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Electrical Transmission Lines as Models for Soliton Propagation in Materials: Elementary Aspects of Video Solitons

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Distributed electrical and mechanical transmission lines are useful models for nonlinear wave motion with dispersion in many interesting physical systems. Nonlinear wave motion with dispersion produces solitons in optical fibers. A soliton will propagate along an appropriate transmission line with constant velocity and without change in shape. The shape remains the same because the nonlinearity of the medium creates higher harmonics and a steeper pulse, whereas dispersion tends to broaden the pulse. A balance between the two is reached, and a stable pulse results. Thus, a communication system using solitons might be advantageous. Electrical distributed lines have an obvious direct application to integrated circuit parametric amplifiers, harmonic generators, and shock-wave generators for pulse shaping. They also have applications to secret, or secure, coding systems using two soliton interactions, and to data transmission using solitons.

I. INTRODUCTION

The study of nonlinear wave propagation along distributed electrical and mechanical transmission lines is important because these lines serve as useful models for nonlinear wave motion with dispersion in many interesting physical systems. Recently, nonlinear wave motion with dispersion has been shown to produce solitons in optical fibers.[1]

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To design devices with exotic properties, one often needs exotic materials. It is very costly in time and money to try to produce these by strictly trial and error methods. Thus the usefulness of models becomes clear. In addition, they permit us to check quickly the accuracy of our computer solutions to certain partial differential equations that describe propagation in nonlinear materials.

Of course, the electrical distributed lines have an obvious direct application to integrated circuit parametric amplifiers, harmonic generators, and shock-wave generators for pulse shaping.[2] They also have applications to secret, or secure, coding systems using two soliton interactions, and to data transmission using solitons.[3,4,5] In this paper we study soliton propagation in materials by means of electrical and mechanical models, and numerical solutions to the appropriate, nonlinear, partial differential equations. Particular attention is paid to multisoliton interactions and the Fermi, Pasta, and Ulam problem.

To a large extent, the material considered in this paper is that which seems appropriate to the materials scientist, and we extract liberally from the literature. This paper is largely concerned with elementary videosolitons, while a proposed paper, with modulated waves in nonlinear dispersive media, is related to the interesting works of Hasegawa and Tappert,[6,7] and to more advanced videosoliton topics.

1.1 The Fermi, Pasta, and Ulam study of nonlinear problems

In 1955 Fermi and co-workers Pasta and Ulam did a remarkable analysis of nonlinear problems. They studied a one-dimensional dynamical system of 64 particles with nonlinear forces between neighbors. In particular, they examined quadratric, cubic, and broken linear types of forces. The behavior of the system was studied for times long compared to the oscillation periods of the related linear problem. Their primary aim was to establish, by means of a computer analysis, the rate of approach to equipartition of energy among the various modes. The nonlinear terms they chose were quite small compared to the linear ones, usually about 10 percent smaller. Basically, they had a string with fixed ends, whose restoring force contained higher-order terms.

For the familiar linear problem, if the initial position of the string is sinusoidal, it will oscillate in this mode forever. Their interest was to watch the string get into complicated shapes because of the nonlinear forces, and eventually, after a sufficient time, get into shapes where all the Fourier modes would be equally important. We now quote from their paper:

> Let us say here that the results of our computations show features which were, from the beginning, surprising to us. Instead of a gradual, continuous flow of energy from the first mode to the higher modes, all of the problems show an entirely different behavior. Starting in one problem with a quadratic force and a pure sine

wave as the initial position of the string, we indeed observe initially a gradual increase of energy in the higher modes as predicted (e.g., by Rayleigh in an infinitesimal analysis). Mode 2 starts increasing first, followed by mode 3, and so on. Later on, however, this gradual sharing of energy among successive modes ceases. Instead, it is one or the other mode that predominates. For example, mode 2 decides, as it were, to increase rather rapidly at the cost of all other modes and becomes predominant. At one time, it has more energy than all the others put together! Then mode 3 undertakes this role. It is only the first few modes which exchange energy among themselves and they do this in a rather regular fashion. Finally, at a later time mode 1 comes back to within one percent of its initial value so that the system seems to be almost periodic. All our problems have at least this one feature in common. Instead of gradual increase of all the higher modes, the energy is exchanged, essentially, among only a certain few. It is, therefore, very hard to observe the rate of 'thermalization' or mixing in our problem, and this was the initial purpose of the calculation.[8]

Figure 1a is taken from this remarkable paper. The horizontal axis is time, the vertical axes is energy, and the number specifies a particular mode. Certainly there is little, if any, tendency towards equipartition of energy among the modes at a given time, and thus there is no mixing.

Figure 1b, which is also taken from the same paper, shows the actual shapes of the string at various cycles of oscillation. (The number is the total number of oscillation cycles.) The initial displacement, as before, is sinusoidal. What is interesting here is that after many, many cycles the sinusoidal shape again appears.

