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Conditional Variable-Length Coding for Gray-Level Pictures

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This paper presents a new variable-length coding algorithm for bit-rate reduction of digital images. The coding scheme is based on the fact that by taking into account the local conditional statistics of the picture, the coding efficiency can be improved considerably. Results of computer simulations are presented in terms of picture quality and required bit rate. Simulations show that for various Differential Pulse Code Modulation (DPCM) coded pictures, coding efficiency is such that a bit rate below the entropy of the DPCM quantizer output is attained. For a 3-bit DPCM encoded picture, a bit rate of 1.64 bits/picture element is achieved.

I. INTRODUCTION

Two popular methods for reducing bit rates of pictures are Differential Pulse Code Modulation (DPCM)[1] and transform coding.[2] When these methods are applied, their reconstructed signals are not exactly the same as the original Pulse Code Modulation (PCM) input because of the quantization noise. Consequently, these encoders are often referred to as nonreversible encoders. Bit-rate reduction can also be achieved by reversible encoders. These methods are based on the fact that it is not necessary to assign an equal number of bits to quantized samples that are not equiprobable. Therefore, it may be advantageous to design variable-length code words in which the length of the code

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words are matched to the Probability Distribution Function (PDF) of the source by using very simple procedures. One such procedure has been devised by Shannon[3] and Fano and another by Huffman.[4] Another reversible coding method is run-length coding, in which a sequence of identical symbols is replaced by a code indicating the length and symbol of the sequence. Run-length codes are useful whenever the long runs are highly probable. This method has been widely used for compression of bilevel pictures.[5]

With the above-mentioned reversible coding methods, the entropy obtained from the PDF of a digital image gives a target for the minimum bit rate necessary to describe the source, ignoring the conditional statistics of the picture. If, by some efficient scheme, conditional statistics can be considered, it may be possible to lower this target to a figure that can be described by conditional entropy. One such method has recently been devised by Gharavi et al. for encoding the gray-level pictures using CCITT one-dimensional run-length codes.[6] In this scheme the run-length statistics of the picture are changed in such a way as to provide a signal that has long bit runs.

In this paper we introduce a similar strategy by developing a scheme in which the picture can be directly coded by the variable-length codes. The code words are designed according to the local conditional statistics of the picture using the Huffman procedure. For example, the picture element (pel) values that are expected to occur most frequently, given the local PDF, are assigned to the shortest code words in accordance. The mathematical model and the details of the coding algorithm are given in the following section.

II. BASIC MODEL

The n-bit PCM coded picture can be described mathematically as a random sequence of 2^n possible outcomes with certain statistical properties, which are given in terms of probabilities. From these probabilities we can calculate the entropy of one pel, which represents a lowest theoretical bound for the one-dimensional memoryless model. The entropy of pictures can be presented as:

$$H_{\mathrm{pel}} = -\sum_{i=1}^{2^n} P_i \mathrm{Log}_2 P_i,$$

where P_i is the probability of the ith outcome (ith level). In the case of an mth-order Markov model, the luminance level of each pel, x_i, is considered to be dependent on the level of m previous surrounding pels. Therefore, the conditional entropy based on the m previous elements is given by:

$$H_c = -\sum_{x_{i-m}}^{2^n} \cdots \sum_{x_{i-1}}^{2^n} \sum_{x_i}^{2^n} P(x_{i-m}x_{i-m-1} \cdots x_i)$$

$$\cdot \text{Log}_2 P(x_i/x_{i-1}, x_{i-2} \cdots x_{i-m}), \quad (1)$$

where $P(x_{i-m}x_{i-m-1} \cdots x_i)$ is the joint probability of $x_i \cdots x_{i-m}$ and $P(x_i/x_{i-1}, x_{i-1} \cdots x_{i-m})$ is the conditional probability of x_i, given m previous pels $x_{i-1}, x_{1-2} \cdots x_{i-m}$. Equation (1) can be rewritten as

