A CROSSTALK MODEL FOR BALANCED DIGITAL TRANSMISSIONS IN MULTIPAIR CABLE

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A new model is developed to examine the crosstalk problem of digital transmission in multipair cables. The traditional assumptions of Gaussian crosstalk loss and Gaussian noise distributions are abandoned in favor of a technique aimed at predicting the peak noise levels that will be observed in a multidisturber environment. The method is designed to encompass bursty transmissions as well as those which are continuous. This technique is then applied to the problem of determining the compatibility of distinct digital systems sharing a common multipair cable in an inside building environment. The result is a procedure to determine peak transmission levels which will allow distinct digital systems to transmit over a common cable without performance being degraded by excessive crosstalk.

Introduction

One of the major concerns with digital transmission systems sharing a multipair cable is crosstalk interference generated by the various systems within the cable. One of the earliest digital systems to address this problem was T1, which transmits a 1.544-Mb/s alternate mark inversion (AMI) bit stream over twisted pair. With two pairs the system will support 24 two-way voice channels, with each channel at 64 kb/s. At the time of its introduction the primary concern with respect to interference was near-end and far-end crosstalk generated by other T1 systems within the same cable, especially from those in the same binder group. This problem was minimized in part by placing opposite directions of transmission in physically separate binder groups, and whenever possible on opposite sides of the cable core.[1]

Today there is a general trend toward all-digital transmission systems. In this rapidly growing digital environment there is a great deal of digital traffic which circulates within a single building location. In this inside building environment the strategy of reducing crosstalk by separating directions of transmission is at best less feasible than it was when T1 was introduced and most likely it is unrealizable. Clearly a method of determining digital system compatibility is needed.

The focus of this paper is to define such a procedure. The motivation for this work was the interest in having PBX and network signals share common inside-building cabling behind a network interface. Signals entering a building from the outside plant would first terminate on a network interface located somewhere in the building. Beyond this point it was hoped that these network signals could circulate throughout the building in cables which would be shared with various PBX systems. The modeling described here was used to determine voltage limits which would ensure system compatibility. Since the time of this original work, the model described here has been successfully used to examine compatibility issues for a variety of different systems. As the primary interest of this work is intrabuilding transmission, all of the cable data used are obtained from measurements on cable intended for use inside a single location. By appropriately modifying the data, the techniques developed may be easily extended to other situations as well.

Objectives

The subject of crosstalk has been studied extensively in the past. Early work by G. A. Campbell has shown that the crosstalk loss of a specific pair combination could be predicted with a knowledge of the capacitance and inductance unbalance of the particular pairs.[2] Building upon this work and making use of Gaussian loss and noise distributions, H. Cravis and T. V. Crater developed a crosstalk analysis for the T1 carrier system.[1]

J. C. Isaacs, Jr., and N. A. Strakhov have extended crosstalk analysis by considering crosstalk in the framework of coupled transmission lines.[3] However, they restricted their work to a single disturber system and considered both transmission lines to be unbalanced and identical in nature. Their results were expressed in the frequency domain as a function of the primary constants of the line and one would have to account for the frequency dependence of these constants before extending their results to the time domain.

G. J. Foschini has developed a very elegant transmission line model for examining crosstalk.[4] However, he concentrated on balanced transmission and capacitance unbalance resulting in the prediction of crosstalk loss in the voice frequency range. No attempt was made to go beyond the prediction of crosstalk loss. Foschini's results were extended to unbalanced circuits by G. Miller who was able to show that crosstalk loss could be predicted by a quantity which is analogous to capacitance unbalance in metallic circuits.[5]

P. M. Lapsa has devised a multidisturber crosstalk analysis which was used for voice frequency applications.[6] The model also included provisions for examining situations where the disturbing and disturbed circuits were not simultaneously active. Building upon actual crosstalk measurements he has shown that the Gaussian description of crosstalk loss was overly restrictive. Improved accuracy was obtained with non-Gaussian distributions. This is attributed to the fact that the tail of loss distribution is of utmost importance and it is here where the Gaussian approximation fails. He concluded that a truncated Gaussian loss distribution was more appropriate.

These results concerning crosstalk loss distributions were further investigated by S. H. Lin.[7,8] He also concluded that a Gaussian loss distribution was overly restrictive. The reason was found to be the fact that in the region of the tail of the distribution, crosstalk performance was determined by a few dominant disturbers which did not exhibit Gaussian characteristics. He concluded that a truncated Gaussian could be a better fit; however, the best agreement with actual data occurred with a γ distribution.

Miller and T. C. Spang have examined the problem of determining voltage limitations which would allow different systems to share a common cable.[9] The primary emphasis was the protection of analog systems which are primarily sensitive to rms noise power. They only considered a single disturber system and did not extend their results to the protection of digital systems which may be more sensitive to peak noise excursions than to rms power.

The objective of this analysis is to define a procedure for determining peak transmission levels which will prevent crosstalk generated by one digital system from interfering with the performance of another. Previous crosstalk work has been largely concerned with the prediction of crosstalk loss, voice-frequency applications, or higher frequency analog systems. An early attempt to establish guidelines for digital system compatibility relied heavily on Gaussian statistics and a power sum analysis. Since this resulted in very restrictive transmission levels and it was unclear how to modify the procedure to accommodate bursty disturbers, it became necessary to reexamine the available crosstalk models and the assumptions on which they were based. (To characterize the bursty nature of the signals of interest we define duty cycle as the fraction of time a disturber is transmitting a nonzero voltage.)

In any given cable it is clear that the crosstalk path between any two wire pairs is dependent upon the nature of the insulating materials, the geometry of the wire pairs, and the electrical terminations. For any two pairs the amplitude and phase characteristic of the crosstalk loss will not only vary with frequency but will also be different for different pair combinations. Within one 25-pair binder group there are 300 completely different crosstalk paths which may be characterized by 300 different transfer functions. The situation is correspondingly worse for larger cables. However, one must realize that the quantity of interest is the peak noise which will result in a multidisturber environment and not the variability of the detailed shapes of all possible crosstalk pulses.

The desire of a tractable analysis for this situation leads to a consideration of models with the following property. All crosstalk noise pulses from a given disturbing signal will have the same shape and differ from one another only by a scaling factor which is dependent upon the particular choice of disturbing and disturbed pairs. Furthermore, the collection of scaling factors is dependent only upon the crosstalk loss distribution at the frequency of most interest. (This is usually taken to be a frequency near the center of the passband of the equalizer in the disturbed system under consideration.)

The crosstalk models in this paper are defined by five quantities. They are

1. A particular disturbed system
2. A particular disturbing signal
3. The number of disturbing wire pairs
4. A crosstalk transfer function
5. A distribution of near-end crosstalk loss at a frequency near the center of the equalizer passband in the disturbed system.

Only when these five quantities are known is a crosstalk model completely defined. In addition, the disturbed system is assumed to consist of a transmitting device, twisted pair transmission medium, an equalizer whose function is to remove some distortion arising from dispersion and attenuation introduced by the twisted pair, and lastly a receiver whose function is to detect incoming signals. In addition, it will be assumed that all disturbing transmissions are bursty and unsynchronized with respect to one another as well as with respect to the disturbed service. It must be emphasized that no claim or attempt is made to predict the variability of pulse shapes resulting from arbitrary choices of disturber and disturbed pair. The only quantity of interest is the peak noise which will be observed in a multidisturber environment. Our goal then is to determine as a function of the burstiness of the disturbing signal (the duty cycle) peak transmission limits which will prevent excessive harm to specific digital systems when these systems share a multipair cable.

Crosstalk Analysis

Crosstalk Transfer Function and Distribution of Scaling Factors. The previous section defined the quantities considered necessary to define a crosstalk model. Two of these are a transfer function, and a distribution of scaling factors to account for the variability in the amplitude of the pulses from different disturbing wire pairs. This section describes the process of creating the transfer function and scaling factors.

