CENTER-TAP TRACKING ALGORITHMS FOR TIMING RECOVERY

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Fractionally spaced adaptive equalizers can readily compensate for timing-phase variations between the transmitter and receiver clocks that are on the order of several symbol intervals. When there is a frequency difference between these clocks, the equalizer compensates for the accumulated timing-phase difference by shifting the equalizer tap weights in the appropriate direction along the delay line. Performance, as measured by the mean-squared error, is not degraded until the center (largest) tap migrates toward one end of the equalizer.

In this paper, we demonstrate how the spatial distribution of the equalizer coefficients can be used to lock the receiver sampling frequency to that of the transmitter. Replacing the standard envelope-derived timing recovery system with a closed-loop center-tap tracking algorithm offers significantly improved performance for narrow roll-off or severely-attenuated systems, reduced complexity, and a structure appropriate for all-digital receivers.

Introduction

Fractionally spaced equalizers (FSE) have been considered for application to voice-grade data transmission.[1,2] Such equalizers are tapped delay line structures whose tap spacing is less than the reciprocal of twice the bandwidth of the baseband data signal. In contrast to the conventional synchronous equalizer, in which taps are spaced a symbol interval apart, sufficiently long FSEs can provide significantly improved performance in the presence of severe delay distortion.

Furthermore, such an FSE can synthesize any fractional delay within its time span, and consequently is insensitive to the choice of receiver timing (sampling) *phase*—assuming that the transmitter and receiver have essentially the same timing *frequency*. In practice, these frequencies typically differ by several parts in 10^5, and it is necessary to provide a means of tracking and eliminating this difference.

Because this frequency difference is so small, significant timing drift only occurs after many symbol intervals have elapsed. Under these circumstances, the timing recovery problem can be viewed as one of adjusting the timing phase. In this paper, we will describe an alternative to the conventional envelope-derived timing recovery technique.[3] The envelope method is known to be plagued by considerable timing jitter, either when the channel bandwidth is fully used by a narrow roll-off system or when the channel is severely attenuated at the band edge.

The purpose of this paper is to describe how the receiver's timing frequency can be locked to that of the transmitter in a closed-loop manner by observing the distribution of the equalizer tap weights. The timing phase is adjusted so as to maintain the dominant tap weights in the center of the equalizer. Several such "center-tap tracking" techniques will be proposed and discussed. Related prior proposals for center-tap timing-frequency tracking have been made by Ungerboeck[1] and Forney.[4]

Three requirements in the design of a closed-loop timing algorithm are:

- Decoupling the timing-frequency tracking algorithm from any equalizer tap rotation (which provides partial carrier-phase tracking).[5]
- Ensuring that any tap drift is controlled by a suitable tap-leakage algorithm, because of the random nature of optimum equalizer tap settings in an FSE.[6]
- Ensuring that the timing recovery method and tap-leakage algorithm drive the system toward the same equilibrium state.

These requirements ensure that, apart from differences in the transmitter and receiver clocks, a stable, steady-state distribution of tap weights exists. Hence, equalizer tap weights are subject to continuous adjustment by the conventional LMS tap-adjustment algorithm,[2] the tap-leakage algorithm,[6] and the center-tap tracking timing-recovery algorithm. It is somewhat remarkable that these coupled algorithms can all function cooperatively to provide an extremely powerful adaptive equalizer that performs an essential part of the timing recovery function, as well as providing effective channel compensation.

It is interesting to note that a sufficiently long fractionally spaced equalizer can adaptively realize the optimum linear receiver,[2] and consequently can provide carrier-phase compensation via the tap rotation property of bandpass equalization,[5] as well as compensation for any timing-phase error (up to some fraction of the time span of the equalizer). However, because of the conflicting bandwidth requirements between the equalizer, timing and carrier-tracking adaptation loops, the carrier synchronization functions are best handled respectively by data-directed carrier-tracking techniques[7] and a center-tap timing-frequency tracking algorithm of the sort described in this paper.

QAM Systems

This section describes the background material in quadrature-amplitude-modulated (QAM) data communication systems employing fractionally spaced equalization needed to develop the timing-frequency tracking algorithm described later.

QAM Data Transmission. High-speed [≥4.8 kilobits per second (kb/s)] QAM voice-grade modems are generally configured as in Figure 1. The transmitted signal is given by

$$s(t) = Re\sum_{n} d_n p(t - nT)\, e^{jw_c t} \qquad (1)$$

where $d_n = a_n + jb_n$ is a complex data signal whose real (in-phase) and imaginary (quadrature) components assume discrete multilevel values, $p(\cdot)$ is the transmitter pulse shape, T^{-1} is the symbol rate, and w_c is the radian carrier frequency.

The received signal at the bandpass filter output is expressed compactly as

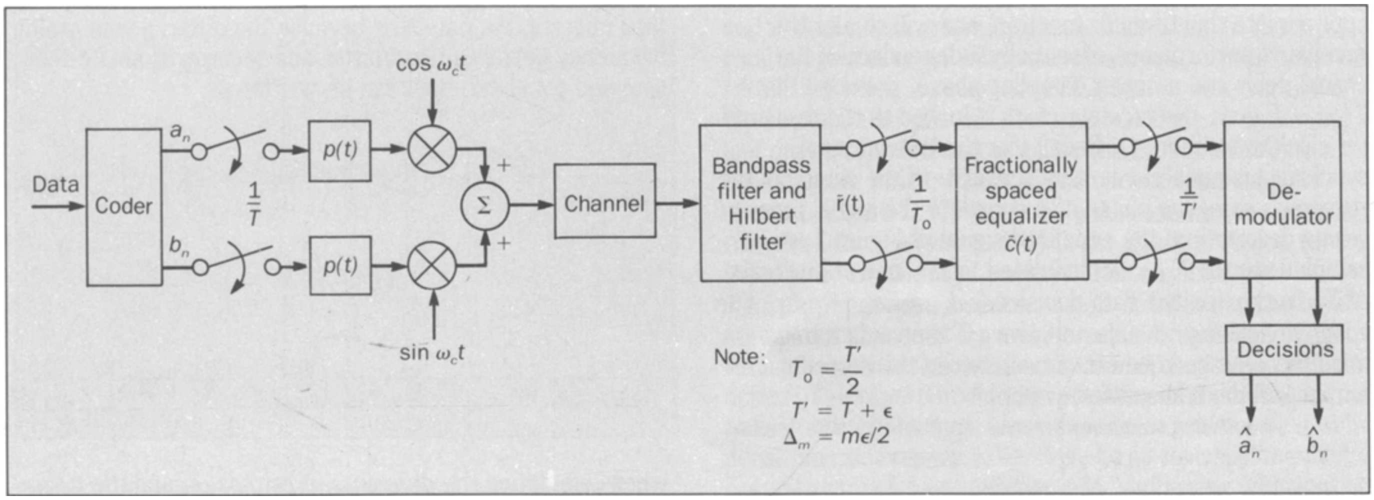

Figure 1. In-phase and quadrature data transmission system.