$$H_c = -\sum_{x_{i-m}}^{2^n} \cdots \sum_{x_{i-1}}^{2^n} P(x_{i-m}, x_{i-m-1} \cdots x_{i-1})$$

$$\cdot \sum_{x_i}^{2^n} P(x_i/x_{i-1}, x_{i-2} \cdots x_{i-m}) \text{Log}_2 P(x_i/x_{i-1}, x_{i-2} \cdots x_{i-m}) \quad (2)$$

and, on defining H_L as a local entropy,

$$H_L - \sum_{x_i}^{2^n} P(x_i/x_{i-1}, x_{i-2} \cdots x_{i-m}) \text{Log}_2 P(x_i/x_{i-1}, \cdots x_{i-m}). \quad (3)$$

We express the conditional entropy as

$$H_c = \sum \sum \cdots \sum P(x_{i-m}, x_{i-m-1} \cdots x_{i-1}) H_L. \quad (4)$$

Equation (4) shows that the conditional entropy depends not only on the local entropy but also on its distribution over $(2^n)^m$ independent states. This model leads to the new coding scheme proposed here.

2.1 Coding algorithm

The first step is to use the m previous pels to construct a table of $(2^n)^m$ independent states of picture statistics. For each state, a set of code words is designed according to the probability distribution of the 2^n outcomes of the quantized values. These code words can be derived by employing optimum coding procedures, such as Huffman, or Shannon and Fano, in which a shorter code is assigned to a more probable level. Then, for a given state, each incoming pel is coded by selecting an appropriate code word corresponding to its quantized value.

III. DPCM CODING

In the previous section it is apparent that the total number of states is heavily dependent on the number of quantization levels. For example, for input pictures of 256 levels corresponding to 8 bits/pel, there are 256^m (m is the number of reference elements) states. This figure is rather high, and a simple reduction of the number of quantizing levels would lead to annoying contouring effects. Thus, to convert an 8-bit picture into fewer bits while maintaining approximately the same quantizer noise, a DPCM encoder is employed. In the DPCM encoder used here, a linear predictor constructs its predicted pel as the weighted summation of previous pels (see Fig. 1); thus

$$\bar{x} = 0.5A + 0.25(B + C), \tag{5}$$

where $\bar{x}$ is the prediction of the present pel. The prediction error is then quantized by a symmetric seven-level quantizer with the transfer characteristics given in Fig. 2. Applying the above coder to the 512 × 512, 8-bit/pel picture shown in Fig. 3 results in the seven-level DPCM coded picture shown in Fig. 4.

3.1 Subsampling and interpolation

Subsampling at a factor of 2:1 gives approximately a 2:1 bit-rate reduction. This results in some degradation of pictures, which can be minimized by an efficient interpolation scheme. Figure 5 shows the 2:1 horizontal subsampling pattern employed here. The interpolation is based on averaging the luminance levels of the four surrounding pels. The subsampled picture is DPCM encoded, in which the present pel D (see Fig. 5) is predicted by

$$\text{Prediction of } D = 0.3\text{B} + (C + F)0.35.$$

Figure 6 shows the interpolated picture where 2:1 subsampling was employed.

IV. COMPUTER SIMULATIONS

Experiments were carried out by means of computer simulation. Figure 7 shows the transmitter and receiver block diagrams. In our

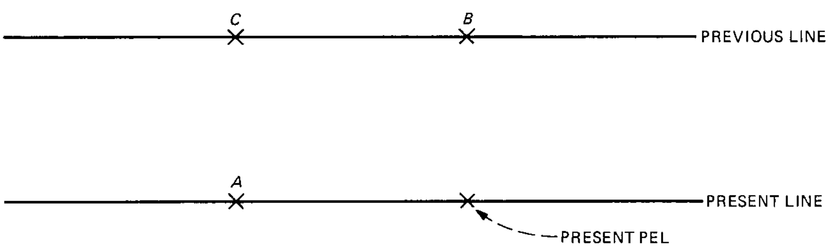

Fig. 1—Configuration of pels used for prediction.