Previous attempts to predict crosstalk loss have largely been based upon a theory of coupled transmission lines. This invariably results in crosstalk being a function of the primary constants of the line. To use these results in the time domain, the frequency dependence of these constants must be known. In order to avoid this complication, the model developed here will be based upon actual crosstalk loss data measured at a variety of frequencies. We begin by assuming that a crosstalk data base derived from near-end crosstalk loss measurements is known. In particular, it is assumed that the mean loss is a continuous piecewise linear function and varies with the logarithm (base 10) of the frequency. This is very common with such data bases. With $\bar{L}$ denoting the mean loss in decibels and f the frequency in hertz, a typical formulation is

$$\bar{L} = A_i - B_i \log_{10}(f) \quad (1)$$

where $f_i \leq f \leq f_{i+1}$ and $i = 0, 1, \cdots, n-1$. Since existing data bases show that the standard deviation is a very weak function of frequency, the standard deviation used here is the value at f_0, the center frequency of the equalizer of the disturbed system.

Let x denote a normalized random variable (zero mean and unit variance) which describes the spread of the crosstalk loss about the mean. The loss is now expressible as

$$L(f) = \bar{L}(f) + x\sigma \quad (2)$$

A transfer function describing crosstalk coupling is obtained by assuming that it is defined by a constant percentile of the frequency-dependent cumulative loss distribution. Thus

$$-20 \log_{10} |H(f)| = \bar{L}(f) + x\sigma \quad (3)$$

where $H(f)$ is a transfer function associated with cable crosstalk loss. It should be noted that the above expression is exact at f_0 and should be a good approximation as long as the frequency range of interest is concentrated about the center frequency of the equalizer. Equation (3) may be equivalently expressed as

$$|H(f)| = 10^{-x\sigma/20}\, 10^{-\bar{L}(f)/20} \quad (4)$$

or

$$|H(s)| = 10^{-x\sigma/20} |H_0(s)| \quad (5)$$

where $H_0(s)$ is the transfer function defined by mean crosstalk loss. $H_0(s)$ will be the transfer function used to define the shape of the crosstalk noise pulses observed on the disturbed pair. The quantities $10^{-x\sigma/20}$ define the scaling factors describing the variability in pulse amplitudes and are dependent only upon the distribution of loss at f_0, the center frequency of the equalizer.

Since in this model only the magnitude of the transfer function is known, it is necessary to obtain information describing the phase characteristic before $H_0(s)$ can be used to compute pulse responses. The method used to accomplish this is to derive a complex function (analytic except for a branch cut on the nonpositive real axis) whose magnitude closely approximates the mean loss characteristic and use this complex function to completely determine the transfer function of the crosstalk path.

Let

$$w_i = 2\pi f_i \quad i = 1, 2, \cdots, n-1 \quad (6)$$

$$w_c = 2\pi f_c \quad (7)$$

where f_c is the frequency of zero mean loss, that is,

$$A_{n-1} - B_{n-1} \log_{10}(f_c) = 0 \quad (8)$$

Then the transfer function is approximated by the following complex function:

$$H_0(s) = Ks^{\gamma_0}\left(1 + \frac{s}{w_c}\right)^{\gamma_c}\prod_{i=1}^{n-1}\left(1 + \frac{s}{w_i}\right)^{\gamma_i} \qquad (9)$$

where

$$\gamma_c = -\sum_{i=0}^{n-1}\gamma_i \qquad (10)$$

and K is chosen to produce unity gain at $s = j\infty$. Equivalently,

$$H_0(s) = s^{\gamma_0}(s + w_c)^{\gamma_c}\prod_{i=1}^{n-1}(s + w_i)^{\gamma_i} \qquad (11)$$

With $H_0(s)$ in this form γ_i, $i = 0, 1, \cdots, n - 1$ are chosen so that the slope of $-20\log|H_0(s)|$ closely approximates the slopes of the piecewise linear mean loss, $\bar{L}(f)$, in each of the linear regions. The values used in conjunction with (1) are

$$\sum_{i=0}^{j}\gamma_i = B_j/20 \qquad j = 0, 1, \cdots, n - 1 \qquad (12)$$

where the B_j are defined by equation (1). The combination of transfer function and scaling factors derived in the above manner will be referred to as an analytical cable model. To summarize, the model consists of a transfer function $H_0(s)$ and a collection of scaling factors derived from a distribution of loss at a single frequency. Properties of the transfer function include

1. $\lim_{s->j\infty} |H_0(s)| = 1$; that is, no gain is predicted.
2. $H_0(s)$ defines a causal transfer function.
3. $-20\log_{10}|H_0(s)|$ has the same shape as the mean crosstalk loss.

Multidisturber Crosstalk Model. This section extends the analytical cable model developed in the previous section to encompass a multidisturber environment. Let $H_e(s)$ denote the transfer function of the receiver from the input to the eye of the detector on the longest loop considered in the analysis. Then the transfer function between the disturbing pair and the eye of the detector in the receiver becomes $10^{-x\sigma/20}H_0(s)H_e(s)$. The quantity $x\sigma$ is a measure of the difference between the loss incurred by an arbitrary disturber and the mean loss. For a disturber of loss L_i (defined at the center frequency of the equalizer) the transfer function corresponding to this particular loss becomes

$$H(s) = 10^{-(L_i-\bar{L})/20}H_0(s)H_e(s) \qquad (13)$$

In a multidisturber model let N denote the number of independent noise waveforms adding to yield an overall noise voltage. All disturbers are taken to be unsynchronized and result in unsynchronized noise pulses at the eye of the receiver. Let V_i $i = 1, 2, \cdots, N$ denote noise pulses [determined by $H_0(s)H_e(s)$], each emanating from a mean loss disturber which will be scaled to become the noise voltage from the ith disturber. Also let

$$\mathbf{V} = (V_1, V_2, \cdots, V_N) \qquad (14)$$

denote a vector of these noise voltages and

$$\mathbf{A} = \left(10^{-(L_1-\bar{L})/20}, 10^{-(L_2-\bar{L})/20}, \cdots, 10^{-(L_N-\bar{L})/20}\right) \qquad (15)$$

be a vector of scaling factors associated with a particular choice of N disturbing wire pairs. Then, with S denoting the allowable transmission voltage of the disturbing signals in order to achieve an overall error rate of ϵ (10^{-6} typically) and τ denoting the receiver detection threshold we have

$$P_E(S\mathbf{A}\cdot\mathbf{V},\tau)=\epsilon \tag{16}$$

where $P_E(\ )$ denotes the expression for the error rate of the disturbed system.

The value of S that satisfies (16) is, however, a function of $\mathbf{A}$ (the N particular disturbing pairs) and is therefore a random variable. The allowable transmission voltage, S_0, for the disturbing signals is chosen so that no more than β percent of the systems will experience error rates in excess of ϵ. The problem of finding S_0 may be stated as follows:

$$P_E(S\mathbf{A}\cdot\mathbf{V},\tau)=\epsilon$$

This implies

$$S=S(\mathbf{A},\epsilon,\tau) \tag{17}$$

Choose S_0 such that

$$\text{Prob}\,(S\leq S_0)=\int_R P(\mathbf{A})\,d\mathbf{A}=\beta/100 \tag{18}$$

where R is the region containing all vectors $\mathbf{A}$ which yield a value of S less than or equal to S_0, and $P(\mathbf{A})$ is the probability density of $\mathbf{A}$.

In practice the integral is not evaluated. A computer simulation is performed which

1. Chooses N loss values from an appropriate distribution and computes the vector $\mathbf{A}$.
2. Computes, given $\mathbf{A}$ and the independent noise voltages $\mathbf{V}$, the voltage distribution of $\mathbf{A}\cdot\mathbf{V}$. (This is accomplished by quantizing the distributions of noise from the N disturbers with the largest being quantized into 60 levels and convolving numerically.)
3. Determines, from the distribution of $\mathbf{A}\cdot\mathbf{V}$, the value of S (the allowable transmission voltage) to achieve an error rate of ϵ (typically 10^{-6}).
4. Repeats this process a large number of times (typically 5000)
5. Forms a distribution of S and determines the β percentile point of the computed transmission voltages.

This process is repeated for various values of duty cycle with the result being a graph of the β percentile of the transmission voltage distribution as a function of duty cycle.

