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Quarter-Wave Corrugated Transformer for Broadband Matching of a Corrugated Feed

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This paper describes a broadband technique for obtaining the transformation $TE_{11} \rightarrow HE_{11}$ at the input of a corrugated feed. Usually, this transformation is obtained by operating the feed in the vicinity of a frequency, ω_1, corresponding to one of the zeroes of the input surface reactance since the feed input reflection, ρ_t, vanishes at ω_1. Here we use a matching transformer to produce $\rho_t \simeq 0$ at two frequencies simultaneously, i.e., at ω_1 and a much lower frequency, ω_0. Using two sections of corrugated waveguides, with a total of eight teeth and $\omega_0 \simeq 0.63\ \omega_1$, the measured reflection, ρ_t, is about −40 dB in the vicinity of both frequencies ω_0 and ω_1, and it remains less than −30 dB over the entire frequency range $0.60\ \omega_1 < \omega < 1.04\ \omega_1$. This makes the corrugated feed suitable for simultaneous operation at the 4- and 6-GHz terrestrial microwave radio bands.

I. INTRODUCTION

Corrugated feeds are usually characterized by a relatively large input reflection,[1-3] which vanishes only at certain frequencies corresponding to the zeroes of the surface reactance due to the input corrugations. In this article we describe a simple technique for reducing considerably this reflection over a wide range of frequencies. The reflection in question arises at the input of the feed as shown in Fig. 1, where a corrugated waveguide is directly connected to an uncorrugated waveguide of circular cross sections. The reflection, ρ_t, due to the discon-

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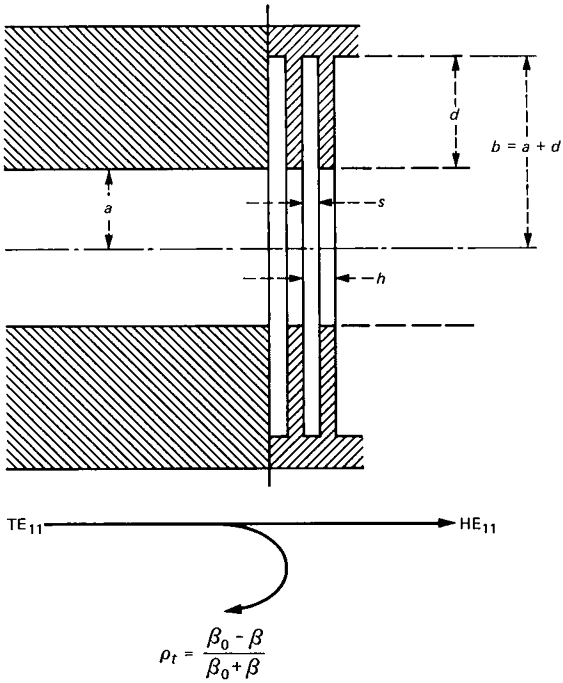

Fig. 1—Junction between two circular waveguides, one with corrugations of depth, d. The reflection, ρ_t, is given accurately by eq. (1).

tinuity in surface reactance between the two waveguides, is given accurately by the formula[2]

$$\rho_t = \frac{\beta_0 - \beta}{\beta_0 + \beta}, \tag{1}$$

relating ρ_t to the propagation constants (β_0 and β) of the dominant modes (TE_{11} and HE_{11}) of the two waveguides. The two propagation constants coincide only at certain isolated frequencies, corresponding to the zeroes of the surface reactance of the input corrugations. The feed is normally operated in the vicinity of the first zero, which is the frequency, ω_1, determined by the condition

$$d = \frac{\lambda_r(\omega_1)}{2}, \tag{2}$$

where d is the depth of the corrugations and $\lambda_r(\omega)$ is the wavelength for the radial waves excited in the grooves at an input frequency, ω. Using eq. (1), one can readily determine, as in Refs. 2 and 3, the variation of ρ_t with frequency, in the vicinity of ω_1.

