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The Overload Performance of Engineered Networks With Nonhierarchical and Hierarchical Routing

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We report the results of a study of the performance of engineered nonhierarchical and hierarchical routing networks under overloads. This study was motivated by results obtained from mathematical models for small, symmetric, uniformly loaded, nonhierarchical networks with transparent switching systems, showing the existence of network instabilities. We extend the mathematical models to more general nonhierarchical networks, and show with analysis and an extant simulation model that such instabilities are also found in nonsymmetric, nonhierarchical networks. We then use our models to consider whether engineered nonhierarchical networks exhibit such unstable behavior. No instabilities are found in the engineered nonhierarchical networks considered here. However, the nonhierarchical networks consistently demonstrate a drop in carried load between 10- and 15-percent overloads. Our analysis of comparably engineered hierarchical networks shows that these networks do not exhibit a drop in carried load under overloads (in the absence of switching system dynamics). Finally, we show that using trunk reservation for first-routed traffic allows the formulation of a control strategy that provides a high level of network carried load during overloads.

I. INTRODUCTION

Hierarchical routing has been used since the early days of the toll network. Before the early 1960s, hierarchical routing was very advantageous for a number of reasons. First, it allowed switching systems

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to determine the path for a call very simply and quickly, using only the call's destination. Furthermore, the call did not loop back to a switching system previously traversed. Second, hierarchical routing combined small traffic parcels on efficient final trunk groups. Third, hierarchical routing networks were relatively easy to engineer.[1]

Recent advances in switching and signaling technology, as well as the tremendous growth in toll traffic, have provided new incentives to increase network design efficiency. Since the mid-1970s, efforts have been under way at AT&T Bell Laboratories to develop a new method of engineering large-scale nonhierarchical networks. These efforts culminated in the unified algorithm,[2] which takes advantage of traffic noncoincidence and routes calls over least-cost paths. This results in nonhierarchical networks, which are less expensive than hierarchical networks. In the nonhierarchical networks, a call can theoretically use any path connecting its origination and destination (although in the unified algorithm only 1- and 2-link paths are allowed). A call blocked at an intermediate switching system is cranked back to its origin so it can take the next path in its route. In addition, the unified algorithm allows routing to take advantage of time-sensitive load variations.[3]

The nonhierarchical networks give service comparable to that of hierarchical networks under engineered traffic conditions. However, it was necessary to promote a better understanding of the performance of nonhierarchical networks under other traffic conditions. In particular, there was concern that network instabilities that existed in small symmetric nonhierarchical networks might also exist in nonhierarchical networks engineered using the unified algorithm.

To help provide this understanding, we studied the performance of engineered nonhierarchical and hierarchical networks under general network overloads, using mathematical and simulation models. We also investigated the effect on these networks of one control, namely, trunk reservation for first-routed traffic.

II. BACKGROUND

The concern about the stability of nonhierarchical networks was stimulated by the work of Krupp,[4] and Nakagome and Mori.[5] They carried out approximate analyses of nonhierarchical routing applied to small, uniformly loaded networks, which are easily analyzed because of their simple symmetric designs. Krupp also considered a nonsymmetric 3-node model. Their models did not include switching system dynamics. Their analyses revealed, in some cases, the existence of network instabilities when trunk-group (or network) blocking probability is considered as a function of offered load. That is, for certain loads the network has two realizable states: (1) a low network blocking state, in which almost all calls use their shorter first-choice path; and

(2) a congested state, in which a large proportion of calls use longer alternate paths, and many calls are blocked.

For example, Fig. 1 shows the relation between the offered load (per point-to-point pair) and trunk-group blocking obtained for a 10-node symmetric network with 100 trunks per trunk group, using their analyses. Each point-to-point pair has a direct first-choice path and eight 2-link alternate paths. The curve shows a range of offered loads that correspond to multiple blocking probabilities, indicating the potential for unstable behavior in this load range. To understand the instability more clearly, we used a simulation model developed by Krupp[4] to simulate the 10-node network.

We first observed the number of calls carried by the network as offered load increased. Figure 2 shows the results of a simulation of the network with an initial load of 84 erlangs (starting with an empty network). The load was increased to 85 erlangs after 65 holding times. These loads are near the top of the multiple-valued region in Fig. 1. As Fig. 2 illustrates, a drastic change in carried load occurs after the offered load is increased. With the initial load, there is a fairly constant throughput of approximately 3750 calls. The trunk-group blocking is low, and few calls are alternate-routed. After the load change, the number of alternate-routed calls increases—first only slightly, and

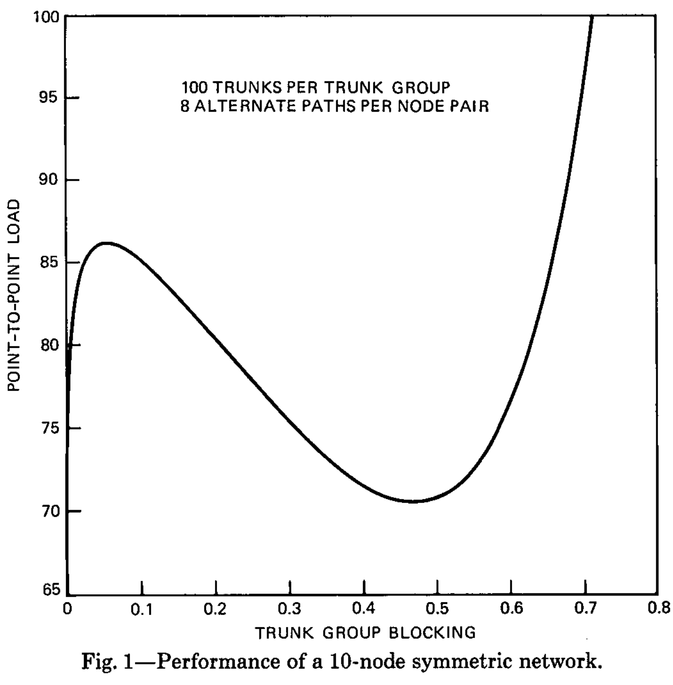

Fig. 1—Performance of a 10-node symmetric network.