Model Validation

The previous sections described the assumptions used in the derivation of the analytical cable model while this section explains the procedure used to validate this model. The objectives here are twofold:

- To examine the accuracy of an analysis relying upon the transfer function $H_0(s)$
- To investigate the importance of the type of distribution used to model near-end crosstalk (NEXT) loss, in particular, examine the usefulness of a Gaussian loss assumption for above-voice-frequency digital transmission.

This is achieved by comparing results predicted by the approach described in the previous sections with an analysis based upon a complete knowledge of the crosstalk behavior for one specific sample of cable. This test cable consists of one 25-pair binder group of an 880-foot section of type ABAM-100 cable. Pair-to-pair loss information was measured as a function of frequency for each of the 300 pair-to-pair combinations possible within the 25-pair binder group. All crosstalk measurements were made with the disturbing and disturbed pairs operating in a balanced mode. Both magnitude and phase information was measured. [Figure 1 shows the magnitude of the loss for the particular pairs (10,13).] Thus, for each combination of pairs a transfer function describing the crosstalk path linking these pairs is known precisely.

In this comparison the disturbing signals are taken to be bursts of widely spaced dipulses (Figure 2) and may

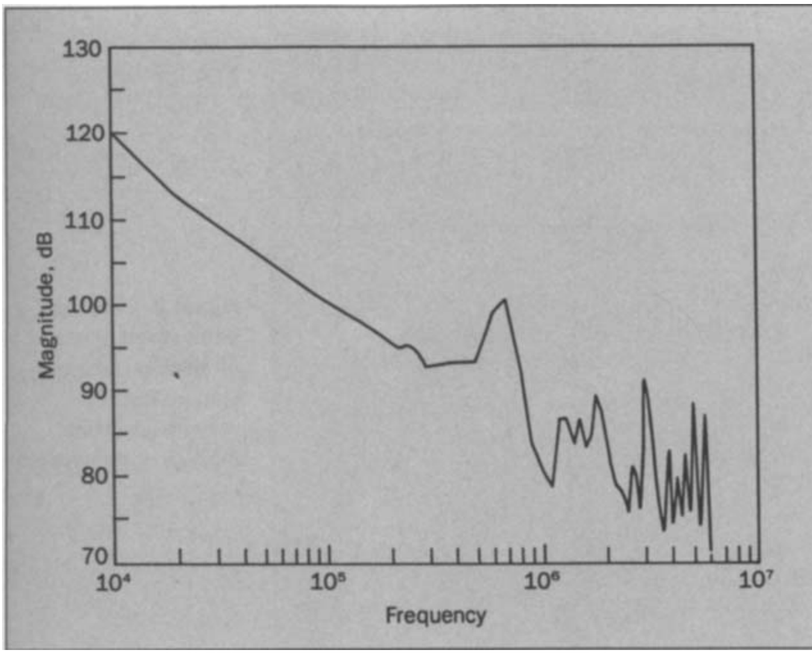

Figure 1. Measured crosstalk data on a 25-pair binder group, wire pairs 10 and 13.

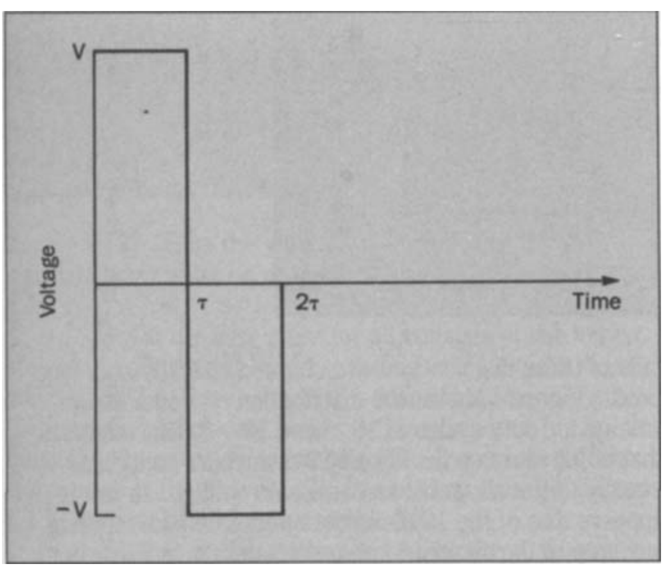

Figure 2. A single dipulse.

be thought of as modeling some of the signals appearing in certain electronic telephones. As previously described, the number of disturbers must be determined by the specific situation being examined. For the purpose of model validation it will be assumed that there are 12 unsynchronized bursty dipulse disturbers transmitting in the 25-pair binder group of the test cable. For disturbing signals from a two-wire system, this represents 12 disturbing signals being active while the disturbed system is attempting to detect incoming signals. One could equally well examine the 24-disturber case, however, this is felt to be unrealistically conservative as it will seldom be the case that all wire pairs in any given binder group are active at the same time. The 12-disturber assumption is felt to be a reasonably conservative assumption.

There will be two distinct disturbed systems used here. Both are actual systems which transmit an alternate mark inversion (AMI) bit stream. The first transmits at 144 kb/s and is assumed to operate on a 45-dB loop (which is the maximum operating range for this system) and the second transmits at 1.544 Mb/s and operates on a 12.5-dB loop. (The 12.5-dB loop was an initial estimate of a distance limitation resulting from NEXT of PBX-like signals. The resulting analysis indicated that a 12.5-dB loop was conservative and the loop length could actually be extended.) The use of two disturbed systems will allow a validation of the assumptions in the multidisturber crosstalk model over a broad range of frequencies.

The validation process will begin with the examination of the compatibility of the dipulse disturbers and the 144-kb/s system. Using the 300 measured transfer functions from the test cable, a computer simulation is created to determine acceptable transmission levels for the dipulse disturbers. This simulation is very similar to the one already described with the most significant differences

