength for a fixed total cochannel interference power corresponds to a maximum average error probability.

Having the moments of x_l we can find the moments of x by a similar method, as described for z. Once the moments of x and z are available, we can use their independence property and solve for the moments of $(x + z)$.

IV. DISCUSSION OF NUMERICAL RESULTS

In this section the average probability of error as a function of signal-to-noise ratio is evaluated for various channel configurations.

We will first discuss our spread-spectrum code-generation method, and then we will exhibit and discuss the behavior of the average probability of error.

4.1 Code-generation method

Pseudonoise (PN) sequence codes applied in our numerical evaluations are Gold sequences[6] obtained from multiplying two primitive polynomials,

$$h_1(x) = x^7 + x^3 + 1 \tag{50}$$

and

$$h_2(x) = x^7 + x^3 + x^2 + x + 1, \tag{51}$$

represented by octal numbers 211 and 217, respectively. Hence, the resulting sequence is

$$h(x) = x^{14} + x^9 + x^8 + x^6 + x^5 + x^4 + x^2 + x + 1, \tag{52}$$

represented by 41567, in octal notation.

The number of shift register stages required to generate the codes from $h_1(x)$ and $h_2(x)$ is $n = 7$, and the codes period is $N = 2^n - 1 = 127$.

To find the actual codes, we used initial loadings of Ref. 23. These initial loadings are shown to generate a class of Gold codes known as Auto-Optimal with Least Sidelobe Energy (AO/LSE). In general, with a generator polynomial of the form

$$h(x) = h_0x^n + h_1x^{n-1} + \cdots + h_{n-1}x + h_n$$

and for an initial loading of

$$\underline{\alpha}_0 = (\alpha_0, \alpha_1, \cdots, \alpha_{n-1}),$$

we can use the following recursive formula to generate the codes:

$$\alpha_{j+n} = h_1\alpha_{j+n-1} \oplus \cdots \oplus h_{n-1}\alpha_{j+1} \oplus h_n\alpha_j, \qquad j \geq 0, \tag{53}$$

where $\oplus$ stands for modulo-2 addition. Notice that in eq. (53) for simplicity we have represented the chips by α instead of a_i^k as introduced in eq. (2). The generated codes have three-valued autocorrelation function sidelobes and a three-valued cross correlation taking on values from the set $\{+15, -1, -17\}$.

In our numerical evaluation we used ten initial loadings.[23] Hence, this covers generating ten periodic code sequences for a hypothetical community of users sharing the common channel band on a spread-spectrum multiple-access basis. Once the code sequences are obtained, we compute the partial correlation coefficients of eq. (37), which are used in conjunction with eqs. (35) and (48) in finding the moments of x and z as described in Section 3.1.

4.2 Numerical results

In what follows we assume the signal communicated between the desired transmitter/receiver pair is received via up to ten distinguishable paths, that is, L in eq. (4) is assumed to be deterministic and at most equal to ten. Also, unless otherwise specified, we assume the transmitters maintain some form of average power control so that the signals from different transmitters arrive at the receivers with the same average power. This kind of average power control in a wireless PBX application is not too difficult, because the users are connected via a star network.

We consider two separate cases:

Case 1—Suppose a terminal in the IWC environment can be moved slightly so that in the case of strong fading of the acquired path, another path, hopefully stronger, can be acquired and then the terminal remains stationary. In this case, if there is not much movement in the channel environment, one may assume β_1 is fixed and perhaps set β_1 to some constant value, d_0, and use eq. (33) to evaluate the error probability.

Case 2—All the desired signals arriving via different paths at the receiver have Rayleigh distributed random gains. This is a scenario in which the transmitter terminals are mobile and multipath gains are Rayleigh with respect to geographic position of the terminals. Therefore we have to use eq. (32) to compute the average error probability.

In all our computations 15 moments of $(x + z)$ were found to be quite adequate in accurately computing the average error probability. All the average path gains between the desired transmitter and receiver, ρ_{0_l}'s, were assumed to be equal. This assumption also applies to the average path gain of the links between the $K - 1$ interfering transmitters and the receiver, ν_{0_k}'s. As stated earlier, this assumption will result in conservative average error probability values for a fixed total interference power.

