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Performance of Low-Complexity Channel Coding and Diversity for Spread Spectrum in Indoor, Wireless Communication

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The application of selection diversity in conjunction with simple channel coding is considered for a multiuser, slowly fading, Spread-Spectrum Multiple Access (SSMA), digital radio system. For the most part, the index of performance for our study is the average bit error probability; we also give some consideration to multipath outage as a performance measure. All subscribers are assumed to communicate to a central station; that is, a star network architecture is assumed. *Average* power control is also assumed. The average mentioned in this context includes averaging over the channel fading statistics. The modulation is direct-sequence, spread-spectrum, binary phase-shift keying. We assume perfect timing and carrier recovery in our coherent receiver, and a slowly varying, Rayleigh fading, discrete multipath model is used. Previous analyses have found that SSMA can tolerate few simultaneous users for fading radio channels. We find that the combination of spread-spectrum modulation with low-complexity diversity and/or channel coding can restore fading-channel user levels to an acceptable figure. In addition, selection diversity plus channel coding is more effective than either method by itself. Finally, it turns out that SSMA is less sensitive to a change in the value of delay spread of a fading channel than, say, time-division multiple access. The method of moments is used to accurately assess the system error probability. Using this technique, we also assess the accuracy of assuming that the multiuser interference has a Gaussian distribution, which allows it to be analyzed by a simple method. Using this assumption, we compare selection

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diversity plus channel coding with the maximal-ratio-combining technique for diversity reception. Except for a high order of diversity, the former is more efficient and is always less complex than the latter.

I. INTRODUCTION

In a recent paper Kavehrad has presented a technique to evaluate the performance of direct-sequence, spread-spectrum binary phase-shift-keying modulation for an Indoor, Wireless Communication (IWC) channel.[1] The analysis uses the method of moments,[2] which gives accurate estimates of error probability for many digital communication systems. Kavehrad did not consider diversity in his study. We find that Kavehrad's formulas are only slightly modified when selection diversity (see pages 313 through 316 of Ref. 3) is included in his reception model. We also determine the effect on system performance of simple channel-coding techniques. Our use of channel coding in spread-spectrum systems is similar to the case reported in Ref. 4, which involved frequency hopping. Both the (7, 4) Hamming code and (15, 7) BCH code are considered in our analyses. We assume hard decisions are made by the demodulator and that its error-producing mechanism results in independent error events. The latter assumption requires interleaving at the transmitter and de-interleaving at the receiver as a slowly fading channel model is considered. As the intended application is to packet transmission, interleaving does not present a severe system problem. A discussion of interleaving is given in Appendix 3A of Ref. 5.

The references to the channel-coding aspects of our study are important, as channel coding is found to be an effective method of combating multiuser interference in Spread-Spectrum Multiple Access (SSMA) systems. This was earlier found by Livine for no signal fading.[6] In an earlier study Turin[7] found that SSMA can tolerate considerably less multiuser interference in a fading channel than can be allowed in an additive white Gaussian noise channel. Adopting a Rayleigh fading model that seems less severe than the model used by Turin[7] for mobile communication applications, it is found that channel coding plus selection diversity performs well in a multiuser environment because the combination can be optimized. Channel coding used with selection diversity is found to perform better than selection diversity alone for the same spread-spectrum system bandwidth. In this sense it is both power and bandwidth efficient, as can be deduced from the similar study of Milstein et al.[4] in the absence of fading. This is true for the simple block codes mentioned above. Using more powerful codes and/or soft-decision decoding would give even greater gains in performance. Our approach has been to adopt a simple

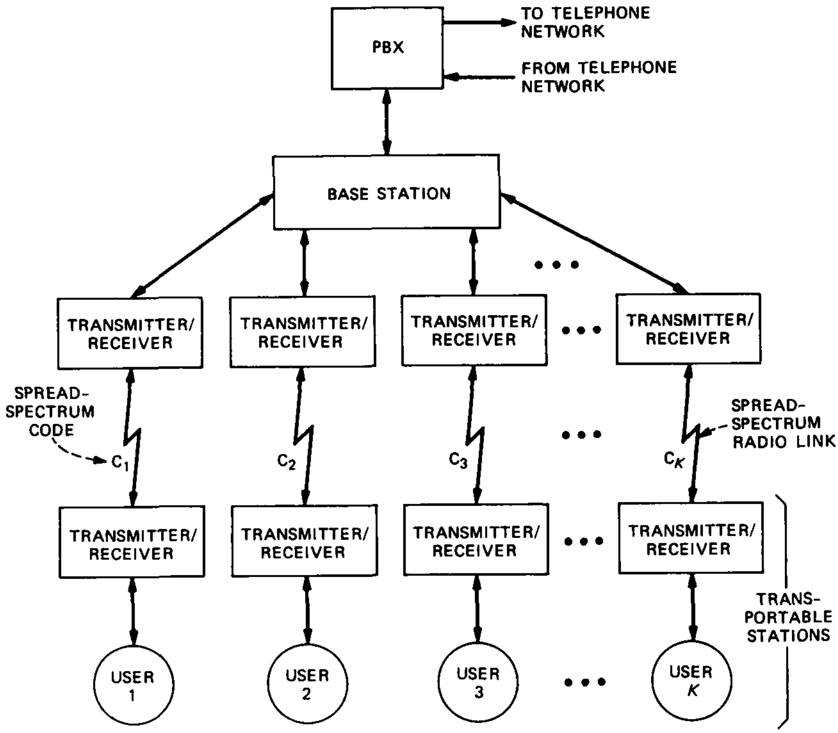


Fig. 1—A star-connected, indoor, wireless, local area network. Each user has a unique spread-spectrum code.

detection and decoding system to observe how a relatively simple system performs.

For this study we assume an indoor, wireless communication channel offering both voice and data services. The speech transmission rate for each user assumed in our system parameter study for IWC systems is 32 kb/s. Packet transmission is assumed and all users communicate through a central station in a star network architecture. Figure 1 is a simple block diagram of the system we analyze. Each active user in the system depicted in Fig. 1 has a unique spread-spectrum code, which is used for communication to the central station. The central station contains a bank of spread-spectrum receivers, one for each active user. Its function is to determine which subscribers are active and to detect the digital information sent in each case. The basis of the spread-spectrum receiver for each active user that exists in the central station is a Surface Acoustic Wave (SAW) device. Such devices have been found to be effective in such a role in the earlier study of Freret et al.[8] A tutorial on SAW devices can be found in Ref. 9. We note that the study in Ref. 8 proposed the use of spread-spectrum diversity in IWC systems. This diversity is inherently supplied by the spread-spectrum modulation as long as the spread-spectrum bandwidth exceeds the coherence bandwidth of the slowly fading channel (see Ref. 10, page 480). More discussion on this point is presented in Section II. We note that spread-spectrum modulation can provide both asynchronous multiple access and also diversity reception.[7,10] Other advantages of spread-spectrum in IWC systems are discussed by Kavehrad.[1]

We present an analysis of the bit error probability for the link between any active user and its receiver in the central station. This link is shown in Fig. 1. As such, we are only considering the communication upstream from an active user to the central station. The downstream communication path is much simpler and will not be considered. We assume that average power control is used by all active users in that, on the average, all active user signals are assumed to arrive at the central station with the same power (in the upstream communication mode). The average here includes the fading statistics. Thus, the power control that must be used by each active user just depends on the distance and power law exponent for the link from a user to the central station and also on the static, shadow fading that is encountered. The sources of signal fading are nicely summarized in Section II of Ref. 11. We do not consider the dynamics of average power control in this paper.

The main contribution of the memorandum is to show that Kavehrad's analyses[1] can be extended to include selection diversity, and that this form of diversity can be used in conjunction with simple channel coding to give an SSMA system with an acceptable number of active users. For instance, for an IWC system having a multipath delay spread, T_m, of 100 nanoseconds, we find that a spread-spectrum code length of 255, a source data rate of 32 kb/s, and a (15, 7), double-error-correcting BCH code can support approximately 75 simultaneous users. If we envision a low-traffic office environment with a 10-percent channel utilization, a total of 750 subscriber terminals can be supported using the aforementioned method. This assumes that each code is shared among a group of subscribers in a contention mode of operation.

An outline of the paper is as follows. The system-fading and multipath model is described in Section II. Section III outlines the use of the computational technique from Ref. 1 to compute the average system error probability. Section IV considers an approximate computational technique based on a Gaussian assumption for the multiuser interference. Section V considers simple channel codes and Section VI contains our numerical results. Section VII presents an application of our computational results to two IWC scenarios for local area

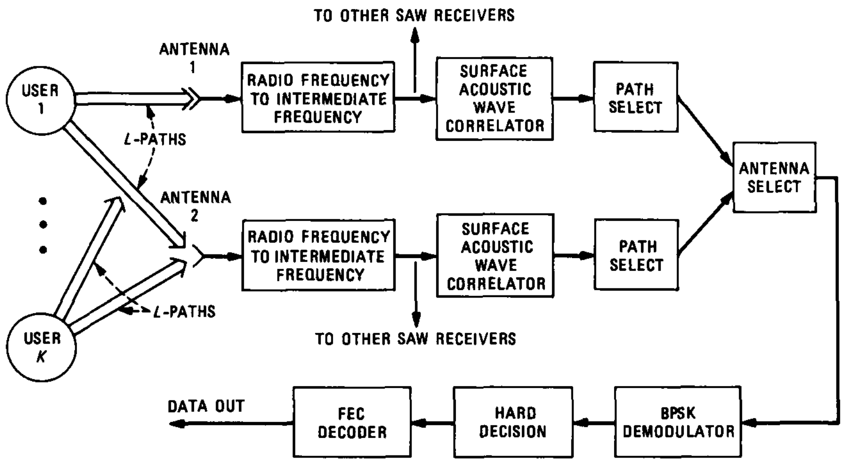


Fig. 2—Per-user receiver for spread-spectrum, direct-sequence receiver using selection diversity.

networks, as well as our consideration of multipath outage as a performance criterion.

II. SYSTEM MODEL

2.1 Transmission model

Our mathematical model will depend heavily on the model developed by Kavehrad.[1] We will use the same notation and borrow heavily from Kavehrad's earlier analysis. Consider the block diagram of the multiuser, IWC channel shown in Fig. 2. The structure is exactly as in Fig. 1. However, the receiving systems for a single reference user, taken as user 1, are shown; for simplicity only a two-antenna system is depicted in Fig. 2.

In Fig. 1 each active user has a code waveform that consists of a periodic (period T) sequence of N, nonoverlapping rectangular waveforms (called chips), each of the duration T_c seconds. The length of the code waveform is T seconds, the reciprocal of the symbol transmission rate, where $T = NT_c$. The sequence of chip waveforms is the spread-spectrum code waveform, which for the kth user is denoted by $a_k(t)$. The data signal is binary with data symbols b_j^k, where the subscript denotes the jth time slot and the superscript denotes the data symbol for the kth user. If we let $P_T(t)$ denote a rectangular pulse of unit height and duration T, the transmitted signal for the kth user is

$$S_k(t) = Aa_k(t)b_k(t)\cos(\omega_c t + \theta_k) \tag{1a}$$

$$a_k(t) = \sum_{i=-\infty}^{\infty} a_i^k P_{T_c}(t - iT_c) \tag{1b}$$

and

$$b_k(t) = \sum_{j=-\infty}^{\infty} b_j^k P_T(t - jT), \tag{1c}$$

where a_i^k is the ith chip amplitude for the kth user, where $\omega_c T = 2\pi$ times an integer, ω_c is the carrier frequency in rad/s, A is the signal amplitude, and θ_k is the signal phase. In our analysis we shall have $k = 1, 2, \cdots, K$, where K will denote the number of simultaneous users.