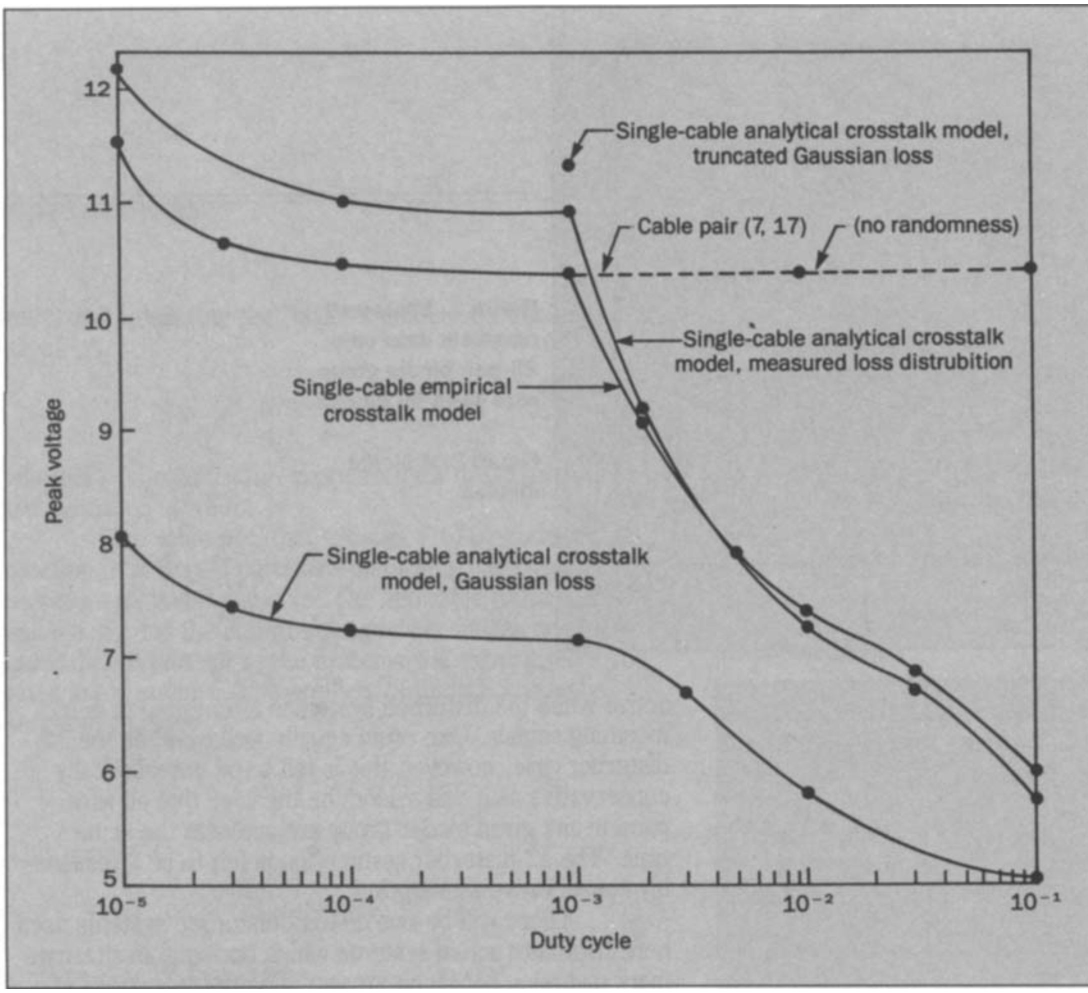

Figure 3. Plots of peak voltage versus allowable transmission voltage for various analytical models of crosstalk.

being that the noise from a given disturber is computed with the appropriate measured transfer function and the pulse width of each disturbing transmission is chosen to maximize the peak noise. (This simulation was repeated for the situation where all disturbing circuits transmitted dipulses of a common pulse width felt to be the most interfering, 4.8 μs. There were no significant differences observed in the results associated with these two approaches.) This model will be referred to as the single-cable empirical crosstalk model. Results of this simulation are shown in Figure 3, and will be used for comparison with transmission levels predicted by an analytical crosstalk model described later in this section.

A close examination of the distributions of the allowable transmission voltages associated with each value of duty cycle indicates that for duty cycles below 10^{-3} the tails of these distributions are characterized by step functions. (Figure 4 shows the distribution of transmission voltage for duty cycles of 10^{-1} and 10^{-3}.) This suggests that at low duty cycles not all 12 disturbers contribute equally to the allowable transmission voltage. In fact it appears that of the 12 disturbers under consideration at any step of the simulation there is a dominant disturber and only this strongest interferer is of importance in determining the transmission level.

Thus, for low duty cycles, the 12-disturber model reduces to a single-disturber model, with this single disturber being the worst offender of the 12 under consideration. This hypothesis is confirmed by a close examination of the simulation output. Each point on the allowable voltage distribution can be associated with a single disturbed pair, i, and 12 disturbing pairs, $\{j_k, k = 1,$

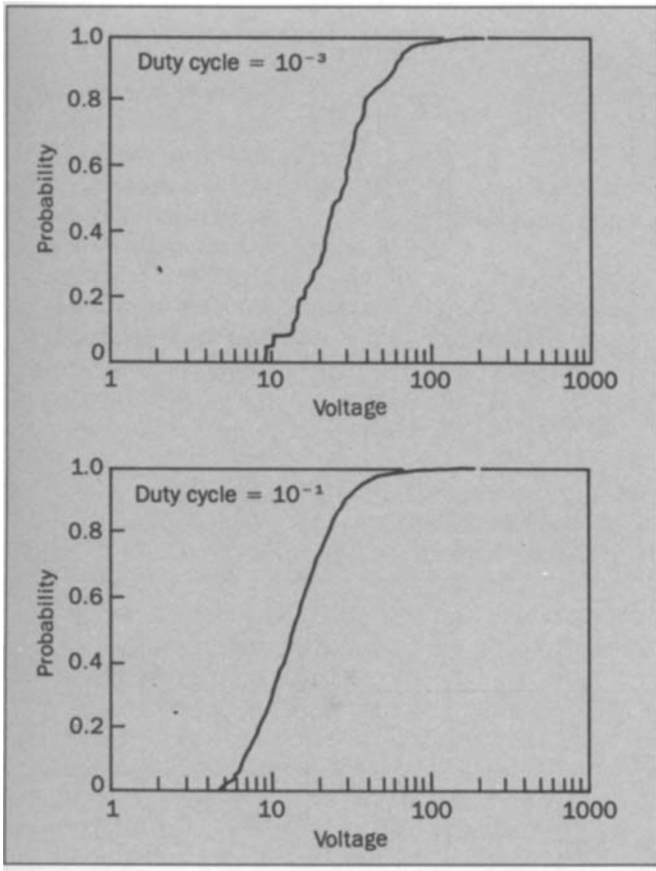

Figure 4. Distributions of transmission voltage for a single-cable empirical crosstalk model.

2, · · · , 12}. Thus the wire pairs responsible for a particular data point must be of the form (i, j_1), (i, j_2), · · · , (i, j_{12}).

When the wire pairs for all voltages of the lowest step of the distribution corresponding to a 10^{-3} duty cycle are identified in this manner, it is found that the only pair (i, j) common to all data points is the single pair (4, 5). Similarly, the only pair common to the second step of the distribution is the pair (7, 17). In addition, pairs (4, 5) and (7, 17) appear nowhere else in the data.

Further investigation shows that the peak noise voltage (at the eye of the receiver) resulting from the crosstalk transfer function of the pairs (4, 5) is the largest of all pairs in the cable while that associated with the pairs (7, 17) is the second largest. When the transmission voltage allowed by the single pair (7, 17) is plotted as a function of duty cycle it is seen [Figure 3, curve labeled cable pair (7, 17)] that at low duty cycles this curve coincides with the 5-percentile point of the allowable transmission voltage which was determined by using all the cable data. This supports the hypothesis that for low duty cycles the 12-disturber model degenerates into a single-disturber model.

Using the loss measurements from the 880-foot ABAM-100 cable, the next step in the validation process is to construct, with the techniques given under "Crosstalk Analysis," an analytical cable model describing the data. The cable transfer function is constructed by plotting the mean loss as a function of frequency and approximating the curve with a complex function (Figure 5*a*). This process has already been described and results in a transfer function (corresponding to mean loss) of the form

$$H_0(s) = s^{\gamma_0}(w_1 + s)^{\gamma_1}(w_c + s)^{-(\gamma_0 + \gamma_1)} \tag{19}$$

where

$$\gamma_0 = 1.067 \tag{20}$$
$$\gamma_1 = -0.279 \tag{21}$$
$$w_1 = 2\pi \times 150 \times 10^3 \tag{22}$$
$$w_c = 2\pi \times 7.88 \times 10^{10} \tag{23}$$

From this transfer function, the noise pulses induced into the 144-kb/s system resulting from a unit step and dipulse transmission are plotted in Figure 5*b* and Figure 5*c* respectively. (The width of the dipulse is chosen to maximize interference.) The computation of a noise distribution resulting from this dipulse disturber is examined in Panel 1, while the computation of an error rate for an AMI transmission is shown in Panel 2. The distribution of cable loss used in conjunction with these noise waveforms is that obtained from the detailed data when the loss is interpolated to 72 kHz, the center frequency of the equalizer. The distribution is assumed to be discrete and consists of 300 pair-pair loss measurements.