Again we quote:

It is not easy to summarize the results of the various special cases. One feature which they have in common is familiar from certain problems in mechanics of systems with a few degrees of freedom. In the compound pendulum problem one has a transformation of energy from one degree of freedom to another and back again, and not a continually increasing sharing of energy between the two. What is perhaps surprising in our problem is that this kind of behavior still appears in systems with, say, 16 or more degrees of freedom.[8]

Actually, the motion is stranger than that just described. Further work by Tuck and Menzel[9] has shown that the return of the string to its original state is not complete on the first return cycle, because 1 or 2 percent of the energy remains in higher modes. After eight return cycles the deviation is 8 percent, or worse than a single return cycle. Strangely enough, after 16 return cycles it is nearly in the original state!

1.2 The observation of Scott-Russell

J. Scott-Russell in 1844 published a paper entitled "Report on Waves" in the Proceedings of the Royal Society of Edinburgh. We quote from his paper:

I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped—not so the mass of water in the channel which it had put in motion; it accumulated round the prow of

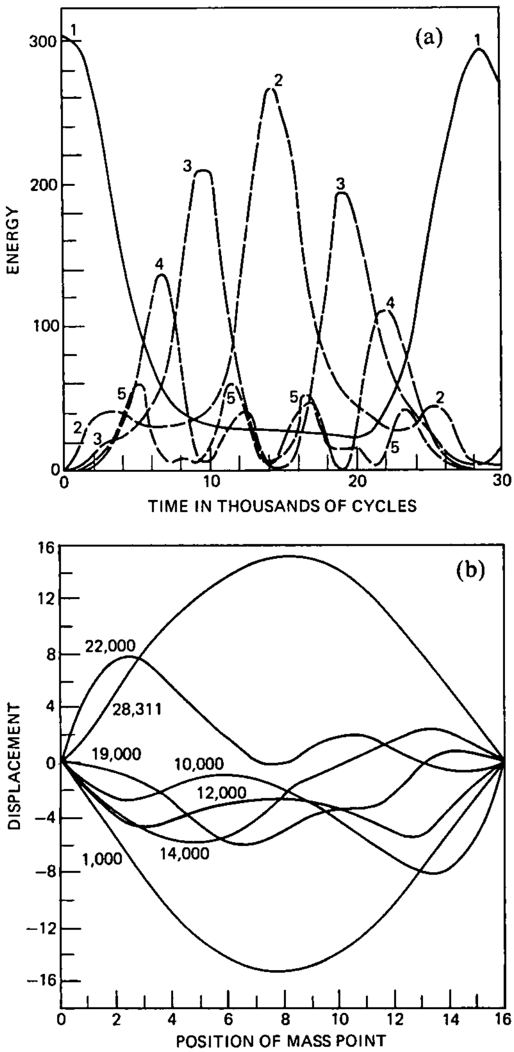

Fig. 1—(a) Energy in each of the first five modes. Initial form of string was single sine wave. Higher modes never exceed 20 units in energy. (b) Actual shape of string at various times.

the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed. I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height. Its height gradually diminished, and after a chase of one or two miles I lost it in the windings of the channel. Such, in the month of August 1834, was my first chance interview with that singular and beautiful phenomenon ....[10]

Thus, we have another example of the remarkable properties of nonlinear wave motion. In 1895 Korteweg and deVries[11] developed their famous equation for shallow-water waves. This equation includes both nonlinear and dispersive effects. One form of it is[12]

$$U_t - 6UU_x + U_{xxx} = 0. \tag{1}$$

This very important wave equation, commonly called the K-dV equation, appears in many branches of physics, including plasma physics, fluid dynamics, and solid-state physics. Solutions to the K-dV equation are the solitary waves described by Scott-Russell.

The variable U represents the perturbation of a physical quantity from its equilibrium value. Its change in time, U_t, is influenced by dispersion, U_{xxx}, which distorts and spreads it, and by nonlinearity, UU_x, which sharpens and steepens it. If these terms balance, we have a soliton solution. This is clearly a special solution to the partial differential equation. Surprisingly, this solution, or soliton, is fairly easy to launch.

II. SOLITONS

We can introduce the concept of a soliton by considering the giant solitary wave described so eloquently by Scott-Russell. We look for traveling-wave solutions to the Korteweg and deVries equation of the form[12]

$$U(x, t) = U(x - ct) = U(\xi). \tag{2}$$

This corresponds to going to a steady moving reference frame with velocity c.

