

OPTIMIZATION: METHODOLOGY, ALGORITHMS, AND APPLICATIONS

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Optimization methods have been applied successfully to complex engineering and operational problems since the 1940s. The topics addressed in this issue represent recent and ongoing work on optimization at AT&T. They encompass generic algorithmic development as well as optimization methodologies directed toward specific applications. The papers on generic work cover recent development and implementation of variants of the Karmarkar algorithm for linear programming problems. The papers on problem-oriented work cover a range of application areas, including telecommunications network planning, product design and manufacturing, and speech processing.

Perspective

Significant advances in computers since the end of World War II have made it possible to develop practical optimization methods for solving large, complex problems in engineering and management.

Computers have changed our way of life. The recent proliferation of personal computers and workstations has made them readily available to engineers and managers. Daily decisions can now be supported by up-to-date information and computer-based decision support tools. At the other end of the spectrum, supercomputers are now solving problems whose solutions were out of reach just a few years ago. This unprecedented increase in computing power lets scientists and engineers take advantage of newly developed optimization techniques and transform them from academic exercises into useful software tools with a significant impact on society.

New discoveries related to optimization have even become the subject of popular interest, highlighted by front-page articles in national newspapers. In 1984, for example, *The New York Times* stated, "The Bell Labs mathematician, Dr. Narendra Karmarkar, has devised a radically new procedure that may speed up the routine handling of such (linear programming) problems by businesses and government

agencies and also make it possible to tackle problems that are now far out of reach.”¹ The article referred, of course, to the Karmarkar algorithm for linear programming problems. Similar articles have appeared in many other newspapers, including *The Wall Street Journal* and *The Washington Post*. This interest and excitement is quite justified: Effective optimization tools can contribute to improving a nation’s productivity and well-being.

Some Uses of Optimization. Representative application areas that have benefited from optimization techniques are described below.

- *Telecommunications Networks.* AT&T and other communication companies invest heavily in expanding their networks. Typically, long-range plans are prepared for, say, ten years. Optimization tools are used, for example, to help determine which links should be expanded, when, and by how much.
- *Transportation.* The profits of airline, railroad, and trucking companies depend on the effective use of their personnel and equipment. Assignment of crews and airplanes to flights, or the design of truck delivery routes, can be formulated as large mathematical models whose solutions require the use of efficient optimization techniques.
- *Emergency Services.* Large-city administrations must provide comprehensive first aid, fire-fighting, and police services. Optimization techniques are often used to allocate limited resources to provide equally effective services throughout a coverage area.

Other applications for which optimization methods have proven beneficial include electric power supply, water resource management, health services, education, and the military. In recent years, significant efforts have also been undertaken by numerous companies covering a wide range of industries (e.g., automobile and “high-tech” companies) to improve design, manufacturing, and distribution processes. In “high-tech” industries, for example, effective application of optimization techniques to integrated circuit design, shop-floor production scheduling, and finished goods distribution can have a significant impact on establishing

or maintaining a competitive edge.

As part of the operations research discipline, a variety of scientific optimization methodologies have been developed, including linear and nonlinear programming, integer and dynamic programming, and network and combinatorial optimization (see, for example, Hillier and Lieberman²). Many optimization algorithms have emerged to solve generic problems in these areas. Some of these algorithms require significant computational effort, while others are ingeniously simple. Among the former, one may include branch-and-bound algorithms; among the latter are labeling algorithms for shortest path problems and the greedy algorithm for the minimum-length spanning tree problem. Since the 1970s, the development of algorithms has been accompanied by the study of computational complexity. Problems have been (roughly) classified as “easy” or “difficult,” depending on whether the effort needed to solve them increases polynomially or exponentially with problem size. For example, linear programs are classified as “easy” (as shown in 1979 by the Russian mathematician Khachian), while integer programs are considered “difficult.”

The effective use of optimization techniques requires much more than the application of sophisticated algorithms to well-formulated problems. It involves, as well, the appropriate representation of a real-life situation within a mathematical model. Too often, models that only faintly resemble real-life problems are optimized. The “art” of mathematical modeling attempts to capture the important ingredients of a real problem without including irrelevant details. And this is a real challenge!

Realistic models are often solved by heuristic methods. These heuristics may incorporate highly efficient optimization algorithms that solve individual modules, taking advantage of special structures in the mathematical formulation. These modules are then tied together through heuristic rules that lead to good, though not necessarily optimal, solutions of the overall model.

In future years, new optimization techniques will

continue to emerge to solve classic as well as nontraditional problems. Many of these techniques will take advantage of new computer technologies to facilitate the solution of larger and more complex problems. The complexity of real-life problems will promote collaboration among existing scientific disciplines and the emergence of new methodologies. For example, new approaches to problem solving may emerge from integrating optimization techniques and artificial intelligence methods³. Further discussion of the role of optimization can be found in "Operations Research: The Next Decade"⁴.