Figure 2 depicts the average error probability as a function of both average faded and unfaded signal-to-noise ratio corresponding to Case 1. In the same figure, performance of an ideal coherent BPSK demodulator is shown. In Fig. 2 we observe two sets of results of eq. (33) corresponding to two different values of d_0. Note that all the interferers are Rayleigh faded with a hypothetical average strength of −14 dB. For $d_0 = 1$, that is, when the desired signal is 14 dB stronger than each interfering signal, the solid curves in Fig. 2 exhibit the performance. The difference in the average strength can be provided by "capture." That is, it can be due to a shorter distance or a higher transmitted

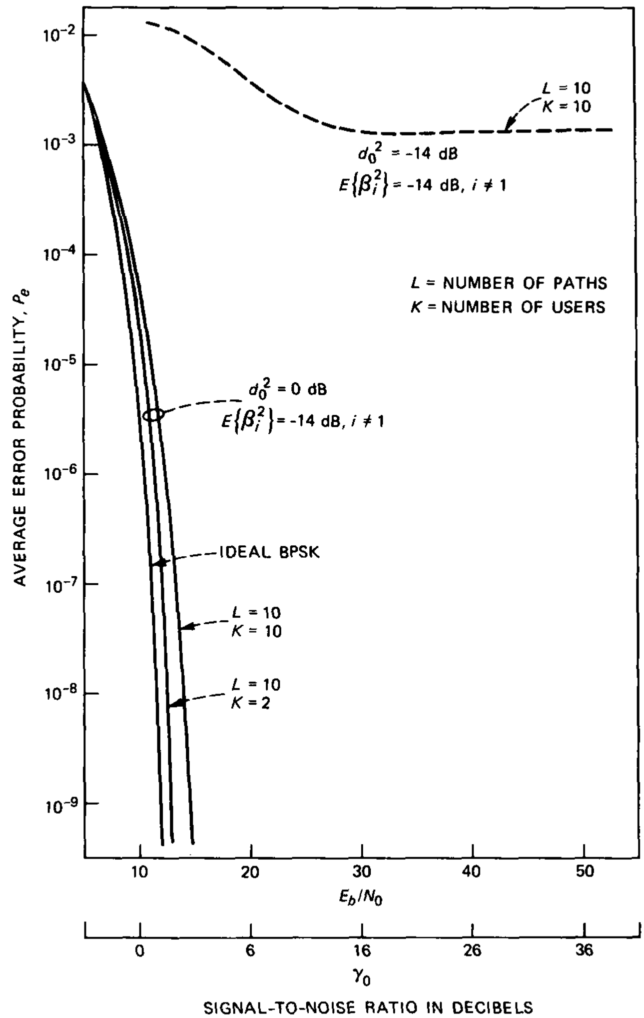

Fig. 2—Error rate performance for a fixed acquired path gain and Rayleigh interferers.

power. As can be observed, the multiple-access interference for at least up to 10 active users can be tolerated, and at an average error probability of 10^{-10} only about 2-dB signal-to-noise ratio degradation is experienced relative to the ideal situation. Therefore the receiver offers an acceptable performance as long as it is operated with capture. Next we demonstrate the performance for when the desired signal is also 14-dB faded, as the interferers are. This is shown by the dashed curve in Fig. 2. As observed, the performance in this scenario is unacceptable.