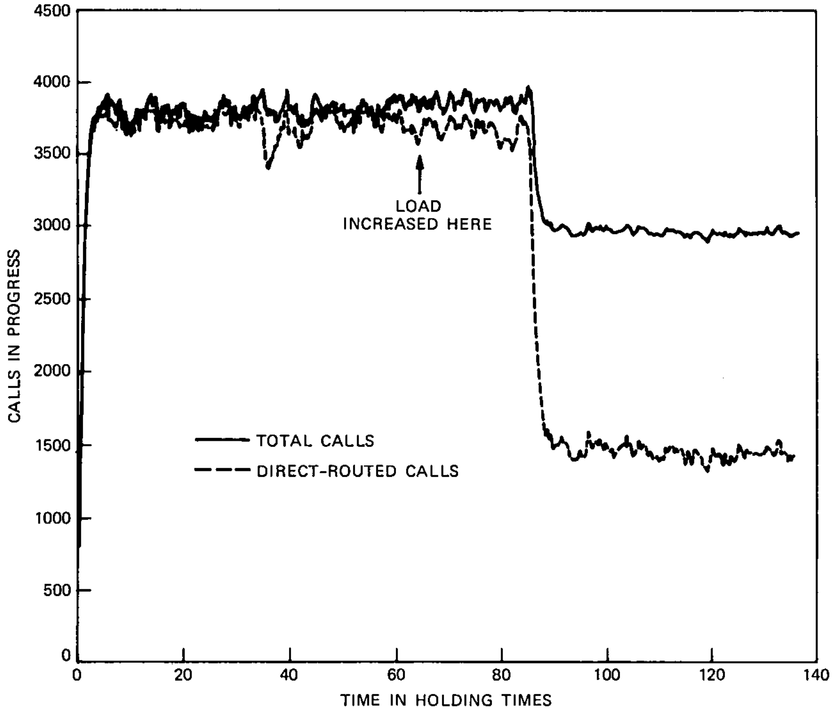

Fig. 2—Simulation of a 10-node symmetric network with load = 84 erlangs/pair; load increased to 85 erlangs/pair.

then dramatically. The average number of calls carried falls to 3000 and half the carried calls are alternate-routed, indicating a high trunk-group blocking. This agrees very closely with the predicted load at which the transition from low to high blocking should occur as offered load increases.

Another consequence of the mathematical solution is illustrated in Fig. 3, which shows the results of a simulation run for the same network with an initial point-to-point offered load of 90 erlangs, starting with an empty network. After approximately 14 holding times, the load is dropped to 80 erlangs. This simulation was conducted to determine the behavior of the network if a load in the multiple-valued region is offered while the network is congested. The initial load of 90 erlangs was used to congest the network, with a resulting carried load of only 3000 calls. When the load is dropped to 80 erlangs, congestion persists for another 16 holding times before the carried load increases to 3600 calls, corresponding to operation in an uncongested state.

These results raised interest in the performance of nonhierarchical routing in more general networks. To address this question, we developed a mathematical model that extends the models in Refs. 4 and 5

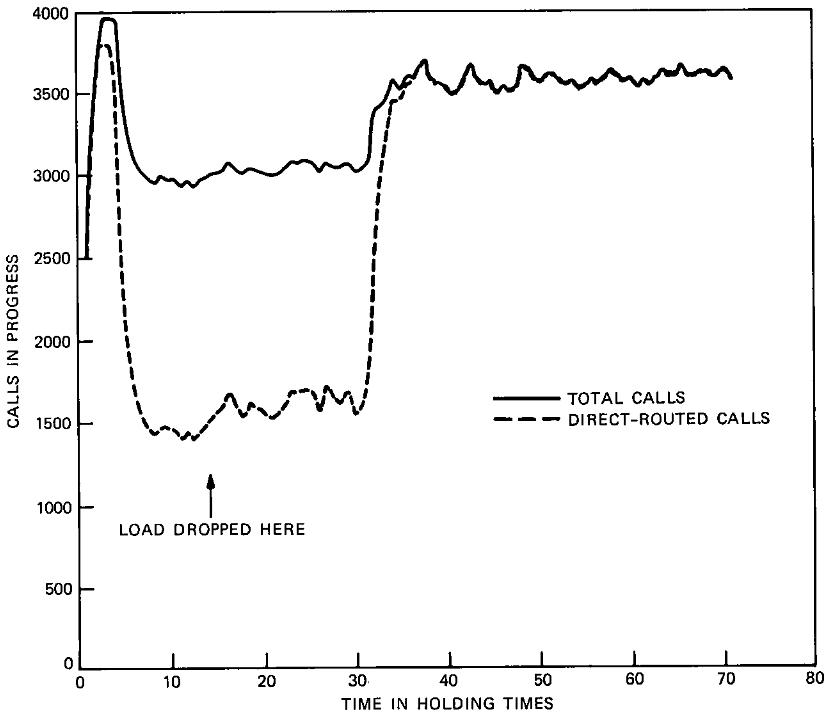

Fig. 3—Simulation of a 10-node symmetric network with load = 90 erlangs/pair; load dropped to 80 erlangs/pair.

to more general nonhierarchical networks. This model, described in the Appendix, analyzes network performance both without controls and with trunk reservation. We also used a simulation model for nonhierarchical networks developed by Krupp. The mathematical model is solved iteratively, starting with an initial estimate of the trunk-group blocking probabilities, to determine the trunk-group offered loads and blocking probabilities in equilibrium. The presence of network instabilities is demonstrated by the existence of multiple solutions for the same parameters (loads, trunk-group sizes, routing) obtained by using different initial estimates of the trunk-group blocking probabilities.