In Fig. 2 we show L discrete multipath links between each user and each receive antenna at the central station. The low-pass equivalent impulse response of the passband channel for the link between the kth user transmitter and central station receiver is

$$h_k(\tau) = \sum_{\ell=1}^{L} \beta_{\ell k} \delta(\tau - \tau_{\ell k}) e^{j\Phi_{\ell k}}, \tag{2}$$

where for the kth user

$\beta_{\ell k} = \ell$th path gain

$\Phi_{\ell k} = \ell$th path phase

$j = \sqrt{-1}$

and

$\tau_{\ell k} = \ell$th path time delay.

We assume $\beta_{\ell k}$ is a Rayleigh random variable; $\Phi_{\ell k}$ is taken as uniform in $[0, 2\pi]$; and $\tau_{\ell k}$ is assumed uniform in $[0, T]$, where T is the data symbol interval. In the sequel the difference between the maximum and minimum values of $\tau_{\ell k}$ will be called the maximum multipath delay spread and will be denoted by T_m. Also, as a slowly fading channel is assumed, the variables in eq. (2) are assumed random but time-invariant.

The impulse response given in eq. (2) is characteristic of a discrete multipath channel and has the same functional form as that given in Ref. 12. The question is, How do we determine L in terms of communication system parameters? The basic result on the time resolution of signals using spread-spectrum signals is given in Section 1.5.3 of the recent textbook by Simon et al.[5] As one would expect, two signals must be separated by one chip time, T_c, in order to be resolved. We illustrate this in Figs. 3 and 4. Figure 3 is a system block diagram plus a diagram depicting the discrete multipath model. Figure 4 shows the response due to $L = 3$ multipath components. We assume that the

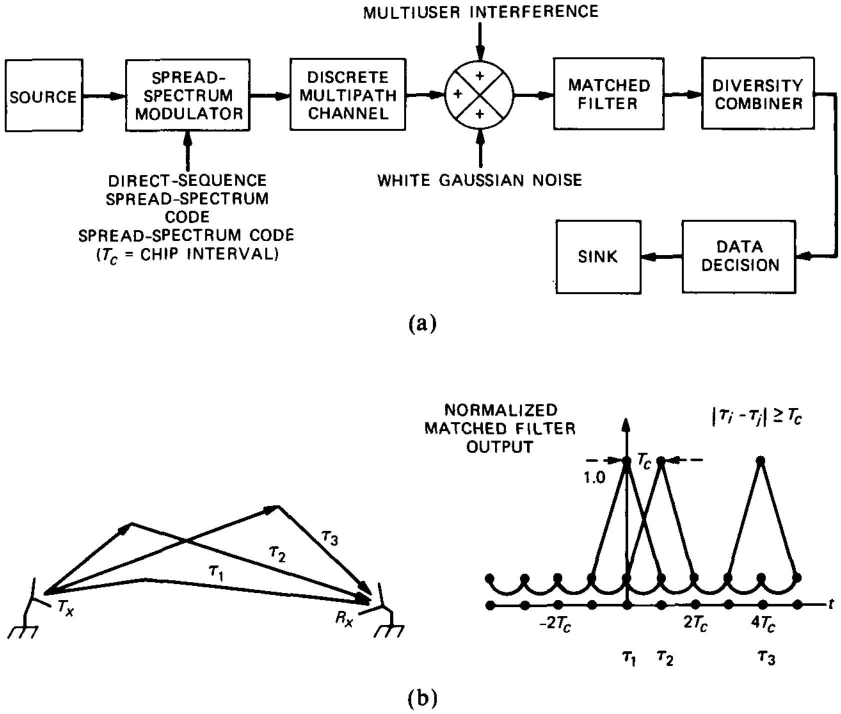


Fig. 3—Multiuser spread-spectrum system for (a) a baseband system and (b) a discrete multipath model.

maximum multipath delay spread, T_m, is less than T, the information bit interval, in order to avoid intersymbol interference. Using the result of time resolution of direct-sequence, spread-spectrum signals given above, one sees that

$$L = \left\lfloor \frac{T_m}{T_c} \right\rfloor + 1 = \lfloor T_m \cdot B_{ss} \rfloor + 1 \tag{3}$$

is the maximum number of resolved paths for a maximum multipath delay spread of T_m seconds. Also, in eq. (3) $\lfloor x \rfloor$ is the largest integer that is less than or equal to x and $B_{ss} = NR_0$, the one-sided bandwidth of the spread-spectrum signal, where $R_0 = T^{-1}$ and $T/T_c = N$ is the sequence length. Copies of the transmitted signal that arrive at unresolvable time differences are assumed to combine to give rise to the Rayleigh path gain of eq. (2). As such, we should assume that the time difference, $\tau_j - \tau_k$, is greater than T_c, where each individual τ is uniform in $(0, T)$. We take $\tau_j - \tau_k > 0$, which approximately is true as $T_c = T/N$ is small relative to T.

Actually, we feel that L in eq. (3) represents the maximum number

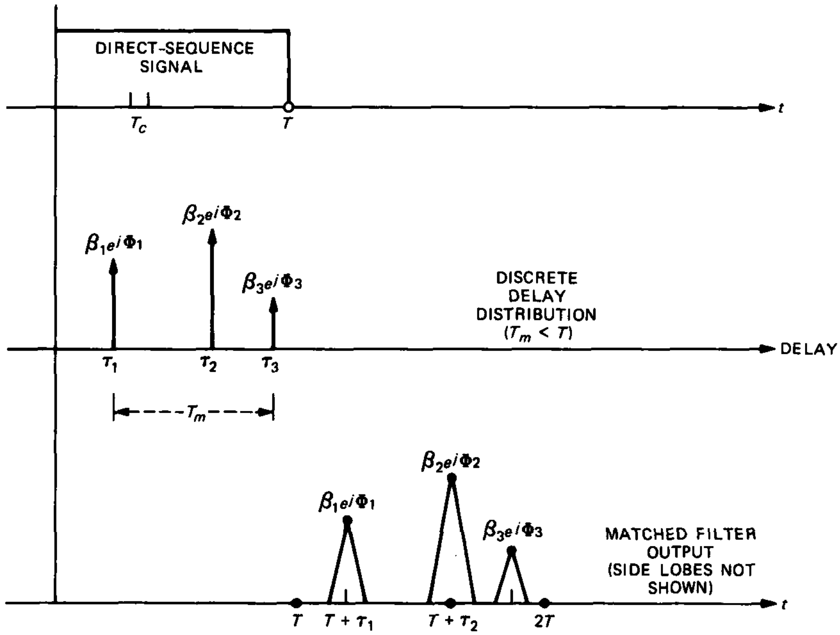


Fig. 4—Signal transmission model for a direct-sequence, spread-spectrum system and a discrete multipath model.

of approximately uncorrelated terms one could have in eq. (2). This is because L is based on the minimum resolution of direct-sequence, spread-spectrum signals. A random model is one in which L varies between unity and the maximum value given in eq. (3).

Our approach to treating L in the paper will be as follows. Up to the IWC parameter study presented in Section VII, we determine system performance in general for any K, L, and M. Here K is the number of simultaneous users, L the number of paths, and M the order of diversity. In Section VII we then adopt two models for L. In one, L is given by eq. (3). In the other, L varies uniformly between unity and the maximum value given in eq. (3). The results turn out to be insensitive to the model used for L. As better models for L evolve, our general results can be used to estimate performance in such cases.

If we combine eqs. (1a) and (2) and use the convolution integral, the received signal at the central station, which will be denoted as $r(t)$, is given by

$$r(t) = \text{Re}\left\{\sum_{k=1}^{K} \int_{-\infty}^{\infty} h_k(\tau)\tilde{S}_k(t-\tau)\exp(j\omega_c t)d\tau\right\} + n(t), \qquad (4)$$

where $\tilde{S}_k(t)$ is the complex envelope of $S_k(t)$ for $\theta_k = 0$ and $\text{Re}\{\cdot\}$

denotes the real part of a complex number. Upon use of eqs. (1), (2), and (4), we have

$$r(t) = A \sum_{\ell=1}^{L} \beta_{\ell 1} a_1(t - \tau_{\ell 1}) b_1(t - \tau_{\ell 1}) \cos(\omega_c t + \Phi_{\ell 1})$$

$$+ A \sum_{\ell=1}^{L} \sum_{k=2}^{K} \beta_{\ell k} a_k(t - \tau_{\ell k}) b_k(t - \tau_{\ell k}) \cos(\omega_c t + \Phi_{\ell k}) + n(t). \quad (5)$$

The white Gaussian noise, $n(t)$, in eq. (5) will have a spectral height of $N_0/2$ W/Hz. In eq. (5) $\tau_{\ell k}$ will be uniform in $[0, T]$, $\Phi_{\ell k}$ uniform in $[0, 2\pi]$, and $\beta_{\ell k}$ will have the Rayleigh probability density function (pdf)

$$f_\beta(x) = \frac{x}{\rho_0} \exp\left(\frac{-x^2}{2\rho_0}\right) u(x), \quad (6)$$

where $u(x)$ is the unit step function, $u(x) = 1$ for $x \geq 0$ and zero elsewhere. As such, the average received signal-to-white-Gaussian-noise ratio is

$$\gamma_0 = E(\beta_{j1}^2) \frac{E_b}{N_0} = 2\rho_0 E_b/N_0, \quad (7)$$

where $E_b = A^2T/2$, the signal energy per bit, and $E(\beta_{j1}^2) = 2\rho_0$, where for user one β_{j1} is the random gain of the jth signal path. In fact $\gamma_0 = E(\gamma)$, where $\gamma = \beta_{j1}^2 E_b/N_0$ has the exponential pdf

$$f_\gamma(y) = \gamma_0^{-1} \exp\left(\frac{-y}{\gamma_0}\right) u(y) \quad (8)$$

with γ_0 as in eq. (7).

The specification of our channel model is now complete. Note that the formulation represented by eq. (5) pertains to only the discrete multipath model whose impulse response is given by eq. (2). We note that the transmission model is similar, except for the specification of the fading parameters, to that used by Pursley.[13] In Ref. 14 multipath diversity reception is considered. However, a Gaussian assumption is used in the performance computations.