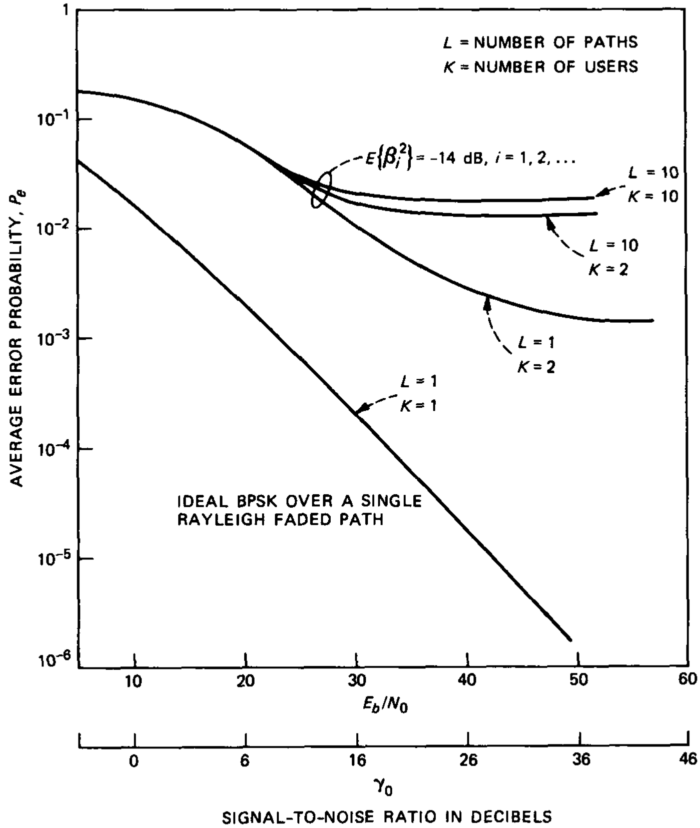

Fig. 3—Error rate performance for Rayleigh acquired path gain and Rayleigh interferers.

We now consider Case 2, where the terminals are to be mobile and the receiver is to cope with the first received Rayleigh faded path it acquires. The average probability of error as a function of faded and unfaded signal-to-noise ratio is depicted in Fig. 3. Again the hypothetical average path strength on all the Rayleigh faded paths was taken to be −14 dB. As can be observed, a simple correlation receiver that is not equipped with any diversity means or error correction capability exhibits a poor performance for the Gold sequences adopted in this work. Needless to say, the ideal performance of such a receiver in the absence of any multiple-access interference—but with a single Rayleigh faded path—is poor to begin with, as depicted in Fig. 3. Comparing the dashed curve of Fig. 2 and the curve in Fig. 3 corresponding to the same parameters reveals that the performance in the latter case is much worse than the former because of the Rayleigh gain of the acquired path, as expected. In a multiple-access environment when a transmitter and a receiver are communicating, as soon as a second transmitted signal comes on the air, the Rayleigh faded path between the interfering transmitter and the desired receiver can be stronger than the one between this receiver and the desired transmitter. This creates a near-far situation owing to the Rayleigh fading channel model. We notice that the degradation between the case of having only 2 or 10 active users is insignificant in this case, since the initial jump in error probability is large with just two users. Such a large jump, as will be seen later, is due to the insufficient processing gain provided by the $N = 127$ period codes for a Rayleigh channel. To improve this situation, longer sequences and/or diversity means are desired. The aforementioned numerical results assume $L = 10$ fading paths of equal average strength between the desired transmitter and receiver. Evidently, the finite cross correlation among the codes, although small in magnitude, can cause cochannel interference limitation due to the Rayleigh fading nature of the environment. Therefore, as the thermal noise tends to zero, the average error probability saturates to an unacceptable value. To gain some insight into this problem, we consider the following example.

Assume that we have a system of two users where there is a single Rayleigh faded path between the desired transmitter/receiver pair and that there is also a single Rayleigh faded path between the interfering transmitter and the desired receiver. A sample of the received signal after correlation and filtering is

$$\xi = \beta \frac{AT}{2} b_0^1 + \frac{AT}{2} \frac{v}{T} [b_{-1}^2 R_{2,1}(\tau) + b_0^2 \hat{R}_{2,1}(\tau)] \cos(\Theta) + \eta, \quad (54)$$

where β and v are Rayleigh gains of the desired and interfering paths, τ is the relative uniform delay experienced by the interferer, and Θ is the relative interferer path phase uniformly distributed over 0 and 2π. Denote

$$u = \frac{1}{T} [b_{-1}^2 R_{2,1}(\tau) + b_0^2 \hat{R}_{2,1}(\tau)] \cos(\Theta). \quad (55)$$