It is clear that

$$\frac{\partial}{\partial x} = \frac{d}{d\xi} \tag{3}$$

and

$$\frac{\partial}{\partial t} = -c\,\frac{d}{d\xi}. \tag{4}$$

Thus we find that

$$U_{\xi\xi\xi} - (6U - c)U_\xi = 0. \tag{5}$$

We see that the partial differential equation has been reduced to an ordinary differential equation. The soliton waves are given by

$$U(x, t) = -\tfrac{1}{2}a^2\operatorname{sech}^2[\tfrac{1}{2}a(x - x_o - a^2t)], \tag{6}$$

which can be checked by substitution. In this expression, x_o is the location of the symmetrical wave at $t = 0$. We note that this wave moves to the right with a velocity a^2. We also note that the amplitude of the wave is proportional to a^2 and that the width is proportional to

a. Since the speed of the wave depends upon its amplitude, a taller wave can overtake a shorter one moving in the same direction.

One might expect that when the two waves interact, because of the nonlinearity, there will be a massive altering of their shapes and speeds after the collison. This is not the case! The surprising result is that they emerge from the interaction completely preserved in shape and speed, with only a shift in position relative to where they would have been had no interaction taken place. Thus, in ferreting out solitons, we first determine whether a solitary wave exists, then we see if these waves retain their shape and velocity after a collision. Scott[13] has given some working definitions for solitary waves and solitons.

Definition: A solitary wave, $\phi_{st}(\xi)$, is a localized traveling wave. More precisely, a traveling wave whose transition from one constant asymptotic state—as $\xi \to -\infty$ (possibly), another as $\xi \to +\infty$—is essentially localized in ξ.

Definition: A soliton $\phi_x(x - ct)$ is a solitary-wave solution of a wave equation that asymptotically preserves its shape and velocity upon collision with other solitary waves. That is, given any solution $\phi(x, t)$ composed only of solitary waves for large negative time,

$$\phi(x, t) \sim \sum_j \phi_{st}(\xi_j) \quad \text{as} \quad t \to -\infty, \tag{7}$$

where $\xi = x - c_j t$. Such solitary waves will be called solitons if they emerge from the interaction with no more than a phase shift.
We see that

$$\phi(x, t) \sim \sum_j \phi_{st}(\overline{\xi}_j) \quad \text{as} \quad t \to +\infty, \tag{8}$$

and

$$\overline{\xi}_j = x - c_j t + \delta_j, \tag{9}$$

with

$$\delta_j = \text{constant}. \tag{10}$$

There are, of course, quite a few nonlinear partial differential equations that are interesting from the point of view of soliton theory. We list a few of these, along with the Lagrangian density, L, from which they can be derived. The Lagrangian density is useful because it allows us to calculate certain conserved densities.

1. The generalized K-dV equation:

$$U_t + \alpha U U_x + U_{xxx} = 0 \tag{11}$$

$$L = \frac{1}{2} W_x W_t + \frac{\alpha}{6} W_x^3 + W_x V_x + \frac{1}{2} V^2, \tag{12}$$

where

$$W_x = U \quad \text{and} \quad W_{xx} = V. \tag{13}$$

A two-interacting-soliton solution is the following:[13,14]

$$U(x, t) = \frac{216 + 288 \cosh(2x - 8t) + 72 \cosh(4x - 64t)}{\alpha[3 \cosh(x - 28t) + \cosh(3x - 36t)]^2}. \tag{14}$$

2. The sine Gordon equation:[15]

$$U_{tt} - U_{xx} + \sin U = 0 \tag{15}$$

$$L = \frac{1}{2}(U_x^2) - \cos U. \tag{16}$$

The quantity U often has the meaning of an angle. The following solution represents solitons, U_+, and antisolitons, U_-:

$$U_\pm = 4 \tan^{-1}\left\{\exp\left(\pm \frac{x - ct}{\sqrt{1 - c^2}}\right)\right\}. \tag{17}$$

3. The Nonlinear Schrodinger equation:[1,6,7,16]

$$U_{xx} + iU_t + K|U^2|U = 0 \tag{18}$$

$$L = i/2(UU_t^* - U^*U_t) + |U_x|^2 - \frac{K}{2}|U|^4. \tag{19}$$

4. The Born and Infeld equation:[17]

$$(1 - U_t^2)U_{xx} + 2U_xU_tU_{xt} - (1 + U_x^2)U_{tt} = 0 \tag{20}$$

$$L = (1 + U_x^2 - U_t^2)^{1/2}. \tag{21}$$

We should point out that the simplest partial differential equation that has soliton solutions is

$$U_{xx} - 1/c^2 U_{tt} = 0. \tag{22}$$

It is dispersionless and linear in contrast to those described earlier, which contained dispersion and nonlinear terms. It is important to realize that if the propagating medium is linear and dispersive, or nonlinear and dispersionless, solitons cannot exist.