In the resulting simulation the selection of loss values (corresponding to a particular selection of 12 disturbing circuits) is not completely at random but care is taken to properly match a disturbed circuit with corresponding disturbers. The disturbed and disturbing pairs in

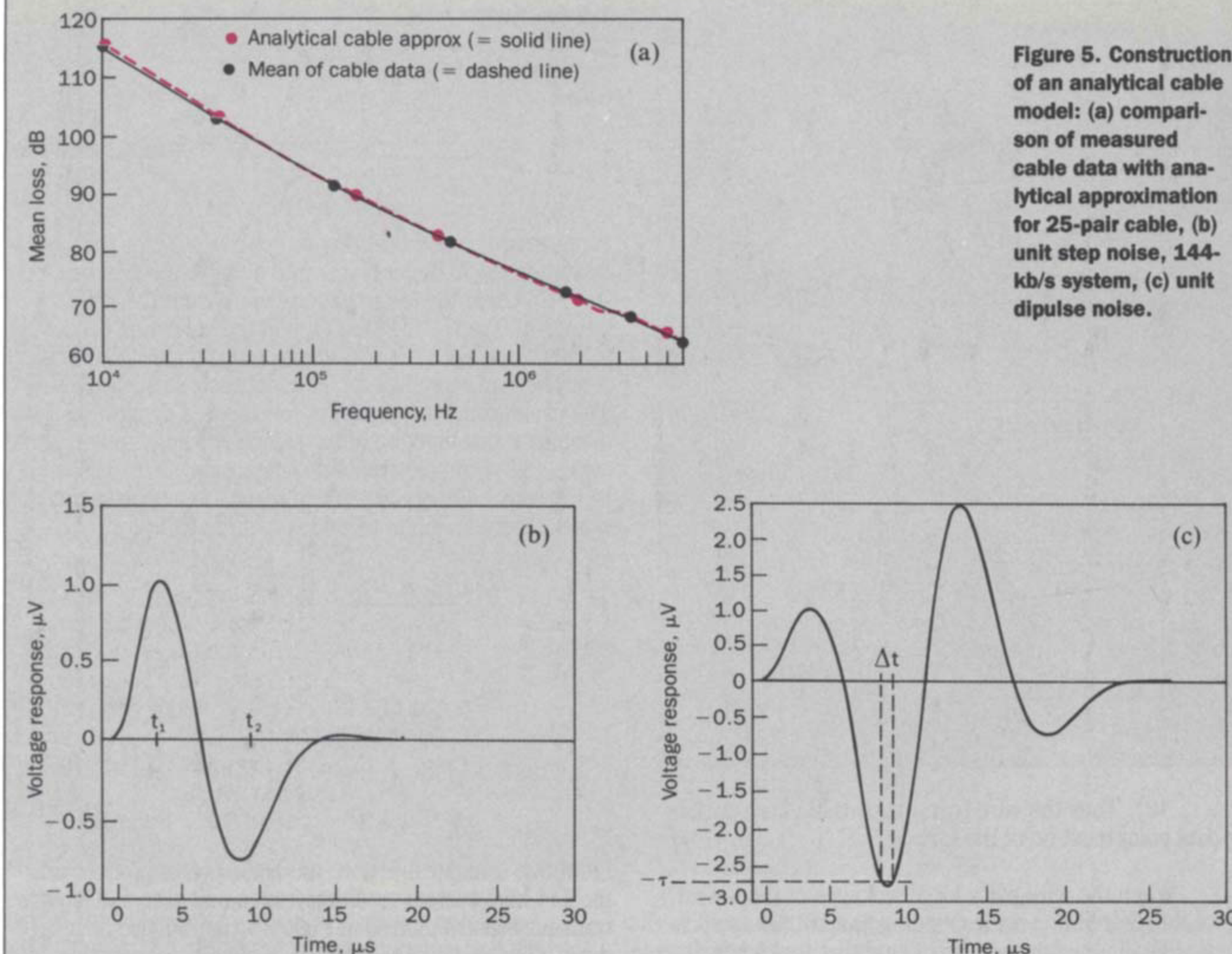

Figure 5. Construction of an analytical cable model: (a) comparison of measured cable data with analytical approximation for 25-pair cable, (b) unit step noise, 144-kb/s system, (c) unit dipulse noise.

the simulation are of the form $(i_0, j_1), (i_0, j_2), \cdots, (i_0, j_{12})$ with no repetition, and the loss values used are those corresponding to these pairs. [In this particular simulation the loss associated with the pair (4, 5) cannot be used simultaneously with the loss from (6, 7), although losses from (4, 5) and (4, 7) can be used simultaneously.] This model shall be called the single-cable analytical crosstalk model, measured loss distribution and the results of this analysis are shown in Figure 3. There is good agreement with those derived from the single-cable empirical crosstalk model, also shown in the figure.

By modifying this simulation so that loss slection is accomplished randomly (without replacement) the significance of the manner of pair selection is found. A comparison of the resulting distributions of peak transmission voltages associated with random loss selection with those distributions obtained from the more careful selection of disturber and disturbers shows good agreement.

The next step in the validation process consists of determining the importance of the form of the loss distribution used to find the scaling factors in the analytical cable model. In particular the effects of a Gaussian loss assumption shall be investigated. This is explored with a simulation incorporating a Gaussian loss distribution together with the analytical cable transfer function [equation (19)]. In this model, which is denoted in Figure 3 as the single-cable analytical crosstalk model, Gaussian loss, all disturbing pulses are identical in shape and differ only by a scaling factor which is derived from the Gaussian loss distribution. The standard deviation of this distribution is taken to be that of the 300 loss values at 72 kHz, 11.62 dBV.

As can be seen in Figure 3, agreement between this result and the single-cable empirical crosstalk model is poor. A detailed examination of the distribution of transmission voltages for a fixed duty cycle indicates that the distributions implied by Gaussian cable loss exhibit a much thicker tail (implying lower allowable transmission levels) than those derived from actual data.

To pinpoint the reason for this divergence of results the distributions of loss at 72 kHz can be plotted for both the observed loss distribution (300 pair-to-pair measurements) and the Gaussian approximation to the observed distribution. The Gaussian distribution approximating the actual data agrees fairly well with the actual data in the vicinity of the mean but exhibits a much longer tail. It should be noted that the actual loss varies between 68.2 dB and 139.4 dB while the Gaussian distribution is unbounded. It is this fact that the actual loss has a much shorter tail than the Gaussian that is responsible for the large discrepancy in the allowable transmission voltage.

The reasoning for this is as follows. For small values of duty cycle it has already been pointed out that the 12-disturber model degenerates into a single-disturber model whose loss is the worst of 12 loss values taken from an appropriate distribution. Although it is relatively unlikely for any single loss from the Gaussian distribution to exceed the limits set by the actual data, it is much more probable that the extreme loss of the 12 disturbers will exceed the range of observed loss values. These extreme loss values translate into very restrictive transmission voltages which skew the entire distribution of allowable voltages in a downward direction. It appears that the tail of the Gaussian distribution has an important effect upon the allowable voltages.

One would expect much better agreement between the Gaussian model and that based upon actual data if the Gaussian loss were truncated in some manner. This hypothesis is tested by a simulation employing the single cable analytical crosstalk model with a truncated Gaussian loss distribution. Since the extremes of the actual cable data are 2.49 standard deviations below the mean and 3.25 standard deviations above, the Gaussian loss is truncated at these values. During the simulation any loss values outside this range are simply discarded and new loss values are chosen.

When an examination of the resulting distribution of allowable transmission voltage associated with a 10^{-3} duty cycle is made, it becomes immediately apparent that the long tail on the distribution of allowable transmission levels is no longer present. In fact the 5-percentile point (Figure 3, point labeled single-cable analytical crosstalk model, truncated Gaussian loss) on this distribution (duty cycle $= 10^{-3}$) compares favorably with those obtained using either the model based upon the 300 pair-pair transfer functions or the model based upon the analytical cable model using the discrete distribution of loss.

One common effect observed in the previous models is the flatness of the transmission voltage curve below duty cycles of 10^{-3}. The following explanation is offered. As already pointed out, at low duty cycles the 12-disturber model degenerates into a single-disturber model. Consider this single disturbing pulse and a situation where the effect of varying the duty cycle is accounted for by choosing an appropriate period for this single pulse (Figure 5*c*). The detection instant in the disturbed system will occur at some unknown (random) instant during the interval [0, Per], where Per represents the time at the end of the period. (The noise transmissions are not synchronized with respect to the disturbed system.) Given this, the error rate is essentially the ratio of the time, Δt, the noise voltage exceeds the threshold τ to the total period and therefore is proportional to Δt/Per. With a 10^{-6} error rate it is quite easy to find a period (and hence a duty cycle) where all that is necessary to achieve this rate is for the tip of the noise voltage to exceed the threshold τ.