From eq. (30) the average error probability conditioned on u and v is

$$P_{e|u,v} = \frac{1}{2} \left\{ \operatorname{erfc}\left[-uv \sqrt{\frac{E_b}{N_0}}\right] - \frac{\sqrt{\gamma_0}}{\sqrt{\gamma_0 + 1}} \exp\left[\frac{-\frac{E_b}{N_0} u^2 v^2}{\gamma_0 + 1}\right] \cdot \operatorname{erfc}\left[-\frac{\sqrt{\gamma_0}}{\sqrt{\gamma_0 + 1}} uv \sqrt{\frac{E_b}{N_0}}\right] \right\}, \quad (56)$$

where

$$\gamma_0 = E\{\beta^2\} \frac{E_b}{N_0}.$$

Let

$$\psi = u^2 v^2 \frac{E_b}{N_0}.$$

Then,

$$\psi_0 = E\{u^2\} \cdot E\{v^2\} \frac{E_b}{N_0}.$$

We assume that v and β have the same mean-square value, that is,

$$\gamma_0 = E\{v^2\} \frac{E_b}{N_0}$$

and

$$\psi_0 = \gamma_0 E\{u^2\}. \tag{57}$$

Denote

$$\epsilon = E\{u^2\},$$

and average the conditional error probability of eq. (56) with respect to v. That is, evaluate

$$P_{e|u} = \frac{1}{\psi_0} \int_0^\infty P_{e|u,v} e^{-(\psi)/(\psi_0)} d\psi. \tag{58}$$

This amounts to

$$P_{e|u} = \frac{1}{2} + \frac{1}{2} \frac{\epsilon\sqrt{\gamma_0}}{\sqrt{1 + \epsilon^2\gamma_0}} - \frac{\sqrt{\gamma_0(\gamma_0 + 1)}}{\epsilon^2\gamma_0 + \gamma_0 + 1} + \frac{\sqrt{\gamma_0(\gamma_0 + 1)}}{2(\epsilon^2\gamma_0 + \gamma_0 + 1)} \left\{1 - \frac{\epsilon\gamma_0}{\sqrt{\epsilon^2\gamma_0^2 + \epsilon\gamma_0 + \gamma_0 + 1}}\right\}. \tag{59}$$

Notice that when $\epsilon = 0$, that is, when there is no interferer, this is the standard Rayleigh faded channel performance.

If we average eq. (59) with respect to Θ and τ and let $\gamma_0 \to \infty$, we get

$$\lim_{\gamma_0 \to \infty} P_e = \frac{1}{12N^3} \sum_{n=0}^{N-1} [C_{1,2}^2(n + 1 - N) + C_{1,2}^2(n - N) + C_{1,2}(n + 1 - N)C_{1,2}(n - N) + C_{1,2}^2(n + 1) + C_{1,2}^2(n) + C_{1,2}(n + 1)C_{1,2}(n)], \tag{60}$$

where $C_{1,2}(\cdot)$ represents the aperiodic cross correlation of eq. (38). The sum in eq. (60) has been approximated by Pursely.[7] When we use his approximation the saturation level of the probability of error in the absence of thermal noise is

$$\lim_{\gamma_0 \to \infty} P_e \approx \frac{1}{6N}, \tag{61}$$

where N is the period of Gold sequences applied here. Therefore, as observed earlier, we can reduce the saturation level by increasing N. In Fig. 4 the average probability of error is depicted for the case in this example. As one can observe, the moment approach yields the same saturation level in the probability of error as predicted by eq. (61).

Finally, as discussed earlier in Section 2.3, we consider the case of having an off period of a T-second period between the adjacent information intervals in order to avoid partial correlation interference from an adjacent bit. In terms of efficiency this is obviously equivalent to reducing the data rate by a factor of 2. The formulation for this case is in Appendix A, and the results for Case 2, where there is no unfaded path available between the transmitter and the receiver, is depicted in Fig. 5. As observed, the return in performance is negligible compared with the results in Fig. 3. To be more specific, the average