2.1 The Toda lattice[18–22]

We are, of course, all familiar with a one-dimensional lattice of mass points. For a vibronic analysis these points are imagined to be connected by springs. In the case of large vibrations they can show nonlinearities. Toda considered a one-dimensional lattice with a potential of the form

$$\frac{a}{b}[\exp(y_j - y_{j-1})] + a(y_j - y_{j-1}) - \frac{a}{b}. \tag{23}$$

Subsequently, Flaschka[23,24] showed that this lattice was a finite-dimensional analogue of the K-dV equation. Also, certain integrals of the Toda equations are the counterparts of the conserved quantities of the K-dV equation.

The Toda-lattice equations can be written as

$$m\frac{d^2y_n}{dt^2} = a[\exp(-br_n) - \exp(-br_{n+1})], \tag{24}$$

where

$$r_n = y_n - y_{n-1}. \tag{25}$$

The Lagrangian is

$$L = \sum_n \frac{m}{2}\dot{y}_a^2 - \frac{a}{b}\exp(-br_n) + \frac{a}{b}\exp(-br_{n+1}). \tag{26}$$

We should remark that a series of mass points coupled by springs has the electrical analogy of a series of inductor-capacitor, low-pass circuits.

Toda obtained analytical solutions to the equations of motion and discovered lattice solitons, i.e., solitons in a nonlinear lattice. He proved that these solitons passed through one another without losing their identity. The solitary-wave solution he found was

$$\phi = \frac{1}{4}\frac{m}{ab}p^2\mathrm{sech}^2\left[\frac{1}{2}(Kn - \beta t) + \delta\right], \tag{27}$$

where ϕ is the force between mass points, K and δ are constants, and

$$\beta^2 = (4ab/m)\sin h^2(K/2). \tag{28}$$

2.2 Solitons in the Morse and other lattices[25–28]

The Morse potential is of the form

$$[\exp(y_j - y_{j-1}) - 1]^2. \tag{29}$$

Rolfe et al.[25] have studied solitons in this lattice by numerical methods. The shape of the solitons is nearly Gaussian. An initial pulse very quickly separates into pairs of pulses that travel in opposite directions and appear to propagate indefinitely. That they are indeed solitons can be verified by studying the collisions between the pulses. As required, there is no change except for a phase shift. In general, the Morse-soliton shape is well approximated by the Toda-soliton shape.

Another family of lattice potentials is the power-law family, which can be specified as M-N, where M is the attractive power and N the repulsive power. For example, 6–12 is the well-known Lennard-Jones potential, which is 6-power attractive and 12-power repulsive. Another one is the 6–32, or "screw", lattice.

It is very striking that the soliton shapes and behavior are all very nearly the same. Thus the Toda soliton is ubiquitous! This may be because of the similarity of the repulsive walls.

III. LINEAR TRANSMISSION LINES

Consider the transmission line shown in Fig. 2. We have all endlessly analyzed this circuit or variations of it in elementary physics, mathematics, or electronics courses. We recognize immediately that the inductance, L, and the capacitance, C, are analogous to mass, M, and force constant, K, of a lattice. Sometimes we look upon such a line as having an inductance-per-unit length, L, and a capacitance-per-unit length, C. Then, instead of a sequence of low-pass filters, we have something akin to a coaxial cable. Likewise, in the mechanical case, instead of a one-dimensional lattice we have a string. We might also say that we have a string with point masses on it, which in the limit becomes a uniform string. It is important to realize that a uniform vibrating string supports an infinity of modes, while a string with point masses on it "gets into trouble" if the frequency is too high. The same is true in the electrical case.

The partial differential equation of a uniform[29] distributed line is

$$-V_x = LI_t \tag{30}$$

$$-I_x = CV_t \tag{31}$$

or

$$V_{xx} = LCV_{tt}, \tag{32}$$

with solutions of the form[29]

$$V = f(\sqrt{LC}\, x \pm t). \tag{33}$$

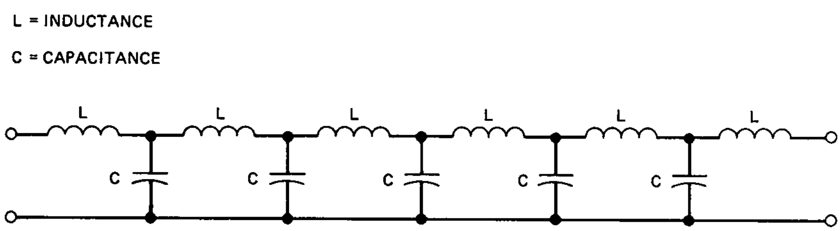

Fig. 2—Linear transmission line consisting of a sequence of inductors, L, and capacitors, C.

This is so familiar that it need not be pursued further. If, however, the capacitance or inductance is nonlinear, then new phenomena arise, for example, shock waves.