One finds a rapid increase in ρ_t, with $|\omega - \omega_1|$ causing, under the

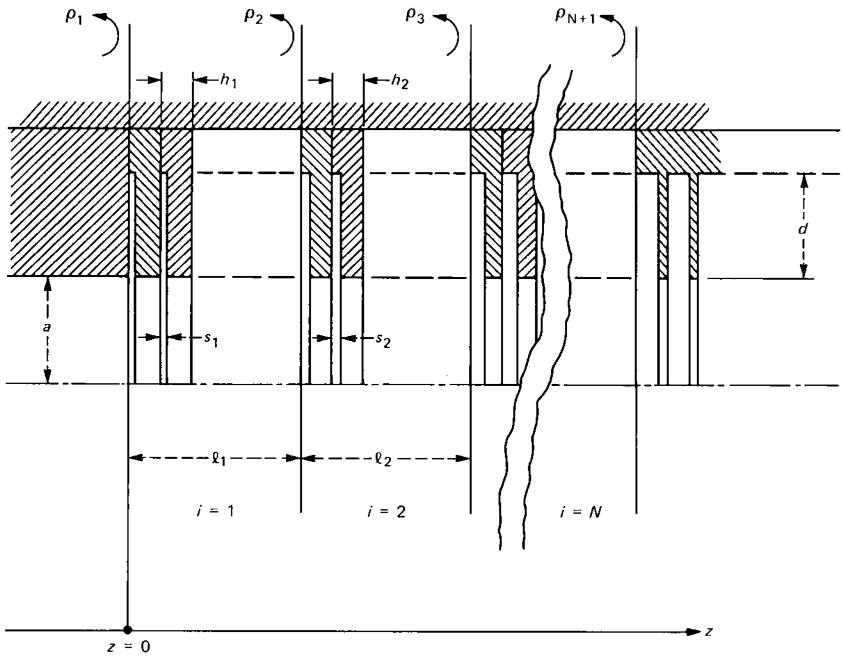

Fig. 2—The two waveguides of Fig. 1 are matched by using N sections of waveguides with corrugations of the same depth, d, but different s_i/h_i.

conditions of Ref. 2,* reflections greater than −30 dB for $\omega < 0.832\ \omega_1$. Thus, the junction of Fig. 1 is unsuitable for applications requiring negligible $|\rho_t|^2$ at widely spaced frequencies, such as, for instance, 4 and 6 GHz, as required in terrestrial radio systems.†

In this article we place a matching transformer between the two waveguides of Fig. 1 to cause $\rho_t \simeq 0$ also in the vicinity of a frequency ω_0 appreciably lower than ω_1. The transformer, shown in Fig. 2, consists of two sections of corrugated waveguide having the same d but different gaps s_i between successive teeth in each section. Since all corrugations have the same depth, d, the total reflection, ρ_t, vanishes for $\omega = \omega_1$. To obtain $\rho_t \simeq 0$ in the vicinity of ω_0, the lengths, ℓ_i, of the various sections and the values of s_i are chosen by the same procedure commonly used for multisection quarter-wave matching transformers.[4] By properly choosing the parameters ℓ_i and s_i, one can cause ρ_t to have N zeroes in the vicinity of ω_0. Here we make these zeroes coincide with ω_0 to obtain a maximally flat characteristic in the

* Notice a misprint in Ref. 2. The frequency $\omega_1 = 17.5$ GHz, given correctly on page 878, is given incorrectly as 19 GHz on pp. 810 and 876.

† The TD and TH bands correspond to the intervals (3.7, 4.2) and (5.925, 6.325) in GHz.

vicinity of $\omega = \omega_0$. As a result, the reflection, ρ_t, is very small at frequencies close to ω_0 and ω_1. Furthermore, we shall see that ρ_t is also small in the entire interval (ω_0, ω_1).

Section III describes a transformer with inside radius a with a measured reflection of less than −30 dB for values of ka in the interval (2.437, 4.22), and less than −35 dB in (2.437, 2.766) and (3.69, 4.22). This performance is obtained using two sections with ordinary teeth, as in Fig. 2. Moreover, it can be further improved by increasing the number of sections or by using special teeth, as in Ref. 5.