To establish the existence of network instabilities in more general nonhierarchical networks, we applied our models to an 8-node, nonsymmetric, nonuniformly loaded, nonhierarchical network. The number of trunks in each trunk group ranged fairly uniformly from 50 to 995 trunks. It should be noted that this was not an engineered network. Point-to-point loads were chosen so that each point-to-point pair would experience a blocking no greater than 0.005 on the direct trunk group in the absence of alternate-routing, and an *overload* means additional load above these loads. Each point-to-point pair was given a direct first-choice path and four 2-link alternate paths.

We considered the performance of this network under general overloads of up to 10 percent, as shown in Fig. 4. The function relating percent overload to network blocking is double-valued for overloads of up to 4 percent, indicating the existence of instabilities for loads in this range. (Solutions corresponding to low network blocking were obtained by starting with low trunk-group blocking estimates, while solutions corresponding to high network blocking were obtained by starting with high trunk-group blocking estimates.) These results were substantiated by simulation. For example, Fig. 5 shows a simulation at 1-percent overload, starting with an empty network. Initially, the network experiences low blocking, with carried load at about 13,500. Then, after about 17 holding times, the network enters the congested state; carried load drops to 11,000, and a large proportion of calls are alternate-routed.

Now that we had developed a methodology, and demonstrated that instabilities occur in more general nonhierarchical networks, we were ready to address the following question: Does the type of instability seen in these small nonhierarchical networks occur in engineered, nonhierarchical networks? To answer this question, we applied our models to three representative network models. These models are described in Section III.

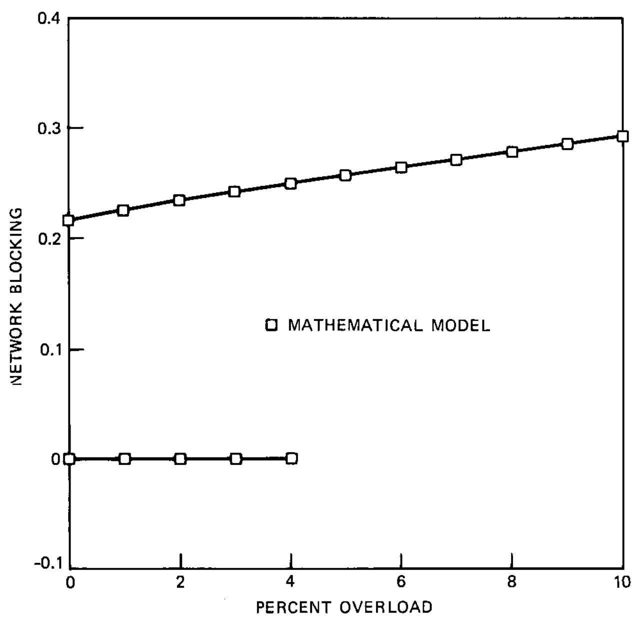

Fig. 4—Network blocking for an 8-node nonhierarchical network.

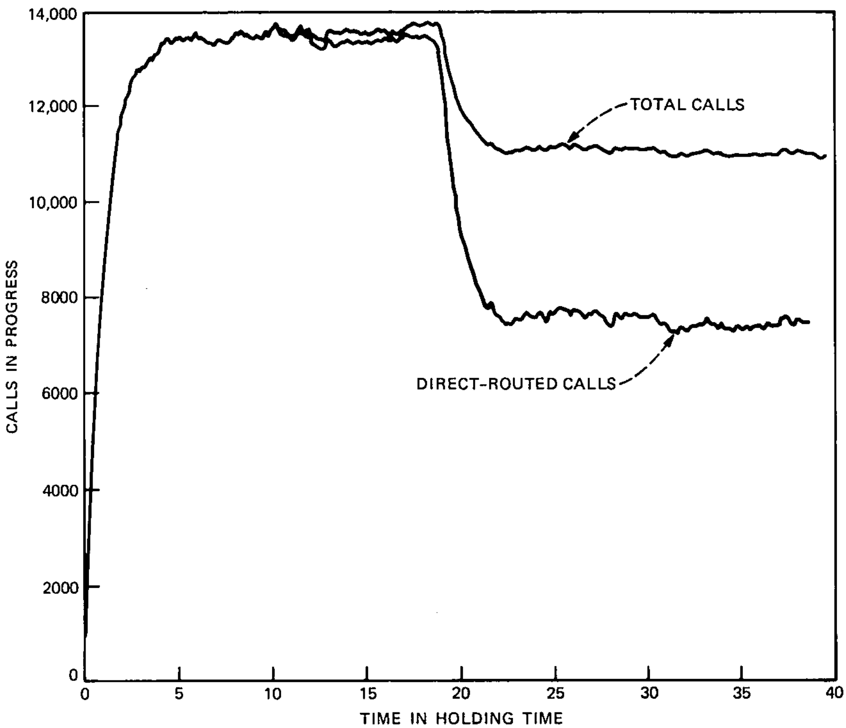

Fig. 5—Simulation of an 8-node network at 1-percent overload.

III. NETWORK MODELS

Three network models were used in our study. The 30-node network model was developed by Gechter and Modarressi at AT&T Bell Laboratories to represent a *4ESS** network. It consists of the 10 regional centers and 20 of the sectional centers in the then existing hierarchical network, and is based on 1977–1978 Trunk Servicing System data. Other characteristics of the network include a wide geographic dispersion of the switching systems, large point-to-point traffic parcels, and fairly uniformly distributed point-to-point loads.