3.1 Shock waves in nonlinear transmission lines

Shock waves are familiar to all of us. We need only think about the sonic boom of the Concord supersonic jet or the firing of a rifle. In fact, even the noise caused by an auto collision on a high-speed highway is a shock wave.

Consider now the nonlinear transmission line shown in Fig. 3. Here the capacitance of c is a function of voltage. Let us again consider the capacitance and inductance to be distributed. Then the partial differential equation becomes

$$-V_x = -LI_t \tag{34}$$

$$-I_x = -C(v)V_t. \tag{35}$$

In analogy, with the solution of the linear line, we guess that the following is the solution:

$$V = f(\sqrt{LC(v)}\; x \pm t). \tag{36}$$

This can be shown to be correct.[30] Thus, we see that the velocity of a disturbance is a function of its amplitude. In fact, we expect that the higher-voltage parts will travel at a different speed than the lower-voltage parts.

One form of a nonlinear capacitance is the reversed bias p-n diode. In this case the larger the voltage, the lower the capacitance. This implies that the higher voltage peak of a waveform will travel faster than the lower voltage bottom. Thus, given enough distance or time, the peak can overtake the bottom and a voltage shock can develop.

Shock-wave formation is one of the more interesting phenomena found in the study of wave propagation through nonlinear media. There are quite a few papers in the literature describing shocks on

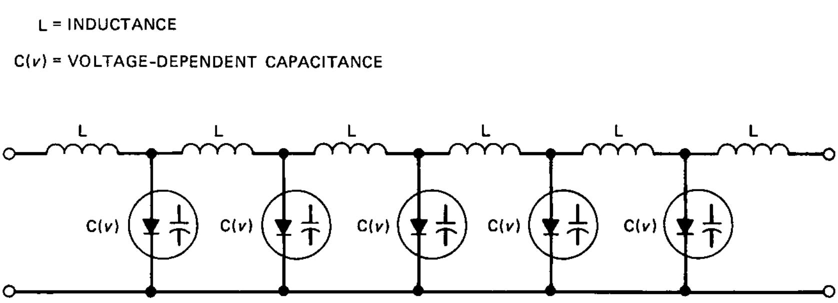

Fig. 3—Nonlinear transmission line, with capacitor, C, a function of voltage.

nonlinear, and nonlinear and dispersive, transmission lines.[31] In fact, there are even books devoted entirely to this topic. One way of looking at shock waves is to imagine that the nonlinearities are constantly building up higher and higher harmonics of the propagating wave.

One of the problems involved in shock-wave studies on transmission lines, and in other nonlinear systems as well, is impedance matching. In general, a line with only one nonlinearity, inductive or capacitive, generates a reflected wave, which reduces the shock amplitude. However, if both the inductance and capacitance are nonlinear functions, and, in particular, if $L(\xi)$ = constant $\times$ $C(\xi)$, then the characteristic impedance is a constant.

Fallside et al. have described a line with[31,32]

$$C = \frac{C_o}{\sqrt{|V|}} \tag{37}$$

and

$$L = \frac{L_o}{\sqrt{|I|}}. \tag{38}$$

In a region of forward simple waves this line has a constant impedance Z_o, given by

$$Z_o = (L_o/C_o)^{2/3} \tag{39}$$

and phase velocity, U, given by

$$U = \sqrt{|V|}\, L_o^{2/3}/C_o^{1/3}. \tag{40}$$

Figure 4 shows typical shock waves formed by this line. We note in particular the sharpness of the shock-wave front.

Our experience with linear transmission lines suggests that the nonlinear partial differential equations should represent forward-traveling and backward-traveling waves. Because of the nonlinearity, the principle of superposition cannot be applied. A very powerful way to solve those partial differential equations is the method of character-

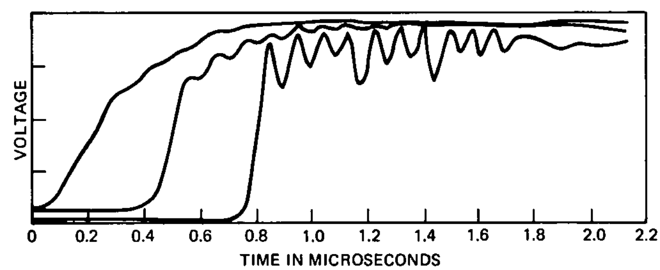

Fig. 4—Typical shock waves on transmission line of Fallside et al.

istics.[33] It is widely used in the study of gas dynamics, and the procedure can be found in the literature.

3.2 Solitons on transmission lines[34–40]

Figure 5 shows four types of wave-propagating media. In a linear-dispersionless media, solitary waves can exist. In a nonlinear-dispersionless media, shock waves exist. In a nonlinear media with dispersion, solitons and solitary waves exist. In a linear media with dispersion, broadening pulses exist. Let us now focus our attention on the electrical analogue of the Toda lattice.