$$\tilde{r}(t) = \sum_n d_n \tilde{f}(t - nT - \tau)e^{j(w_c t + \phi)} + \tilde{v}(t) \qquad (2)$$

where the real and imaginary parts of $\tilde{r}(t)$ denote the in-phase and quadrature (Hilbert-transformed) received signals, $\tilde{f}(t)$ is the received baseband-equivalent pulse,[2] τ is the time delay in the channel, ϕ is the carrier phase shift (or jitter), and $\tilde{v}(t)$ is additive noise of variance N_0. If the baseband transmitter pulse, $p(t)$, has no energy beyond $(1 + \alpha) / 2T$ Hz, then to compensate perfectly for delay distortion,[2] the tap spacing of the FSE, T_0, must be such that $T_0 < T / (1 + \alpha)$. After sampling $\tilde{r}(t)$ at the instants mT_0, the signal is passed through an adaptive mean-squared fractionally spaced equalizer which, if infinitely long, realizes the baseband-equivalent transfer function[2]

$$\tilde{C}_B(w) = \frac{\tilde{F}^*(w)e^{jw\tau}}{\frac{1}{T}\sum_k \left| F\left(w + k\frac{2\pi}{T}\right)\right|^2 + N_0}, \quad |w| \leq \frac{\pi}{T_0} \qquad (3a)$$

where the analytic, or passband, equalizer transfer function is $\tilde{C}_B(w - w_c)$. We denote the equalizer tap weights by $\tilde{c}_n$ where we recall the Fourier relations

$$\tilde{C}_B(w) = T_0 \sum_n \tilde{c}_n e^{-jwnT_0}, \ |w| \leq \frac{\pi}{T_0}$$

$$\tilde{c}_n = \int_{-\pi/T_0}^{\pi/T_0} \tilde{C}_B(w)e^{jwnT_0}\frac{dw}{2\pi} \qquad (3b)$$

Henceforth we will, for convenience, drop the subscript on $\tilde{C}_B(w)$. (This means that we have implicitly interchanged the demodulator and equalizer.) This interchange has no effect on the timing problem under consideration, and, without loss of any generality, we further specialize to the case $T_0 = T' / 2$, where T' is the data symbol interval as seen at the receiver. Also note that at $t = mT'$, the $m/2^{th}$ data symbol is entering the equalizer, and when $T' = T$, synchronization between transmitter and receiver has been achieved.

Note that the FSE compensates exactly for the fixed channel delay, τ, as well as the delay distortion in the channel. Practical finite-length equalizers will closely approximate this transfer function, and will render the receiver's performance essentially independent of the channl delay and receiver sampling phase, provided that $T' = T$ (i.e., the receiver clock is locked to the transmitter clock in frequency). Recall that the conventional synchronous equalizer is very sensitive to the choice of the receiver's sampling phase.[2] To complete the signal processing description, the equalized signal of Figure 1 is sampled at rate T_0^{-1}, demodulated, and sliced at intervals of T' to provide the data decisions, $\hat{d}_n = \hat{a}_n + j\hat{b}_n$. The adaptive equalizer is adjusted every T' seconds to minimize the average squared error between the demodulator output and the transmitted symbol.[2]

The Timing Frequency Problem. In practice, the transmitter and receiver clocks will differ somewhat, and, if nothing else were done, the equalizer would simply track the accumulated delay by building the compensating delay (or advance). As shown in Figure 2, this compensation would be manifest by a slow drift of the equalizer dominant taps toward one end of the equalizer (during which time performance would be initially satisfactory but gradually degrading) until, at some later point in time, the dominant taps would "fall off" the equalizer. Performance as measured by bit error rate or mean-squared error would by then be unacceptable.

That is, even though the FSE can compensate for any *fixed* timing phase (small compared with the time span of the equalizer), it is necessary for the receiver to track the transmitter timing frequency. The fact that this tracking can be done by observing the migration of the equalizer center tap provides an alternative for the conventional envelope-derived timing recovery technique.[3] This is desirable for several reasons. First, the circuitry for constructing the envelope can be eliminated. Second, the envelope technique produces a timing wave form whose jitter increases significantly as the amount of excess bandwidth is decreased to the 12-percent level commonly used for 9.6-kb/s data transmission.[3] Third, the center-tap tracking algorithms may be readily applied to all-digital receivers.

To quantify the manner in which the equalizer taps migrate, we note that because the difference in timing frequency between transmitter and receiver is small, the sampled equalizer input can be written as

$$\tilde{r}\left(\frac{mT'}{2}\right) = \sum_n d_n \tilde{f}\left(\frac{mT'}{2} - nT\right) = \sum_n d_n \tilde{f}\left(\frac{mT}{2} - nT + \frac{m\epsilon}{2}\right) = \sum_n d_n \tilde{f}\left(\frac{mT}{2} - nT + \Delta_m\right) \quad (4)$$

where we define the *difference in periods*, ϵ, and the *accumulated timing-phase error*, Δ_m, via

$$T' = T + \epsilon$$
$$\Delta_m = m\epsilon/2 \quad (5)$$

If the elapsed time since the equalizer was first adjusted is such that the accumulated phase difference, Δ_m, is a small fraction of a symbol interval, then Δ_m can be approximated by the constant, Δ. Thus, in these circumstances the timing-frequency problem has been recast as a timing-phase problem. Note that the equalizer will continuously adapt to compensate for the accumulated timing phase, so that the equalizer transfer function at $t = mT'/2$ is given by

$$C_m(w) = C_0(w)e^{jw\Delta} \quad (6)$$

where $C_0(w)$ is the steady-state (or optimum) transfer function when the equalizer is first adjusted. The goal of the timing-frequency tracking algorithm is to estimate Δ, while it remains a small fraction of a symbol interval, and once Δ is estimated, the frequency of the receiver clock will be adjusted in the direction to coincide with that of the transmitter. Use of the equalizer output is precluded in estimating Δ, because the equalizer has presumably com-

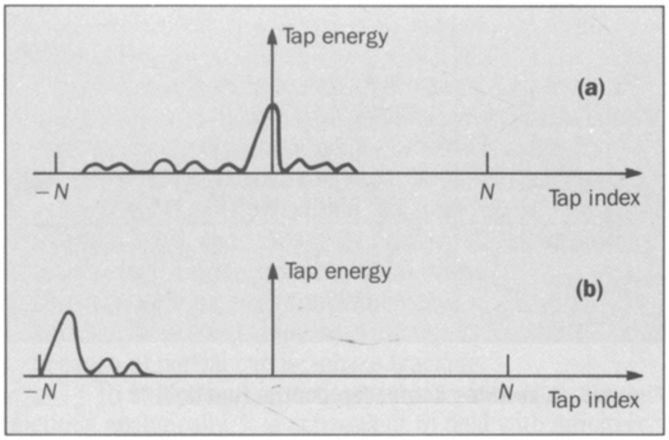

Figure 2. Sum-of-squares of tap weights when receiver clock is slightly faster than transmitter clock; (a) shortly after steady state is reached, (b) after some time has elapsed.

pensated for this small timing phase shift and performance is unaffected. Next we will discuss using equalizer coefficients for providing an estimate of Δ.