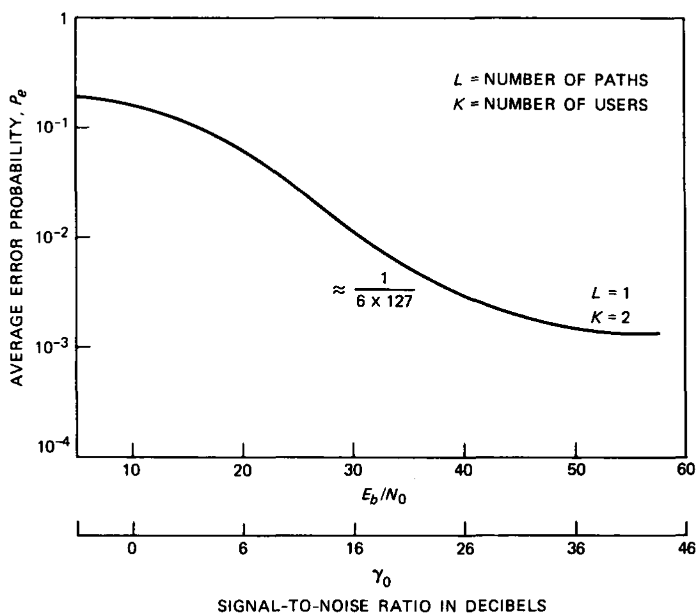

Fig. 4—Error rate performance of two-users system example.

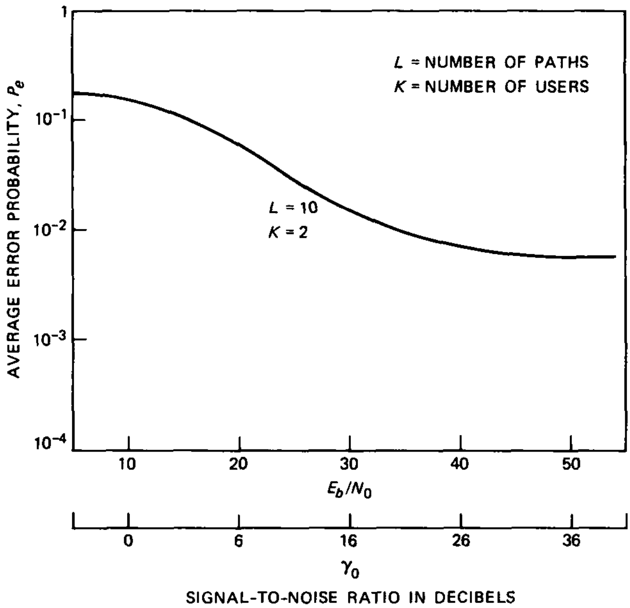

Fig. 5—Error rate performance without adjacent bit overlap.

error probability at large signal-to-noise ratios improves almost by a factor of 2. This improvement can intuitively be predicted considering that the cause, that is, partial correlation interference, is only due to an adjacent bit overlap.

Again the average path strength of all the Rayleigh faded paths in Figs. 4 and 5 was assumed to be −14 dB.

Therefore, without some form of diversity or error-correction coding, the situation as described in Case 2 is hopeless. Of course, other multiple-access fading environments would suffer similar penalties in performance if the access orthogonality could not be maintained. For example, in Frequency-Division Multiple Access (FDMA), any spectral overlap caused by imperfect filtering of adjacent frequency slots can create a similar situation. The same can be said about Time-Division Multiple Access (TDMA), if burst modems used in this application introduce any interburst interference. Consequently, regardless of the mode of access, to overcome the Rayleigh fading in IWC applications a diversity of some form seems necessary.

V. SUMMARY AND CONCLUSIONS

Current work reported herein extends previous results[7-18] in the following respects. Analysis of the average error probability for Direct-Sequence Spread-Spectrum Multiple Access (DS-SSMA) is extended to include the effects of multipath fading, typically experienced in an IWC environment.

For spread-spectrum transmission the IWC environment may be modeled by a discrete number of resolved paths with each path having a Rayleigh distributed gain, a uniformly distributed phase, and a uniformly distributed delay that can vary from zero to one information symbol period. The latter assumption is made to ensure having a negligible amount of intersymbol interference. We assume a coherent receiver that uses *no diversity information* to detect the transmitted symbol. We use average probability of error in our performance evaluation. The method of moments is applied to multipath and multiple-access interference, and Gauss quadrature integration is used in the error probability evaluation.