This network consists of a sequence of LC networks with nonlinear capacitors; and the circuit is dispersive. The K-dV equation, as mentioned earlier, is an asymptotic equation for a weakly nonlinear lattice. A beautiful analysis of the K-dV equation, by Zabusky and Kruskal,[40] gives us insight into what we will see on our transmission line. The expected phenomena can be broken into four time intervals.

1. Initially, the first two terms of the K-dV equation dominate, i.e.:

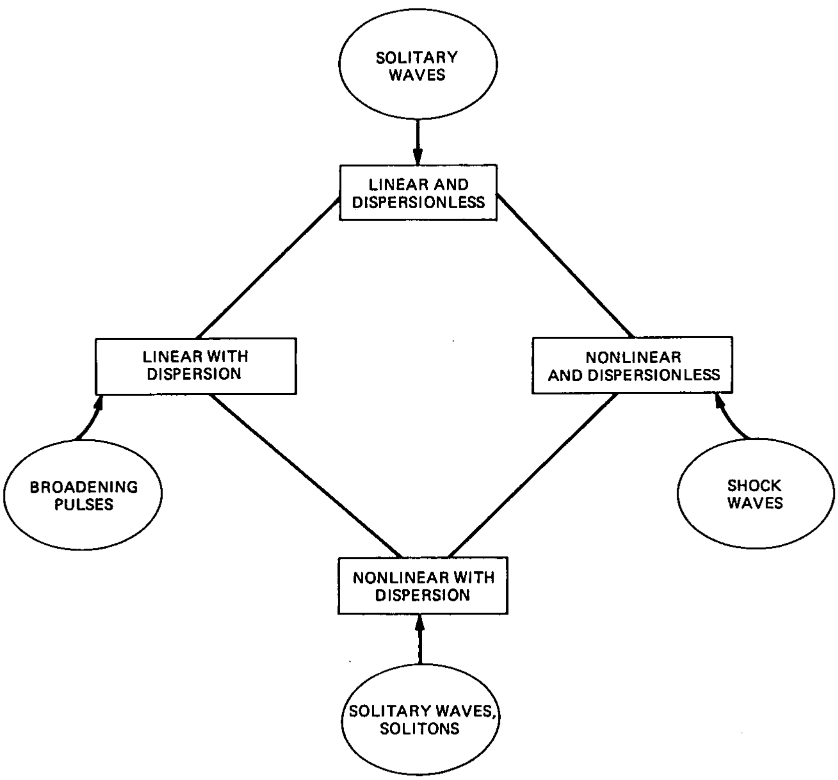

Fig. 5—Four types of wave-propagating media and the types of waves to be expected.

$$U_t + UU_x. \tag{41}$$

A shock starts to develop, and U steepens in regions where it has a negative slope.

2. After U has become quite steep, the third term comes into play and prevents a discontinuity:

$$U_t + U_{xxx}. \tag{42}$$

Oscillations then develop.

3. Solitons are "shaken off", and each propagates with a velocity proportional to its amplitude.

4. After a sufficient length of time, all the solitons come together with a phase that can almost reconstruct the initial waveform.

The propagation equations for the networks are:[35]

$$\frac{\partial}{\partial t} LI_n(t) = V_n(t) - V_{n+1}(t), \tag{43}$$

$$\frac{\partial}{\partial t} Q_n(t) = I_{n-1}(t) - I_n(t), \tag{44}$$

and

$$Q_n(t) = C(V_n(t))\ V_n(t). \tag{45}$$

Here V_n is the voltage across the nth nonlinear capacitor, and I_n is the current through the nth inductor.

3.3 Experiments with a Toda-lattice transmission line

Hirota[35] has reported his experiments with a transmission line having $L = 22\ \mu$H, and $C(v) = 27v^{-0.48}$ pf. Figure 6 shows that applying a pulse to the line results in a train of solitons. In addition, there is a low-amplitude oscillatory tail formed. It is clear from eq. (43) that

$$\int_{-\infty}^{+\infty} V_n(t)dt = \text{constant, independent of } n. \tag{46}$$

Thus, the area under the pulse and the area under the solitons should be the same. This is found to be true to about 2 percent.

Let us now consider the case of two solitons of different amplitudes moving in the same direction on the nonlinear transmission line. The larger one moves faster than the smaller one and eventually swallows it up. The amplitude *decreases* during the overlap period. Finally, the larger soliton emits the smaller one, which then proceeds on its way unaltered.

Suppose we now consider two solitons moving towards each other on the transmission line. In this case both solitons have the same amplitude. During the overlap the amplitude *increases* rather than decreases as in the previous case. After the collision both solitons proceed on their way essentially unaltered.