The 25-node network model is made up of a subset of the switching systems that comprise the 215-node hybrid network model recently developed at AT&T Bell Laboratories. The 215-node network was engineered using loads for October 1989, projected from 1978 Centralized Message Data System data. This network includes the 140 *4ESS*s planned for deployment, as well as 75 stored-program-controlled toll tandem switching systems.

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The 140-node network model consists of the 140 4*ESS*s in the 215-node network.

Versions of the two smaller network models engineered for both nonhierarchical and hierarchical routing were studied, while the 140-node network model was only considered with nonhierarchical routing. The 25- and 140-node networks were designed using the unified algorithm.[2] The 30-node nonhierarchical network was engineered using another nonhierarchical network design developed at AT&T Bell Laboratories. An average of 7-percent reserve capacity was added to the trunk groups of the nonhierarchical networks. For the 30- and 25-node network models, the nonhierarchical network has about 9 percent fewer trunks than its corresponding hierarchical network. The nonhierarchical 25-node network was the initial choice for the 1986 Phase One deployment of dynamic nonhierarchical routing (DNHR). The 140-node nonhierarchical network represented the 1989 DNHR network envisioned in the predivestiture environment.

Comparisons of the trunk-group sizes (number of trunks per trunk group) and route sizes (number of paths per route) for the 25-, 30-, and 140-node nonhierarchical networks are shown in Tables I and II. The 25- and 30-node nonhierarchical networks bound the 140-node nonhierarchical network both in terms of trunk-group sizes and route sizes.

IV. RESULTS WITHOUT CONTROLS

4.1 Introduction

We applied the mathematical and simulation models to the 30-, 25-, and 140-node networks. Network performance was investigated under both uniform and nonuniform general overloads. All three network models were studied under uniform general overloads, which were obtained by multiplying the engineered loads by a constant factor.

Table I—Trunk-group size distributions for the nonhierarchical networks

	Proportion of Trunk-Group Sizes $<=x$		
x	25-Node Network	30-Node Network	140-Node Network
100	0.346	0.928	0.746
200	0.580	0.986	0.868
300	0.708	1.000	0.913
400	0.786		0.938
500	0.847		0.951
600	0.871		0.960
700	0.902		0.967
800	0.932		0.972
900	0.939		0.977
1000	0.949		0.980

Table II—Route size distributions for the nonhierarchical networks

x	Proportion of Route Sizes $<=x$ 25-Node Network	30-Node Network	140-Node Network
1	0.213	0.012	0.028
2	0.385	0.021	0.100
3	0.584	0.045	0.242
4	0.791	0.101	0.407
5	0.929	0.196	0.565
6	0.976	0.297	0.718
7	0.993	0.377	0.816
8	1.000	0.441	0.883
9		0.524	0.926
10		1.000	0.971

In addition, the 30-node network was studied under nonuniform general overloads, which were thought to emulate a more realistic overload situation. To represent the nonuniform overloads, we used a set of high-day loads for the 30-node network. The high-day loads were generated using the distribution derived for the ratio of the high-day loads to the average-business-day (engineered) loads and a Gamma distribution to achieve the proper ratio of the peak-day loads to the average of the ten-high-day loads. The high-day loads, which are on average 5 percent higher than the engineered loads, were used to represent a 5-percent overload. Other overloads were obtained by multiplying these loads by the appropriate factor. Uniform and nonuniform general overloads of up to 200 percent were considered. As we discuss below, the mathematical and simulation models give excellent agreement, indicating that the assumptions in the mathematical model do not distort the fundamental network behavior.

4.2 The 30-node network

Figure 6 presents the results for the 30-node networks under uniform overloads. The symbols in the figures indicate results obtained from the mathematical and simulation models. For a given set of loads, the mathematical model always converged to the same solution, regardless of the initial trunk-group blocking estimates, indicating that no network instabilities exist. This is demonstrated by the single-valued function in Fig. 6. However, both the mathematical and simulation results show a striking difference in the performance of the nonhierarchical and hierarchical networks. The two networks show similar performance up to about a 10-percent overload, with carried load increasing with increasing offered load. At that point the number of calls carried in the nonhierarchical network falls sharply, because of an increase in the number of multilink calls. The drop continues until around 100-percent overload, where the carried load begins to increase

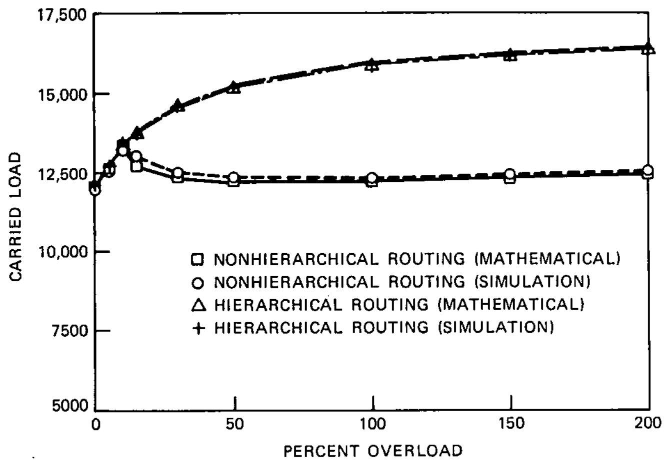

Fig. 6—Performance of the 30-node network under uniform overloads.

gradually. This increase results because the network has become congested to the point that the probability of finding available trunks for a multilink call is very small, so that now 1-link calls begin to be favored over multilink calls. Such behavior is not seen in the hierarchical network, because, intrinsically, it can better limit the number of multilink calls under overloads. This is due to the presence of final trunk groups and of primary high-usage trunk groups that do not carry alternate-routed traffic, the absence of cranking back, and more trunks than are in the nonhierarchical network. The number of calls carried in the hierarchical network increases steadily as offered load increases over the entire range of overloads considered.