Center-Tap Tracking Algorithm

To see how the equalizer tap weights can be used to estimate Δ, we note from Equations (3) and (6) that at $t = mT'/2$

$$C_m(w) = C_0(w)e^{jw\Delta} = \frac{\tilde{F}(w)e^{jw\tau}e^{j\psi}e^{jw\Delta}}{\frac{1}{T}\sum_k \left|\tilde{F}\left(w + k\frac{2\pi}{T}\right)\right|^2 + N_0}$$

$$= \frac{\tilde{G}^*(w)e^{j\psi}e^{jw\Delta}}{\frac{1}{T}\sum_k \left|\tilde{G}\left(w + k\frac{2\pi}{T}\right)\right|^2 + N_0} \qquad (7)$$

where the pulse $\tilde{G}(w)$ incorporates the channel delay, τ, and ψ denotes an arbitrary phase shift owing to the tap-rotation property[5] of bandpass equalizers. At this point we recall that when N_0 is small, FSEs have a wide variety of tap settings that provide performance close to optimum,[6] and digital roundoff errors cause the taps to drift over a considerable range. Under these uncontrolled circumstances, Equation (7) would only approximate the equalizer transfer function (particularly in the spectral region where there is no transmitted energy). The tap-leakage algorithm[6] provides a simple means of inhibiting the building up of extraneous tap weights; thus by using the tap-leakage algorithm, Equation (7) can be used with confidence. The time-domain interpretation of the above is that *without* the tap-leakage algorithm, there will not be a well-defined center tap, and relatively large tap weights will occur at various locations along the delay line. This sort of quasi-random tap drifting would preclude use of the center tap for tracking purposes.

The key ideas in center-tap tracking algorithms are:

1. Construct a control function, like that shown in Figure 3, of the equalizer tap weights, say $D(\underline{c}_n;\Delta)$, that indicates the accumulated timing-phase error, Δ_n at the n[th] sampling instant.
2. Use this control function to adjust the receiver sampling phase to minimize the accumulated timing phase difference between transmitter and receiver clocks.

The desired properties are that over a reasonable fraction of a symbol interval, $D(\cdot)$ be a monotonic increasing function of Δ, and $D(\underline{c};\Delta) = 0$ for Δ equal to or near zero. Once a suitable function $D(\underline{c};\Delta)$ is selected, the *receiver sampling phase*, t_n, at the A/D converter becomes

$$t_n = \frac{nT'}{2} + \delta_n \qquad (8a)$$

where the *accumulated phase difference*, Δn, between transmitter and receiver clocks is

$$\Delta_n = t_n - \frac{nT}{2} = \delta_n + \frac{n}{2}(T' - T) = \delta_n + \frac{n}{2}\epsilon \qquad (8b)$$

The timing phase, δ_n, is controlled by the first-order control algorithm:

$$\delta_{n+1} = \delta_n - \beta D(\underline{c}_n;\Delta_n),\ n = 0,1,2,\ldots \tag{9a}$$

Or from Equation (8b):

$$\Delta_{n+1} = \Delta_n + \frac{\epsilon}{2} - \beta D(\underline{c}_n;\Delta_n) \tag{9b}$$

(A second-order algorithm would eliminate any fixed steady-state timing-phase error. However, the *a priori* timing-frequency error, ϵ, is so small that the second-order loop is probably not necessary in most applications.) In Equation (9a), β is a constant, called the step size, and is chosen small enough so that Δ_n will converge to a fixed value; it is clear that when Equation (9b) converges, then Δ_n can settle only at a value, Δ, such that $D(\underline{c}_n;\Delta) = \epsilon / 2\beta$. To establish local convergence, we assume that a two-term Taylor series is reasonaby accurate over several, say L, symbol intervals, i.e.,

$$D(\Delta) \approx D(0) + \dot{D}(0)\Delta\ ,\ |\Delta| \leqslant LT \tag{10}$$

and thus (9b) becomes

$$\Delta_{n+1} = \left[1 - \beta\dot{D}(0)\right]\Delta_n + \frac{\epsilon}{2} - \beta D(0),\quad n = 1,2,3,\ldots \tag{11a}$$

As long as $|1 - \beta\dot{D}(0)| < 1$, it is clear that (11a) converges to the steady-state value

$$\Delta_{opt} = \frac{\frac{\epsilon}{2\beta} - D(0)}{\dot{D}(0)} \tag{11b}$$

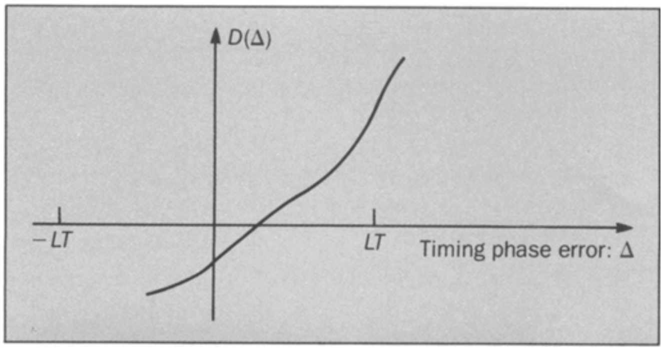

Figure 3. A suitable center-tap control function.

Thus the steady-state timing offset has two components: one because of the *a priori* difference in transmitter and receiver timing frequencies, and the other because of the bias in the control function (i.e., $D(0) \neq 0$). The size of the latter component is dependent on the particular control function, $D(\Delta)$, selected. The former component could be eliminated by use of a second-order control algorithm, but the magnitude of the error is generally negligible. [If $T^{-1} = 2400$ and the clocks are accurate to within 50 parts in a million, then $(T')^{-1} = 2400\,(1 \pm 50 \times 10^{-6})$, and $\epsilon = T - T' \approx 2.08 \times 10^{-8}$.]

It is important to realize that the tap weights will generally be continually adapted, perhaps to track a slow drift in the channel characteristics, and thus the control function cannot simply attempt to restore the tap weights to some fixed set of values. Rather, the control function must sense some property of the tap weights that reflects the mismatch of the transmitter and receiver clocks, and which also allows the tap weights to compensate continuously for channel variations.

There are various functions $D(\Delta)$ that might be considered for controlling the timing offset, Δ. In this discussion, we assume that $|T' - T|$ is quite small and that the frequency error can be conveniently treated as a timing-phase error.

Timing Error Functions. There are several considerations governing the choice of $D(\underset{\sim}{c};\Delta)$, and we list these below:

1. $D(\underset{\sim}{c};\Delta)$ should be a monotone function in Δ over a small number of symbol intervals. Because an FSE can compensate for timing-phase errors of several symbol's duration, it is only necessary that the algorithm converge to a time instant within this interval.
2. $D(\underset{\sim}{c};\Delta) = 0$ when $\Delta = \Delta_{opt}$, where Δ_{opt} is no more than a small number of symbol intervals.
3. $D(\underset{\sim}{c};\Delta)$ should be easily implementable.
4. $D(\underset{\sim}{c};\Delta)$ should be independent of any tap rotation because of partial carrier-phase tracking.

To study the properties of the various control functions analytically, it is convenient to deal with infinitely long equalizers, and we use Equation (3a) freely in the sequel.