The Fermi, Pasta, and Ulam phenomena can easily be observed on the Toda-lattice transmission line. A sinusoidal signal is introduced onto the line. By observing the waveform at various points along the line, we can examine the influence of nonlinearity and dispersion on the signal. These results are shown in Fig. 7. Figure 7a shows the input sine wave. In Figs. 7b and c the signal is progressively decom-

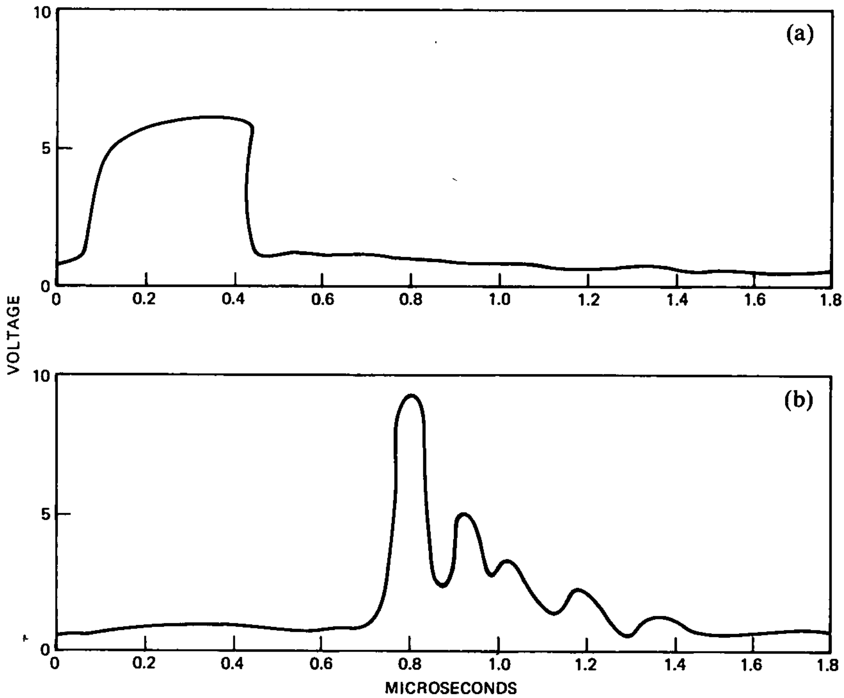

Fig. 6—Decomposition of a pulse into a finite train of solitons.

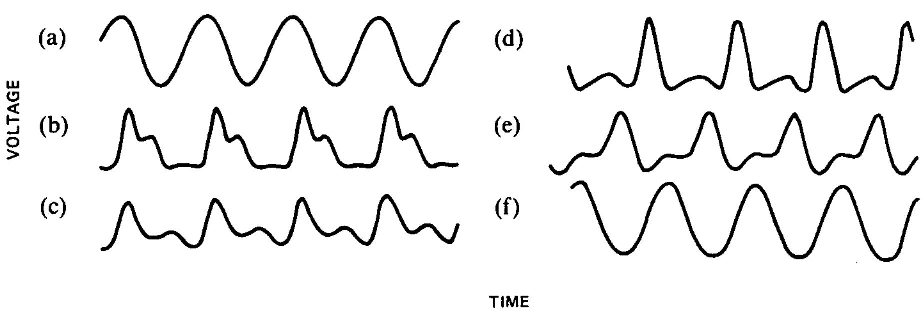

Fig. 7—Fermi, Pasta, and Ulam recurrence phenomena.

posed into higher harmonics, or solitons. In Figs. 7d and e the decomposition is reversing itself. Finally, Fig. 7f is reached, where the original sinusoidal signal has been reconstructed. The recurrence period (or distance) depends very strongly on the amplitude and frequency of the input sine wave. The higher the frequency and amplitude, the shorter the recurrence distance.

3.4 Other transmission lines

Besides the Toda-lattice transmission line, there are a variety of other electrical transmission lines on which solitons can readily be studied. Figure 8 shows four such transmission lines. Two of these have nonlinear inductances, and two have nonlinear capacitances. By a proper choice of parameters, we can simulate the dispersion relations for the electron wave functions in the Kronig and Penney band theory of semiconductors, ion acoustic waves in plasmas, or Trivelpiece and Gould waves.[41] Recently, Jäger and Tegude[42] have reported a nonlinear transmission line that has a cutoff frequency of about 500 mHz. This permits very interesting studies to be performed on a short length of line.