Figure 7 demonstrates this point. Here we have plotted the ratio of the number of multilink calls to the number of 1-link calls as determined from the mathematical model for various overloads. This ratio grows sharply for the nonhierarchical network for overloads of up to 50 percent, then levels off and begins to decline. On the other hand, for the hierarchical network, this ratio rises slowly before leveling off at around 30-percent overload. These results are consistent with a simulation study by Weber using 3-, 4-, 5-, and 6-node networks, which showed that hierarchical networks perform more efficiently under overloads than nonhierarchical networks.[6]

Figure 8 displays the performance of the 30-node network under the nonuniform overloads. Qualitatively, the results are the same as those obtained under uniform overloads. In fact, the difference in the number of calls carried under uniform and nonuniform overloads is very small at the higher overloads. The nonhierarchical network under nonuniform overloads also exhibits a drop in carried load at about 10-percent

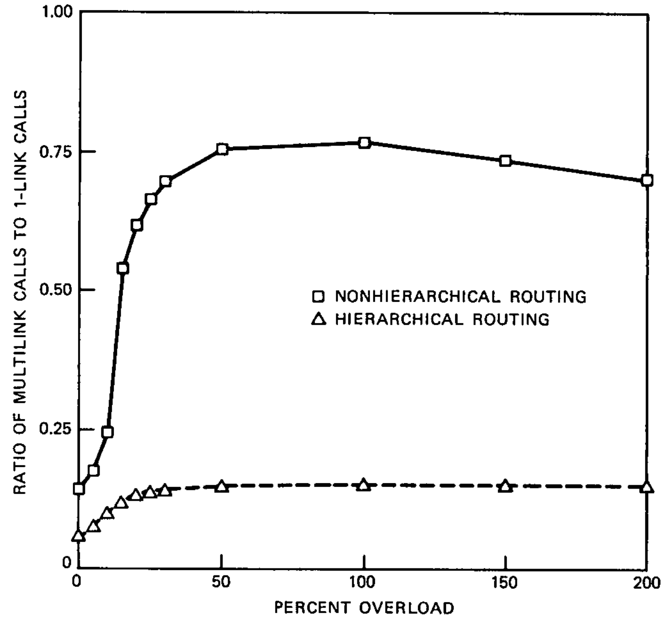

Fig. 7—Ratio of multilink calls to 1-link calls for the 30-node network.

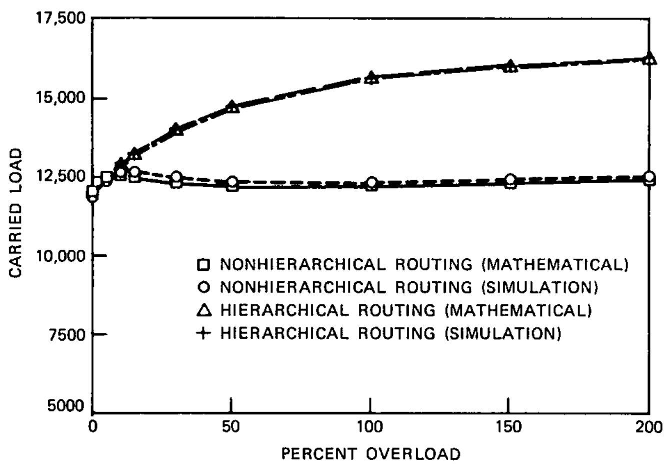

Fig. 8—Performance of the 30-node network under nonuniform overloads.

overload, though not as severe. The drop is attenuated because, with the nonuniform overloads, carried load cannot increase to the maximum level seen with the uniform overloads.

4.3 The 25-node network

Figure 9 presents the results for the 25-node network under uniform

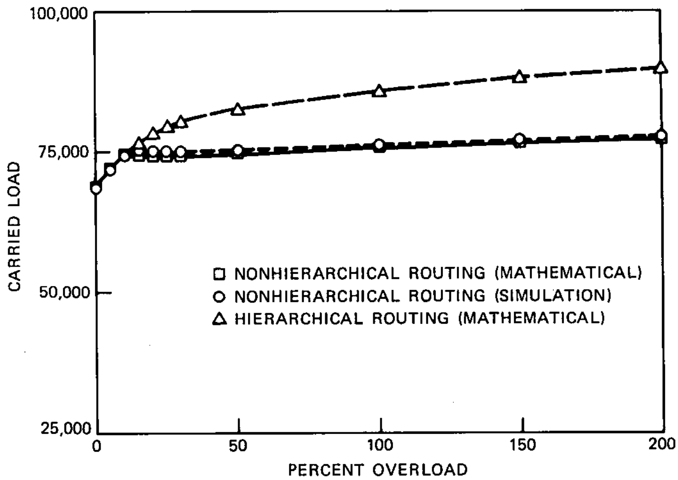

Fig. 9—Performance of the 25-node network under uniform loads.

overloads. As with the 30-node network, the 25-node nonhierarchical network shows a drop in carried load: at about 10-percent overload in the mathematical model and at about 15-percent overload in the simulations. However, the drop is not as sharp as that seen in the 30-node nonhierarchical network. We attribute this to the smaller number of paths per route in the 25-node nonhierarchical network (see Table II), which limits the potential for a large number of multilink calls. The hierarchical network exhibits a continuous increase in carried load with increasing offered load.

4.4 The 140-node network

Figure 10 displays the performance of the 140-node nonhierarchical network under uniform overloads. These results were obtained from the mathematical model. No simulations of this network were made because of its large size. The results agree qualitatively with those derived for the 30- and 25-node nonhierarchical networks. Again, carried load declines at around 10-percent overload and increases at the larger overloads. Figure 10 also shows the number of 1-link calls in the network as derived from the mathematical model. The direction of change in the number of 1-link calls is almost always the same as the direction of change in the total number of calls. Again we conclude that the degree to which network capacity is efficiently used is related to the network's ability to favor 1-link calls over multilink calls.