Equation (1) was derived in eq. (2) by a perturbation analysis that assumes that the difference $\beta_0 - \beta$ is small. Recently, ρ_t has been derived[6] without this restriction for the special case where one of the two waveguides is uncorrugated, as in Fig. 1, and a more general derivation is given in Ref. 7. By letting $\beta_0 - \beta \rightarrow 0$ in the results of Refs. 6 and 7, we obtain eq. (1). For the purpose of this analysis, however, eq. (1) is more than adequate since all reflections, ρ_i, are small.

Reflections of less than −30 dB were reported in Ref. 8 over a band $2.7 < ka < 3.8$, using only five slots. A mode-matching technique was used to accurately determine the transformer scattering matrix. Here, however, we are interested in a ratio ω_1/ω_0, which is much greater than 3.8/2.7. Experimental and theoretical results on the problem of minimizing the input reflection of a feed using slots of varying depth were reported in Ref. 9, and excellent performance was obtained using ring-loaded slots, which greatly reduced excitation of the HE_{12} mode. We also mention the very low cross-polarization levels obtained in Ref. 10 using a curved-aperture corrugated horn.

II. APPROXIMATE DERIVATION OF ℓ_i, s_i

In this section we minimize the total reflection, ρ_t, in Fig. 2 in the vicinity of ω_0. We assume that only the HE_{11} mode propagates for $z > 0$ and that the TE_{11} mode is incident from the left. Let M_i be the number of corrugations in the ith section, and let λ_{gi} be the wavelength for the HE_{11} mode. The reflection, ρ_i, at the ith junction will be determined approximately using eq. (1), which is strictly valid only if ρ_i is small and

$$M_i \gg 1, \tag{3}$$

implying $h_i \ll \lambda_{gi}$, where h_i is the teeth separation in the ith section.

The propagation constant, β, of a mode is related to its transverse wave number, σ,

$$\beta a = \sqrt{(ka)^2 - u^2}, \tag{4}$$

where $u = \sigma a$ and k is the free-space propagation constant. For the TE_{11} mode,

$$u = u_0 = 1.8411. \tag{5}$$

For the HE_{11} mode in a corrugated waveguide, u is determined by s/h and d, and it can be calculated as in Refs. 1 and 3.

Assume the reflections, ρ_i, in Fig. 2 are small, and let the frequency dependence of ρ_i and β_i/k be neglected. Then, for a maximally flat passband characteristic,[4]

$$\rho_i = \frac{1}{2^N} \frac{N!}{(N-1)!i!} \rho_t, \tag{6}$$

where N is the number of sections and

$$\rho_t = \Sigma \rho_i. \tag{7}$$

The reflection, ρ_i, is related[3] to the propagation constants β_{i-1} and β_i of the waveguides at the ith junction,

$$\rho_i = \frac{\beta_{i-1} - \beta_i}{\beta_{i-1} + \beta_i}, \tag{8}$$

and, therefore, using eqs. (4), (6), and (8) one can determine u_1, u_2, etc., if u_0 and u_{N+1} are given. Once the u_i are known, the values of s_i/h_i can be determined approximately as in Refs. 1 and 3. Notice the length, ℓ_i, of the ith section must be chosen so that

$$\ell_i = \frac{\lambda_{gi}}{4}, \qquad \text{at} \quad \omega_0. \tag{9}$$

In the following section the corrugated waveguide at the input of the feed (see Fig. 1) will be characterized by

$$\frac{b}{a} = 1.789, \qquad \frac{h}{s} = 1.256, \tag{10}$$

which can be shown to give

$$ka = 4.03, \qquad \text{at} \quad \omega = \omega_1. \tag{11}$$

Then, if one chooses

$$\omega_0 = .63\ \omega_1, \tag{12}$$

one finds

$$u_N = u_3 = 2.114, \qquad \text{for} \quad \omega = \omega_0. \tag{13}$$

Therefore, eqs. (4), (6), and (8) give

$$u_1 = 1.928, \qquad u_2 = 2.056, \qquad \text{for} \quad \omega = \omega_0.$$

This requires

$$\frac{h_1}{s_1} = 16.8, \qquad \frac{h_2}{s_2} = 3.15, \tag{14}$$

taking into account that $u_0 = 1.841$ in the input guide.