Spectral measurement functions: frequency domain. J. E. Mazo has noted that because the equalizer transfer function, in the nonexcess bandwidth region (i.e., where signal energy is present) initially has the form

$$C_0(w) = e^{j\psi_0} / G(w) \qquad (12)$$

and after some time is given by

$$C_m(w) = \frac{e^{j\psi_m}}{G(w)} e^{jw\Delta} \qquad (13)$$

it is possible to estimate Δ by examining $C_m(w) / C_0(w)$ at two frequencies. Consider,

$$D_1(\Delta) = \operatorname{Im}\left[\frac{C_m\left(\frac{\pi}{4T}\right)C_m^*(0)}{C_0\left(\frac{\pi}{4T}\right)C_0^*(0)}\right] = \sin\frac{\pi\Delta}{4T} \qquad (14)$$

which is a sinusoidal function of the error Δ. Note that properties (1), (2), and (4) relating to timing error functions and discussed above are satisfied, and that $D_1(\Delta)$ can be written from Equations (3b) and (14) in terms of rather simple operations on the current and *stored* tap weights.

Median type of functions: time domain. Generally, when the equalizer is first adjusted, there will be a small number of dominant (i.e., largest in magnitude) tap weights, and the remaining tap weights will be significantly smaller. One approach to maintaining this equilibrium is to ask for the sampling phase that equates the tap energy to the left and right of the center tap. Toward this end we define the function,

$$D_2(\Delta) = \sum_{n>0} |\tilde{c}_n|^2 - \sum_{n<0} |\tilde{c}_n|^2 = \sum_{n\geq 0} |\tilde{c}_n|^2 - \sum_{n\leq 0} |\tilde{c}_n|^2 \qquad (15)$$

which measures the asymmetry of the tap-magnitude distribution. In order to investigate the properties of Equation (15), we note the following identities[8] involving "one-sided" summations:

$$\sum_{n=1}^{\infty} e^{jwnT} = \frac{j}{2T}\cot\frac{wT}{2} - \frac{1}{2} + \frac{\pi}{T}\sum_{n=1}^{\infty}\delta\left(w - n\frac{2\pi}{T}\right) \qquad (16a)$$

$$\sum_{n=1}^{\infty} e^{jwnT} - \sum_{n=-\infty}^{-1} e^{jwnT} = \frac{j}{T}\cot\frac{wT}{2} \qquad (16b)$$

From Equation (7) we know that the equalizer transfer function is given by

$$C(w) = \tilde{H}(w)e^{j\psi}e^{jw\Delta} \qquad (17)$$

where

$$\tilde{H}(w) \equiv \frac{\tilde{G}^*(w)}{\frac{1}{T}\sum_k \left|G\left(w + k\frac{2\pi}{T}\right)\right|^2 + N_0} \qquad (18)$$

Thus from Equations (15) and (16b), we have

$$D_2(\Delta) = \left[\sum_{n>0} - \sum_{n<0}\right] \cdot \iint_{-2\pi/T}^{2\pi/T} C(w)C^*(v)e^{jwn\frac{T}{2}}\, e^{-jvn\frac{T}{2}}\, \frac{dw}{2\pi}\frac{dv}{2\pi}$$

$$= \frac{j}{T/2}\iint_{-2\pi/T}^{2\pi/T} \cot\frac{(w-v)\,T}{4}$$

$$\times \tilde{H}(w)\tilde{H}^*(v)\, e^{j(w-v)\Delta}\, \frac{dw}{2\pi}\frac{dv}{2\pi} \qquad (19)$$

Note that if $\tilde{H}(w)$ is *real* then

$$D_2(\Delta) = \frac{4}{T}\iint_{-2\pi/T}^{2\pi/T} \cot\frac{(w-v)T'}{2}\,|\,\tilde{H}(w)\,| \cdot |\,\tilde{H}^*(v)\,|\,\sin(w-v)\,\Delta\, \frac{dw}{2\pi}\frac{dv}{2\pi} \qquad (20)$$

which is a monotonic function of Δ that goes through zero when $\Delta = 0$. This is not surprising, because when the channel has no phase distortion and the amplitude distortion is symmetric about the carrier, it is well known that the taps are such that $\tilde{c}_n = \tilde{c}_{-n}$, and an asymmetric tap distribution (relative to $\tilde{c}_n = \tilde{c}_{-n}$) will be an accurate measure of the accumulated timing delay. To consider the realistic case of a complex channel distortion, we let

$$\tilde{H}(w) = |\,\tilde{H}(w)\,|\, e^{j\theta(w)} \qquad (21)$$

then

$$D_2(\Delta) = \frac{4}{T}\iint_{-2\pi/T}^{2\pi/T} \cot\frac{(w-v)T'}{2} \cdot |\,\tilde{H}(w)\,|$$

$$\times\, |\tilde{H}(v)| \cdot \sin\Big(\theta(w) - \theta(v) + (w-v)\Delta\Big)\frac{dw}{2\pi}\frac{dv}{2\pi} \qquad (22)$$

and we observe that the zero crossing of the function $D_2(\Delta)$ will be shifted away from $\Delta = 0$. This can be demonstrated by expanding $D_2(\Delta)$ in a Taylor series, for small Δ, and noting that $D_2(0) \neq 0$. To within the approximation afforded by the first two terms in the Taylor series, we have that the zero-crossing of $D_2(\Delta)$ is

$$\Delta_{opt} = \frac{D_2(0)}{\dot{D}_2(0)} = \frac{j\iint_{-2\pi/T}^{2\pi/T} \cot\frac{(w-v)\,T'}{2}\, \tilde{H}(w)\tilde{H}^*(v)\,\frac{dw}{2\pi}\frac{dv}{2\pi}}{\iint_{-2\pi/T}^{2\pi/T} (w-v)\cot\frac{(w-v)\,T'}{2}\, \tilde{H}(w)\tilde{H}^*(v)\,\frac{dw}{2\pi}\frac{dv}{2\pi}} \qquad (23)$$

As we have previously noted, all that is important in stabilizing the receiver timing is that Δ_{opt} be well within the span of the equalizer. We will demonstrate that this is indeed the case by noting that, to a first approximation, the cotangent function behaves like the negative derivative of an impulse, i.e.,

$$\cot x \approx -\dot{\delta}(x) \qquad (24a)$$

As suggested by G. J. Foschini, to improve this approximation, we would have to introduce a parameter that "speeds up" the decay of the cotangent near the origin. We recall that

$$\int_{-\infty}^{\infty} f(x)\dot{\delta}(x-a)\,dx = -\dot{f}(a) \qquad (24b)$$

With the above approximation for cotangent we see that

$$D_2(\Delta) \sim j\int_{-2\pi/T}^{2\pi/T} \tilde{H}^*(v)\,\frac{dv}{2\pi} \cdot \int_{-2\pi/T}^{2\pi/T} \tilde{H}(w)\dot{\delta}(w-v)\, e^{j(w-v)\Delta}\,\frac{dw}{2\pi}$$

$$\sim j\int_{-2\pi/T}^{2\pi/T} \tilde{H}^*(v)[\dot{\tilde{H}}(v) + j\Delta\tilde{H}(v)]\,\frac{dv}{2\pi} \qquad (25)$$

$$= j \int_{-2\pi/T}^{2\pi/T} \tilde{H}^*(v)\, \dot{\tilde{H}}(v) \frac{dv}{2\pi} - \Delta \int_{-2\pi/T}^{2\pi/T} |\tilde{H}(v)|^2 \frac{dv}{2\pi} \tag{26}$$