4.5 Comparison of the nonhierarchical networks

Our analysis of the engineered nonhierarchical networks without controls shows that the type of instability discussed in Section II does

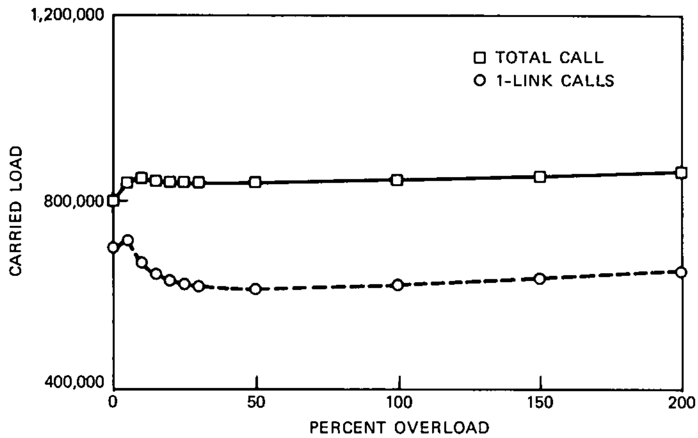

Fig. 10—Performance of the 140-node network under uniform overloads.

not occur—the offered load versus carried load functions are all single-valued. We have seen, however, that the nonhierarchical networks without controls demonstrate a drop in carried load under overload but with different degrees of severity. The most severe drop occurs in the 30-node network, where carried load drops 8.37 percent below the level at 10-percent overload before leveling off (based on the mathematical model). The 25- and 140-node networks demonstrate respective drops of 0.58 and 1.13 percent. Krupp has shown that, for symmetric nonhierarchical networks, all other things being equal, performance under heavy loads worsens as the number of trunks per trunk group or the number of paths per route increases (see also Weber[6]). Thus, we might expect the difference in the severity of the poor performance in our networks to be related to these variables. From Table I, which shows the distribution of trunk group sizes for the three nonhierarchical networks, it is clear that network overload performance does not worsen as trunk group size increases. In fact, the 30-node network, which shows the worst overload performance, has an average of only 40.4 trunks per trunk group, while the 25- and 140-node networks have, respectively, 320.7 and 132.6 trunks per trunk group.

However, this should not be interpreted as a contradiction of Krupp's results, since there are other significant differences in the networks. In particular, the networks vary widely in their route sizes (number of paths per route), as shown in Table II. The 30-node network provides on average 6.91 alternate paths per route, the 25-node network provides 2.12, and the 140-node network provides 4.41. Thus, we see a strong relationship between route size and poor overload performance and, we can conclude that route size is a better predictor of network overload performance than trunk group size. This further substantiates the conclusion that the reason for the poor overload performance of the nonhierarchical networks is the large amount of alternate routing under overloads made possible by the large number of alternate paths.

V. RESULTS WITH TRUNK RESERVATION FOR FIRST-ROUTED TRAFFIC

As we know from the study of hierarchical networks, appropriate network controls are an effective way to mitigate poor network performance under nonengineered conditions. Since the poor network performance seen above is related to inefficient use of trunks for alternate routing, trunk reservation for first-routed traffic appeared to be an effective way to control our test networks. On each trunk group we reserved 5 percent of the trunks, with a minimum of one trunk, for first-routed traffic. Only the effect on network carried load was considered. Other variants of the trunk-reservation control, as well as the impact on point-to-point blockings, have been investigated but are not discussed here.

The effects of trunk reservation on the 30-node networks are typical of all the networks considered. The results under uniform overloads are shown in Fig. 11. By comparing Figs. 6 and 11, we see that trunk reservation has a very beneficial effect with nonhierarchical routing under large overloads. The drop in carried load with increasing offered load without controls disappears with the use of trunk reservation. Trunk reservation limits the number of multilink calls, allowing more efficient use of the trunks. Similarly, the performance of the hierar-

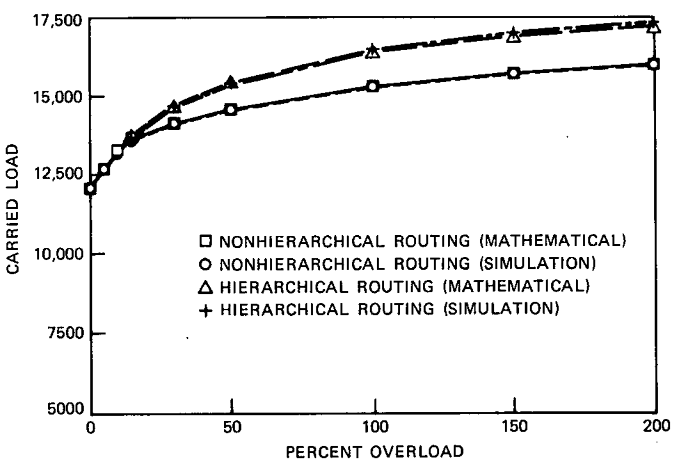

Fig. 11—Performance of the 30-node network under uniform overloads with trunk reservations.