III. EXPERIMENTAL RESULTS

The values of eq. (14) will not produce exactly a maximally flat characteristic because of the frequency dependence of ρ_i and β_i/k. Furthermore, in the experiment each section was realized using only four corrugations, in which case the values calculated for ρ_i and u_i in the preceding section for $\omega = \omega_0$ are not expected to be accurate since condition (3) is violated (a consequence of this is pointed out in the appendix of Ref. 3). Thus, taking into account the difficulty involved in accurately determining the dependence of ρ_i on h_i/s_i, the optimum values of h_i/s_i were determined experimentally, and they are

$$\frac{h_1}{s_1} = 15.23, \qquad \frac{h_2}{s_2} = 2.492,$$

assuming the conditions in (10).

The experiment was carried out choosing

$$a = 0.651'', \qquad d = 0.514'', \tag{15}$$

which implies $\omega_0 = 7.18$ GHz and $\omega_1 = 11.455$ GHz. The two sections were constructed using eight rings assembled as indicated in Fig. 2. The output corrugated waveguide was realized by the technique of Ref. 2. The input circular waveguide was connected to a rectangular waveguide using a long transition, and the input reflection, ρ_t, was measured as in Fig. 3, using a commercial directional coupler. The measured reflection, shown in Figs. 4(a), (b), and (c), includes a component due to small reflections from the directional coupler and the transition. This component, of about −45 dB, is responsible for the fast ripples in Fig. 4. Also shown in Fig. 4 are the values of ρ_t, calculated using eq. (8), for h_i/s_i given by eq. (8), assuming condition

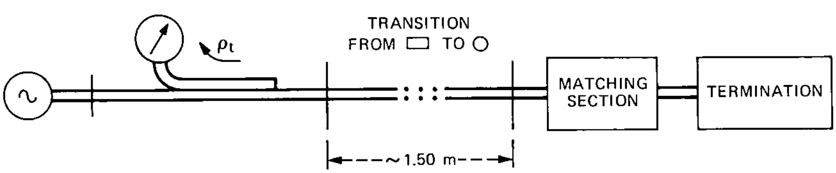

Fig. 3—Apparatus used to measure the reflection, ρ_t.