2.2 Receiver model

The input to the receiver for the reference user is given by the right-hand side of eq. (5). The first term in this equation represents all the copies of the transmitted, spread-spectrum signal that are available for detection. Let us assume that the receiver can ideally lock on to the term at delay τ_{j1} and phase Φ_{j1}. Then the jth decision variable for detection is given by

$$\xi_j = \int_0^T r(t)a_1(t - \tau_{j1})\cos(\omega_c t + \Phi_{j1})dt. \tag{9}$$

If the order of diversity used in the receiver is M, we will have ξ_j, $j = 1, 2, \cdots, M$ decision variables available for detection purposes. Substituting eq. (5) into eq. (9) yields the result

$$\begin{aligned}\xi_j = b_0^1 \frac{AT}{2} \beta_{j1} &+ \frac{A}{2} \sum_{\substack{\ell=1 \\ \ell \neq j}}^{L} \beta_{\ell 1} \cos[\Phi_{\ell 1} - \Phi_{j1}] \\ &\cdot \int_0^T a_1(t - \tau_{\ell 1})b_1(t - \tau_{\ell 1})a_1(t - \tau_{j1})dt \\ &+ \frac{A}{2} \sum_{\ell=1}^{L} \sum_{k=2}^{K} \beta_{\ell k} \cos[\Phi_{\ell k} - \Phi_{j1}] \\ &\cdot \int_0^T a(t - \tau_{\ell k})b_k(t - \tau_{\ell k})a_1(t - \tau_{j1})dt \\ &+ \int_0^T n(t)a_1(t - \tau_{j1})\cos[\omega_c t + \Phi_{j1}]dt, \end{aligned} \tag{10}$$

where b_0^1 is the data bit to be detected. If one consults Ref. 1, it is clear that our eq. (10) is equivalent to eq. (8) of that reference with one difference; that is, in our eq. (10), L fading paths are assumed for each interferer, as it is important in this work. Following this same reference, one can express eq. (10) in the form

$$\begin{aligned}\xi_j = \beta_{j1} \frac{AT}{2} b_0^1 &+ \frac{A}{2} \sum_{\substack{\ell=1 \\ \ell \neq j}}^{L} \beta_{\ell 1} \cos[\Phi_{\ell 1} - \Phi_{j1}] \\ &\cdot [b_{-1}^1 R_{11}(t_{\ell 1}) + b_0^1 \hat{R}_{11}(t_{\ell 1})] \\ &+ \frac{A}{2} \sum_{\ell=1}^{L} \sum_{k=2}^{K} \beta_{\ell k} \cos[\Phi_{\ell k} - \Phi_{j1}] \\ &\cdot [b_{-1}^k R_{k1}(t_{\ell k}) + b_0^k \hat{R}_{k1}(t_{\ell k})] + \nu, \end{aligned} \tag{10a}$$

where $t_{\ell k} = \tau_{\ell k} - \tau_{j1}$, ν is Gaussian with zero mean and variance $N_0 T/4$,

$$R_{k1}(\tau) = \int_0^\tau a_k(t - \tau)a_1(t)dt, \tag{10b}$$

and

$$\hat{R}_{k1}(\tau) = \int^T a_k(t - \tau)a_1(t)dt. \tag{10c}$$

Although eq. (10) is rather long, each term in this equation can be interpreted with respect to the block diagram shown in Fig. 2. The first term in eq. (10) represents the desired signal to be detected. The second term in eq. (10) is the self-interference for the reference user (say, User 1 in Fig. 2) due to sidelobes of the autocorrelation function of the spread-spectrum code of User 1. The third term in eq. (10) is the $L(K-1)$ multiuser interference terms from the $K-1$ other simultaneous users of the system. Finally, the last term in eq. (10) is the Gaussian random variable due to additive white Gaussian noise. Note that there are $L-1$ self-interference plus $L(K-1)$ multiuser interference terms in eq. (10). Thus, the total number of interference terms is $\eta = L(K-1) + L - 1 = LK - 1 = LK$ for a large LK. We will find in our computations that the per-user average error probability is, for all practical purposes, a function of $\eta = LK$. We note that ξ_j in eq. (10a) could correspond to any diversity term for the receiving system shown in Fig. 2. For instance, if there are two antennas and $L = 2$ (see Fig. 2), we have two antennas and two paths per antenna to give a total order of diversity of $M = 4$.

As we have noted previously, our eq. (10) is equivalent to eq. (8) of Ref. 1. Also eq. (10a) is equivalent to eq. (10) of this reference. However, in our detection procedure we assume that selection diversity is used. That is, we can find ξ_j in eq. (10) such that β_1 is the largest path gain relative to User 1, so that

$$\beta_1 = \mathrm{Max}(\beta_{11}, \beta_{21}, \cdots, \beta_{M1}).$$

Let ξ be that value of ξ_j corresponding to path gain β_1. Then our eq. (10) with ξ_j replaced by ξ is equivalent to eq. (8) of Ref. 1, except that β_1 has the pdf of the maximum of the path gains $\beta_{j1}, j = 1, 2, \cdots, M$. As shown by Jakes,[3] pages 313 through 316, or Papoulis,[15] pages 139 and 140, the pdf of β_1^2, the maximum of the β_{j1}^2, is easily found. It is this pdf that will be needed in our error probability analyses.

III. ERROR PROBABILITY

We will return to the subject of the pdf of β_1^2 later. Recall that our eq. (10) is equivalent to eq. (8) of Ref. 1. If we mimic the development through to eq. (22) of this reference, we find that the probability of error, conditioned on β_1^2, the self-interference, and the multiuser interference is given by

$$P(e \mid \beta_1, x, z) = \frac{1}{2}\,\mathrm{erfc}\left\{\sqrt{\frac{\beta_1^2 E_b}{N_0}} - \sqrt{\frac{E_b}{N_0}}\,(x+z)\right\}, \quad (11)$$

where

$$\operatorname{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^{\infty} \exp(-y^2)dy$$

$$x = \frac{1}{T} \sum_{\substack{\ell=1 \\ \ell \neq j}}^{L} \beta_{\ell 1}\{b_{-1}^{1} R_{11}(t_{\ell 1}) + \hat{R}_{11}(t_{\ell 1})\}\cos\nabla_{\ell} \tag{12}$$

$$z = \frac{1}{T} \sum_{\ell=1}^{L} \sum_{k=2}^{K} \beta_{\ell k}\{b_{-1}^{k} R_{k1}(t_{\ell k}) + b_0^k \hat{R}_{k1}(t_{\ell k})\}\cos\theta_{\ell k}, \tag{13}$$

where $t_{\ell k} = \tau_{\ell k} - \tau_{j1}$, $\nabla_{\ell} = \Phi_{\ell 1} - \Phi_{j1}$, and $\theta_{\ell k} = \Phi_{\ell k} - \Phi_{j1}$. The index j in, say, τ_{j1}, is taken as the delay for the path having the largest β_{j1}. The largest β_{j1} is denoted as β_1 and this random variable is independent of β_{i1}, $i \neq j$. Thus x and z, which are mutually independent, are also both independent of β_1.

Kavehrad's[1] technique of finding the average error probability, $P(e)$, was to integrate $P(e \mid \beta_1, x, z)$ with respect to the pdf of β_1^2 and then remove the conditioning on (x, z) using the method of moments. Actually our result for $P(e)$ in the case of selection diversity can be deduced from Kavehrad's mathematics. Let the pdf for β_1 be the Rayleigh pdf given in eq. (6). The pdf for β_1^2 is

$$f_{\beta_1^2}(y) = \frac{1}{\rho_0'} \exp\left\{-\frac{y}{\rho_0'}\right\} u(y) \tag{14}$$

with $\rho_0' = 2\rho_0 = E(\beta_1^2)$. In eq. (11) we have

$$P(e \mid x, z) = \int_0^{\infty} f_{\beta_1^2}(y) P(e \mid \beta_1, x, z)dy = K\left(\frac{E_b}{N_0}, \gamma_0, D\right), \tag{15}$$

where $\gamma_0 = \rho_0' E_b/N_0$, $D = x + z$,

$$K(v, \gamma_0, D) = \frac{1}{2} \operatorname{erfc}[-\sqrt{v}D] - \frac{1}{2} \sqrt{\frac{\gamma_0}{\gamma_0 + 1}} \exp\left[\frac{-vD^2}{\gamma_0 + 1}\right] \cdot \operatorname{erfc}\left[-\sqrt{\frac{\gamma_0}{\gamma_0 + 1}} D\sqrt{v}\right] \tag{16}$$

and $v = E_b/N_0$. This result is the same as in eq. (30) of Ref. 1.

It turns out that a result similar to eq. (14) is obtained when β_1^2 is the maximum of the β_{j1}^2, $j = 1, 2, \cdots, M$. If all the β_{j1}'s are Rayleigh with $E(\beta_{j1}^2) = 2\rho_0 = \rho_0'$,

$$f_{\beta_1^2}(y) = \frac{M}{\rho_0'} \left(1 - \exp\left[\frac{-y}{\rho_0'}\right]\right)^{M-1} \exp\left[\frac{-y}{\rho_0'}\right] u(y), \tag{17}$$

as follows from eq. (5.2-7) of Jakes.[3] Using the binomial theorem in eq. (17),

$$f_{\beta_1^2}(y) = M \sum_{k=0}^{M-1} \binom{M-1}{k} \frac{(-1)^k}{(k+1)\rho'_{0k}} \exp[-y/\rho'_{0k}]u(y), \tag{18}$$

where $\rho'_{0k} = \rho'_0/(k+1)$ and

$$\binom{N}{k} = \frac{N!}{(N-k)!k!}. \tag{19}$$

The evaluation of the integral in eq. (15) for the pdf in eq. (18) will just involve a summation of the $K(\cdot,\cdot,\cdot)$ function of eq. (16). This follows as the pdf in eq. (18) is just a summation of the exponential pdf's, $(\rho'_{0k})^{-1}\exp(-y/\rho'_{0k})$. Thus, use of eqs. (14) and (15) in the evaluation of the integral in eq. (15) for $f_{\beta_1^2}(y)$ in eq. (18) yields

$$P(e\,|\,x, z) = M \sum_{k=0}^{M-1} \binom{M-1}{k} \frac{(-1)^k}{k+1} K\left(\frac{E_b}{N_0}, \frac{\gamma_0}{k+1}, D\right), \tag{20}$$

where the $K(\cdot,\cdot,\cdot)$ function is as specified in eq. (16). This is the main mathematical result of this study. Kavehrad[1] showed how to average the $K(\cdot,\cdot, D)$ function with respect to $D = x + z$. To remove the dependence of $P(e\,|\,x, z)$ on $D = x + z$, one just carries out Kavehrad's procedure once, to get the moments of $D = x + z$, and then evaluates the resulting $K[\cdot, \gamma_0/(k+1), D]$ function for all $\gamma_0/(k+1)$. In mathematical terms,

$$P(e) = M \sum_{k=0}^{M-1} \binom{M-1}{k} \frac{(-1)^k}{k+1} \sum_j w_j K\left(\frac{E_b}{N_0}, \frac{\gamma_0}{k+1}, \zeta_j\right), \tag{21}$$

where (w_j, ζ_j) are the weights and nodes, respectively, of the Gauss-Quadrature algorithm (see Appendix C of Ref. 1). For $M = 1$ the result in eq. (21) reduces to eq. (32) of Kavehrad's analysis in Ref. 1.

IV. GAUSSIAN ASSUMPTION

The Gaussian assumption is to take all the multiuser interference as Gaussian noise. We will base our calculation of the average error probability on eq. (10a), which is equivalent to eq. (10) of Ref. 1. The last term in eq. (10a), ν, is a Gaussian random variable having zero mean and variance $N_0T/4$. The first term in eq. (10a) is the signal term and it has average power, $\beta_1^2A^2T^2/4$ for a fixed β_1. The rest of the terms in eq. (10a) are all mutually independent. To calculate the total power of this term, we must evaluate a term like

$$\epsilon^2 = E\left\{\frac{[\alpha_{-1}R(t_1) + \alpha_0\hat{R}(t_1)]^2}{T^2}\right\}, \tag{22}$$

where $\alpha_i = \pm 1$, $i = -1, 0$, are independent binary variables. In eq. (22), ϵ^2 was shown by Pursley[16] to have the value $2/(3N)$, where N is the sequence length of the Gold codes considered in Ref. 16.

There are approximately $\eta = LK$ such expectations in eq. (10a). Hence, for a fixed β_1 we have

$$\text{signal power} = \left(\frac{AT}{2}\right)^2 \beta_1^2$$

$$\text{interference power} = \eta \left(\frac{AT}{2}\right)^2 \epsilon^2 E(\beta^2)/2$$

and

$$\text{noise power} = N_0 T/4,$$

where β denotes a Rayleigh random variable and is any of the independent identically distributed random variables $\beta_{\ell k}$'s excluding β_1. The term $E(\beta^2)/2$ occurs in the interference power as $\beta \cos \theta$ is Gaussian with zero mean and variance $E(\beta^2)/2$ as θ is uniform in $[0, 2\pi]$. Thus, with the Gaussian assumption, the error probability conditioned on β_1 is given by

$$P(e \mid \beta_1) = \frac{1}{2} \operatorname{erfc}(\sqrt{\gamma}). \tag{23}$$

Note that in eq. (23) γ is equal to half the signal-to-noise plus interference power ratio; hence

$$\gamma = \frac{\beta_1^2 E_b}{(LK)\epsilon^2 E_b E(\beta^2) + N_0} \tag{24}$$

with $\epsilon^2 = 2/(3N)$ given by eq. (22) and $E_b = A^2T/2$. The average value of γ is

$$\gamma_0 = \frac{\overline{E_b}}{\eta \epsilon^2 \overline{E_b} + N_0}, \tag{25}$$

where $\overline{E_b} = E(\beta^2)E_b$ and $\eta = LK$. For $N_0 = 0$ we have $\gamma_0 = 3N/2\eta$, which is a result to be used in what follows.