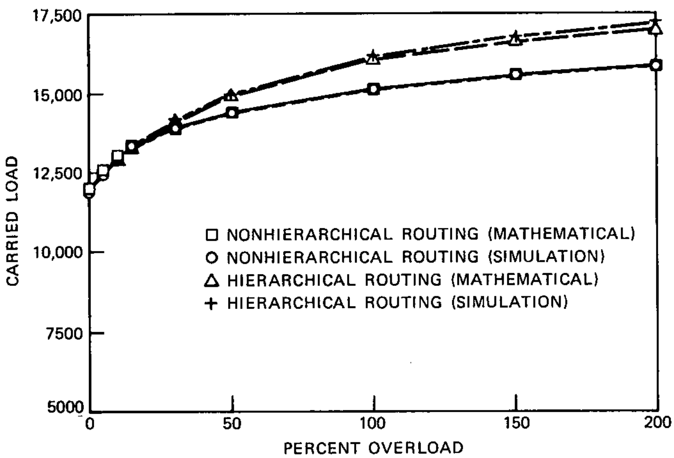

Fig. 12—Performance of the 30-node network under nonuniform overloads with trunk reservation.

chical network under large overloads is improved, although the improvement is not as dramatic as for the nonhierarchical network. At overloads of less than 10 percent, though, two negative effects appear: (1) carried load actually decreases slightly since some alternate-routed calls are prevented from accessing paths that have been engineered to carry them, and (2) some point-to-point blockings are increased, distorting servicing measurements used to schedule trunking augmentations. These effects can be eliminated by using a triggering mechanism to allow the trunk-reservation control only when overloads are large enough to preclude these effects.[7]

Figure 12 shows the trunk reservation results for the 30-node networks under the nonuniform overloads. The effect of the control under large overloads for both the nonhierarchical and hierarchical networks is essentially the same as that seen under uniform overloads. One contrast to the uniform overload case is that now trunk reservation is usually beneficial even at the lighter overloads. Reduction in carried load with trunk reservation is extremely small.

VI. SUMMARY

Analysis of small, symmetric, uniformly loaded nonhierarchical networks has shown the existence of network instabilities for certain networks when only trunking behavior is considered. We have demonstrated by example that such instabilities persist even when the assumptions of symmetry and uniformly distributed loads are removed. Using a mathematical model that we developed, together with a simulation model, we have studied the performance of three realistic network models under overloads. The 25- and 140-node nonhierarchical networks were designed using the unified algorithm. The 30-node nonhierarchical network was engineered using another design algorithm. In addition, two of the models were studied with hierarchical routing. Results obtained demonstrate that, without controls, the performance of nonhierarchical networks is inferior to that of hierarchical networks under overloads. No instabilities of the type described in Refs. 4 and 5 are seen in any of the engineered nonhierarchical networks, but a drop in carried load consistently occurs at about 10-percent overload because of the tendency of the nonhierarchical networks to alternate-route calls when overloaded. The severity of this drop appears to be correlated with the number of paths per route; nonhierarchical networks with more paths per route exhibit greater throughput degradation under overloads. Such behavior is not seen in the hierarchical networks, which intrinsically can better limit the amount of alternate routing under overloads. An important assumption in our models is the absence of switching system effects. Other studies have shown that, when these effects are included, the performance of networks under large overloads changes significantly from that given by our analysis. However, our results indicate that engineered nonhierarchical networks without controls exhibit unsatisfactory performance even at overloads of 10 to 15 percent, where switching system dynamics are not likely to be an important factor.

We also have applied a control, namely, trunk reservation, for first-routed traffic, to the network models. This control improves the performance of the nonhierarchical and hierarchical networks. By diminishing the amount of alternate routing in the networks under large overloads, trunk reservation permits 1-link calls to use the trunks more efficiently. For the nonhierarchical networks, this results in a continuous increase in carried load with increasing offered load over the entire range of overloads considered. However, at very light overloads, carried load may drop when trunk reservation is activated. This suggests the use of a triggering mechanism to impose trunk reservation only at larger overloads.

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APPENDIX

Mathematical Models

A.1 Models without controls

The model for the nonhierarchical networks without controls requires that a network be specified, together with trunk-group sizes, point-to-point offered loads, and a fixed route for each point-to-point pair. It is assumed that (1) the mixture of traffic offered to a trunk group is Poisson, (2) trunk-group blocking probabilities are independent, and (3) the time needed to make connections (setup time) is small enough, relative to the average holding time of calls, to be ignored. The effect of the first two assumptions can be gauged by comparison with the simulation results, since these assumptions do not occur in the simulator.

We permit the trunk-group sizes and the offered load and the number of paths for each point-to-point pair, as well as the number of trunk groups in a path, to be arbitrary. We also assume that a call blocked on a trunk group of a path can always be cranked back to the originating office so that the call can access the next path in its route.

In the discussion that follows, the term *path* means a set of distinct trunk groups that form a connection between two nodes. A *route* is an ordered collection of paths connecting the same point-to-point pair, specifying the paths used for routing calls between the pair in the order that seizure of the paths is attempted.

Before giving the details of the model, we introduce some notation. Let L^j be the offered load for point-to-point pair j and let $L = \sum_j L^j$ be the total offered load. Let p_i, n_i, and a_i denote, respectively, the blocking probability, trunk-group size, and offered load for trunk group i (in erlangs), and let $q_i = 1 - p_i$. We denote a path by r, a route by R, the route for point-to-point pair j by R^j, and the route formed by the first k paths of R^j by

$$R_k^j = (r_i^j, \ldots, r_k^j).$$

Finally, we define $D(R)$ to be the probability that route R is blocked, i.e., each path in R has at least one blocked trunk group.