Note that if $\tilde{H}(w)$ is real, then for a pulse with less than 100-percent excess bandwidth the first term in (26) is zero, because

$$\int_{-2\pi/T}^{2\pi/T} \tilde{H}(v)\dot{\tilde{H}}(v) \frac{dv}{2\pi} = \int_{-2\pi/T}^{2\pi/T} \frac{d}{dv} \cdot \tilde{H}^2(v) dv = \tilde{H}^2 \left[\frac{2\pi}{T}\right] - \tilde{H}^2 \left[\frac{-2\pi}{T}\right] = 0 \tag{27}$$

(Because the excess bandwidth is less than 100 percent, $H[2\pi / T] \equiv 0$.) Consequently, Equation (26) has the solution $\Delta_{opt} = 0$. When there is delay distortion, the first term in (26) provides an offset, because $D_2(\Delta) = 0$, when $\Delta = \Delta_{opt}$ where

$$\Delta_{opt} = \frac{\mathrm{Im} \int_{-2\pi/T}^{2\pi/T} \tilde{H}^*(v)\dot{\tilde{H}}(v) \frac{dv}{2\pi}}{\int_{-2\pi/T}^{2\pi/T} |\tilde{H}(v)|^2 \frac{dv}{2\pi}} = \frac{\int_{-2\pi/T}^{2\pi/T} |\tilde{H}(v)|^2 \dot{\theta}(v) \frac{dv}{2\pi}}{\int_{-2\pi/T}^{2\pi/T} |\tilde{H}(v)|^2 \frac{dv}{2\pi}} \tag{28}$$

Thus the bias, Δ_{opt}, can be interpreted as a weighted delay distortion, where the delay distortion is weighted by the channel magnitude. To estimate Δ_{opt}, we note that the maximum channel delay distortion, $\dot{\theta}_{max}$, is generally limited to several symbol intervals. Thus, from Equation (28), we see that $\Delta_{opt} \leq \dot{\theta}_{max}$, and the bias is generally confined to several symbol intervals. We again emphasize that as long as Δ_{opt} is well within the span of the equalizer, a root-seeking algorithm, of the type described by Equation (9), will converge to a satisfactory value.

If one wishes to eliminate the bias (i.e., the fact that $\Delta_{opt} \neq 0$), then it is necessary to form the difference of the control function at two time instants. (If the bias is too large, then the ability of a finite-length equalizer to compensate for the channel distortion can become impaired.)

If we use $D_2(0)$ to denote the value of D_2 when the equalizer is first adjusted (i.e., when $\Delta = 0$), then consider the difference.

$$D_3(\Delta) = D_2(\Delta) - D_2(0) = \frac{j}{T/2} \iint_{-2\pi/T}^{2\pi/T} \cot \frac{(w - v)T'}{4} \tilde{H}(w) \tilde{H}^*(v) [e^{j(w-v)\Delta} - 1] \frac{dw}{2\pi} \frac{dv}{2\pi} \tag{29}$$

where we have used Equation (19). Expanding $D_3(\Delta)$ in a Taylor series for small Δ, we see that

$$D_3\Delta \approx -\frac{2\Delta}{T} \iint_{-2\pi/T}^{2\pi/T} (w - v) \cot \frac{(w - v)T'}{4} \tilde{H}(w) \tilde{H}^*(v) \times \frac{dw}{2\pi} \frac{dv}{2\pi} \tag{30}$$

and the double integral is easily shown by symmetry arguments to be a real function. Thus, for small Δ, $D_3(\Delta)$ is the desired zero-intercept linear function and $D_{opt} = 0$. However, as we have mentioned, it may be neither possible nor desirable to store the original set of tap weights. Moreover, the bias will typically be less than a small number of symbol intervals, as indicated by Equation (28).

A Practical Control Function. For practical applications with finite-length equalizers and where implementation complexity is an important factor, it is necessary to reduce the range of the summations in Equation (15); typically, we would be interested in using as few as two terms, and constructing the control function

$$D_4(\Delta) = |c_0|^2 - |c_1|^2 \qquad (31)$$

which, in the frequency domain, is given by

$$D_4(\Delta) = \frac{j}{T/2} \iint_{-2\pi/T}^{2\pi/T} \tilde{H}(w)\tilde{H}^*(v) \left[1 - e^{j(w-v)\frac{T}{2}}\right] e^{j(w-v)\Delta} \frac{dw}{2\pi}\frac{dv}{2\pi} \qquad (32)$$

which will again be biased whenever there is delay distortion. However, as with the previous function, if we construct

$$\begin{aligned} D_5(\Delta) &= D_4(\Delta) - D_4(0) \\ &= \frac{j}{T/2} \iint_{-2\pi/T}^{2\pi/T} \tilde{H}(w)\tilde{H}^*(v) \left[1 - e^{j(w-v)\frac{T}{2}}\right] \\ &\times \left[e^{j(w-v)\Delta} - 1\right] \frac{dw}{2\pi}\frac{dv}{2\pi} \end{aligned} \qquad (33)$$

then, expanding $D_5(\Delta)$ in a Taylor series about $\Delta = 0$ gives the unbiased function

$$\begin{aligned} D_5(\Delta) &= D_4(0) + \Delta\dot{D}_4(0) - D_4(0) \\ &= \Delta \frac{4}{T} \iint_{-2\pi/T}^{2\pi/T} (w - v)\, \tilde{H}(w)\tilde{H}^*(v) \\ &\times \left[1 - e^{j(w-v)\frac{T}{2}}\right] \frac{dw}{2\pi}\frac{dv}{2\pi} \\ &= \Delta\frac{4}{T} \iint_{-2\pi/T}^{2\pi/T} (w - v)\, \tilde{H}(w)\tilde{H}^*(v) \\ &\times e^{j(w-v)\frac{T}{4}} \sin(w - v)\, \frac{T}{4}\, \frac{dw}{2\pi}\frac{dv}{2\pi} \end{aligned} \qquad (34)$$

To summarize, we have demonstrated several suitable control functions for the timing-recovery loop. We have also indicated that to eliminate the bias or offset, it is necessary to store (or freeze) the tap weights achieved when the equalizer has first converged. This is undesirable (a) because of increased complexity and (b) because it does not permit the possibility that the equalizer might have to track variations in the channel characteristics. It is reassuring to know that the bias is generally less than several symbol intervals, and, consequently, quite acceptable.

Required Tap-Weight Precision

In this section, we will indicate the tap-weight precision in the computation of $D(\Delta)$ required to keep the timing-phase jitter to a fraction of a symbol interval. The results reported in previous sections presume that the equalizer coefficients possess infinite precision, and that this precision is used to compute the various timing control functions.