From our numerical work, exhibited in a sequence of graphs, we draw the following conclusions:

1. If a non-Rayleigh faded path exists in an IWC environment, a simple receiver can operate with DS-SSMA in a capture mode with a graceful performance degradation caused by multiple-access interference.

2. If all the discrete paths have Rayleigh gains and guaranteed low average error probability is expected at all times, the simple non-diversity coherent receiver considered in this work will not be able to cope with the Rayleigh channel fading with spread-spectrum codes of period $N = 127$. Therefore some form of diversity seems absolutely necessary. Otherwise, very long code sequences are needed to decrease the error probability of the interference-limited system.

3. The results of this work indicate that in the absence of diversity even small amounts of multiple-access interference can be harmful in a Rayleigh fading IWC environment. Therefore, if a channelized access such as frequency-division or time-division multiple access is to be employed, then careful channelization, that is, tight filtering in the case of FDMA and isolated transmitted bursts in the TDMA case, is necessary to maintain a low probability of error, given that the synchronization problem of a channelized access in a multipath environment can be solved.

VI. ACKNOWLEDGMENT

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APPENDIX A

Performance Evaluation in the Absence of Partial Correlation

Consider the case of having T-seconds off periods between adjacent information bits. The decision variable of eq. (10) is changed to

$$\xi = \beta_1 \frac{AT}{2} b_0^1 + \frac{AT}{2} \sum_{l=2}^{L} \frac{\beta_l}{T} b_0^1 \hat{R}_{1,1}(t_l)\cos(\psi_l)$$

$$+ \frac{AT}{2} \sum_{k=2}^{K} \frac{V_k}{T} b_0^k \hat{R}_{k,1}(\tau_k)\cos(\Theta_k) + \eta. \quad (62)$$

So now

$$x = \sum_{l=2}^{L} \frac{\beta_l}{T} \hat{R}_{1,1}(t_l)\cos(\psi_l) \quad (63)$$

and

$$z = \sum_{k=2}^{K} \frac{V_k}{T} \hat{R}_{k,1}(\tau_k)\cos(\Theta_k). \quad (64)$$

To find the moments of x and z we follow a method similar to the one in Section III:

$$E\{x_l^{2m}\} = \frac{1}{T^{2m+1}} (2^m)(\rho_{0_l}^{m})(m!) \frac{\binom{2m}{m}}{4^m} \sum_{n=0}^{N-1} \int_{nT_c}^{(n+1)T_c} \hat{R}_{1,1}^{2m}(t_l)dt_l, \quad (65)$$

where

$$x_l = \frac{\beta_l}{T} b_0^1 \hat{R}_{1,1}(t_l)\cos(\psi_l) \quad (66)$$

and

$$\hat{R}_{1,1}(t_l) = \hat{A}_{n_{1,1}} T_c + \hat{B}_{n_{1,1}}(t_l - nT_c). \quad (67)$$

After proper change of variables we can define

$$H = T_c^{2m+1} \sum_{n=0}^{N-1} \int_0^1 [\hat{A}_{n_{1,1}} + \hat{B}_{n_{1,1}} x]^{2m} dx \quad (68)$$

$$= \frac{1}{(2m+1)} \cdot \frac{1}{\hat{B}_{n_{1,1}}} \{[\hat{A}_{n_{1,1}} + \hat{B}_{n_{1,1}}]^{2m+1} - [\hat{A}_{n_{1,1}}]^{2m+1}\}. \quad (69)$$

Hence,

$$E\{x_l^{2m}\} = 2^m \cdot \rho_{0_l}^{m} \cdot (m!) \cdot \frac{\binom{2m}{m}}{4^m} \cdot \frac{1}{N^{2m+1}} \sum_{n=0}^{N-1} \Gamma_{m,n},$$

where

$$\Gamma_{m,n} = \frac{1}{2m+1} \cdot \frac{1}{\hat{B}_{n_{1,1}}} \{[\hat{A}_{n_{1,1}} + \hat{B}_{n_{1,1}}]^{2m+1} - [\hat{A}_{n_{1,1}}]^{2m+1}\}; \quad (70)$$

from here on the problem is identical to the one solved in Section III.