The basic idea of our analysis is to determine the offered load a_i for trunk group i as a function of the trunk-group blockings. For each route containing trunk group i, we determine the contribution that the route makes to the total trunk-group offered load. Suppose trunk group i is in path ^{j}, the kth path in the route for point-to-point pair

j. The load carried by r^j_k, c^j_k, is given by

$$c^j_k = L^j[D(R^j_{k-1}) - D(R^j_k)], \tag{1}$$

which is the point-to-point load for pair j that overflows the first $k-1$ paths but not the kth path. The load c^j_k contributes to the carried load for each trunk group $i \in r^j_k$. The total carried load for trunk group i, K_i is obtained by considering all paths containing trunk group i and is given by

$$K_i = \sum_{\substack{j,k \\ i \in r^j_k}} c^j_k.$$

From the relation

$$K_i = a_i q_i$$

for Poisson traffic, we immediately obtain

$$a_i = \sum_{\substack{j,k \\ i \in r^j_k}} c^j_k / q_i. \tag{2}$$

In addition to the relations given by (2), based on our assumption of Poisson trunk-group offered loads, we relate the trunk-groups offered loads to the trunk-group blockings by the Erlang-B formula:

$$p_i = B(n_i, a_i). \tag{3}$$

The equations given by (2) and (3) can be solved iteratively, starting with an initial estimate of the trunk-group blockings, to determine the trunk-group offered loads and blocking probabilities in equilibrium. [The calculation of $D(R)$ is discussed below.] Once a solution has been obtained, several quantities of interest can be calculated. In particular, network blocking, z, is given by

$$z = \frac{\sum_j L^j D(R^j)}{L},$$

and network carried load, C, by

$$C = L(1 - z).$$

We now consider the calculation of $D(R)$, $R = (r_1, \ldots, r_k)$. If the paths of R are disjoint, then, by our independence assumption, the blocking probabilities for the paths are independent, so that

$$D(R) = \prod_{t=1}^{k} \left(1 - \prod_{i \in r_t} q_i\right). \tag{4}$$

For the nonhierarchical networks studied here, the paths have all been restricted to contain either one or two trunk groups. This implies that all paths in a route are disjoint, and $D(R)$ can be calculated using the formula above.

The model used for the hierarchical networks without controls was taken from Ref. 8. We used only the trunk-group portion of the model, omitting the parts dealing with switching system dynamics, retrials, and DABY (don't answer, busy). In fact, this abbreviated model differs from the nonhierarchical model only in the way in which trunk-group carried load is calculated. In the nonhierarchical-network model, this calculation incorporates crankback, while in the hierarchical model, final trunk groups are taken into account.

A.2 Models with trunk reservation for first-routed traffic

Network performance was also modeled with trunk reservation for first-routed traffic. Under this control, a threshold is specified for each trunk group, and alternate-routed calls attempting to seize a trunk on the trunk group are refused if the number of busy trunks on the trunk group has reached the threshold. For the nonhierarchical networks, this control was implemented by subjecting a call to trunk reservation on all legs of an alternate path. In the hierarchical networks, the classification of a call as first-routed or alternate-routed was made at each switching system that the call traversed. Each call was classified as follows: On the first-choice trunk group out of a switching system the call is considered first-routed, whereas on any other trunk group it was considered alternate-routed. All alternate-routed calls offered to a trunk group were subjected to this control. A call overflowing a trunk group because of trunk reservation was offered to the next path in its route. When the first path of a route uses fewer trunk groups than the alternate paths (e.g., the first path is a direct path), then under large loads this control has the effect of decreasing the average number of trunks per call and thus increasing network carried load.

For the trunk reservation model, let a be the total trunk-group offered load, p the trunk group blocking probability, and $q = 1 - p$. Also, let m be the trunk-reservation threshold on the trunk group, $\hat{q}$ the probability that no more than $m - 1$ trunks in the trunk group are busy, $\hat{a}$ the trunk-group offered load that is subject to the trunk-reservation control (i.e., the alternate-routed traffic), $\hat{K}$ the trunk-group carried load subjected to trunk reservation, and $r = \hat{a}/a$. Using a birth-death model for the behavior of n servers with offered load a when less than m servers are busy and offered load $a(1 - r)$ when at least m servers are busy, we obtain the probabilities P_j that exactly j trunks on the trunk group are busy:

$$P_j = \frac{a^j}{j!} P_0, \qquad j = 0, \ldots, m-1,$$

$$= \frac{a^j}{j!} (1 - r)^{j-m} P_0, \qquad j = m, \ldots, n.$$

Here

$$P_0 = \left[\sum_{k=0}^{m} \frac{a^k}{k!} + \sum_{k=m+1}^{n} \frac{a^k}{k!} (1 - r)^{k-m} \right]^{-1}.$$

It follows that

$$p = \frac{a^n}{n!} (1 - r)^{n-m} P_0 \tag{5}$$

$$\hat{q} = \sum_{j=0}^{m-1} \frac{a^j}{j!} P_0. \tag{6}$$

The quantities p and $\hat{q}$ are easily calculated using recursive formulas.

We also need formulas for the calculation of a and $\hat{a}$, which we obtain by relating these quantities to the corresponding carried loads. The total carried load K is given by

$$K = \sum_{j=0}^{n} j P_j$$

$$= ar\hat{q} + a(1 - r)q,$$

so that the offered load that is not subject to trunk reservation is given by

$$a - \hat{a} = \frac{K(1 - r)}{r\hat{q} + (1 - r)q}. \tag{7}$$

We relate the offered load $\hat{a}$ to the carried load $\hat{K}$ by

$$\hat{a} = \frac{\hat{K}}{\hat{q}}. \tag{8}$$

Carried load K is still calculated as in the previous section with $\hat{K}$ being the portion of K that was subjected to trunk reservation.

The nonhierarchical model with trunk reservation for first-routed traffic is obtained by replacing eq. (2) with eqs. (7) and (8) and replacing eq. (3) with eqs. (5) and (6). The calculation of $D(R)$ in (4) is now given by

$$D(R) = \left(1 - \prod_{i \in r_1} q_i\right) \prod_{t=2}^{k} \left(1 - \prod_{i \in r_t} \hat{q}_i\right).$$

The hierarchical model with trunk reservation for first-routed traffic is obtained by combining eqs. (5) through (8) with the Franks and Rishel formulas for trunk-group carried load. Separate calculations of total carried load and alternate-routed carried load are made for each trunk group.

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