It seems quite clear that analogues can be made for other partial differential equations besides the K-dV and the Toda lattice. For example, Scott[43] has given both electrical and mechanical transmission lines that simulate the Klein and Gordon equation. This equation is particularly interesting because it describes the motion of a block wall between ferromagnetic domains, motion of a slide dislocation in a

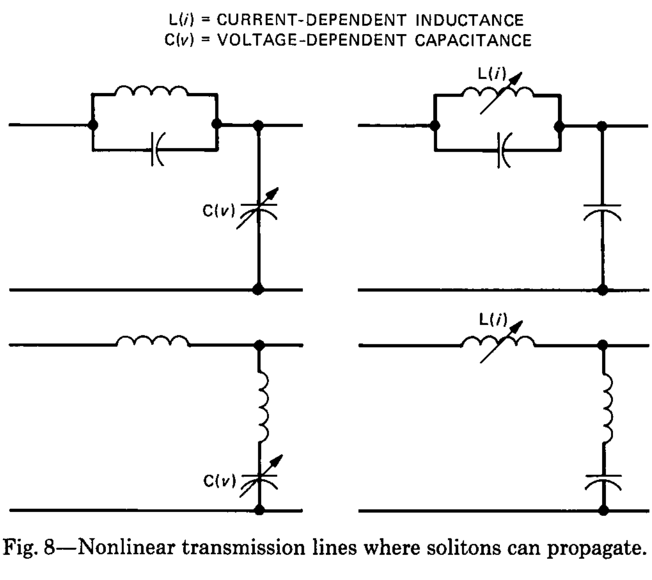

Fig. 8—Nonlinear transmission lines where solitons can propagate.

crystalline structure, and one-dimensional models for elementary particles.

IV. APPLICATIONS OF SOLITONS TO COMMUNICATION SYSTEMS[44–47]

Since a soliton will propagate along an appropriate transmission line with constant velocity and without change in shape, a communication system using solitons might be advantageous. We remind the reader that the shape remains the same because the nonlinearity of the medium is creating higher harmonics and a steeper pulse, whereas dispersion is constraining the harmonics and tending to broaden the pulse. A balance between the two is reached, and stable pulse results.

Chu and Whitbread[46] have reported their experiments on a Pulse Code Modulation (PCM) system using solitons. A soliton is excited by a rectangular pulse in the transmitter. In the receiver the incoming pulse is sent directly to a threshold detector. *No equalizer is needed,* because a soliton suffers no distortion.

One problem involved in a soliton transmission system is that the soliton sometimes has an oscillatory tail, which influences the velocity of the soliton following it. For an optimum system this tail should be suppressed. Another problem with such a system is impedance matching of the nonlinear transmission line. Chu and Whitbread solve this problem by terminating the line with a piecewise simulation of a nonlinear resistance. Figure 9 shows a block diagram of their system. They conclude that:

1. It is practical to use solitons as signal carriers in a PCM communication system.
2. The main source of jitter is an oscillatory tail, which can be removed.
3. A properly designed system would incur a lower bit error rate

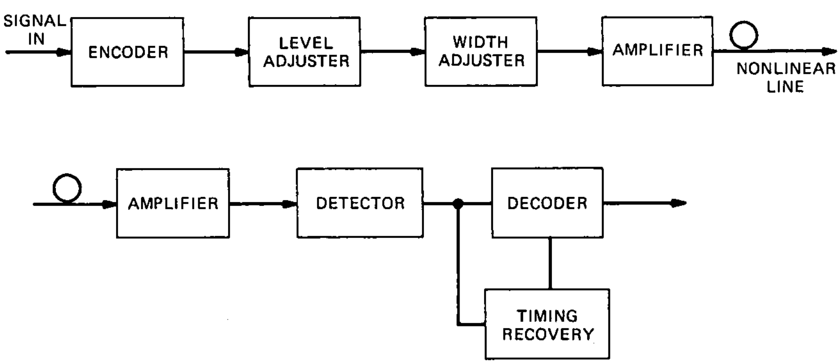

Fig. 9—Pulse code modulation transmission system employing solitons.

than the corresponding system using a linear, dispersive transmission line.

Suzuki, Hirota, and Yoshikawa[44,45] have described a multiplex system based upon the unique properties of solitons. If a sinusoidal signal is introduced into an electrical equivalent of the Toda lattice, it is converted into solitons. Proper choice of lattice and frequency will produce two solitons of different amplitudes. These individual solitons can then be amplitude- or phase-modulated.

Suppose now these two *modulated* solitons are introduced into a nonlinear network. The solitons meet, overlap, and combine into a single carrier, which can be transmitted. This is just an example of the Fermi, Pasta, and Ulam phenomena. The modulation on the combined signal appears to be quite arbitrary, and it is very difficult to distinguish the two original modulating signals that comprise it.

If the receiver is equipped with the same nonlinear network, the signal is converted back into the original pair of separately modulated soliton trains. Strange as it seems, the feasibility of this multiplex system has been successfully demonstrated.

The proposed paper will discuss modulated waves in nonlinear dispersive media—for example, the work of Karpman and Krushkal.[47] In addition, some of the more advanced video soliton topics will be examined.

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