When β_1^2 in eq. (24) has the pdf in eq. (14) with $E(\beta_1^2) = 2\rho_0 = \rho_0'$, Proakis[10] in his textbook shows that the average of eq. (23) with respect to β_1 is

$$P(e) = p(\gamma_0) = \frac{1}{2}\left\{1 - \sqrt{\frac{\gamma_0}{\gamma_0 + 1}}\right\}. \tag{26}$$

When selection diversity is used, the pdf for β_1^2 has the form in eq.

(18), which is just a summation of exponential pdf's. Accordingly, $P(e)$ for this case is just a sum of the terms in eq. (26), viz,

$$P(e) = M \sum_{k=0}^{M-1} \binom{M-1}{k} \frac{(-1)^k}{k+1} p\left(\frac{\gamma_0}{k+1}\right), \tag{27}$$

a result given by Sundberg.[17]

A more complicated, but higher performance, form of diversity is Maximal Ratio Combining (MRC). Here the gain and phase of each signal term must be known. These gains and phases are then used to coherently combine individual path terms to form a single decision variable for the data detection process. The decision statistic, assuming perfect coherence, is just the summation of $\beta_j \cdot \xi_j$, $j = 1, 2, \cdots, M$, where ξ_j is given in eq. (10). The error performance of this form of diversity can also be given in terms of γ_0 in eq. (25). If the interference terms are assumed to be uncorrelated from one diversity branch to another, then the result is given in eq. (7.4.15) of Proakis[10] text, viz,

$$P_e = [p(\gamma_0)]^M \sum_{k=0}^{M-1} \binom{M-1+k}{k} [1 - p(\gamma_0)]^k, \tag{28}$$

where $p(\gamma_0)$ is given in eq. (26).

We have not yet determined the error performance for MRC using the method of moments. Therefore, for this form of diversity we will have to rely on the Gaussian assumption. We will estimate the accuracy of the Gaussian assumption for the case of selection diversity and then apply this to the MRC case.

V. CHANNEL CODING

We shall be interested in the performance of two simple block channel codes used in conjunction with selection diversity. These are the (7, 4) Hamming code and the (15, 7) BCH code. The former corrects one channel error while the latter corrects two channel errors in a coded block. Such codes are discussed in the introductory textbooks by Pless[18] and Lin and Costello.[19]

For a channel code that corrects t-errors, the bit error probability is given in eq. (25) of Milstein et al.[4] as

$$P_{bt} = \frac{1}{n} \sum_{i=t+1}^{n} i \binom{n}{i} p_e^i (1 - p_e)^{n-i}, \tag{29}$$

where for simplicity we denote the channel error probability in eq. (21), $P(e)$, by p_e and n is the coded block length. This is an approximation and the assumption is made that channel errors are independent.

We have done some calculations with eq. (29) for the (7, 4) Hamming code and the (15, 7) BCH code. However, the formulas given below are more precise, as they are based on the weight distribution of the codes used in our study. The two approaches never gave a difference in bit error rate of more than 66 percent. For the (7, 4) Hamming code we show in Appendix A that

$$P_{b1} = 9p_e^2(1 - p_e)^5 + 19p_e^3(1 - p_e)^4 \tag{30}$$

for small p_e. As this is a perfect code the result in eq. (30) is exact for small p_e. The result in eq. (29) gives $6p_e^2$ not $9p_e^2$ for the first term in eq. (30) when p_e is small.

For the (15, 7) BCH code we show in Appendix A that for small p_e,

$$P_{b2} = 150p_e^3(1 - p_e)^{12} + 512p_e^4(1 - p_e)^{11}. \tag{31}$$

Some approximations are involved here, as the code is not perfect. However, the weight distribution of the code is considered. The formula in eq. (29) gives $P_{b2} = 91p_e^3$ for small p_e.

VI. NUMERICAL RESULTS

The computer programs developed by Kavehrad[1] for the method of moments were modified to incorporate the moments of interference terms in the new form in eq. (10a). The new program was simply adapted to perform the computations needed to evaluate eqs. (20) and (21). Our computations will be for the Gold sequences of length 127 and the Kasami sequences of length 255. Initial loadings to generate these codes were taken from Ref. 20.

Before discussing our numerical results let us just review the main parameters of our model. They are

N = spread-spectrum sequence length

L = number of multipath links

M = number of terms used for diversity

K = number of simultaneous users

and

γ_0 = average signal-to-noise ratio.

In our computations we are interested in the case in which L is small, M is moderate, and K is large. For the most part, we will concentrate on the so-called[5] noise floor average error probability. This is the error probability when the thermal noise is absent; that is, when $N_0 = 0$. Our computations were most easily done for $1 \le K \le 15$ and $1 \le L \le 30$.

We will examine the three hypotheses listed below. The first one follows.

6.1 Error probability is approximately a function of only $\eta = KL$

Our hypothesis is that the error probability is approximately a function of only $\eta = KL$ and not of K and L themselves. This is certainly true in the case when a Gaussian assumption is made, as can be seen by an examination of eqs. (25), (26), (27), and (28) of Section IV. The goal of our system analysis is to estimate performance for, say, $K = 90$ and $L = 2$. We wish to do this by computing the error probability for $\eta = KL$, with, for instance, $K = 15$ and $L = 12$, which also gives $\eta = 180$.

The results of our computations for $N = 255$, the Kasami code, are shown in Table I. For error probabilities in the channel-coded cases, that is, P_{b1} and P_{b2}, of around 10^{-4} we see that, for the most part, we are in error at most by a factor of 2 or 3. This is an acceptable error factor for practical error probabilities. An error factor of 6 to 10 would not be acceptable in our view. This point takes us to our next hypothesis.

6.2 Coding plus selection diversity is power efficient

To evaluate the error performance when channel coding is used we take the result of the computation represented by eq. (21) and substitute it into (30) or (31). Equation (30) is for single-error correction and eq. (31) is for double-error correction.

We will compare channel coding plus selection diversity of order M versus selection diversity alone of order $2M$. This will be done for the single-error-correcting (7, 4) Hamming code. Thus P_{b1} is given in eq. (30). For large γ_0 it is easily shown that P_e in eq. (27), for selection diversity and a Gaussian assumption, is given by $c\gamma_0^{-M}$, where c is a constant. For various constants, c, see Table 1 of Ref. 17. Thus, using the (7, 4) Hamming code plus diversity takes $M \rightarrow 2M$, since the channel error probability is squared to give the decoded error proba-

Table I—The data to test the $P_e = f(\eta)$, $\eta = LK$, hypothesis for $N = 255$

M	η	K	L	P_e	P_{b1}	P_{b2}
4	60	10	6	0.41E-02	0.15E-03	0.98E-05
4	60	6	10	0.50E-02	0.22E-03	0.18E-04
6	90	15	6	0.85E-03	0.65E-05	0.91E-07
6	90	6	15	0.19E-02	0.32E-04	0.10E-05
6	150	15	10	0.41E-02	0.15E-03	0.98E-05
6	150	10	15	0.50E-02	0.22E-03	0.18E-04
6	180	15	12	0.69E-02	0.41E-03	0.45E-04
6	180	12	15	0.91E-02	0.57E-03	0.72E-04
8	180	15	12	0.35E-02	0.11E-03	0.62E-05
8	180	10	18	0.44E-02	0.17E-03	0.12E-04
8	240	15	16	0.88E-02	0.67E-03	0.91E-04
8	240	12	20	0.97E-02	0.80E-03	0.12E-03

bility and thus doubles the order of diversity. One more point is important in our comparison. Diversity and channel coding are similar in nature, since both try to exploit the redundancy in the transmitted signal. Our codes require bandwidth expansion to achieve this redundancy, but space diversity does not. Now the number of discrete multipath links between transmitter and receiver is given by eq. (3). Since the signal bandwidth, B_{ss}, is smaller for selection diversity alone, we evaluate its performance when L is replaced by $L - 1$ for $L = 2$ or 3, where the channel-coded system is taken to produce L multipath terms. Of course, for larger L's, the former should be replaced by $L - 2$, and so on. In any case, the uncoded system is subject to less interference than the coded system. The results of our computations are shown in Fig. 5. Also shown in Fig. 5 are two isolated points for

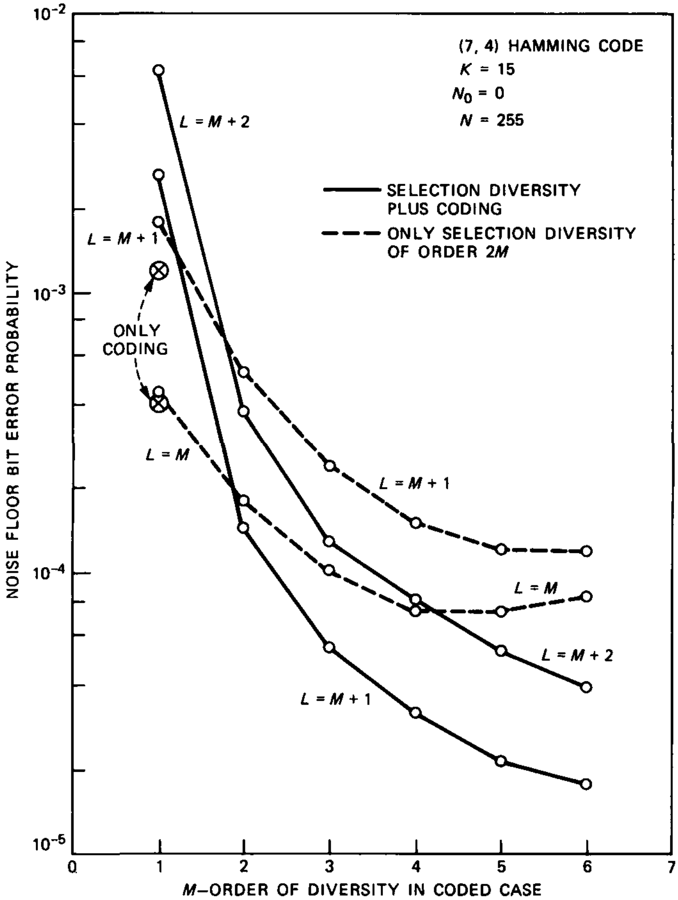


Fig. 5—Comparison of selection diversity plus coding versus selection diversity alone.

coding alone, that is, $M = 1$. One notes that, except for low M, the combination of channel coding plus selection diversity is significantly better than using selection diversity alone.