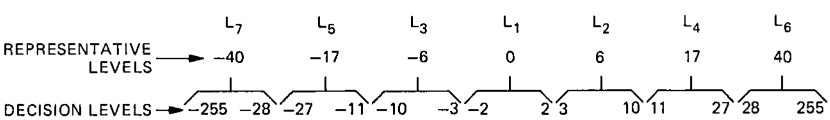

Fig. 2—Transfer characteristics of the quantizer.

Fig. 3—Original picture.

simulation, for a set of seven-level DPCM coded pictures, tables of 49 and 343 states were constructed by observing the quantized values of two and three previous elements, respectively (A, B and A, B, C of Fig. 8). As a matter of convenience only, the table of 49 states with the corresponding code words is shown in Table I. For each state, the seven code words are obtained from the probability distribution function of the seven outcomes, using the standard Huffman procedure.

As Fig. 7a shows, the input signal is first DPCM encoded to generate a 3-bit DPCM coded picture. The coded picture is then applied to the conditional variable coder, which uses the line and sample delay to represent the values of neighbors B and A, respectively. These values, along with current pel value, comprise the input for the encoder Look Up Table (LUT). The encoder LUT uses this information to give the bits of the appropriate variable-length code word, as shown in Table I. The output is smoothed by a buffer to allow for the uniform bit rate in the transmission channel. In the receiver, the transmitted variable-length code words are received from the channel and applied to a buffer and then to the decoder LUT (see Fig. 7b). The conditional variable-length decoder LUT also receives the decoded sample and line-delayed signals to represent the values of neighbors B and A, respectively. Based on this information, the decoder determines

Fig. 4—Three-bit DPCM coded picture.

X	O	X^C	O	X^F
O	X^B	C^A	X^D	O
X	O	X^E	O	X
O	X	O	X	O

X : SAMPLE SELECTED FOR TRANSMISSION
O : SAMPLE DROPPED

INTERPOLATION OF $A = (B+C+D+E) / 4$

Fig. 5—Subsampling pattern used for interpolation and prediction.

whether the current pel code word is the one expected to occur most frequently, second most frequently, and so on, as specified in Table I. The output of these is then applied to the DPCM decoder, which serves as the receiver output signal.

Fig. 6—Subsampled and interpolated picture.

4.1 Results

Table II presents the bit-rate results of the variable-length encoder with the corresponding entropies and conditional entropies. The results of variable-length coding based on the statistics of the quantizer output (ignoring the conditional statistics) are also included in Table II as a reference.

By looking at the tabulated results, it is clear that a considerable improvement is obtained by using this conditional variable-length algorithm compared with a nonconditional variable-length encoder. An even better improvement is obtained by increasing the number of reference elements from two to three (third row of Table II) at the expense of complexity, as the number of states is increased from 49 to 343. In the case of 2:1 subsampling, a 2:1 bit-rate reduction is unobtainable. This is due to the lower spatial correlation as a result of an increased distance between neighboring pels, as compared to a nonsubsampled case. However, the most important conclusion from Table II is that bit rates below the entropy of the DPCM quantizer output are achievable.