Modeling Tap-Weight Quantization. When the tap weights are quantized to a given number of bits, the receiver timing phase is adjusted according to

$$\delta_{n+1} = \delta_n - \beta\,[D(\Delta_n) + Q_n] \qquad (35)$$

where Q_n represents the fluctuation in the control function because of tap weight quantization. For example, if we denote the quantized tap weights by $[c_m]$, the quantization error by γ_m then

$$c_m = [c_m] + \gamma_m \qquad (36)$$

and the quantized control function $D_2(\cdot)$ is given by

$$\begin{aligned} D_2([c_m], \Delta) &= \sum_{m>0}^{N} |\tilde{c}_m + \tilde{\gamma}_m|^2 - \sum_{m<0}^{N} |\tilde{c}_m + \tilde{\gamma}_m|^2 \\ &= \sum_{m=-N}^{N} (sgn\, m)\cdot\left[|\tilde{c}_m|^2 + 2\mathrm{Re}\,\tilde{\gamma}_m\tilde{c}_m^* + |\tilde{\gamma}_m|^2\right] \end{aligned}$$

$$= D_2(\Delta) + Q \tag{37}$$

where

$$Q = \sum_{m=-N}^{N} (sgn\ m) \cdot \left[2\text{Re}\ \tilde{\gamma}_m\ \tilde{c}_m^* + |\tilde{\gamma}_m|^2 \right] \tag{38}$$

and a control function, D_2, with the arguments $[c_m]$ and Δ denotes a function with quantized tap weights.

An exact evaluation of Equations (35) through (38) to determine the timing jitter is quite formidable. We will make some rough approximations to estimate the additional timing jitter because of limited tap-weight precision.

Worst-Case Timing Offset. Here we specialize to the control function $D_5(\Delta)$, thus

$$\begin{aligned} D_5([c_m], \Delta) &= D_5(\Delta) + 2\text{Re}\left\{ \tilde{c}_1\ \tilde{\gamma}_1 + \tilde{c}_0\ \tilde{\gamma}_0 \right\} \\ &\quad + |\tilde{\gamma}_0|^2 + |\tilde{\gamma}_1|^2 \\ &\leq D_5(\Delta) + 4|\tilde{\gamma}| \cdot |\tilde{c}|_{max} + 2\gamma^2 \end{aligned} \tag{39}$$

We have assumed a Δ such that D_5 is positive. In Equation (39), γ denotes the maximum tap-weight quantization error and $|\tilde{c}|_{max}$ is the maximum tap value.

Note that $\gamma^2 << |\tilde{\gamma}|\ |\tilde{c}|_{max}$, and thus

$$D_5([c_m], \Delta) \leq D_5(0) + \Delta\dot{D}_5(0) + 8|\tilde{\gamma}|\ |\tilde{c}|_{max} \tag{40}$$

Consequently, because $D_5(0)$ is zero, the peak timing jitter is, from Equation (40), approximately

$$\Delta_{pk} = \frac{8 \cdot |\tilde{\gamma}| \cdot |\tilde{c}_{max}|}{\dot{D}_5(0)} \tag{41}$$

The peak quantization error is given by

$$|\tilde{\gamma}| = \Gamma 2^{-B} \tag{42}$$

where B is the number of bits and Γ is the full-scale tap range. To avoid overflow, typically the ratio of Γ^2 to the received signal power is on the order of four. We also note that $|\tilde{c}_{max}|^2$ is on the order of one over the received signal power. With unity received power, the above considerations imply that Equation (41) reduces to

$$\Delta_{pk} = \frac{32 \cdot 2^{-B}}{\dot{D}_5(0)} \tag{43}$$

In order to estimate $\dot{D}_5(0)$, we will use Equation (34) to evaluate the control function $D_5(\Delta)$ for an ideal channel. For this control function, an ideal channel has the transfer function $H(w) = Te^{-jw\frac{T}{4}}$ (i.e., there is a $T/4$ delay that should force $\tilde{c}_0$ and $\tilde{c}_1$, which are spaced $T/2$ apart, to be nearly equal in magnitude). Under these circumstances, differentiation of Equation (34) gives

$$\dot{D}_5(0) = T^2 \iint_{-2\pi/T}^{2\pi/T} (w-v)\sin(w-v)\frac{T}{4}\frac{dw}{2\pi}\frac{dv}{2\pi} \tag{44}$$

Recalling the formula

$$\int_{-A}^{A}\int C(w-v)\,dw\,dv = \int_{-A}^{A} [2A - w]\, C(w)dw \tag{45}$$

aids in our evaluation of (44), and we find that

$$\dot{D}_5(0) = \frac{128}{\pi^2 T} \tag{46}$$

Combining Equations (43) and (46), we have

$$\frac{\Delta_{pk}}{T} = 2.46 \cdot 2^{-B} \tag{47}$$

and for $B = 7$, peak jitter extends for only 2 percent of a symbol interval. Thus, the control function can be computed using far less tap-weight precision than the sixteen bits needed for normal operation of the equalizer[9]. Consequently, using this particular center-tap tracking algorithm does not impose any special precision requirements on the equalizer.

Implementation of Center-Tap Tracking on a *T/2* Equalizer

In this section, we describe using a center-tap tracking algorithm for a QAM fractionally spaced *T/2* equalizer. The algorithm essentially identifies the (complex) equalizer tap of greatest magnitude, and notes its position relative to an imaginary dividing line positioned along the equalizer tap structure to evenly partition the taps. A clock pulse addition-deletion circuit operates periodically at a rate sufficient to overcome worst-case relative transmitter-receiver clock drift. The correction is based on whether adding or deleting clock pulses will push the largest tap in the direction of the dividing boundary. The equilibrium condition occurs when two taps, one on each side of the boundary, contend to be the largest tap. This control function is closely related to the control function in Equation (31).

A convenient configuration for an experimental *T/2* equalizer is shown in Figure 4. Two conventional synchronous (T-spaced) equalizers operate concurrently on interleaved sets of *T/2*-spaced data samples. Associated with each tap is a complex coefficient with real and imaginary components c_i and d_i. To emphasize that the structure consists of two coordinated complementary T equalizers, we have two "ith" taps labeled i_A and i_B. The "A" taps are referred to as the primary taps; the "B" taps as alternate taps. It is important to recognize that the data sample components stored at tap i_B are *T/2* seconds older than those at tap i_A.

So that the equilibrium condition is not a function of the carrier phase angle, the magnitudes (or the squared magnitudes) of the complex tap coefficients (rather than any individual component) must be compared to establish the largest tap weight. In consideration of this, the eight most significant bits each of c_i and d_i address squaring ROMS whose outputs are added so as to generate a 12-bit representation of the quantity $c_i^2 = d_i^2$. For convenience, let $C_{i_A}^2$ represent the 12-bit representation of $c_{i_A}^2 + d_{iA}^2$ thus symbolically

$$c_{i_A}^2 + d_{i_A}^2 \rightarrow C_{i_A}^2$$

and

$$c_{i_B}^2 + d_{i_B}^2 \rightarrow C_{i_B}^2 \qquad (48)$$

Once formed, $C_{i_A}^2$ (or $C_{i_B}^2$) is compared bit by bit to the contents of a 12-bit reference storage denoted as R with the contents of a 12-bit reference storage denoted as R with $C_{i_A}^2$ replacing R if and only if $C_{i_A}^2$ is larger than R. The reference R is cleared at the start of each symbol interval, and the squared magnitudes are examined in the sequence $C_{1_B}^2$, $C_{1_A}^2$, $C_{2_B}^2$, $C_{2_A}^2$, . . . $C_{i_B}^2$, $C_{i_A}^2$, . . . $C_{n_A}^2$. A boundary line is drawn between two consecutive squared magnitudes, say $C_{j_A}^2$ and $C_{(j+1),B}^2$, such that a timing signal T_p goes high after $C_{j_A}^2$ is compared with R, but before $C_{(j+1),B}^2$ is compared with R, where typically $j = N/2$. The replacement of R by a larger C_i^2 at such time the signal T_p is high will set and latch an initially cleared flip-flop F_1 to the "1" state.