We note that selection diversity alone can saturate to a poor value of error probability as M gets large. From Jakes[3] it is well known that the signal-to-noise ratio (s/n) performance of selection diversity becomes poor relative to MRC as M grows. In the case of multipath diversity, performance becomes even worse. To see this point let us first consider MRC. Jakes[3] shows that, for antenna diversity of order M, the average s/n is $M\gamma_0$ (see page 3.19 of Ref. 3), where γ_0 is the average s/n for no diversity. Let the only source of noise for multipath diversity be the self-noise term in eq. (10). Accordingly, for multipath diversity alone γ_0 must be changed to γ_0/M and the average s/n with MRC diversity is γ_0 for all M. Thus, multipath diversity allows for no improvement in average s/n. In general, let there be L_S antennas and L_M multipath diversity terms giving $M = L_S \cdot L_M$. In the sequel we always take $L_M = L$, the number of paths in our multipath model. Then the average s/n is $L_S\gamma_0$ and the improvement is only due to L_S. With selection diversity Jakes[3] shows that the average s/n is

$$E(\gamma) = \gamma_0 \sum_{k=1}^{M} \frac{1}{k}.$$

For $M = 2$ we have $E(\gamma) = 0.75\ \gamma_0$ as for multipath diversity alone, since we must replace γ_0 by $\gamma_0/2$ due to self-interference. For $M = 4$ with $L_S = L_M = 2$ we have $E(\gamma) = 2\gamma_0$ for MRC and we have $E(\gamma) = 25\gamma_0/24$ for selection diversity, a loss of about 3 dB to MRC. Fortunately, the system error probability is not a function of $E(\gamma)$ alone, as it is a polynomial as γ_0^{-1}. In any case, for acceptable performance we will find that selection combining requires both antenna and multipath diversity. However, this may not be necessary for MRC or equal gain combining; only multipath diversity may suffice.

6.3 Coding plus selection diversity is power and bandwidth efficient

Our next comparison involves the performance of coding plus selection diversity versus selection diversity alone for the same system bandwidth. We did this by finding the selection diversity performance of the 127-length code (the Gold codes) with the (7, 4) Hamming code versus the length 255 code (the Kasami codes) with no channel coding. The result is plotted in Fig. 6. As Milstein et al.[4] found earlier, simple error-correcting codes are an effective way to improve the performance of spread-spectrum systems for the same system bandwidth.

6.4 Performance: coding plus selection diversity

The results of our computations using the method of moments are

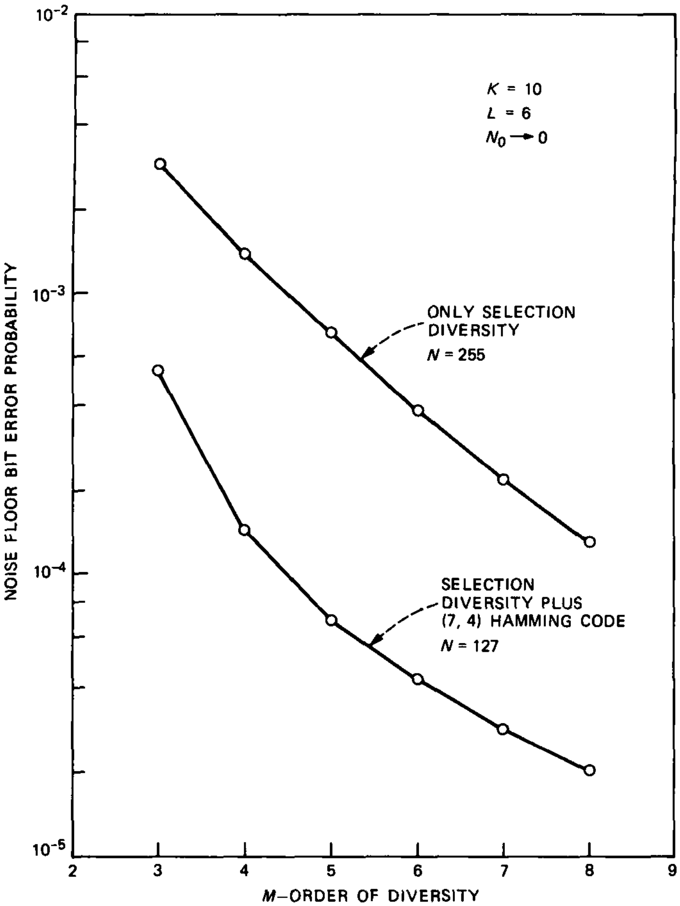


Fig. 6—Comparison of coding versus no coding in the same spread-spectrum bandwidth.

given in Figs. 7, 8, and 9. Figures 7 and 8 depict the noise floor error probability as $N_0 = 0$. Figure 7 is for small L and Fig. 8 is for large L. These computations are for independent values of K, L, and M; M is not a function of L, as will be the case in Section VII. We are interested in large L and $P_b = 10^{-4}$ in our system analysis, which will be discussed in the next section, where extensive use will be made of the results in Figs. 7 and 8. Figure 9 presents the performance with $N_0 \neq 0$, which will not be used in the sequel, since for IWC systems analysis is completely based on the noise floor error probability.

6.5 Computations: Gaussian assumption

It is clear that computation of the system error probability when a Gaussian assumption is invoked is quite simple. This is true for either

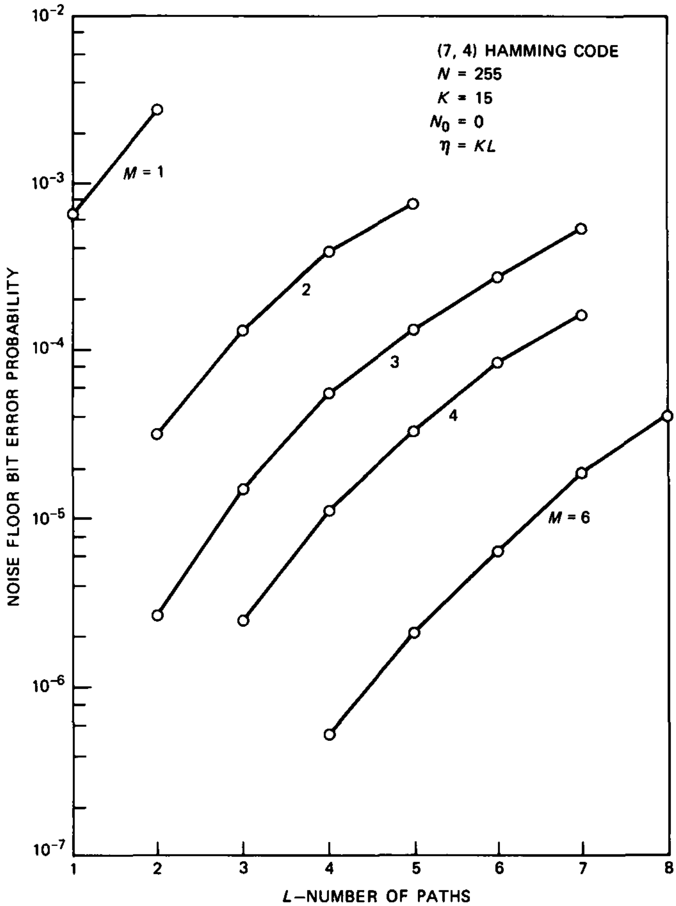


Fig. 7—Noise floor error probability for the (7, 4) Hamming code and various orders of diversity.

selection or MRC diversity, as can be observed by referring to eqs. (27) and (28), respectively. Since the method of moments precisely computes the system error probability, we can assess the goodness of the Gaussian approximation.

Our calculations are plotted in Figs. 10a and b. The Gaussian approximation underestimates the noise floor error probability. This gets worse as the number of interferers increases, a counterintuitive result based on intuition related to the central limit theorem. Around a 10^{-4} error probability, however, the discrepancy is acceptable. Because the Gaussian assumption underestimates the error probability, it will, for a fixed error probability, lead to an overestimation in the number of simultaneous users of a spread-spectrum multiple access system. We present the degree of this overestimation in Table II. In

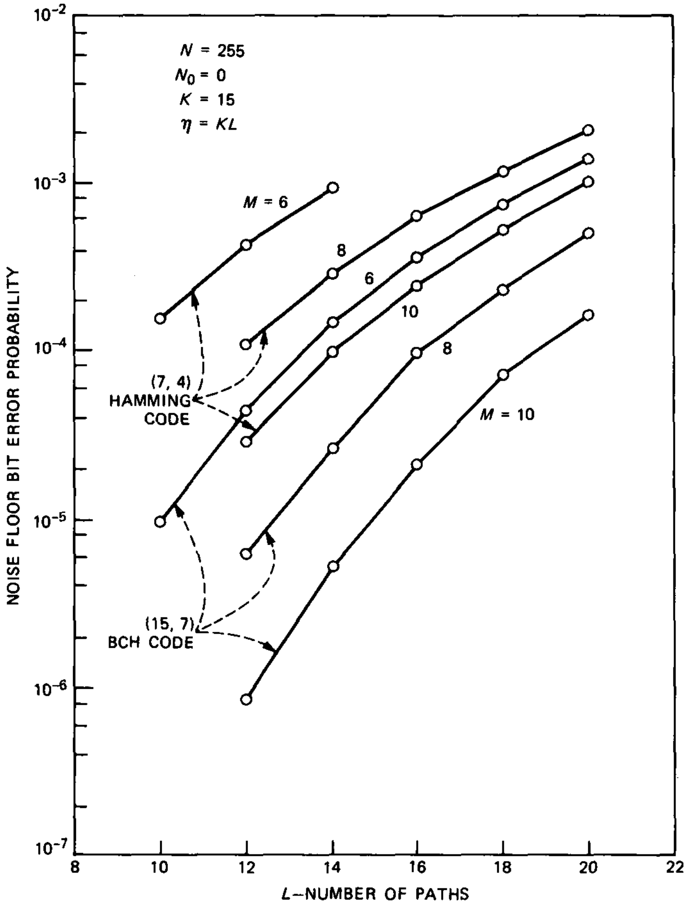


Fig. 8—Noise floor error probability for the (7, 4) Hamming code and (15, 7) BCH code for various orders of diversity.

this table both the Hamming and BCH codes are considered for two cases of diversity, $M = 4$ and 6. In both cases we are interested in that value of $\eta = KL$ that gives a noise floor error probability of approximately 10^{-4}. To do this we consider the Gold code of length 127 and the Kasami code of length 255. In the case of a Gaussian assumption the system performance depends only on $\gamma_0 = 3N/(2\eta)$. Thus, if N is doubled, so should η for the same value of γ_0. The percent error in this assumption is shown in Table II. For the orders of diversity of interest, the error is at most 20 percent.

We have also done computations for MRC by invoking the Gaussian assumption. This procedure uses eq. (28) and just depends on the parameter, $\gamma_0 = 3N/(2\eta)$, $\eta = KL$. We then reduce the value of η produced by this computation by 20 percent for, say, $P_e = 10^{-4}$, as this

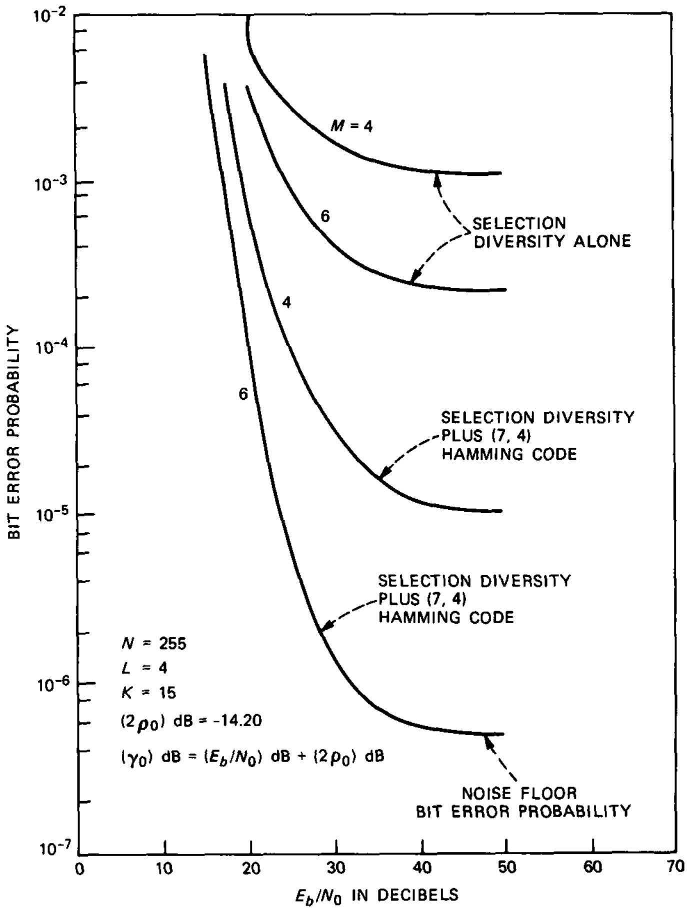


Fig. 9—Bit error probability as a function of E_b/N_0 for $M = 4$ and $M = 6$.

was the error in the case for selection diversity. The results of a limited number of such computations will be discussed in the next section.