In This Issue

The papers in this issue cover recent and ongoing work on optimization at AT&T. The papers on generic optimization activities cover development and implementation of variants of the Karmarkar algorithm. The papers on problem-specific optimization methodologies cover a wide range of application areas, including telecommunications network planning, product design and manufacturing, and speech processing.

The Karmarkar Algorithm and Its Implementation. The discovery of the Karmarkar algorithm for linear programs in 1984 has opened significant opportunities for solving large, complex optimization problems that could not have been solved by other available techniques. Three papers concentrate on variants of the Karmarkar algorithm and their implementations on a parallel-vector computer.

Cheng et al. present an overview of the AT&T KORBX[®] system, which implements certain variants of the Karmarkar algorithm on a parallel-vector computer. The authors describe the underlying methodology and major features of the KORBX system. They also provide computational results for problems encountered in telecommunications facility planning, military personnel and logistics planning, and other applications.

Karmarkar et al. describe the application of power series methods to Karmarkar-type algorithms. Intuitively, such algorithms give rise to continuous tra-

jectories obtained by taking infinitesimal small step sizes. Properly chosen power series expansions that approximate continuous trajectories accelerate the convergence of a wide variety of interior point linear programming algorithms. This is demonstrated for two algorithms implemented in the KORBX system.

Housos, Huang and Liu describe some of the computational schemes that take advantage of the KORBX system hardware architecture. In particular, they show how cache memory and parallel processors can be used to speed up the repeated solution of large symmetric sets of linear equations. Their paper demonstrates the important role of computer science in the development of efficient optimization tools.

Telecommunications Networks. Over the years, optimization techniques have played a major role in the design of cost-effective telecommunications networks. As new sophisticated services are offered to users, these networks become larger and more complex, requiring even more versatile and flexible optimization tools. Three papers describe ongoing work at AT&T on network optimization methods.

Lee et al. present an algorithm for a telecommunications network loading problem. This loading problem attempts to route traffic through a network at minimum cost, subject to satisfying the capacity constraints on the different links. This problem is formulated as a multiperiod, multicommodity network flow problem. Taking advantage of the problem's special mathematical structure, the authors develop an efficient nonlinear programming algorithm for large-scale problems.

Chang's paper presents a heuristic algorithm that maximizes traffic throughput in telecommunications networks with dynamic routing capabilities. In such networks, a call is set up by trying a sequence of paths that connect the originating and destination nodes. Chang's algorithm provides near-optimal path sequences for all pairs of nodes by employing, at each iteration, a linear programming approximation complemented by heuristic improvements.

Survivability issues are important to users who need uninterrupted telecommunications services. The paper by Agarwal presents an algorithm for the design of minimum-cost networks, survivable under any single-link failure. The problem is formulated as an integer program. The author uses a Lagrangean relaxation and partial branch-and-bound approach to develop an algorithm that finds near-optimal backbone network topologies and routing alternatives.

Design and Manufacturing. In recent years, substantial progress has been made at AT&T and other companies toward improving product design and manufacturing processes⁵. Three papers describe optimization tools for these processes.

Singhal et al. describe a user-friendly software system used to optimize integrated circuit designs and semiconductor processing technology designs. The system provides convenient interfaces between optimization tools tailored for design problems and simulators used to evaluate design alternatives. The authors describe the system's functionality and its basic architecture, and offer some illustrative examples of its use for design optimization.

Rajan and Segal present a model of robotic assembly workcells used to insert components into electronic circuit packs. Efficient use of such workcells requires balancing the workloads among the workcells, as well as minimizing wasted time on unproductive processes such as the loading and unloading of circuit pack magazines. The authors describe an integer programming formulation for this problem, solved heuristically by successive applications of a network-flow algorithm.

King's paper describes an interactive optimization tool for the allocation of limited manufacturing resources among competing products. Such limited resources may include integrated circuits used by many "high-tech" products. The tool uses a specially tailored resource allocation algorithm to solve large-scale problems efficiently. The author emphasizes various ele-

ments that must be addressed in order to create an effective, practical optimization tool for manufacturing applications.

Speech and Language Processing. Many readers may find quite fascinating the extent to which optimization techniques have been applied to speech and language processing. Lee presents applications of dynamic programming algorithms to a sample of such problems, including speaker verification, speech recognition, and language modeling. Due to their sequential nature, many speech and language processes can be formulated as sequential decision models. The resulting models have been found useful in enhancing human-machine communications.

AT&T has traditionally been at the forefront of developing and applying new optimization methodologies and algorithms for complex telecommunications problems. Much of this work has also been extended to other application areas in government and industry. The papers in this issue represent recent and ongoing activities on optimization; they cover a wide range of topics and demonstrate significant technical breadth and depth.

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