When the period doubles and the duty cycle is cut in half, the amount of time the curve is above the level τ

must double to maintain the same error rate. However, as the noise voltage is flat in the vicinity of its peak, it only requires a marginal increase in transmission voltage to accomplish this. Thus, one expects a broad range where the allowed transmission voltage is insensitive to duty cycle. At extremely low duty cycles approximately (10^{-6}) this effect no longer occurs and one expects the allowed transmission voltage to increase with decreasing duty cycle. This, in fact, is the case.

The last step in the validation process is to examine the utility of the analytical crosstalk model for high-bit-rate systems, in particular, the 1.544-Mb/s system. This is accomplished by comparing the acceptable dipulse transmission levels predicted by the single-cable analytical crosstalk model, measured loss distribution (loss measured at 800 kHz), with results obtained using the single-cable empirical crosstalk model. In each case the transmitted signal consists of a burst of square waves whose frequency is chosen to maximize interference. The results of the analysis are shown in Figure 6.

With the analytical crosstalk model, two methods of selecting disturbing circuits are used. The first [indicated in Figure 6 by analytical cable (random)] corresponds to a random choice of 12 loss values (without replacement and without regard to wire pair numbering) from the observed discrete distribution. In the curve denoted by Analytical cable (pairwise) the loss values are chosen by selecting a disturbed wire pair and 12 disturbing wire pairs. Thus, the loss values correspond to wire pairs of the form $(i, j_1), (i, j_2), \cdots, (i, j_{12})$.

Figure 6 also shows the results of an analysis based upon the single cable empirical crosstalk model (labeled as detailed cable) when each disturber is assumed to transmit bursts of square waves whose frequency coincides with that used in the analytical cable model. As can be seen, the manner of loss selection used with the analytical cable model is not critical and the approach based upon the analytical cable model is in good agreement with that based upon a detailed knowledge of magnitude and phase for each of the disturbing wire pairs (each disturber transmitting pulses of a common frequency).

The model comparisons contained in this section lead to the following conclusions. The agreement between the analytical crosstalk model, measured loss distribution, and the single-cable empirical crosstalk model based upon complete knowledge of all pair-pair transfer functions strongly suggests that the analytical crosstalk model, measured loss, is adequate and it is not necessary to retain phase information associated with pair-pair transfer functions, nor is it necessary to retain detailed knowledge associating loss values with particular wire pairs. This allows the possibility of using existing larger data bases of loss values (where detailed phase information is unavailable) when determining an appropriate crosstalk model. Furthermore, the assumption of a Gaussian loss distribution is extremely restrictive and inadequate. What is needed is a distribution which describes the extremes of the crosstalk loss more accurately than that predicted by a Gaussian assumption. This is consistent with the findings of Lapsa[6] and Lin.[7,8]

The model validation described here was achieved by applying the modeling techniques described under "Crosstalk Analysis" to a data base consisting of crosstalk measurements of one specific cable. In the next section a more general crosstalk model is obtained by working with a crosstalk data base encompassing several types of cable commonly used for inside building transmission. Instead of a specially developed extensive model for the crosstalk distribution (with particular emphasis on the extremes), a discrete distribution consisting of a large number of crosstalk measurements will be used.

Intrabuilding Crosstalk Model

The data used to define a crosstalk transfer function and a discrete distribution of crosstalk consists of pair-to-pair loss measurements from 25-pair binder groups of seven distinct cables. In total, there are 3576 loss values at each of the two frequencies measured, 150 kHz and 772 kHz. The cables used and the number of measurements taken from each are shown in Table I.

Interpolation may be used to obtain mean loss values at frequencies other than 150 kHz and 772 kHz where

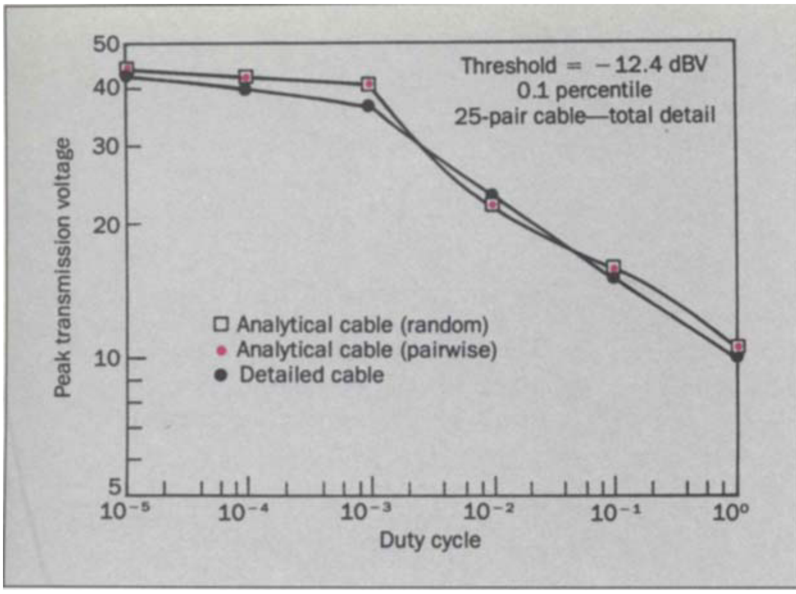

Figure 6. Allowable transmission voltage for repetitive dipulse noise, 1.544-Mb/s system.

it shall be assumed that the mean loss (in decibels) is piecewise linear and varies with the logarithm of the frequency. The slope of this curve above 150 kHz is determined from the mean loss of the two frequencies measured while the slope of the curve below 150 kHz is assumed to be identical with that obtained from the 880-foot cable previously described. This results in a cable transfer function of the form

$$H(s) = s^{\gamma_0}(s + w_1)^{\gamma_1}(s + w_c)^{-(\gamma_0+\gamma_1)} \tag{24}$$

where

$$\gamma_0 = 1.067 \tag{25}$$
$$\gamma_1 = -0.265 \tag{26}$$
$$w_1 = 2\pi \cdot 150 \cdot 10^3 \tag{27}$$
$$w_c = 2\pi \cdot 2.68 \cdot 10^{10} \tag{28}$$

This model is denoted as the seven-cable analytical crosstalk model, measured loss distribution and is felt to be appropriate for inside-building transmission where both the disturbing and disturbed systems are balanced.

The primary emphasis of this paper is to define a crosstalk model along with a model validation procedure. This has been accomplished. What follows now are examples of how this model might be used in actual applications. It must be emphasized that the derived voltage limitations are a function of the specific performance objectives chosen for these examples and will in general be different for different systems. In applying this model to the 144-kb/s and 1.544-Mb/s systems, a distribution of loss is required at the center frequencies of the equalizers. Since measurements are not available at 72 kHz, a distribution of loss is obtained by shifting the data by an amount indicated by the transfer function $H_0(s)$, equation (24).

The result is shown in Figure 7, where the seven-cable model is also compared to a Gaussian loss distribution predicted by certain existing data bases. As can be seen, agreement is relatively good except at the left-hand tail. As expected, the Gaussian distribution is seen to fall off more slowly than the actual data.

The performance objective for the 144-kb/s system is that no more than 5 percent of the systems operating on a 45-dB loop shall experience error rates in excess of 10^{-6}. The crosstalk transfer function and distribution of crosstalk are those derived in this section. The detection threshold of the receiver is taken to be −3 dBV and includes some equalizer margin as well as margin for unknowns. A sample calculation is shown below. (It must be pointed out that the performance objective and detection threshold used here are only for the purpose of illustrating the crosstalk analysis techniques. Other values may be used for a more conservative analysis.)