VII. IWC PARAMETER STUDY

7.1 T_m = 100 nanoseconds

Measurements by Saleh and Valenzuela[21] have established the multipath delay spread in the Crawford Hill building at AT&T Bell Laboratories, Holmdel, New Jersey. The measurements indicate that the maximum delay spread is usually $T_m = 100$ ns. The distance over which these measurements were taken was approximately 300 ft.

The application that interests us is for 32-kb/s digital speech. We take this as the source rate. The service supplied also will include a 9.6-kb/s data service, and such a source would be channel coded up to

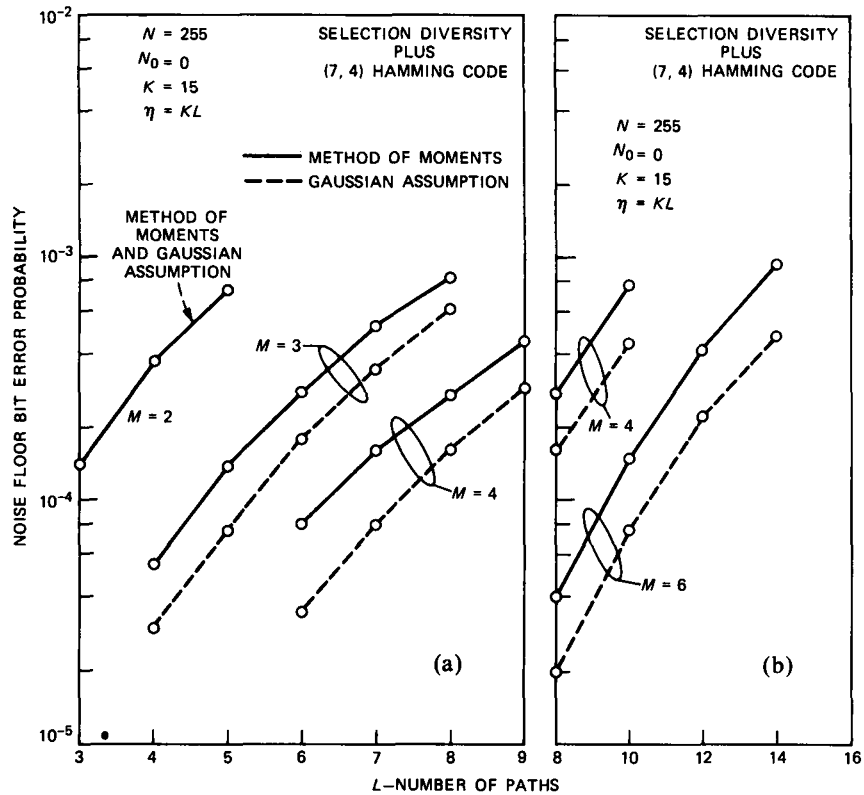


Fig. 10—The Gaussian assumption for (a) low orders of diversity and (b) high orders of diversity.

Table II—Error in the Gaussian assumption when the sequence length is doubled. The calculations of η are for $P_b \approx 10^{-4}$ and $\eta = KL$.

M	Code	η 127	η 255	η Gaussian 255	Percent Error
4	(7, 4)	60	105	120	14.3
4	(15, 7)	90	150	180	20.0
6	(7, 4)	80	150	160	6.7
6	(15, 7)	120	210	240	14.3

the 32-kb/s rate to provide the extra error protection needed for data. We focus on a threshold average error rate of 10^{-4} for speech. For 1000 bit packets this translates into approximately a 10-percent packet error probability. Valenzuela[22] and Wong et al.[23] have developed interpolation schemes to handle such packet loss rates. In any case, for a 32-kb/s source rate, the bandwidths of the various spread-spectrum

Table III—Transmission parameters for different sequence lengths

N	No Coding	(7, 4) Code	(15, 7) Code
(a) Bandwidth in MHz of various spread-spectrum sequence lengths for a source rate of 32 kb/s			
127	4.06	7.11	8.70
255	8.16	14.20	17.50
512	16.40	28.67	35.11
(b) Number of discrete multipath components when $T_m = 100$ ns			
127	1	1	1
255	1	2	2
512	2	3	4
(c) Number of discrete multipath components when $T_m = 250$ ns			
127	2	2	3
255	3	4	5
512	5	8	9

systems that we will consider are given in Table IIIa. The IWC application here presumes overlay signaling,[24] where spread-spectrum users' signal coexists with that of users of other services in a lightly loaded part of the radio frequency band.

A crucial parameter in our study will be the number of discrete paths, L, for a given maximum multipath delay spread and spread-spectrum bandwidth as predicted by eq. (3). Thus, for $N = 255$ in Table III and for $T_m = 100$ ns, we let $L = 2$. Other values of L are given in Table IIIb. From Fig. 8 we note that for the (15, 7) BCH code (double-error-correcting), we can have $L = 14$ for $K = 15$ in order that $P_b \approx 10^{-4}$ when $M = 6$. As $K = 15$ we have $\eta = LK = 210$. We now invoke our assumption about the fact that the system error rate is, to a good practical approximation, just a function of $\eta = KL$. Thus, if $L = 2$ we can get $K = 105$ simultaneous users since $\eta = 210$. The order of diversity $M = L_S \cdot L$, where L_S is the number of antennas and L the discrete order of spread-spectrum diversity. Therefore, $L_S = 3$ antennas at the central station is needed to support 105 simultaneous users.

Let us see now how many simultaneous users the single-error-correction system can support. For $M = 6$ from Fig. 8 we get $L = 10$ and thus $\eta = 150$. As $L = 2$, our assumption $P_e \approx f(KL)$ gives $K = 75$ active users. This is for $L_S = 3$ antennas.

With the sort of computation we have just outlined—through use of Fig. 8—and other related calculations, we can construct Table IV. The bandwidth efficiency measure given in Table IV will be discussed below. Also given are some performance estimates for the MRC form of diversity with no channel coding. When the spread-spectrum se-

Table IV—The number of simultaneous users in terms of sequence length, N, and L_s. The number of users is given in columns 3, 4, and 5. The order of diversity is $L_s \cdot L$, where L is given in Table IIIb. We have set $T_m = 100$ ns, $P_b \approx 10^{-4}$.

N	L_s	(7, 4) Code	(15, 7) Code	MRC at $2N$	Bandwidth Efficiency (15, 7) Code	Bandwidth Efficiency MRC at $2N$
127	2	20	40	6	0.14	0.02
255	1	23	38	6	0.07	0.01
255	2	50	75	60	0.12	0.12
255	3	75	105	132	0.19	0.25
512	1	42	60	—	0.05	—
512	2	80	108	—	0.10	—

quence length, N, is, say, 255, we have set $N = 512$ for MRC in order that the MRC system and selection diversity with channel coding system occupy the same bandwidth. The estimates for MRC were computed using the Gaussian assumption, $\gamma_0 = 3N/(2\eta)$, and eqs. (26) and (28). Such estimates of K were then reduced by 20 percent in accordance with our earlier results on the Gaussian approximation. We have made this reduction in computing the data of Table IV. Subject to such estimates we note that $M = 6$ is needed for MRC to outperform the combination of selection diversity and channel coding.

We note that the estimate for K for an $N = 512$ sequence length was determined as follows. The value of $\eta = KL$ for $N = 255$ was doubled and the result was reduced by 20 percent. This is in keeping with our findings regarding the Gaussian assumption (see Table II).

In Tables Va and b we present results when L is random. We let L vary from unity up to the maximum value, L_{max}, given by the right-hand side of eq. (3). Each value of L is taken to occur with a probability of $1/L_{max}$. We display the average value of K and also the value K corresponding to $L = L_{max}$ in Tables Va and b. Actually, the value of K for each value of L, $1 \leq L \leq L_{max}$, differed only slightly from the average value of K. We find this invariance because as L decreases, the order of diversity, $M = L_s L$, decreases, but so does the maximum number of interference terms, $\eta = KL$, thus giving rise to approximately a fixed P_b. Note that we cannot have only one antenna in the random model, since when $L = 1$ all diversity is lost.

In Fig. 11 we have plotted the bandwidth efficiency of some of our schemes. This is given by

$$\text{BE} = \frac{K \cdot (\text{Code Rate})}{N},$$

where N is the spread-spectrum code length and K is the number of

Table V—A comparison of the estimate of the number of simultaneous users when $L = L_{max}$, as given by eq. (3), and when $L = 1, 2, \cdots, L_{max}$, each with probability $1/L_{max}$. Average K is over such L.

N	L_s	L_{max}	K for L_{max}	Average K for $1 \le L \le L_{max}$
(a) Data are for the (7, 4) code with $T_m = 100$ ns				
127	2	1	20	20
255	2	2	50	50
255	3	2	75	75
512	2	3	80	79
(b) A comparison of the estimate of the number of simultaneous users for the (15, 7) code				
127	2	1	40	40
255	2	2	75	75
255	3	2	105	112
512	2	4	108	115

simultaneous users as estimated using the procedure just described above. We note that the double-error-correcting system is 11 percent more bandwidth efficient than the single-error-correcting system. Tabular data on bandwidth efficiency are given in Table IV. Although the efficiency values in Fig. 11 are rather low, we remember that in overlay signaling the values represent the order of frequency reuse.

We have also placed other points on our bandwidth efficiency plot in Fig. 11. These points are for less severe channel models than the one we consider. If we assume an Additive White Gaussian Noise (AWGN) channel, the performance follows from eq. (24). In eq. (24) let $N_0 = 0$ so that we get the noise floor error probability and assume that the multiuser interference follows the Gaussian model. Furthermore, let $b = \beta_1^2/E(\beta_1^2)$ and $\eta = LK$. Combining eqs. (23) and (24) for the bandwidth efficiency, there follows

$$\mathrm{BE} = \frac{3b}{2L\{\mathrm{erfc}^{-1}(2P_e)\}^2}. \tag{32}$$

In eq. (32) for $P_e = P_b = 10^{-4}$ we have BE = 0.22b/L. For the AWGN channel $\beta_1 = \beta = 1$ and $L = 1$ to give BE = 0.22, which agrees with Turin's[7] result (see Table 1 of Ref. 7; our result is slightly higher, as we use coherent binary phase-shift keying rather than differential binary phase-shift-keying modulation). Thus, for the same bandwidth as used in Table IV, as we have $\eta = 0.22$, $N = 512$ gives $K = 112$ and for no coding or diversity.