APPENDIX B

Integration of Conditional Error Probability

Denote

$$\Gamma_0 = -(x + z)\sqrt{E_b/N_0} \tag{71}$$

and

$$I = \frac{1}{2\gamma_0}\int_0^\infty \operatorname{erfc}(\sqrt{\gamma} + \Gamma_0)e^{-\gamma/\gamma_0}d\gamma. \tag{72}$$

Letting $\sqrt{\gamma} + \Gamma_0 = t$ we have

$$I = -\frac{1}{2}\int_{\Gamma_0}^\infty \frac{-2(t - \Gamma_0)}{\gamma_0}\operatorname{erfc}(t)e^{-(1/\gamma_0)(t-\Gamma_0)^2}dt. \tag{73}$$

Now we integrate by parts in eq. (73) to get

$$I = \frac{1}{2}\operatorname{erfc}(\Gamma_0) - \frac{1}{\sqrt{\pi}}\int_{\Gamma_0}^\infty e^{-(1/\gamma_0)(t-\Gamma_0)^2}e^{-t^2}dt. \tag{74}$$

Furthermore,

$$\int_{\Gamma_0}^\infty e^{-(1/\gamma_0)(t-\Gamma_0)^2}e^{-t^2}dt$$

$$= e^{-\Gamma_0^2/(\gamma_0+1)}\int_{\Gamma_0}^\infty e^{-\{[(\sqrt{\gamma_0+1}/\sqrt{\gamma_0})t]-[\Gamma_0/\sqrt{\gamma_0(\gamma_0+1)}]\}^2}dt, \tag{75}$$

and making the change of variable

$$x = \sqrt{\frac{\gamma_0 + 1}{\gamma_0}}\,t - \frac{\Gamma_0}{\sqrt{\gamma_0(\gamma_0 + 1)}}$$

in the integral in eq. 75, we get

$$\frac{\sqrt{\pi}}{2}e^{-\Gamma_0/(\gamma_0+1)}\frac{\sqrt{\gamma_0}}{\sqrt{\gamma_0 + 1}}\operatorname{erfc}\left(\Gamma_0\sqrt{\frac{\gamma_0}{\gamma_0 + 1}}\right). \tag{76}$$

Using this result in eq. (76) in the second term in eq. (74) gives

$$I = \frac{1}{2}\left\{\operatorname{erfc}(\Gamma_0) - \frac{\sqrt{\gamma_0}}{\sqrt{\gamma_0 + 1}}\right.$$

$$\left.\cdot\exp\left(-\frac{\Gamma_0^2}{\gamma_0 + 1}\right)\operatorname{erfc}\left(\Gamma_0\frac{\sqrt{\gamma_0}}{\sqrt{\gamma_0 + 1}}\right)\right\}, \tag{77}$$

which is the desired result in eq. (30).

APPENDIX C

Formulation of Gauss Quadrature Rules From the Moments of (x + z)

Denote the first $N_m = 2N_c + 1$ moments of $(x + z)$ by the sequence $\{\mu_n\}$, $n = 0, 1, 2, \cdots, 2N_c$. In the problem at hand the random variables are evenly distributed. Therefore, as previously stated, the odd moments are all zero.

Let $\mathbf{M} = [m_{ij}]$, $i, j = 1, 2, \cdots 2N_c + 1$, be the Gram matrix of the system with

$$m_{ij} = \mu_{i+j-2}. \tag{78}$$

Thus,

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & \mu_2 & 0 & \cdot & \cdot & \cdot & \mu_{N_c} \\ 0 & \mu_2 & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ \mu_2 & 0 & \mu_4 & \cdot & \cdot & \cdot & \cdot & \mu_{N_c+2} \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \mu_{N_c} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \mu_{2N_c} \end{bmatrix}, \tag{79}$$

where $\mu_0 = 1$. Also, let $\mathbf{M} = \mathbf{R}^T\mathbf{R}$ be the Cholesky decomposition of $\mathbf{M}$, where $\mathbf{T}$ represents the transpose matrix with

$$r_{ii} = \left(m_{ii} - \sum_{k=1}^{i-1} r_{ki}^2 \right)^{1/2} \tag{80}$$

and

$$r_{ij} = \left(m_{ij} - \sum_{k=1}^{i-1} r_{ki} r_{kj} \right) \Big/ r_{ii}, \qquad i < j. \tag{81}$$