V. CONCLUSION

A new concept for a variable-length coding scheme has been pre-

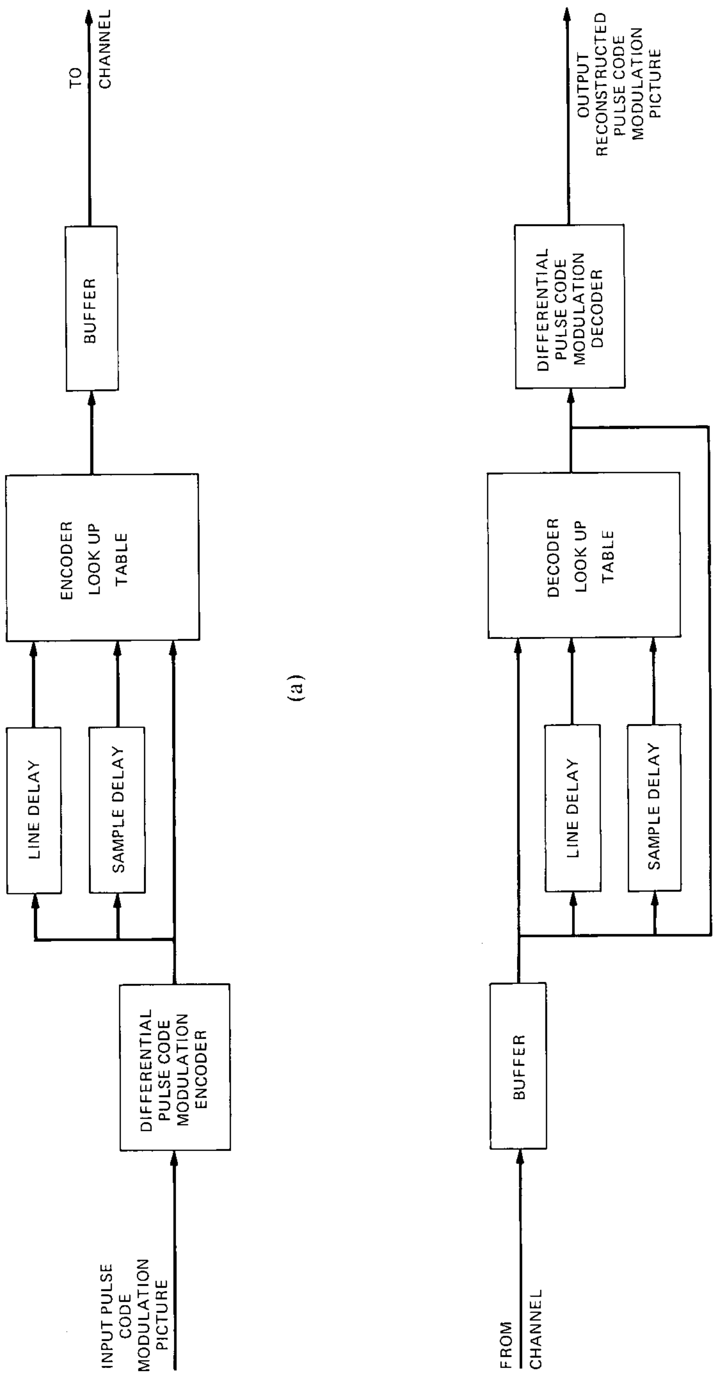

Fig. 7—Block diagram of (a) transmitter and (b) receiver.

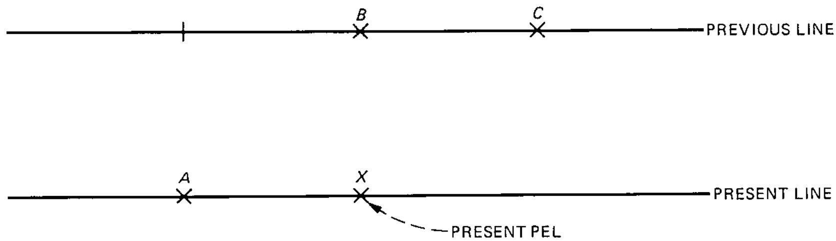

Fig. 8—Configuration of reference elements used for conditional variable-length coding.