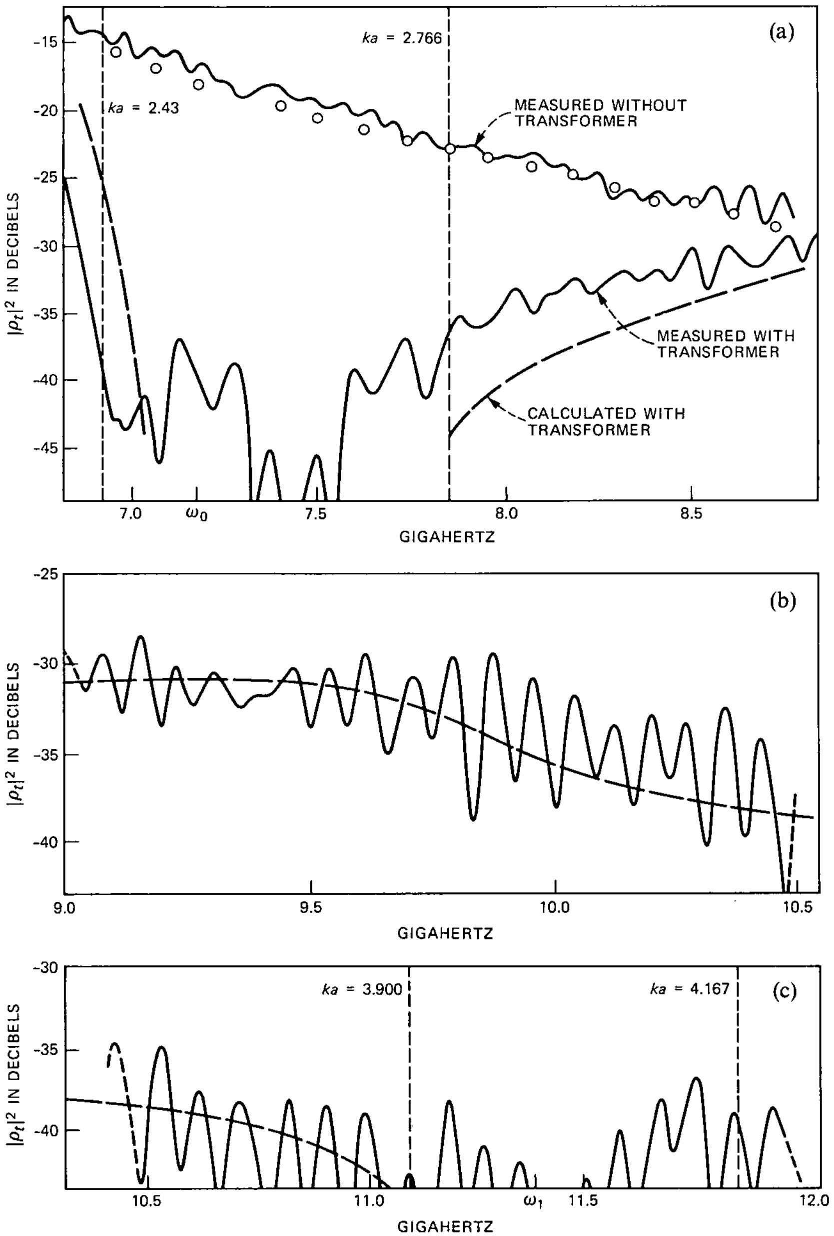

Fig. 4—Reflection, ρ_t, measured with and without transformer from (a) $ka = 2.4$ to 3.0, (b) $ka = 3.0$ to 3.7, and (c) $ka = 3.7$ to 4.2.

(3) but without neglecting the frequency dependence of β_i/k and ρ_i. There is considerable discrepancy for $\omega \simeq \omega_0$, as expected in view of the relatively large values of h_i/λ_{gi}. Also shown in Fig. 4 for comparison is the reflection ρ_t measured without matching section.

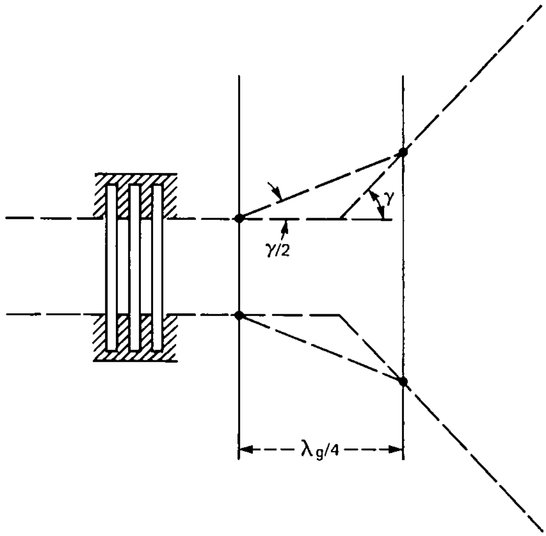

Fig. 5—The reflection at the throat of a horn can be matched by a suitable quarter-wave transformer.

IV. CONCLUSIONS

The input reflection, ρ_t, in Fig. 4 is less than about −30 dB for values of ka in the band (2.43, 4.22) and less than −35 dB over the two bands (2.43, 2.766) and (3.69, 4.22). Over most of these two bands, $|\rho_t|^2$ is less than −40 dB. The transformer is simple to realize, consisting of eight rings assembled as shown in Fig. 2.

Over part of the above frequency range, the input waveguide can be shown to be multimoding because of the TM_{11} mode, which propagates for $ka > 3.8317$. Also, the output waveguide is multimoding over approximately the same frequency range. However, one can show, using the approximate formulae of Ref. 3, that the transformer greatly reduces generation of this mode, which should therefore be negligible. For this reason, and also because of the difficulty in precise measurements of the conversion coefficients, the mode in question was ignored here.

Finally, in a feed, the input reflection includes a contribution from the throat of the horn.[9,10] This contribution is determined by the angle γ in Fig. 5, and it can be eliminated by using a suitable matching transformer as shown in Fig. 5. This is pointed out in Ref. 9.

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