Because all the odd moments are zero, it follows that $r_{ij} = 0$ when $(i + j)$ is odd. We now have an upper triangular matrix $\mathbf{R} = [r_{ij}]$, $i, j = 1, 2, \cdots N_c + 1$. The matrix is used to calculate a set of numbers $\{\delta_j\}$, $j = 1, 2, \cdots, N_c$, where

$$\delta_j = \frac{r_{j+1 j+1}}{r_{jj}}. \tag{82}$$

Now we construct a tridiagonal matrix $\mathbf{J}$ as follows:

$$\mathbf{J} = \begin{bmatrix} 0 & \delta_1 & 0 & \cdot & \cdot & \cdot & 0 \\ \delta_1 & 0 & \delta_2 & \cdot & \cdot & \cdot & 0 \\ 0 & \delta_2 & 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \delta_{N_c-1} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \delta_{N_c-1} & 0 \end{bmatrix}, \tag{83}$$

where $\mathbf{J}$ is an $N_c \times N_c$ matrix.

By finding the eigenvalues and eigenvectors of the matrix $\mathbf{J}$, it is possible to arrive at the weights and nodes of the quadrature rule. Let the eigenvalue equation be

$$\mathbf{J}\tilde{q}_j = \lambda_j \tilde{q}_j. \tag{84}$$

Then the quadrature rule for the sequence (W_j, ζ_j) in eq. (32) is given by the set of numbers $\{q_{1j}^2, \lambda_j\}, j = 1, 2, \cdots N_c$, where q_{1j}^2 is the square of the first element of the eigenvector $\tilde{q}_j$, and λ_j is an eigenvalue in eq. (84). Hence, if the first $2N_c + 1$ moments are calculated, then the resulting quadrature rule will contain N_c weights and nodes.

APPENDIX D

Evaluation of Even Moments

Consider the integral

$$I_0 = \int_0^1 (a + bx)^n \cdot (c + dx)^m dx. \tag{85}$$

This integral can be expanded to

$$I_0 = \left[\frac{(a + b)^n (c + d)^{m+1}}{d(m + 1)} - \frac{a^n c^{m+1}}{d(m + 1)}\right] - \frac{nb}{d(m + 1)} \int_0^1 (a + bx)^{n-1} \cdot (c + dx)^{m+1} dx, \tag{86}$$

where we now have to solve for

$$I_1 = \int_0^1 (a + bx)^{n-1} \cdot (c + dx)^{m+1} dx. \tag{87}$$

If we integrate by parts in eq. (87) and substitute the result into eq. (86), we find that

$$I_0 = \sum_{i=0}^{n} (-1)^i \frac{b^i}{d^{i+1}} \cdot \frac{1}{(i + 1)} \cdot \frac{\binom{n}{i}}{\binom{m + i + 1}{i + 1}} \cdot [(a + b)^{n-i}(c + d)^{m+i+1} - a^{n-i}c^{m+i+1}]. \tag{88}$$

In the problem considered in the main body of the paper,

$$a = A_n, \qquad b = B_n, \qquad c = \hat{A}_n, \quad \text{and} \quad d = \hat{B}_n.$$

Thus, I_0 takes the form

$$I_0 = \Gamma_{m,r,n} = \sum_{i=0}^{2r} (-1)^i \frac{(B_n)^i}{(\hat{B}_n)^{i+1}} \cdot \frac{1}{(i+1)} \cdot \frac{\binom{2r}{i}}{\binom{2(m-r)+i+1}{i+1}}$$

$$\cdot \{(A_n + B_n)^{2r-i} \cdot (\hat{A}_n + \hat{B}_n)^{2(m-r)+i+1} - A_n^{2r-i} \cdot \hat{A}_n^{2(m-r)+i+1}\}. \quad (89)$$

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