Table I—Table of variable-length code words

	Neighbors								
State	*A*	*B*	Code Words						
1	L_1	L_1	L_1 1	L_2 01	L_3 001	L_4 00001	L_5 000001	L_6 000001	L_7 000000
2	L_1	L_2	L_1 0	L_2 11	L_3 101	L_4 1001	L_5 10001	L_7 100001	L_6 100000
3	L_1	L_3	L_1 0	L_3 11	L_2 101	L_5 1001	L_4 10001	L_6 100001	L_7 100000
4	L_1	L_4	L_2 11	L_1 10	L_4 01	L_3 001	L_7 0001	L_5 00001	L_6 00000
5	L_1	L_5	L_3 11	L_5 10	L_1 01	L_2 001	L_6 0001	L_4 00001	L_7 00000
6	L_1	L_6	L_5 1	L_6 01	L_3 0011	L_2 0010	L_7 0000	L_1 00011	L_4 00010
7	L_1	L_7	L_7 1	L_1 01	L_2 0011	L_6 0010	L_5 0000	L_3 00011	L_4 00010
8	L_2	L_1	L_1 0	L_3 11	L_2 101	L_4 1001	L_5 10001	L_7 100001	L_6 100000
9	L_2	L_2	L_1 0	L_2 11	L_3 101	L_4 1001	L_5 10001	L_7 100001	L_6 100000
10	L_2	L_3	L_3 0	L_1 11	L_2 101	L_5 1001	L_4 10001	L_6 100001	L_7 100000
11	L_2	L_4	L_2 11	L_4 10	L_1 01	L_3 001	L_7 0001	L_5 00001	L_6 00000
12	L_2	L_5	L_3 11	L_5 10	L_1 01	L_2 001	L_6 0001	L_4 00001	L_7 00000
13	L_2	L_6	L_6 0	L_5 01	L_7 0011	L_3 0010	L_4 0000	L_1 0011	L_2 00010
14	L_2	L_7	L_7 0	L_4 01	L_6 0011	L_1 0010	L_2 0000	L_3 0011	L_5 00010
15	L_3	L_1	L_1 0	L_2 11	L_3 101	L_4 1001	L_5 10001	L_6 100001	L_7 100000
16	L_3	L_2	L_1 0	L_2 11	L_3 101	L_4 1001	L_5 10001	L_7 100001	L_6 100000
17	L_3	L_3	L_1 0	L_3 11	L_2 101	L_5 1001	L_4 10001	L_6 100001	L_7 100000
18	L_3	L_4	L_2 0	L_1 11	L_4 101	L_3 1001	L_7 10001	L_5 100001	L_6 100000
19	L_3	L_5	L_3 11	L_1 10	L_5 01	L_2 001	L_6 0001	L_4 00001	L_7 00000
20	L_3	L_6	L_6 0	L_5 01	L_3 0011	L_2 0010	L_1 0000	L_7 00011	L_4 00010
21	L_3	L_7	L_7 0	L_6 111	L_4 110	L_3 100	L_2 1010	L_5 10111	L_1 10110