Another case of interest is when the signal term has a deterministic

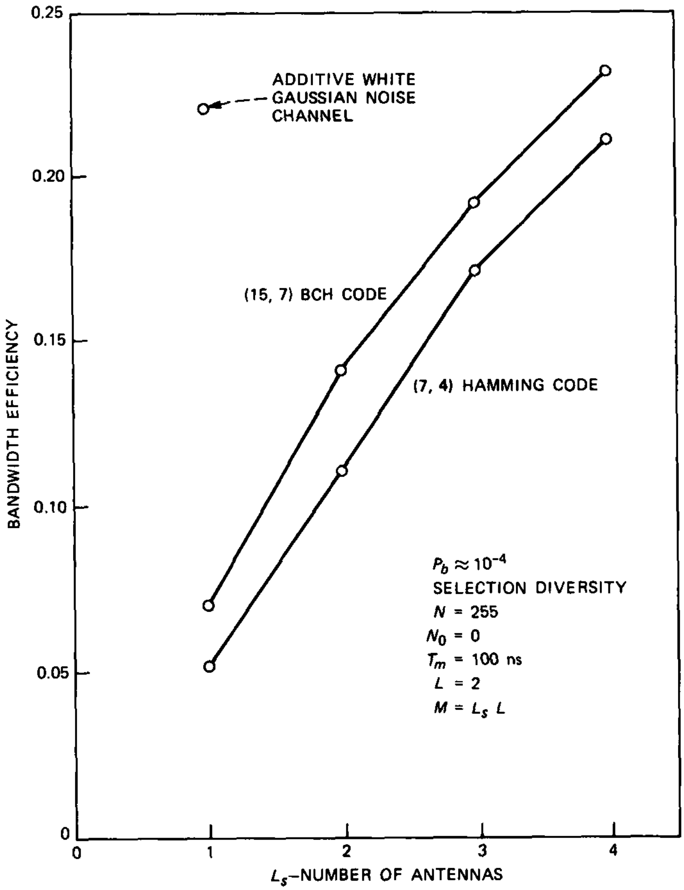


Fig. 11—Bandwidth efficiency of error-correction-coded spread-spectrum systems. For $N = 255$, selection diversity alone requires $L_s = 8$ to get an efficiency of 0.11.

gain but the multiuser interference is subject to Rayleigh fading (see Case 1, Section 4.2 of Ref. 1). As stated in Ref. 1, this is the case that the reference transmitter is stationary and there is not much movement in the indoor medium. For $L = 2$, that is, $T_m = 100$ ns, $N = 255$ and $R_0 = 32$ kb/s, we have BE = $0.11b$, where $b = \beta_1^2/E(\beta^2)$. If $b = 2$, meaning that the average, faded interference power is 3 dB less than the average, unfaded, signal power, BE = 0.22 as for the AWGN channel. Of course, BE grows linearly with b.

For the case just treated we can do a more exact analysis, as was done by Kavehrad in Ref. 1 (see Case 1). Let us assume that the result of the selection diversity process is deterministic, not random, whereas all the rejected path gains are Rayleigh faded. The method of moments can be applied to get the exact solution for the error probability in

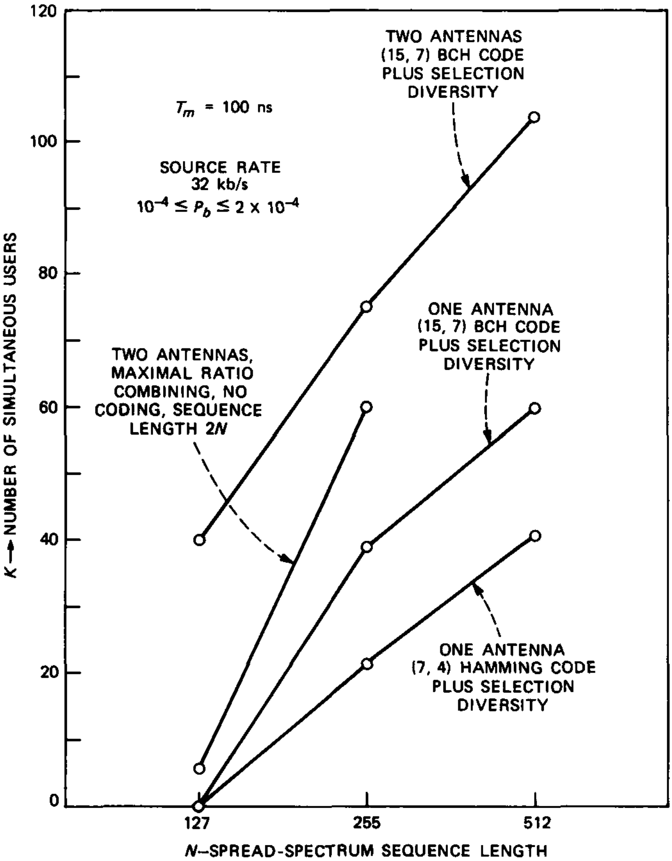


Fig. 12—Number of simultaneous users versus spread-spectrum sequence length when $T_m = 100$ ns.

this case. The theory is similar to that given by Kavehrad[1] and will not be repeated here.

We have summarized the results of all our computations in Fig. 12. The results are all for the Rayleigh faded, discrete multipath model. Note that for a single antenna, the double-error-correction system will support around 40 simultaneous users with a spread-spectrum code length of 255.

7.2 T_m = 250 nanoseconds

We now consider the larger multipath delay spread reported in Ref. 25, which was characteristic of the Holmdel building at AT&T Bell Laboratories. Now $T_m = 250$ ns, a Root Mean Square (RMS) value, a figure that should be used in a larger building. Now for $N = 255$, $R_0 =$ 32 kb/s and one gets $L = 4$ in eq. (3) for the discrete multipath model. Other values of L for $T_m = 250$ ns are given in Table IIIc. Note that

Table VIa—The number of simultaneous users in terms of sequence length, N, and the number of base station antennas, L_s. The number of users is given in columns 3, 4, and 5. The order of diversity is $L_s \cdot L$, $T_m = 250$ ns, $P_b \approx 10^{-4}$.

N	L_s	(7, 4) Code	(15, 7) Code	MRC at $2N$	BE (15, 7) Code	BE MRC at $2N$
127	1	10	20	9	0.07	0.04
127	2	30	40	44	0.15	0.17
255	1	26	36	30	0.07	0.06
255	2	48	60	120	0.11	—
512	1	40	50	—	0.05	—

Table VIb—A comparison of the estimate of the number of simultaneous users with $T_m = 250$ ns

N	L_s	Code	L_{max}	K for L_{max}	Average K
127	2	(7, 4)	2	30	27
255	2	(7, 4)	4	48	49
127	2	(15, 7)	3	40	42
255	2	(15, 7)	5	60	68

$N = 512$ would give $L = 8$ or 9 for the coded system, and thus a diversity order that is too large. In this sense our spread-spectrum system has an optimum sequence length or bandwidth. Results similar to those in Table IV are given in Table VIa and are also plotted in Fig. 13. Note that the (15, 7) coded system can support just under 38 simultaneous users with the same antenna diversity and code length (that is, $L_s = 1$ and $N = 255$) as used when $T_m = 100$ ns (see Table IV). However, the multipath diversity is now of order 5 (see Table IIIc). In any case, the number of simultaneous users is about the same. In a simple Time-Division Multiple Access (TDMA) system, one-half the number of users would be lost as the maximum multipath delay spread has been increased by approximately a factor of two. As such, a SSMA system is less sensitive to a change in maximum multipath delay spread than a simple TDMA system would be. However, if more than 40 simultaneous users are needed, the SSMA system also loses. For instance, compare the performance of the double-error-correcting system at $L_s = 2$ when $N = 512$ for $T_m = 100$ ns (Table IV) and when $N = 255$ for $T_m = 250$ ns (Table VIb); the loss is from 108 to 60 simultaneous users.

As in the case when $T_m = 100$ ns we include estimates of K for the random model for L in Table VIa. The trend in the results is essentially the same as was observed in Tables Va and b.

7.3 Multipath outage performance estimate

Up to now we have used the average error probability as the

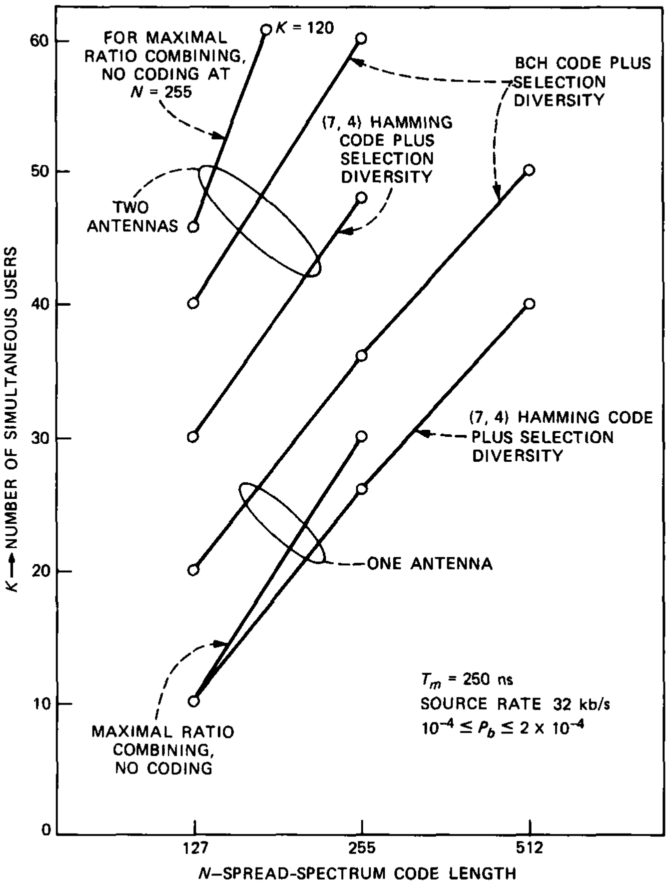


Fig. 13—Number of simultaneous users versus spread-spectrum sequence length when $T_m = 250$ ns.

performance measure. Another measure is to find the distribution of the error probability. This has been used in other radio studies, for example, in Refs. 26 and 27. In the latter study if the probability that the average error probability exceeds the value X is, say, 0.10, the multipath outage is said to be 10 percent. In keeping with our earlier work we take $X = 10^{-4}$.

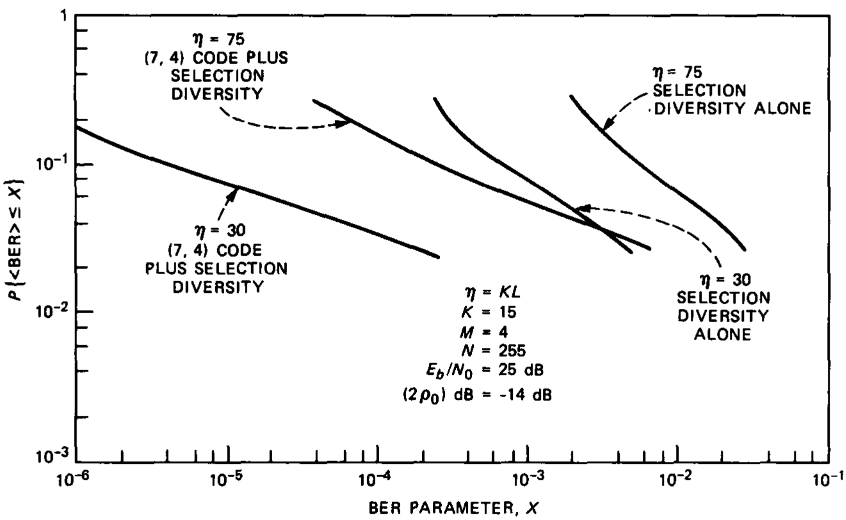
We will determine the multipath outage for the reference user path gain, β_1, with all other path gains having the Rayleigh statistics as assumed earlier. The computation of the multipath outage follows closely Case 1 of Ref. 1. First β_1 is taken to be fixed, and the error probability is found by averaging the right-hand side of eq. (11) with respect to the sum of the self-plus multiuser interference, using the method of moments. Let us call the result of this calculation $\overline{P_e}(\beta_1)$, since it depends on the path gain β_1. We then vary β_1 from zero to β_0, where $\overline{P_e}(\beta_0) = X$, the bit error rate parameter. Then

$$P\{\overline{P_e}(\beta_1) \geq X\} = P\{0 \leq \beta_1^2 \leq \beta_0^2\}.$$

The probability $P(0 \leq \beta_1^2 \leq \beta_0^2)$ is easily obtained by integrating the pdf for selection diversity as given in eq. (17).