Table I. Cables Used to Determine Crosstalk Distribution

Cable	Number of 25-pair binder groups measured	Number of loss values per frequency
1. ABAM-100	2	600
2. ABAM-100	2	600
3. ABAM-100	1	300
4. ARTM-1800	2	600
5. ARTM-1800	2	600
6. ARAM-200	2	576
7. ARAM-200	1	300

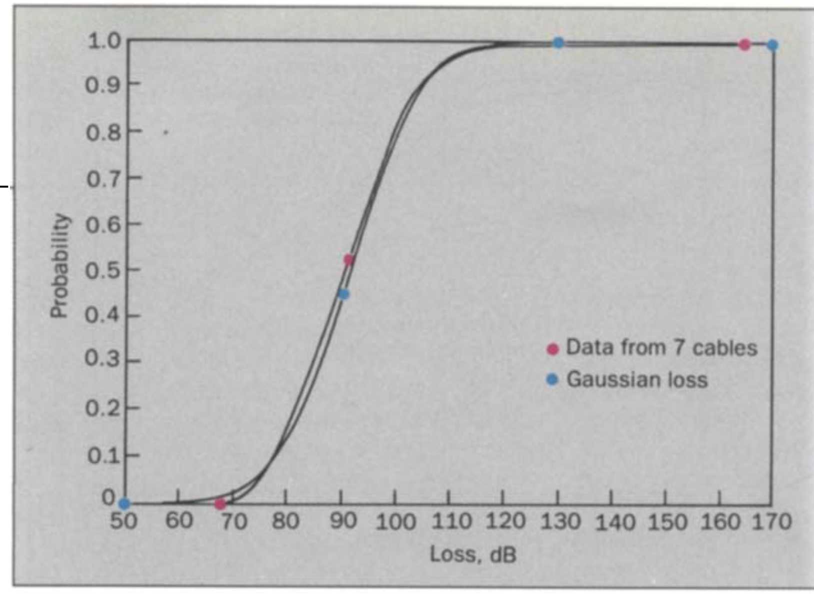

Figure 7. Cumulative distribution of loss, 25-pair cable, at a frequency of 72 kHz.

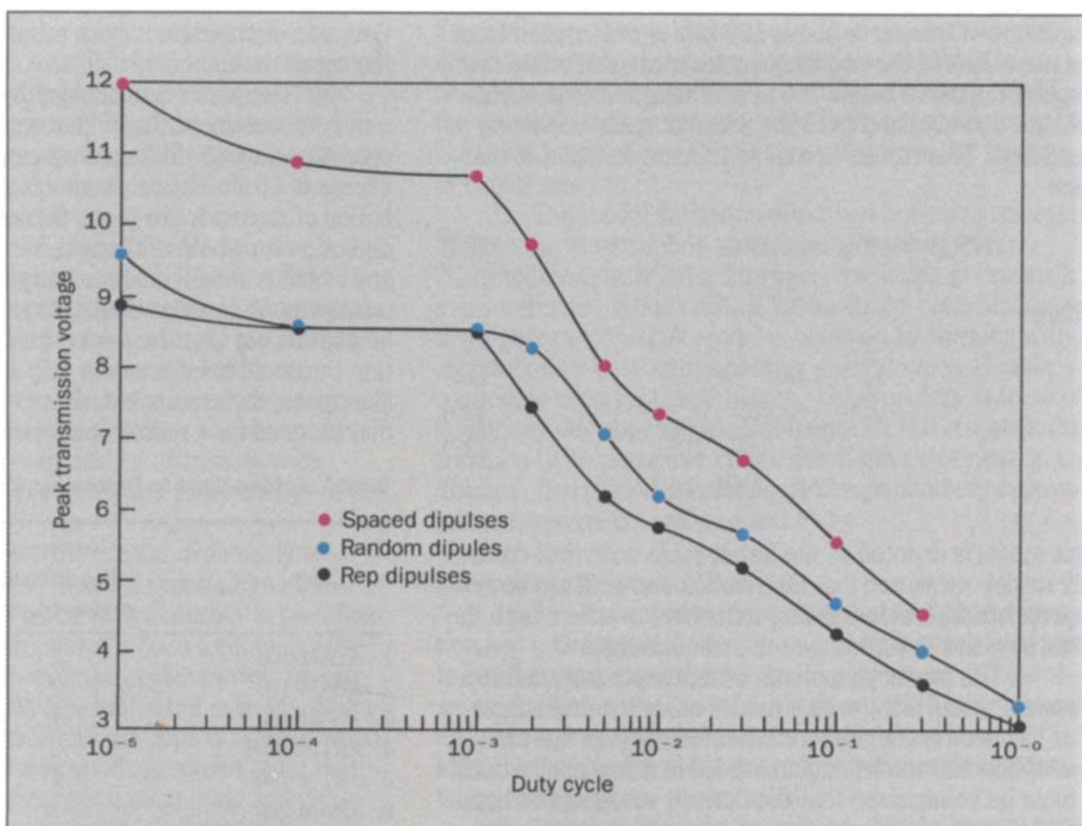

Figure 8. Allowal transmission vol for the seven-cal analytical model, measured loss di bution. The 5 percentile thresh –3 dBV.

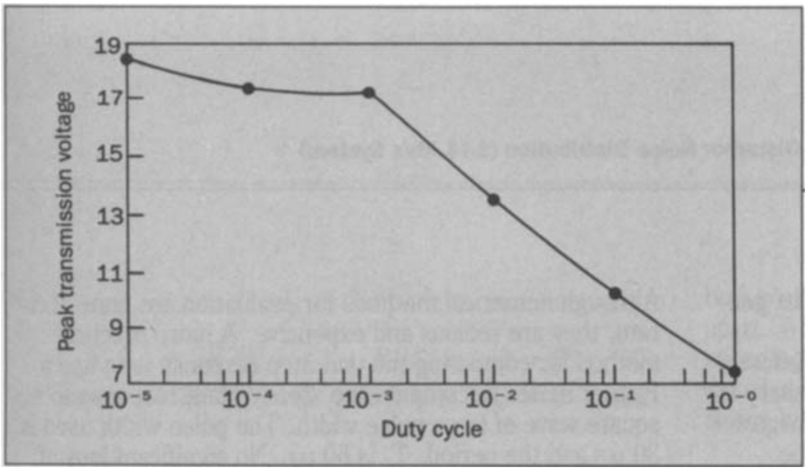

Figure 9. Allowable transmission voltage for 1.544-Mb/s-system performance objectives.

7.5	dBV	peak signal
−6.	dB	decision point adjustment (half of peak)
−1.5	dB	equalizer margin
−3.	dB	unknowns
−3.	dBV	detection threshold

As before, there are 12 unsynchronized bursty dipulse disturbers. Assuming the disturbers are widely spaced dipulse transmissions results in the peak dipulse transmission levels shown in Figure 8. Repeating this analysis under the assumption that the disturbing signal consists of a burst of square waves tuned to the most damaging frequency results in an allowable transmission voltage curve (labeled rep dipulses). Finally, assuming the transmissions to be bursts of dipulses corresponding to a random data sequence where there is no spacing between the individual pulses within a burst yields the allowable transmission voltage (labeled random dipulses).

In the 1.544-Mb/s system, the performance objective is to have no more than 0.1 percent of the systems operating on a 12.5-dB loop experience an error rate in excess of 10^{-6}. There are 12 unsynchronized disturbers consisting of bursts of square waves whose frequency is chosen to maximize interference. The detection threshold within the receiver is taken to be −12.4 dBV. For the assumption that the crosstalk is described by the seven-cable analytical crosstalk model, measured loss distribution, the acceptable dipulse transmission levels are shown in Figure 9.

Summary

This paper has presented a new crosstalk model specifically designed to examine the situation of digital systems sharing a common multipair cable. This approach relies heavily upon a crosstalk transfer function which is used to determine the crosstalk waveform appearing in a disturbed system and generated by another system within the cable. The assumption of a Gaussian distribution of crosstalk loss is shown to be restrictive and is abandoned in favor of empirical distributions which more carefully model the extremes of the crosstalk loss. This model is designed to examine bursty transmissions which are fairly common in many digital systems in use today.