Table I—*Continued*

	Neighbors								
State	*A*	*B*	Code Words						
22	L_4	L_1	L_3 0	L_1 11	L_2 101	L_5 1001	L_4 10001	L_7 100001	L_6 100000
23	L_4	L_2	L_2 11	L_1 10	L_3 01	L_4 001	L_5 0001	L_7 00001	L_6 00000
24	L_4	L_3	L_3 0	L_1 11	L_5 101	L_2 1001	L_4 10001	L_7 100001	L_6 100000
25	L_4	L_4	L_4 0	L_2 11	L_1 101	L_7 1001	L_3 10001	L_5 100001	L_6 100000
26	L_4	L_5	L_5 0	L_3 111	L_1 110	L_2 100	L_6 1010	L_4 10111	L_7 10110
27	L_4	L_6	L_6 11	L_7 10	L_5 01	L_4 001	L_2 0001	L_1 00001	L_3 00000
28	L_4	L_7	L_7 0	L_4 11	L_2 101	L_3 1001	L_6 10001	L_1 100001	L_5 100000
29	L_5	L_1	L_2 11	L_1 10	L_3 01	L_5 001	L_4 0001	L_6 00001	L_7 00000
30	L_5	L_2	L_2 0	L_1 111	L_4 110	L_3 100	L_5 1010	L_6 10111	L_7 10110
31	L_5	L_3	L_3 11	L_1 10	L_2 01	L_5 001	L_4 0001	L_6 00001	L_7 00000
32	L_5	L_4	L_4 11	L_2 10	L_7 01	L_1 001	L_3 0001	L_5 00001	L_6 00000
33	L_5	L_5	L_5 11	L_3 10	L_6 01	L_1 001	L_2 0001	L_4 00001	L_7 00000
34	L_5	L_6	L_6 0	L_5 11	L_3 101	L_1 1001	L_2 10001	L_7 100001	L_4 100000
35	L_5	L_7	L_6 0	L_7 11	L_4 101	L_5 1001	L_2 10001	L_1 100001	L_3 100000
36	L_6	L_1	L_1 11	L_6 10	L_7 011	L_3 010	L_2 000	L_5 0011	L_4 0010
37	L_6	L_2	L_6 11	L_7 01	L_2 00	L_4 1011	L_5 1010	L_1 1001	L_3 1000
38	L_6	L_3	L_1 0	L_6 110	L_2 101	L_3 1111	L_5 1110	L_7 1001	L_4 1000
39	L_6	L_4	L_7 11	L_6 10	L_4 01	L_2 001	L_1 0001	L_3 00001	L_5 00000
40	L_6	L_5	L_5 11	L_6 10	L_3 01	L_1 001	L_2 0001	L_4 00001	L_7 00000
41	L_6	L_6	L_6 1	L_5 01	L_3 0011	L_7 0010	L_1 0000	L_4 00011	L_2 00010
42	L_6	L_7	L_6 0	L_7 11	L_5 101	L_4 1001	L_2 10001	L_3 100001	L_1 100000
43	L_7	L_1	L_7 0	L_5 110	L_3 101	L_4 1111	L_6 1110	L_2 1001	L_1 1000
44	L_7	L_2	L_7 11	L_2 10	L_3 011	L_4 010	L_1 000	L_5 0011	L_6 0010
45	L_7	L_3	L_7 0	L_5 111	L_6 110	L_4 100	L_3 1010	L_1 10111	L_2 10110
46	L_7	L_4	L_4 11	L_7 10	L_2 01	L_3 001	L_1 0001	L_6 00001	L_5 00000
47	L_7	L_5	L_7 11	L_6 10	L_5 011	L_3 010	L_4 000	L_2 0011	L_1 0010
48	L_7	L_6	L_7 1	L_6 01	L_4 000	L_5 00111	L_3 00110	L_2 00101	L_1 00100
49	L_7	L_7	L_7 1	L_4 01	L_2 000	L_1 00111	L_3 00110	L_6 00101	L_5 00100

Table II—Performance of variable-length coding algorithms for 512 x 512 pictures

Variable-Length Encoder	DPCM Encoder	Entropy of Quantizer Output (bits/pel)	Conditional Entropy (bits/pel)	Bit Rate (bits/pel)
Based on quantizer output	Nonsubsampled	1.84	—	1.94
Based on conditional statistics ref. elements A, B	Nonsubsampled	1.84	1.64	1.73
Based on conditional statistics ref. elements A, B, C	Nonsubsampled	1.84	1.53	1.64
Based on conditional statistics ref. elements A, B	Subsampled	1.05	0.92	1.01
Based on conditional statistics ref. elements A, B, C	Subsampled	1.05	0.89	0.97

sented in which the structure of the variable-length code word changes from pel to pel, depending on the local conditional statistics of the picture. In this scheme 3-bit DPCM words are coded by assigning a variable-length code word to represent each pel as a function of its expected frequency of occurrence, given the neighbor pel values (state). We have shown that for subsampled and nonsubsampled 3-bit DPCM coded pictures, bit rates of 0.97 bits/pel and 1.64 bits/pel, respectively, are achievable. We therefore conclude that by using the conditional variable-length encoder it is possible to obtain bit rates below the entropy of the quantizer output.

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