The presence of a "1" at F_1 at the close of the symbol interval is taken as an indication that the receiver timing is running fast relative to the transmitter and must be retarded. The presence of a "0" indicates the reverse situation. Periodic pulse stuffing corrections are made to the master timing countdown chain in the direction specified by the state of flip-flop F_1 at the end of the symbol intervals during which the corrections are called for. In the experimental modem we have been describing, the symbol interval is lengthened or shortened by approximately 0.1 percent once each 15 symbols to overcome a maximum relative clock drift of 50 parts per million, which is typical of

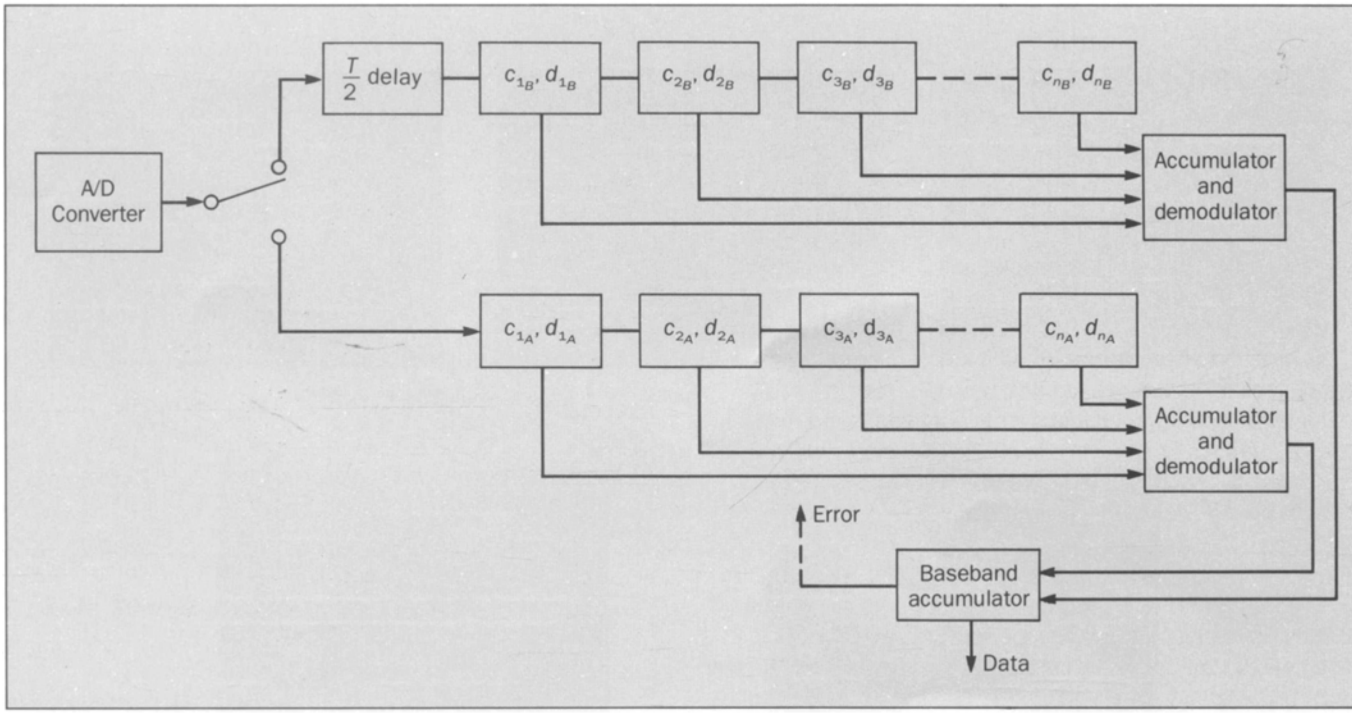

Figure 4. Experimental modem equalizer configuration.

the numbers experienced in modem design.

The stability of the receiver timing thus acquired is indicated by the oscilloscope pictures shown in Figures 5(a-e). Each trace is a one-minute time exposure of the transmitter symbol rate clock of the experimental modem in a line signal loopback mode, as viewed on an oscilloscope triggered by the counterpart receiver symbol rate clock. Because a common oscillator is used in this mode of operation, it is possible to display, for the purpose of measuring jitter, a stationary pattern by inhibiting the timing recovery operation. This is shown in the top trace of each picture. The second trace shows the relative excursion of the symbol rate clocks with conventional envelope-derived timing recovery, and the third trace shows the excursion using the tap tracking procedure. Each photograph is for a different simulated channel as specified. The jitter is reflected in the amount of overlap of the two levels shown in each trace.

The striking feature of these photographs is the relative insensitivity of the performance of the tap tracking procedure in the face of increasingly severe impairments as compared to the highly variable and often severely degraded envelope-based timing recovery performance. In fact, envelope-derived timing recovery is seen to fail totally in Figures 5(c) and 5(e). The relative insensitivity of the tap tracking scheme can be understood in terms of the plots of distribution of squared tap weights for the various simulated channels shown in Figures 6(a-e). The nature of the distributions, in terms of having pronounced dominant taps, is not significantly influenced by the degree of the