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The first type of crosstalk to consider is that generated by a sequence of widely spaced dipulse transmissions (Figure 2) from several disturbers. Before actually computing the noise distribution, it is necessary to determine the pulse width, τ, which produces the largest peak noise voltage. Since a unit dipulse, $M(t)$, can be expressed in terms of unit step functions

$$M(t) = U(t) - 2U(t-\tau) + U(t-2\tau), \quad (29)$$

its crosstalk response can be obtained in terms of the response of a unit step function. Letting $H_e(s)$ denote the transfer function of the equalizer of the 144-kb/s system for a 45-dB loop (the longest loop considered in this analysis), the transform of the unit step response, $U(s)$, (with respect to mean loss) becomes

$$U(s) = \frac{H_0(s)H_e(s)}{s} \quad (30)$$

and the time waveform becomes

$$R(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} s^{(\gamma_0 - 1)} (s + w_1)^{\gamma_1} (s + w_2)^{\gamma_2} (s + w_c)^{-\gamma_0 - \gamma_1 - \gamma_2} H_e(s) e^{st} \, ds \quad (31)$$

For $\gamma_0 > 0$ the integrand may have a singularity but not a pole at $s = 0$. As a result the integral reduces to a Fourier integral by taking $\sigma = 0$.

$$R(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} (jw)^{\gamma_0 - 1} (jw + w_1)^{\gamma_1} (jw + w_2)^{\gamma_2} (jw + w_c)^{-\gamma_0 - \gamma_1 - \gamma_2} H_e(jw) e^{jwt} \, dw \quad (32)$$

Although numerical methods for evaluation are quite accurate, they are tedious and expensive. A more practical method for computing the unit step response is to use a Fourier series to compute the steady-state response to a square wave of large pulse width. The pulse width used is 30 μs and the period, T, is 60 μs. No significant loss of accuracy results by using a pulse 30 μs wide as the time response decays to zero long before 30 μs. The pulse response becomes

$$P(t) = c_0 H_0(0) H_p(0) + 2 \cdot \text{REAL} \sum_1^{\infty} c_n H_0\left(\frac{n2\pi}{T}\right) H_e\left(j\frac{n2\pi}{T}\right) e^{j2\pi nt/T} \quad (33)$$

where $\{c_n\}$ are the Fourier coefficients of the square wave input. By truncating the series at an appropriate number of terms, the above expression may be computed very easily using Fast Fourier Transform (FFT) techniques. A comparison of the above two approaches indicates that there is excellent numerical agreement between the two.

Let $M(t, \tau)$ denote the crosstalk noise resulting from a single dipulse transmission. Then

$$M(t,\tau) = R(t) - 2R(t - \tau) + R(t - 2\tau) \quad (34)$$

Choosing τ to maximize this expression reduces to finding values of t_0, τ_0 which maximize a function of two variables. Thus it is necessary to find t_0, τ_0 such that

$$\frac{\partial M}{\partial t}(t_0, \tau_0) = 0 \quad (35)$$

$$\frac{\partial M}{\partial \tau}(t_0, \tau_0) = 0 \quad (36)$$

or

$$R'(t_0) - 2R'(t_0 - \tau_0) + R'(t_0 - 2\tau_0) = 0 \quad (37)$$
$$2R'(t_0 - \tau_0) - 2R'(t_0 - 2\tau_0) = 0 \quad (38)$$

Intuitively, one would like to choose τ so that the shifted unit step response $-R\,(t_0 - \tau_0)$ has its first peak (at t_1) coinciding with the second peak (at t_2) of the unshifted waveform $R(t)$ (Figure 5). That is,

$$t_0 = t_2 \quad (39)$$
$$\tau_0 = t_2 - t_1 \quad (40)$$

In this case equations (37) and (38) both reduce to

$$R'(2t_1 - t_2) = 0 \quad (41)$$

However, when $t < 0$, $R(t) = 0$ and $R'(t) = 0$. Thus, the above choices for t_0 and τ_0 are optimal whenever

$$t_2 > 2t_1 \quad (42)$$

When this is not the case a simple gradient technique is used to optimize the function $M(t,\tau)$. [The unit step response is approximated with a cubic spline fit to the points obtained from an FFT. This spline is used to compute the gradient of $M(t,\tau)$. Once τ_0 is obtained, the worst-case dipulse response is easily computed.]

From the computed unit step response, the noise resulting from a unit dipulse applied to a mean loss disturber is computed for the time interval $[0,\ 30 \times 10^{-6}]$. Beyond 30 μs this response is essentially zero. The cumulative distribution $F(x) = \text{Prob}(M(t_i,\tau_0) \leq x)$ (where t_i is uniformly distributed in the interval $[0,\ 30 \times 10^{-6}]$) is easily obtained by first solving the equation $M(t_i,\tau_0) = x$ and then determining all intervals such that $M(t_i,\tau_0) \leq x$. The total length of these intervals divided by 30×10^{-6} yields the number $F(x)$ and corresponds to a voltage distribution of a sequence of identical pulses spaced 30 μs apart. When it is desired to obtain a distribution corresponding to pulses spaced L μs ($L > 30$) apart the above distribution must be modified. For this situation the distribution becomes

$$F_L(x) = \frac{30}{L}F(x) + \left(1 - \frac{30}{L}\right)U(x) \quad (43)$$

where $U(x)$ is the unit step function.

As duty cycle is related to L by

$$DTC = \frac{2\tau}{L} \quad (44)$$

with 2τ being the dipulse width (in microseconds), the cumulative distribution associated with an arbitrary value of duty cycle is

$$F_{DTC}(x) = \frac{15DTC}{\tau}F(x) + \left(1 - \frac{15DTC}{\tau}\right)U(x) \quad (45)$$

A second method can be used to eliminate the necessity of solving the equation $M(t_i,\ \tau_0) = x$. A FFT involving 512 points is used to compute the crosstalk noise. These 512 points are then sorted in ascending order with the kth point representing the voltage V such that

$$P(M(t_i,\ \tau_0) \leq V) = k/512 \quad (46)$$

The cumulative distribution is then known at 512 points and linear interpolation is used to determine the distribution at other values. Agreement between the two methods is good.

The above procedures for computing the cumulative noise distribution are also applied to the situation where the disturbing circuit is assumed to transmit bursts of square waves at the frequency that is most harmful to the system under consideration.

The procedure for determining the noise distribution for a circuit transmitting a random sequence of dipulses is as follows. Let *M(t)* denote the crosstalk response of a single isolated dipulse. The voltage at an arbitrary instant of time, *t*, in the random sequence of transmissions then becomes

$$r(t) = \sum_{k=0}^{\infty} \eta_k \cdot M(t + kP) \qquad (47)$$

where P = period of the dipulse transmissions and η_K is a random variable with

$$\text{Prob}(\eta_K = 1) = 0.5 \qquad (48)$$

$$\text{Prob}(\eta_K = -1) = 0.5 \qquad (49)$$

As *M(t)* decays very rapidly, the above expression is truncated after a few terms with no significant loss of accuracy. Thus, for computational purposes

$$r(t) = \sum_{k=0}^{n} \eta_K \cdot M(t + kP) \qquad (50)$$

It is now easy to compute the distribution of $r(t_i)$ where t_i is uniformly distributed in the interval [0, Per], Per being the time at which the period ends.

When $\eta_k = 1$, $k = 0, 1, \cdots, n$ or $\eta_k = -1$, $k = 0, 1, \cdots, n$, it is seen that the noise voltage reduces to that resulting from square wave transmissions. Thus the square wave response must be included among the possible noise waveforms observed in the interval [0, Per]. Since the square wave response produces the largest peak voltage of all waveforms observed in [0, Per], choosing P to maximize the peak noise results in selecting P as the period of the square wave most harmful to the system under consideration.

Panel 2: Error Rate Calculation for Alternate Bipolar Transmission

Let

P_{+1}	=	probability of a positive voltage being transmitted in the disturbed system
P_{-1}	=	probability of a negative voltage being transmitted in the disturbed system
P_0	=	probability of zero voltage being transmitted in the disturbed system
N	=	noise voltage from all bursty disturbers
τ	=	detection threshold

The probability of error, P_E, becomes

$$P_E(N,\tau) = P_{+1}P(N < -\tau) + P_0[P(N > \tau) + P(N < -\tau)] + P_{-1}P(N > \tau)$$

This analysis shall assume that $P_0 = 1/2$, $P_{+1} = P_{-1} = 1/4$, whereby the above expression reduces to

$$P_E(N, \tau) = (3/4)[P(N > \tau) + P(N < -\tau)]$$

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