If coding is involved, $\overline{P_e}(\beta_1)$ changes. For instance, for $X \leq 10^{-2}$, $\overline{P_e}(\beta_1)$ is well approximated by $9\overline{P_e^2}(\beta_1)$, say, for the (7, 4) Hamming code. To find β_0^2 one solves the equation $X = 9\overline{P_e^2}(\beta_0)$ for a given X. Of course, for the same X, β_0 for the coded case is smaller than β_0 for diversity alone, which leads to a lower outage probability.

The results of our computations for the manner just described are shown in Fig. 14. Note that coding is effective in reducing the outage probability for increasing $\eta = KL$. We found that, to a good approximation, the outage probability was only a function of the product, KL. This allows us to estimate the number of users for a fixed multipath outage as we did earlier for the average error probability. The results are shown in Table VIIa for a 10-percent multipath outage and T_m = 100 ns. Note that the results are given for E_b/N_0 = 25 dB. We found that for up to 15 moments, outage probabilities for larger E_b/N_0's were not reliable in these computations. No such problem occurred in computing the average error probability. The number of simultaneous users is about 10 percent less than it would be if the average error probability for E_b/N_0 = 25 dB were used as a performance measure.



BER – BIT ERROR RATE

Fig. 14—Distribution of error probability.

Table VIIa—The number of simultaneous users for a multipath outage probability of 10 percent when $T_m = 100$ ns, $E_b/N_0 = 25$ dB, $2\rho_0 = -14$ dB, and the error probability is parameter $X \approx 10^{-4}$

N	L_s	K (7, 4) Code	K (15, 7) Code	MRC at $2N$
127	2	5	15	0
127	3	20	36	24
255	1	5	15	0
255	2	38	53	53
255	3	45	68	112

Table VIIb—The reduction in outage probability as L_s, the number of antennas, is increased. This reduction is relative to $L_s = 2$, where $K = 38$ for the (7, 4) code and $K = 53$ for both the (15, 7) code and the MRC system.

N	L_s	System	P_o
255	3	Coded	3.5×10^{-2}
255	4	Coded	1.1×10^{-2}
512	3	MRC	10^{-2}
512	4	MRC	4.7×10^{-4}

We note that the Gaussian assumption had the same accuracy as we stated earlier for the average error probability. Accordingly, the MRC results were computed by invoking a Gaussian assumption on the interference. Table VIIb shows results on how the multipath outage can be reduced by increasing antenna diversity. For a large order of diversity, MRC is stronger than selection diversity plus coding in this task.

Our work to date has assumed an average power control so that all user signals arrive at the central station with the same average power. We now let the static attenuation of the reference user be higher than each member for the whole multiuser population. The results for a 10-percent multipath outage are shown in Table VIII. Note that the loss in user population in percent is about the same as the static power loss of the reference user. Thus, SSMA is not sensitive to small deviations in static power control.

VIII. CONCLUSION

Our main conclusion is that direct-sequence, spread-spectrum modulation can give a quite respectable number of simultaneous users for

Table VIII—The percentage reduction in the number of simultaneous users in Table VII as the reference user is subjected to increased attenuation. This is for T_m = 100 ns, $X \approx 10^{-4}$, and the multipath outage probability of 10 percent.

$\delta\gamma_0$ dB	$\delta\gamma_0$ Percent	δK (7, 4) Code Percent	δK (15, 7) Code Percent
−0.5	12	21	15
−1.0	26	36	28
−2.0	58	58	47

communication over fading, multipath channels. This is for communication from transportable stations to a base station in a star network using average power control that depends only on the power law exponent and static shadow fading. The inherent diversity of spread-spectrum modulation can be combined with antenna diversity using the simple selection diversity rule to give efficiencies that are comparable to those attained with maximal ratio combining. Fairly high orders of diversity are needed for the latter to be better than selection diversity used with channel coding.

We also have found that a Gaussian assumption regarding the multiuser interference can lead to a maximum error of 20 percent in predicting the number of simultaneous users for a noise floor average error probability of 10^{-4}. This is for a system using selection diversity.

We conclude that spread-spectrum modulation can be less sensitive to a change in maximum multipath delay spread than, say, time-division multiple access would be. Thus the same spread-spectrum modem could possibly be used in either large or small buildings for indoor radio communication.

The main assumptions used in the paper are that demodulation errors are independent and that the multipath model is discrete. The former can be overcome with interleaving. However, the latter must be verified experimentally for indoor, wireless, local area network application.

We have also considered multipath outage as a performance criterion. For an outage of 10 percent we find that the number of simultaneous users is reduced by approximately 10 percent over that predicted by using average error probability as a performance measure.

Although our analysis was for selection diversity or maximal ratio combining and coherent binary phase-shift keying, in practice we would suggest equal gain combining and differential binary phase-shift-keying modulation. We make this suggestion as usually equal gain combining falls in performance between that for selection diversity and maximal ratio combining. Also, differential phase-shift keying is less troublesome to demodulate in a fading environment such as occurs in IWC.

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APPENDIX A

Channel Coding Formulations

In this Appendix we derive the formula for the bit error probability used in the paper. Independent channel errors are assumed and only single-error-correcting, (7, 4) Hamming code and double-error-correcting, (15, 7) BCH codes are considered.

A.1 The (7, 4) Hamming code

We first determine the bit error probability for a small channel error probability $p_e = 1 - q_e$. The first term in the power series in p_e for P_{b1} will be denoted as P_{b1}^1. A well-known approximation [see eq. (1-27) of Ref. 28] gives $P_{b1}^1 = dP_s/n$, where $P_s = \binom{7}{2} p_e^2 q_e^5$. Here P_s is the probability, for small p_e, that a code vector is in error and d is the minimum Hamming distance. As $n = 7$ and $d = 3$ we have $P_{b1}^1 = 9p_e^2q_e^5$ for small p_e for the first term in eq. (30).

The weight of a code word is the number of its nonzero symbols. Let A_i be the number of code words of weight i. For the (7, 4) Hamming code $A_0 = A_7 = 1$ and $A_3 = A_4 = 7$ with all other $A_i = 0$. Since the (7, 4) Hamming code is a perfect code, two channel errors always produce a weight-3 code word where we assume the all-zero code word is transmitted. As the code is linear we have no loss in generality. In the weight-3 code words, three code words contain one bit error, three contain two bit errors, and one contains three bit errors, where a bit error refers only to erroneous information bits. Thus, as there are four information bits,

$$P_{b1}^1 = \frac{\binom{7}{2}}{7} p_e^2 q_e^5 \left\{\frac{3}{4} + \frac{6}{4} + \frac{3}{4}\right\},$$

which gives $9p_e^2q_e^5$, as before.

We now find a correction term to P_{b1}^1, which we call P_{b1}^2. When three channel errors result, either a weight-3 or a weight-4 code word is decoded. Now, P_{b1}^2 is the sum of the probabilities of these two cases. The weight-4 code words can be chosen in 28 ways, since each of the seven weight-4 code words has four correctable error patterns. Thus,

$$P_{b1}^{2}(\text{weight-4}) = \frac{28}{7}\, p_e^3 q_e^4 \left\{\frac{9}{4} + \frac{6}{4} + \frac{1}{4}\right\} = 16 p_e^3 q_e^4,$$

as among the weight-4 code words three have three bit errors, three have two bit errors, and one code word contains a single bit error.

To consider the second component of P_{b1}^2, we note that a weight-3 code word is chosen when the channel errors combine with the transmitted all-zero code word to produce a weight-3 code word. Thus,

$$P_{b1}^{2}(\text{weight-3}) = p_e^3 q_e^4 \left\{\frac{3}{4} + \frac{6}{4} + \frac{3}{4}\right\}$$

to give $3p_e^3q_e^4$. Adding the results for P_{b1}^2, we get the correction term in eq. (30) of the paper. We get exactly the same result by applying the approximation dP_s/n to the weight-3 and weight-4 error events. That is, for the weight-3 bit error probability we have $7 \times 3p_e^3q_e^4/7$ and for weight-4, $28 \times 4p_e^3q_e^4/7$ to get $P_{b1}^2 = 19p_e^3q_e^4$.

A.2 The (15, 7) BCH code

We generated the (15, 7) BCH code with generator polynomial

$$g(x) = 1 + x^4 + x^6 + x^7 + x^8.$$

This gave $A_0 = 1$, $A_5 = 18$, and $A_6 = 28$, which are the only components of the weight distribution we will need. Now A_5 has five code words with one bit error, five with two, six with three, and two with four. Thus,

$$P_{b2}^{1}(\text{weight-5}) = \frac{455}{18}\, p_e^3 q_e^{12} \left\{\frac{5}{7} + \frac{10}{7} + \frac{18}{7} + \frac{8}{7}\right\} = \frac{2470}{18}\, p_e^3 q_e^{12},$$

since there are $\binom{15}{3} = 455$ error patterns with three channel errors. Also,

$$P_{b2}^{1}(\text{weight-6}) = \frac{455}{28}\, p_e^3 q_e^{12} \left\{\frac{2}{7} + \frac{16}{7} + \frac{39}{7} + \frac{16}{7} + \frac{5}{7}\right\} = \frac{5070}{28}\, p_e^3 q_e^{12},$$

since in A_6 two code words have one bit error, eight have two, thirteen have three, four have four, and one code word has one bit error.

To combine our two values of P_{b2}^1, we use the density of the code words. For each of the 128 code words there are $\binom{15}{2} = 105$ correctable double-error patterns and 15 correctable single-error patterns. Thus the number of channel outputs within a distance two of code words is

121 × 128. There are 2^{15} channel outputs in all, to give a probability of 0.47 of falling within a decoding sphere of radius two. We assume 47 percent of the space around the zero code word is filled with weight-5 code words. In the remaining 53 percent we assume weight-5 and weight-6 code words are chosen with equal probability. Thus,

$$P_{b2}^1 = P_{b2}^1(\text{weight-5}) \times 0.735 + P_{b2}^1(\text{weight-6}) \times 0.265 = 150 p_e^3 q_e^{12},$$

which agrees well with $P_{b2}^1 = P_s d/15$, as now $d = 5$ and $P_s = \binom{15}{3} p_e^3 q_e^{12}$.

To get the correction term we assume that the four channel errors produce either a weight-5 code word or a weight-6 code word. The weight-5 codes have five correctable error patterns of weight-4 per code word and thus 18 × 5 = 90 weight-4 channel outputs in their decoding spheres. For weight-6 code words we have $\binom{6}{2} = 15$ correctable error patterns and thus 28 × 15 = 420 channel outputs in weight-6 decoding spheres. This leaves $\binom{15}{4} - 510 = 855$ error patterns and we assume they are equally divided between weight-5 and weight-6 code words. Averaging over the bit errors in weight-5 and weight-6 codes gives

$$P_{b2}^2 = \left(\frac{518 \times 41}{18 \times 7} + \frac{848 \times 78}{28 \times 7}\right) p_e^4 q_e^{11} = 506 p_e^4 q_e^{11},$$

where the first term is for weight-5 code words. Use of the dP_s/n approximation gives

$$P_{b2}^2 = \left(\frac{518 \times 5}{15} + \frac{848 \times 6}{18}\right) p_e^4 q_e^{11} = 512 p_e^4 q_e^{11},$$

which is in good agreement with the result just given above.

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