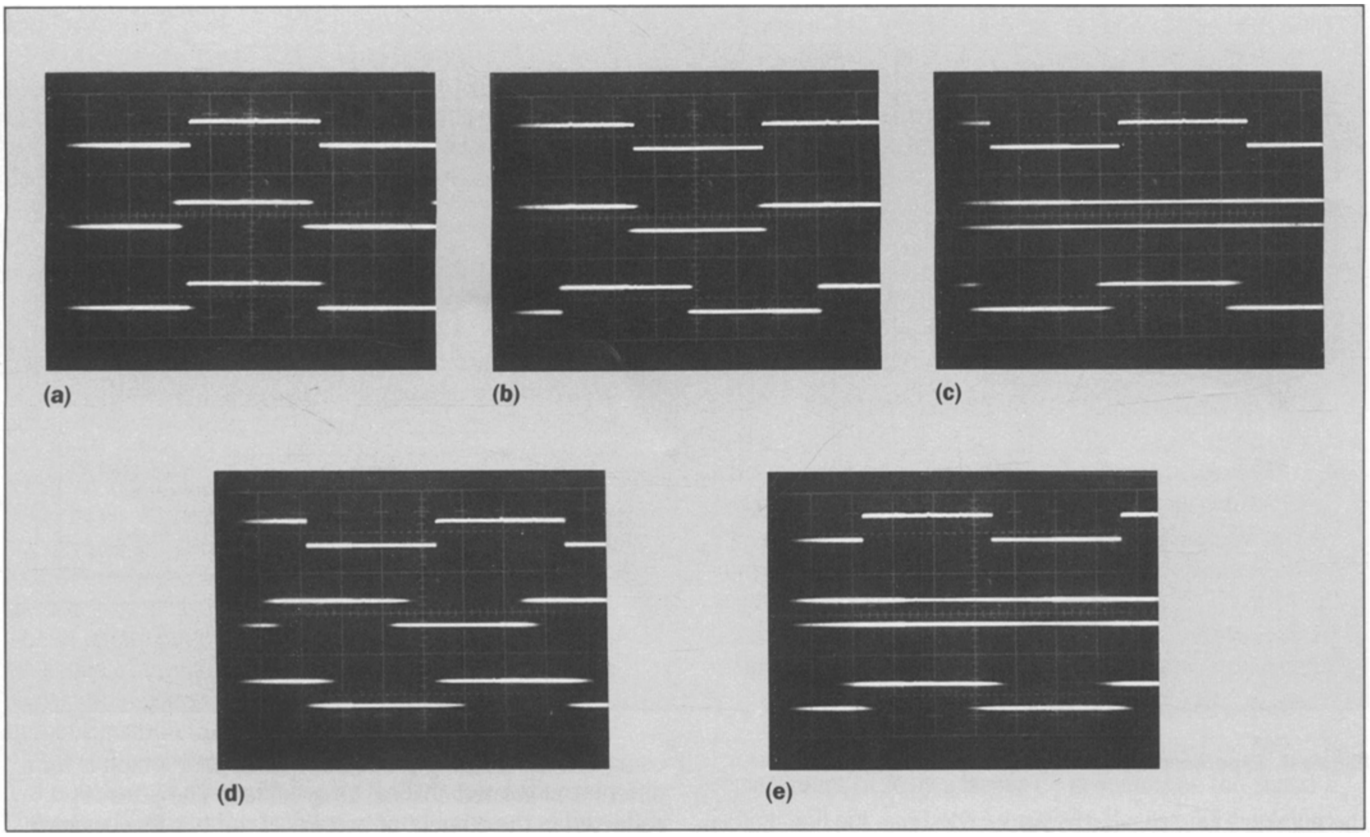

Figure 5. One-minute time exposures of transmitter symbol rate clock. Each oscillator photograph shows a different simulated channel: (a) back to back; (b) simulated C4 channel; (c) simulated C1 channel; (d) simulated C2 channel; (e) simulated 3002 channel.

linear impairments. In fact, the ability of tap tracking to successfully acquire stable timing on a given line is strongly coupled with the ability of the receiver to equalize that line. Tap tracking and equalization work in close harmony. By way of contrast, the envelope-based timing recovery is usually the weak link in receiver operations, denying operation over channels that even a synchronous equalizer could equalize. Thus, tap tracking may extend the operation of 9.6-kb/s modems to channels that have amplitude distortion severe enough that the envelope circuitry fails, but that are capable of being satisfactorily equalized when synchronization is provided.

There have been recent proposals [such as that surrounding Equation (15)] for timing recovery schemes that attempt to achieve certain relationships among the tap weights such as equality between coefficients on either side of the presumed center tap, or perhaps equality between sums of taps on either side of a specified bound-

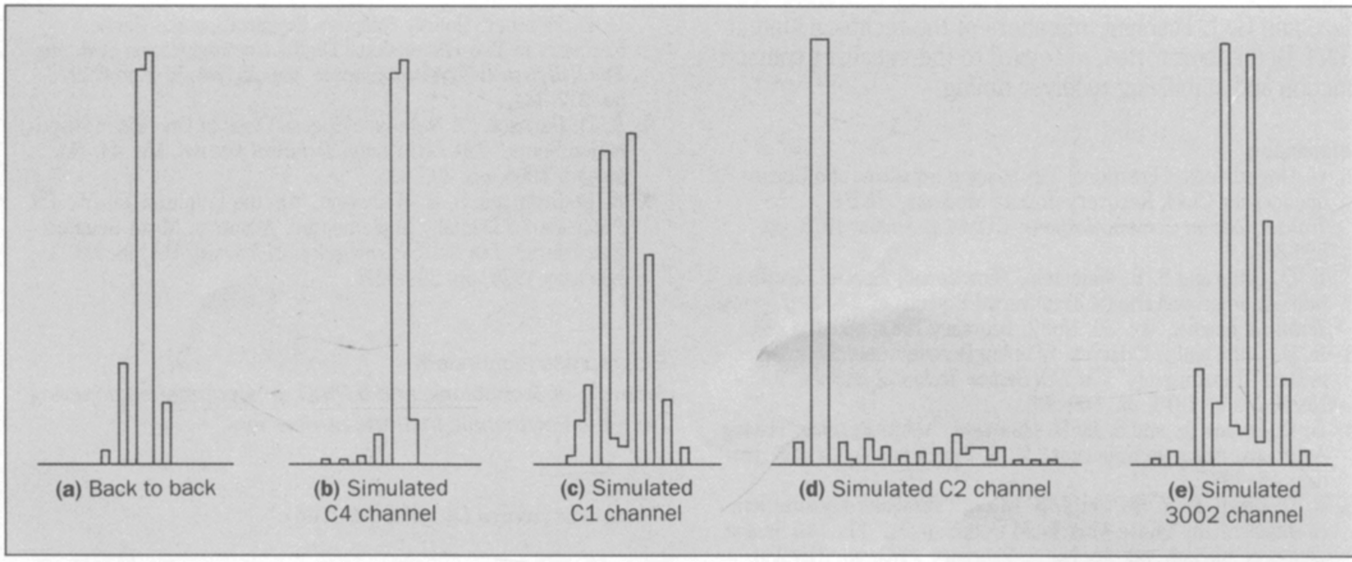

Figure 6. Squared tap distribution.

ary.[1] These procedures would seem to be particularly vulnerable in light of the tap build-up or drift phenomenon associated with oversampled equalizers.[7] Tap build-up is enhanced by timing drift, and it is conceivable that numerous widely spaced equilibrium conditions might exist for these alternate proposals.[1] The situation would be compounded by the introduction of tap leakage[7] to suppress tap growth, for leakage would tend to make the equilibrium states unstable. The dominant tap might drift off the end of the equalizer during the search for equilibrium. Leakage, on the other hand, enhances the operation of the tap-tracking method proposed herein by suppressing spurious large taps. Essential large taps, that is, taps that contribute response essentially in the signal energy band and that are not suppressed by leakage, are constrained to the center of the delay line. Thus, it has been observed in the laboratory that a simple technique, in terms of complexity, may be used to lock the receiver and transmitter clocks.

Conclusion

We have shown that an efficient, powerful method for synchronizing the transmitter and receiver clocks in an adaptively equalized data receiver can be realized by monitoring the tap-weight distribution of a fractionally spaced equalizer. Novel timing recovery techniques based on the tap weight distribution can replace the conventional envelope-derived timing circuitry and achieve the following advantages:

- Improved performance for severely amplitude-attenuated or narrow roll-off systems.
- Reduced complexity.
- A structure appropriate for an all-digital receiver.

The tap-tracking technique appears capable of extending ≥9.6-kb/s service to those band-edge attenuated channels in which the envelope-derived timing circuitry fails, but that are capable of being satisfactorily equalized when synchronization is provided.

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Biographies (continued)

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